Fluid Mechanics

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Lecture No. #24

Welcome to this lecture number 24 on this NPTEL course on fluid mechanics for under graduate chemical engineering students. The topic of our discussion for today is differential momentum balances, and the derivation of the so called navier stokes equations. In the last lecture, we derived the differential momentum balance in terms of the components of the stress tenser as follows.

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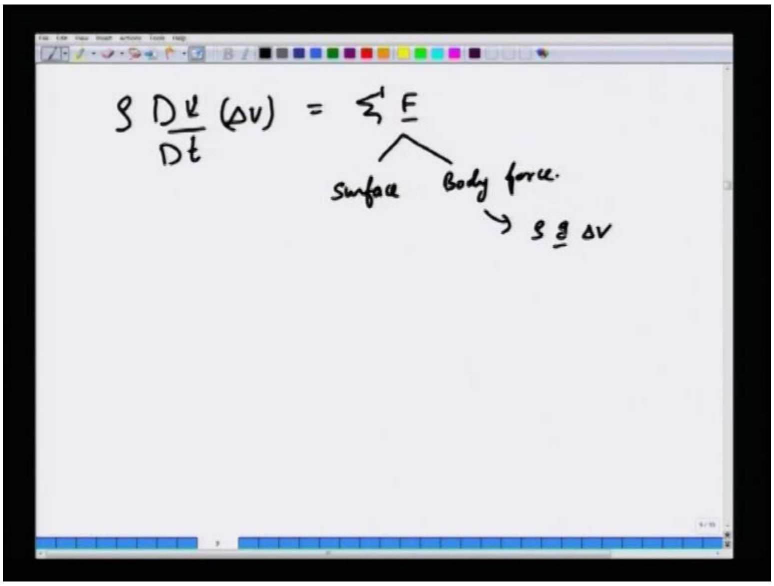

So, essentially we had rho times Dv Dt times del v is some of all forces which as the surface force, and body force. And the surface forces where written in terms of the stress tensor, and the body force in our cases generally due to gravity, which is simply written as rho, g, delta v.

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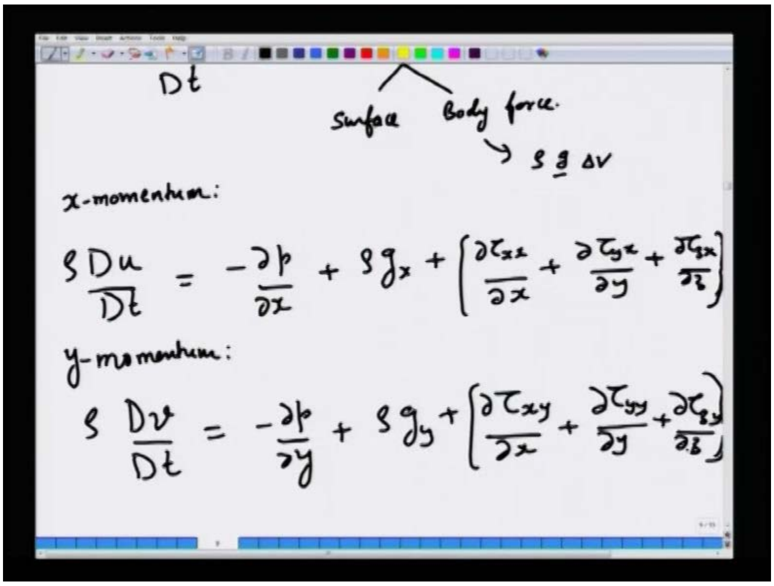

And then by doing deferential balance about cubic volume element, we came up with the following three relation for the x, y and z momentum. So, the x momentum equation will look like this, is minus partial p partial x plus rho g x plus partial tau x x by partial x plus partial tau y x by partial y plus partial tau z x by partial z. Similarly, you have the y momentum equation or the y component of the momentum equation rho partial, rho substantial derivative of v with respect to time is minus partial p partial y plus rho g y plus partial tau x y partial x plus partial tau y y partial y plus partial tau z y partial z.

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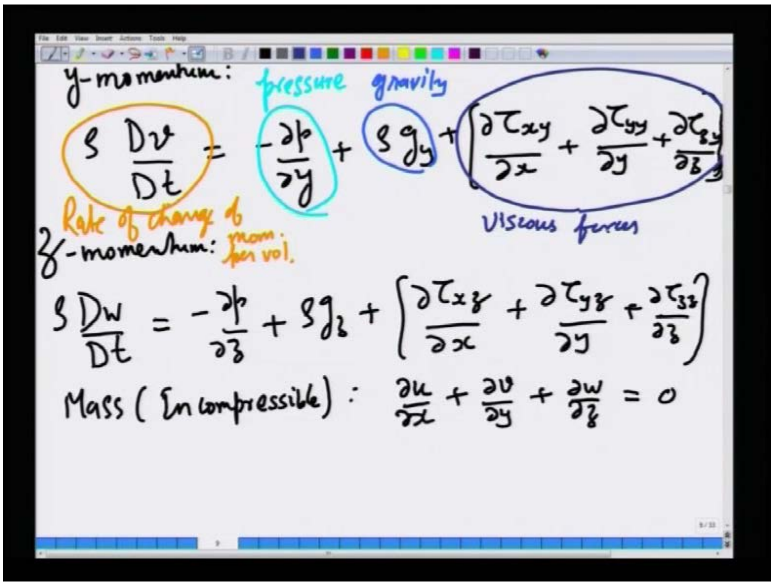

And we finally have z momentum equation. z component of the momentum equation rho times substantial derivative of the z comportment of velocity with respect to time is minus partial p partial z plus rho g z plus partial tau x z by partial x partial tau y z by partial y plus partial tau z z by partial z. In addition, we have the continuity or the mass conservation equation for an incompressible fluid, as partial u partial x plus partial v partial y plus partial w partial z is zero. The interpretation of any component of this momentum balance is fairly simple; it's essentially a consequence of Newton second law of motion. So, it is says that the mass per rho times the substantial derivative of the velocity. The substantial derivative velocity is the acceleration, rho times that is the rate of change mass times acceleration per unit volume. It is a rate of change of momentum per unit volume is equal to some of all forces. So, this is the rate of change of momentum per unit volume.

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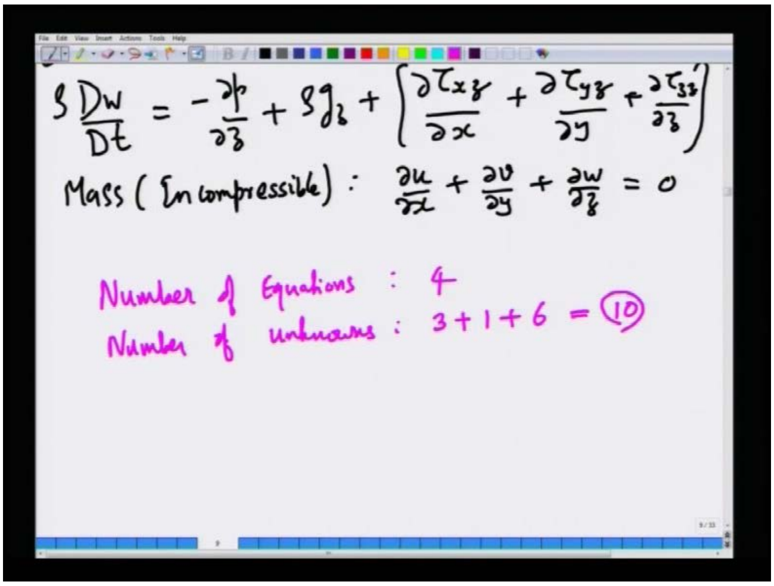

So, these are various forces that act on the fluid, these are the pressure forces that act on the infinitesimal control volume. This is the body force, it is a gravitational body force and these are the viscous forces that act on an infinitesimal control volume. Now, in order to able to solve these four equations, we commented in the last lecture that the number of equations is four, while the number of unknowns is far too many. Is essentially three plus one three components of velocity plus one component of pressure plus six component of the stress tensor, because the stress tensor is symmetric. So, you have about ten unknowns, but we do not have as many equations.

So, we also commented that this mathematical problem tends from the physical reason that we have still not prescribed our stipulated, what the material is? We are nearly carried out a momentum balance by applying Newton second of control volume, and by taking the control volume to be infinitesimally small. We have merely stated the Newton second law of motion for a substance. But, we have not told what the nature of substance is, this is at equally well for a solid as it is for a fluid. So in some sense, that information is what is lacking so far. And once we provide that information, as to how a given material is going to respond to applied stress then we will be able to complete the set of equations. Such relations are called Constitute Relations.

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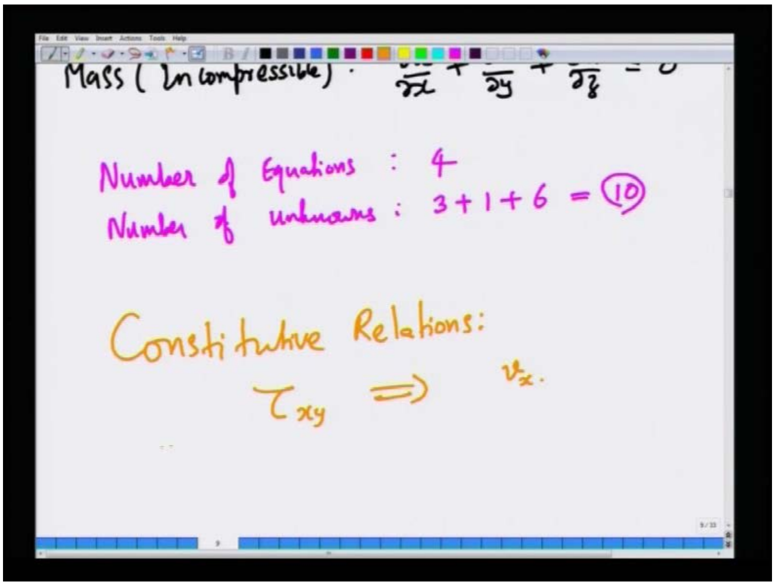

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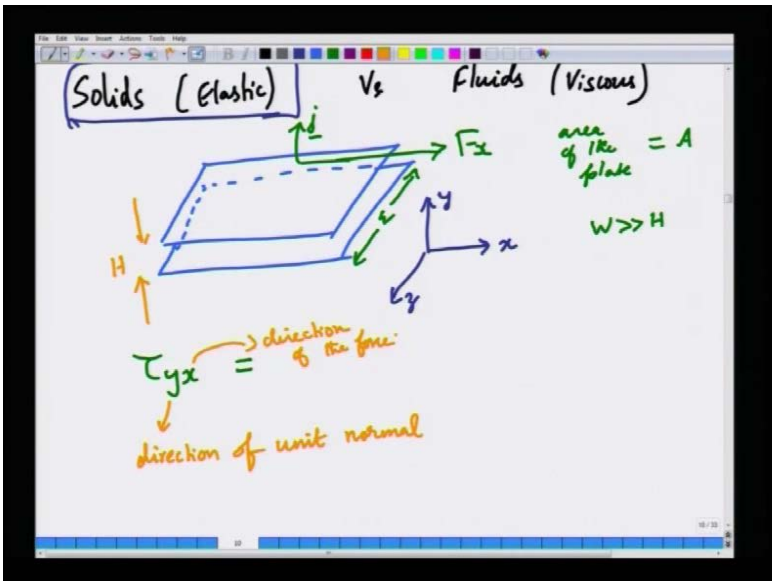

The Constitutive Relations relate the stresses the various components of the stress to the velocities. So, we need that relation in order to able to say how a given substance is going to respond. Now, in order do this, let us take a very simple context. So we will try to understand constitutive relations of two simple materials, elastic solids, which are elastic in nature verses fluids, liquids and gases, which are viscous in nature. We will try do a very simple thought experiment. Imagine we have two slabs separated by a distance. The distance that separates these two slabs is let say H. And let us put a coordinate system x, y, z. Imagine keeping, lets first worry about solids. Imagine keeping a piece of elastic solid in between these two plates and let us apply a force F x in the x direction. Let the area of this plate be A. So, you are applying in essence a stress F x divided by the area on the top plate. Let further assume that the third dimension W along the z direction is very large compare to H.

So essentially, we do not have to worry about any variations in the third direction z. So, let us now understand the moment you put this force, you are going to generate stress. The stress is acting on a surface whose unit normal is in the y direction j and the direction of the stress is x, so as per convention, this is the direction of the unit normal to the surface over which stress is acting this is the direction of the force. So this is the stress acting on the top surface in the x direction, the force per unit area. So, this tau x y is nothing but, is equal to F x by A.

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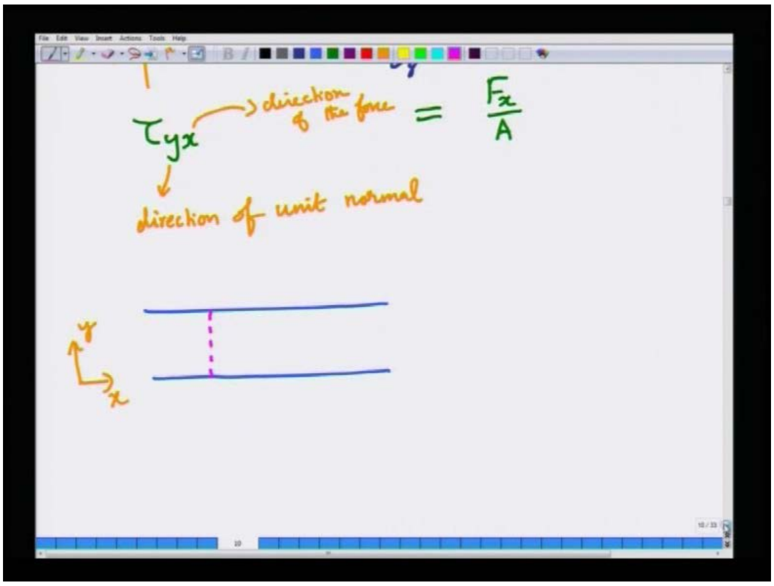

Now, if I keep elastic solid and since I have told you that the third dimension is irrelevant. Now we can just take any cross section along the z axis and look at the material. So, we are now going to just tick to only x y direction, in the x y plane. So, in the unstressed state if you keep a solid, now the solid may suppose you are drawing your marking a line in the solid, this is the unstressed state, before applying unstressed state before applying stress.

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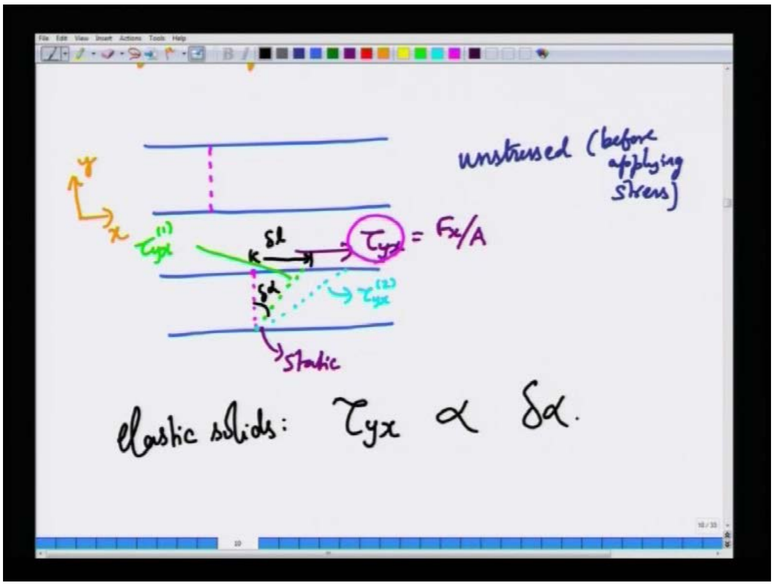

Now, I am going to draw the evolution of this points, the set of points upon application of a shear stress tau y x is F x by divided by A. What will happen to these points? Let us assume that the bottom plate is stationary, so and the material is rigidly bonded to the bottom play. So, if I were draw the suppose in the unstressed state, all this points were like this upon application of shear stresses in the x direction, you may get a deformation like this. That is a point which was here would be now here, a point which was here would be now here, a point which was here would be now here and so on.

So in general, a material elastic material deforms of up on application of shear stresses. But the material the points do not continue to keep deform. After achieving a certain deformation, a material stops deforming. So, if you apply a given stress tau y x, you do not find that the material keeps on deforming, the points move somewhat and then internal stresses develop within the elastic material that resists further deformation. So, if we characterize the amount of deformation by either this length delta l or the angle delta alpha that the deform line makes with the original line, then you would anticipate that for elastic materials, elastic solids. If you apply more stress than the amount of deformation will be more. Suppose, I were to draw another line with blue color. If you apply more stress, this is for a stress, lets a tau y x two, while the green point it is for a stress tau y x.

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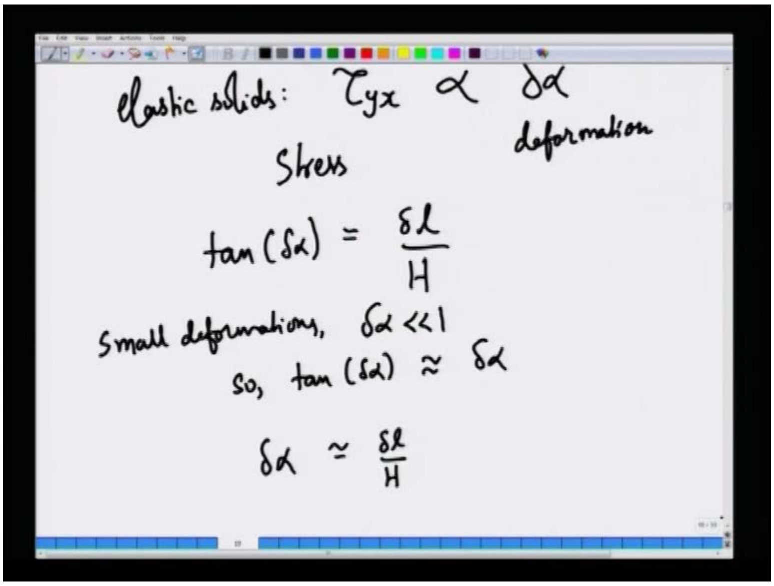

The green point, let me draw it separately it is for a stress tau y x one. If you apply more stress then the alpha, the angle alpha you will get will be more. So for elastic solids, the shear stress that you apply is directly proportional to the amount of deformation that you get in the material. The stress is proportional to the deformation in the material. Now, using simple geometry, we can find that tan delta alpha is nothing but, delta l by h, but we are looking at small deformations. For small deformations, the angle delta alpha is small compare to one, so we can write tan of delta alpha is approximately equal to delta alpha itself.

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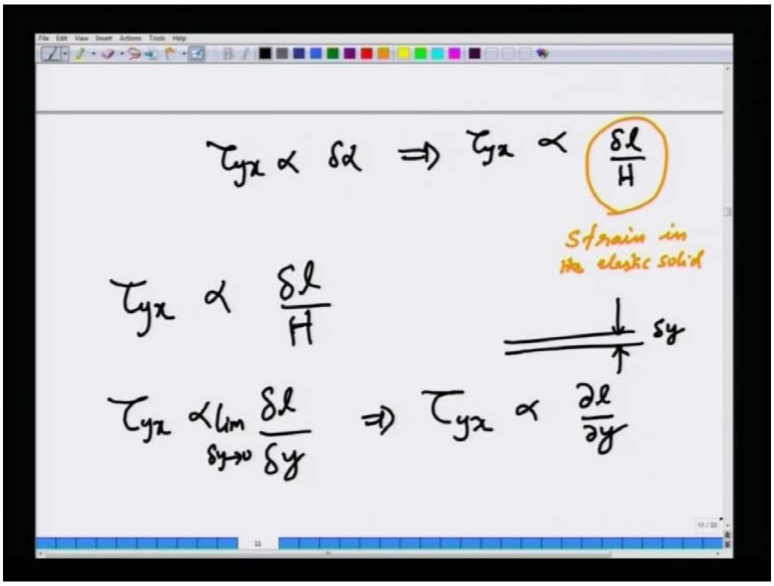

When the angle is a very small, tan of the angle is approximately equal to the angle itself. So we can write, delta alpha is approximately equal to l by H. So the stress in a solid, the shear stress in a solid is directly proportional to delta alpha or the shear stress in a solid is proportional to delta l divided by H. Delta l is the amount of deformation in the x direction and H is the gap width between the two plates. So this quantity is called the strain, this is called the strain in the solid; this is the non-dimensional deformation in the elastic solid. Delta l has dimensions of length, H has dimensions of length, and if I divide the two it will be non-dimensional quantity.

So, tau y x is directly proportional to the strain, the stress is directly proportional to the strain in the elastic solid. Now, the constant of proportionality… Now before I do that, instead of taking a finites slab H, I can consider an infinitesimal thickness delta y. And then you will find that the stress within the continuum hypothesis is still proportional to delta l by delta y. Instead of considering the entire slab of length thickness H, we can consider an infinitesimal slab. And the stress will be proportional in the limit as delta y tends to zero, you will find that tau y x is proportional to d l dy, this is the strain in the differential the limit.

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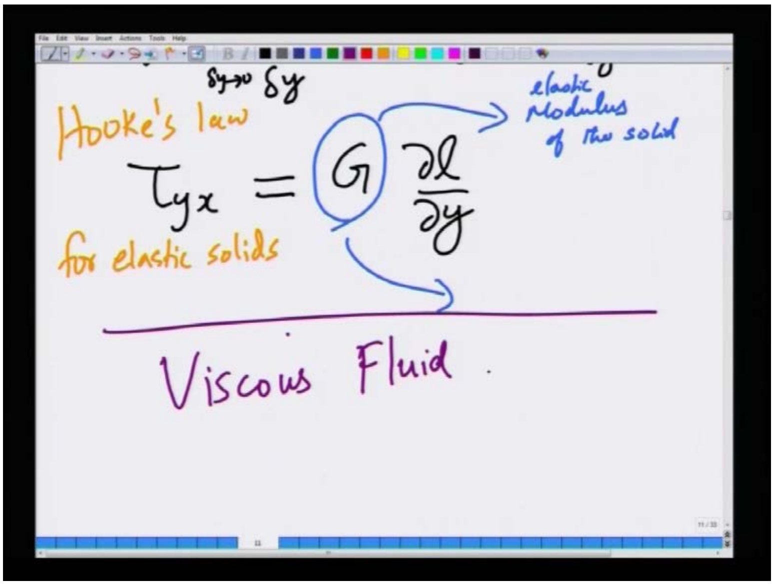

And the constant of proportionality is the shear modulus which is related to young's modulus of the solid. So you can convert this proportional to equality, when introducing constant proportionality this is the elastic modulus of the solid. So in a solid experimentally, the stress is directly proportional to the amount of deformation present in the solid and the constant of proportionality is the elastic modulus. The elastic modulus is very high for materials like steel, while it is a lower for materials like rubber. It tells you how hard this material is, so this is the celebrated Hooke's law of elasticity, for elastic solids.

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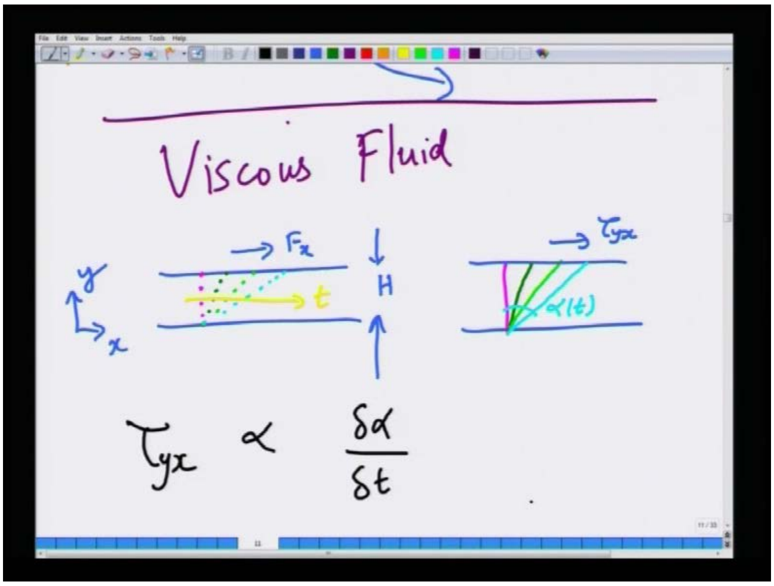

Now, will now do the same thing for a fluid for a viscous fluid. I am going to the same thought experiment; take two slabs and the distance between them H. Now, I am going to mark points before the application of stress with pink. The moment I apply a stress that is moment I apply a force in the x direction, let us keep the coordinates like this x, y. The moment I apply a force, this points let say the bottom plate is stationary, at a later time we will move here, but if you still wait long enough this will move here. At various times, these points will be kept on involving in this fashion that is indicated here qualitatively.

So as time proceeds, is with increasing time the points will continue to deform upon application of shear stress in a viscous fluid. So for example, if you to call, if you were to draw line instead of set of points at various times, the angle that the deforming lines make with the indicial pink line will continue to increase with time in a viscous liquid. So the angle alpha is actually a function of time, it is not a constant. For given amount of stress that you apply on the top plate, the angle continues increase with time. So if you wait for longer time, you will find that the angle is going to be, if you the angle is going to be more. So, we cannot say that the stress is proportional to the angle itself, because if you wait long enough you can get more deformation. The angle is a measure of deformation suffered by a fluid element present in between the two plates.

Now for, if you apply a given stress that, if I given force F of x. And if you apply given stress tau by x which is force divided by unit area of the top plate, then these lines will continue to deform. So we cannot say that the stress is proportional to the angle, because if you can get a larger angle provided you your willing to wait long enough. So clearly, this is what we mean by saying that the fluid cannot resist any deformation upon the application of shear stresses. Because, you can get as much angle you want by weighting long enough, regardless of how much stress you apply.

Suppose you apply a different stress, you can get the same amount of deformation by waiting long enough or short enough depending on the magnitude of the stress. So the stress cannot be clearly proportional to the deformation present in the fluid. The stress has to be proportional to; the shear stress has to be proportional to the rate of deformation, because the angle will continue to increase with time for a given stress. If you change the stress, if you increase the stress then the fluid deform more quickly. While, if you decreases stress, the fluid deform more slowly. So the stress is not proportional to the deformation itself, but it is proportional to the rate at which the fluid deforms. And the rate of deformation is simply delta alpha by delta t, the rate at which the angle is going to change with time.

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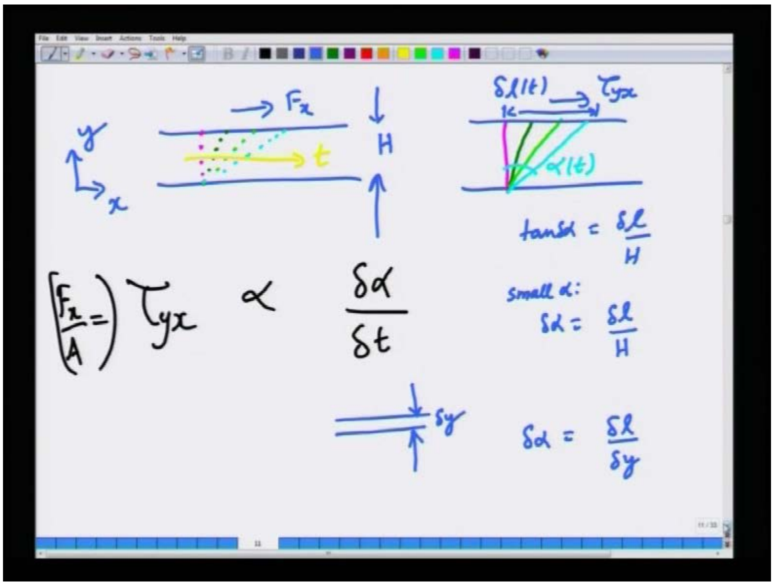

So tau y nothing but, F x divided by A. Now from geometry, we can find that this angle, suppose I call this distance l of t. Now tan alpha is l over H, but for small alpha then you can find that in the limit of infinitesimally small, if you look at a given instant of time, so the deformation will be small. For small alpha, tan alpha is simply tan delta alpha, simply delta alpha is delta l, so let us call this delta divided by H. So, you can also write this instead of writing it for a bigger thickness H, you can write it for a smaller thickness as y, so delta l will be delta, delta alpha will be delta l by delta y.

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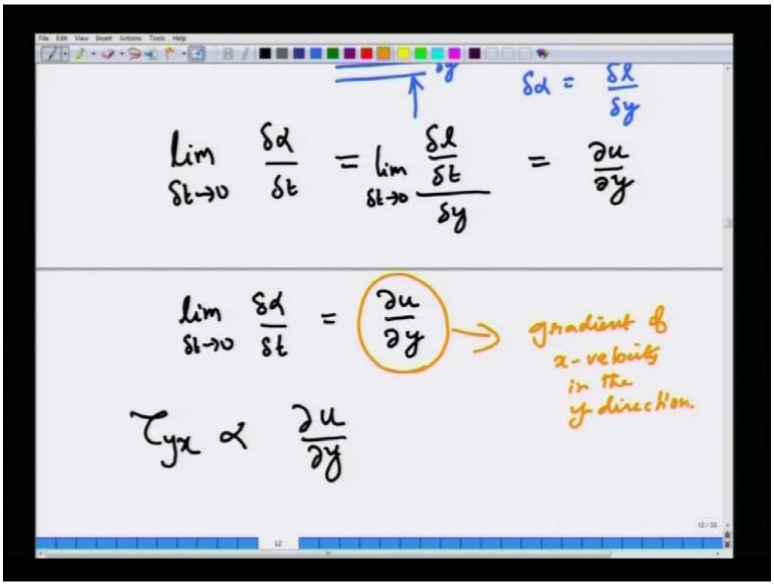

So, in the limit as delta goings to zero, delta alpha by delta t will become delta l by delta t divided by delta y. So, as delta t goes to zero delta l by delta t is nothing but, the velocity delta is delta u delta t, so this is the velocity divided by the differential distance delta y. So in the limit, delta t going to zero, the rate of deformation as measured by the rate of change of angle is nothing but, the partial derivative of the velocity in the x direction with respect to y. so the shear stress tau y x is proportional to the this is nothing but, the velocity gradient as we mentioned, this is the gradient of x velocity, x component of velocity in the y direction.

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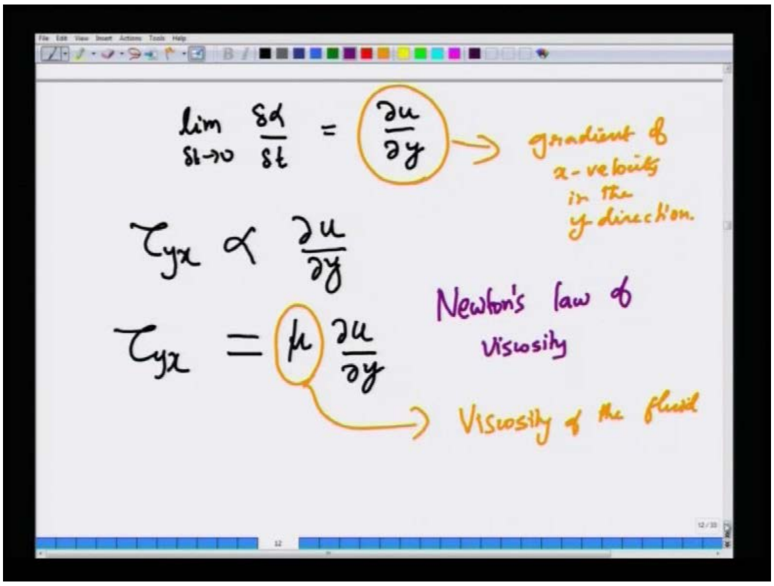

So tau y x, so in a fluid in a liquid or a gas the amount of stress that you exert will not be proportional to the deformation itself, it will be proportional to the rate at which deformation. Or in simple words, a fluid does not care how much you deform rather it cares about how fast you deform, while a solid merely cares about how much you deform. Because in a solid, elastic solid the stress directly proportional to the strain, the amount of deformation. Where as in a fluid, the stresses are proportional to the rate at which you deform the fluid. So if you deform the fluid quickly, the stresses generated will be more, while if you deform the flow more slowly, the stress is generated will be less. So, you can of course convert the constant of proportionality to an equality by introducing, you can introduce you can convert this proportionality to an equality by introducing constant of proportionality.

I use the symbol mu for viscosity, so that constant of proportionality is the viscosity of the fluid. So, if you have higher liquids of very high viscosity, then it is rather difficult for us to deform them quickly, because they will generate high internal shear stresses. Whereas, fluid fluids of lower viscosity will deform rather very easily, because you can generate a higher rates of deformation and then the stresses generated developed inside the liquid will be lower. So viscosity tells you, how difficult it is for us to carry out rates of deformation present in the fluid. Just as the modulus of the solid tells is elastic modulus of the solid tells us about how difficult it is deform the flow as solid.

Here the shear viscosity tells us how difficult or easy it is to make the fluid flow, because if something is continuing to deform we say it is flow. That is what a fluid cannot resist any amount of shear stress, because it continues deform and therefore it flows. So, the viscosity tell you about the resistance to flow in a fluid, just as the elastic model modulus of a solid tells you, how difficult it is to deform or deform a given solid.

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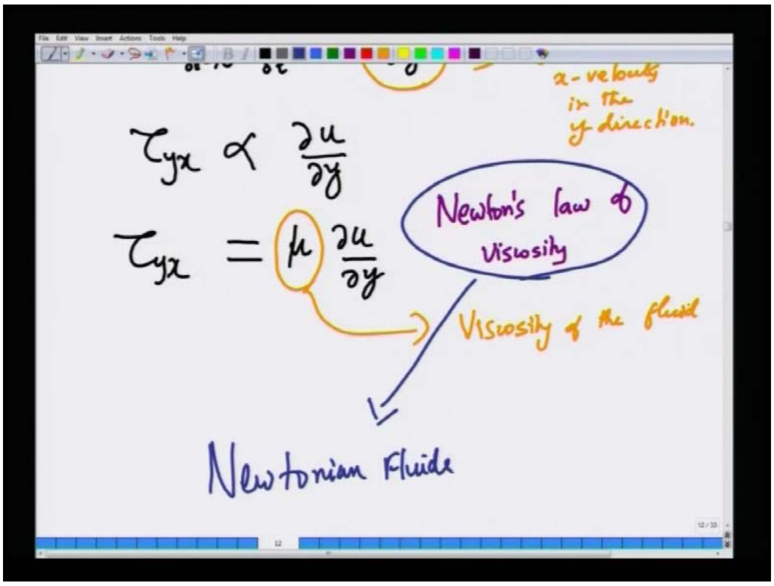

So this simple equation is called the Newton’s law of viscosity. And this the fluids obey, that obeys the Newton’s law viscosity or called Newtonian fluids. Now, this is not to say that all fluids should obey this all liquids or all gases should obey this relation. But it turns out that a majority of simple fluids such as air, water and glycerol and many oils they reasonably obey this relation. There are other fluids such as solutions of polymer or molten polymers or emulsions, which do not obey this behavior and such fluids are called non Newtonian fluids. But there the way in which they respond to a given applied stress is much more complex than a simple Newtonian fluid. So for now we are going to restricts restrict our attention only to Newtonian fluid in this course.

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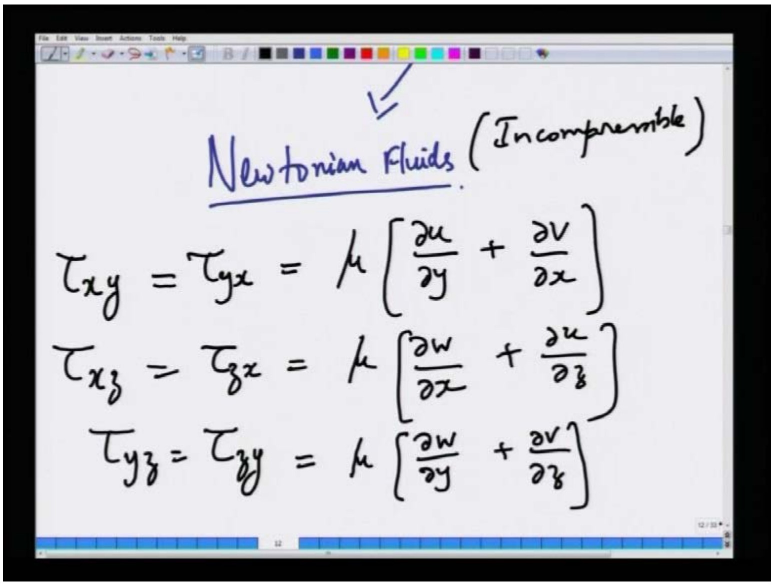

But one must keep in mind that in many applications, we do encounter liquids that are very different that behave very different from this simple Newtonian constitutive relation. Now, this constitutive relation is applied is derived only for a simple one dimensional approximation. As I told you in the last lecture, in general fluid flow can occur in all three dimensions, so we have to worry about stresses in all the directions. So once we do that, so the general constitutive relation for a Newtonian fluid is they can be written as follows.

Now remember that the stress tensor is symmetric tenser, so tau y x is tau x y is mu partial u partial y is partial v partial x and this is only for an incomparable Newtonian fluid. If we have a compressible Newtonian fluid, you will have some additional contributions. So I am going to write down the relations only for an incompressible fluid for which the continuity equation is simply the mass conservation equation reduce to del dot u equal to zero. Similarly, tau x z is tau z x is mu partial w partial x plus partial u partial z. Similarly, tau y x, y z is tau z y is mu partial w partial y plus v partial z. But you also have the three normal stresses tau x x is 2 mu partial u partial x tau y y is 2 mu partial v partial y tau z z is 2 mu partial w partial z.

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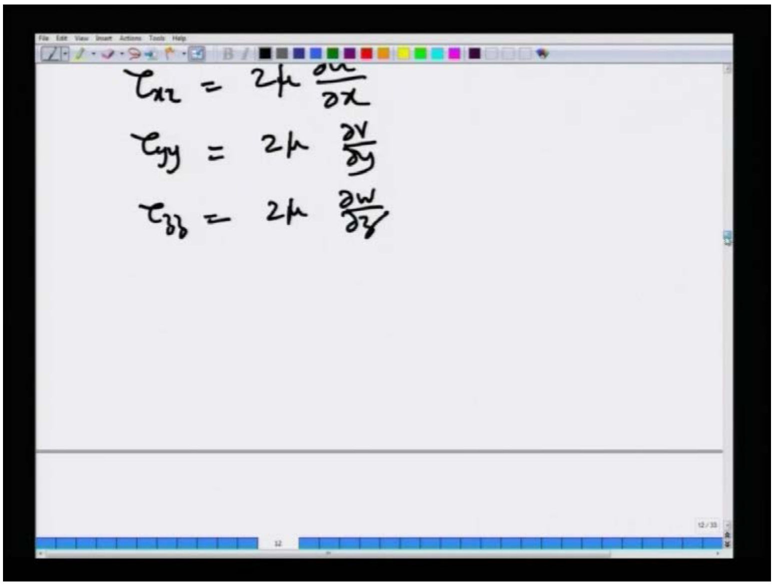

When we substitute this set of equations in the momentum equations like here, we have to substitute for the each value of stress, the constitutive relations that we just wrote down for a Newtonian fluid. Then you will get the following equations called the navier stokes equations. The navier stokes equations are simply the differential form of the momentum balance for a Newtonian fluid. We are going to specialize for incompressible fluids as well as Newtonian fluids.

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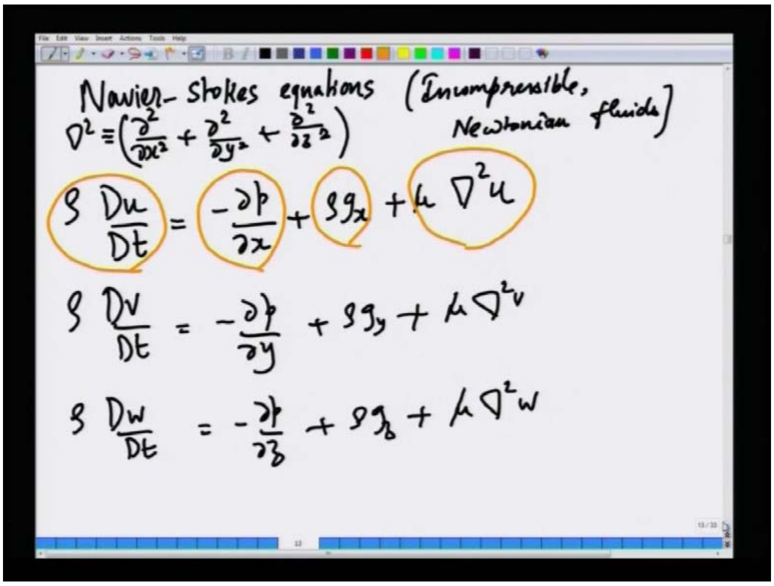

Now, this we can write now rho times substantial derivative of the velocity partial p partial x plus rho g x plus mu del square u, where del square is the laplacian operator given by partial square plus partial x square partial square by partial y square partial x partial square by partial x square plus partial square by partial y square plus partial square by partial z square.

Similarly, you have the y component of the momentum balance plus mu del square v. Similarly, the z component of the momentum balance del square w. Now, the interpretation is again similar, these are the inertial terms. That is, the mass times acceleration per unit volume of the fluid or the rate of change of momentum per unit volume of the fluid. These are the pressure forces acting on the unit volume of the fluid is the body forces due to gravity acting per unit volume of the fluid.

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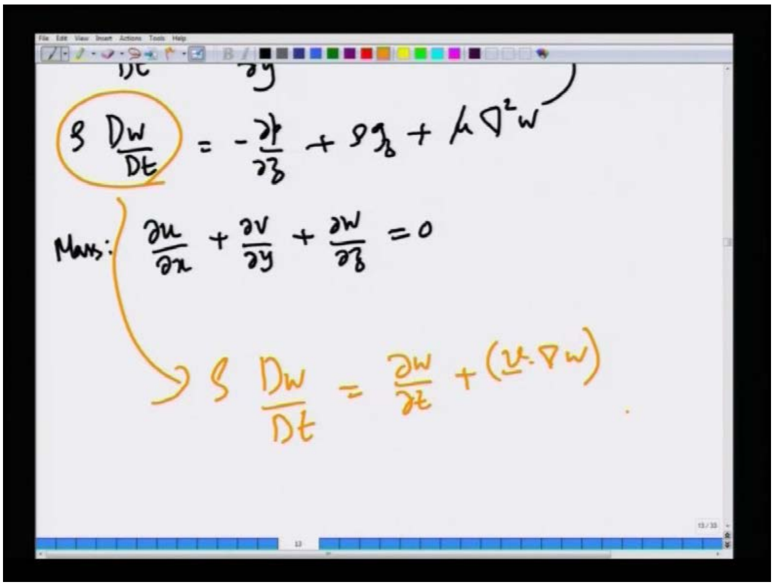

These are the viscous forces acting per unit volume of the fluid that is the interpolation of the each term. Of course you also have the mass conservation equation for an incompressible fluid. So you have the three momentum equations plus one mass conservation equation. And the number of unknowns are four, the number of unknown are the three components of velocity and one component of one pressure, so four equations and four unknowns.

So in principle, the problem is well set mathematically speaking, because you have the required number of differential equations for the same set of, for the required number of unknown set, we have four equations and four unknowns.

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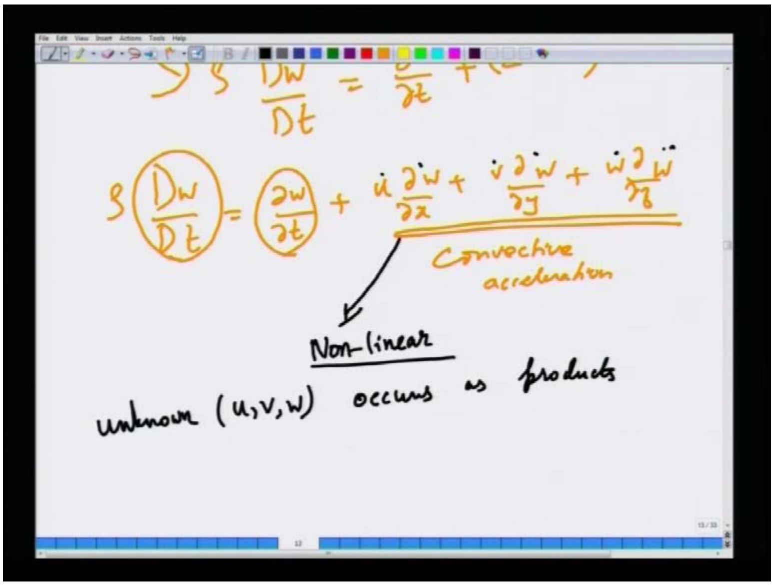

But it is also useful to appreciate the complexity of these equations, because if you take the substantial derivative of any quantity, rho Dw Dt is partial w partial t plus v dot del w. But what is that, rho Dw Dt is partial w partial t plus u partial partial x of w plus v partial partial y w plus w partial partial z of w. so each of this equations is coupled to the other equations through these terms, because if you want to write an equation for the z momentum. If you look at it just like that, it appears like w id only a function of w and pressure. But, that is not simple that not so, because the total acceleration substantial derivative to tells you what is the total acceleration of a of a fluid particle that is equal to local acceleration.

The partial derivative of w with respect to time plus the convicted convictive acceleration. Now, that couples a given component of velocity to the other component of the velocity. And what is more important is to realize that this is non-linear. That is, what we mean by this is that, the unknown, let say u, v or w occurs as products. You have one unknown multiplying another unknown and so on.

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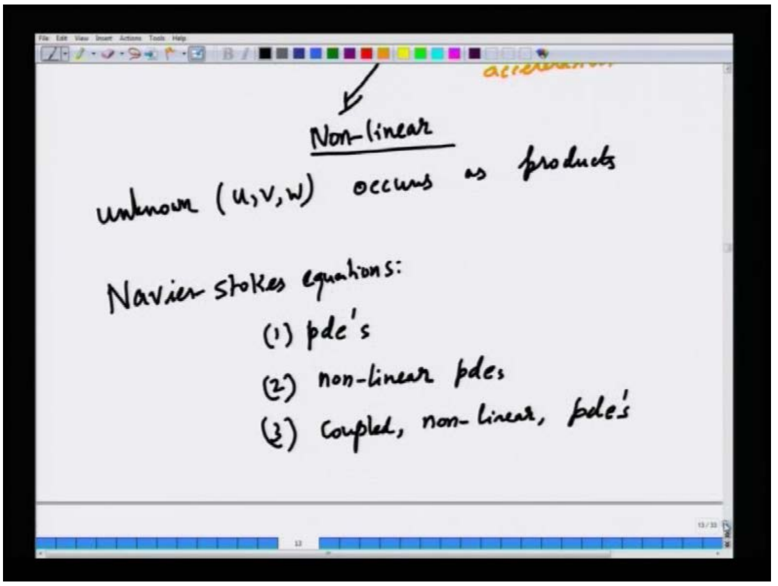

So the navier stokes equations are partial differential equation, because you have partial derivatives. These are pde's; as they are called they are partial differential equations. And they are non-linear partial differential equations, because of the convector non-linear pde's. And they are also coupled, because one equation is not independent of the other they are linked by the convector acceleration term. So they are coupled non-linear pde's. So, they are not in general very easy to solve. Because in mathematics, they are well defined well developed techniques for solving linear partial differential equations such as separation of variables. But, when you go to the realm of non-linear partial differential equations, there are no straight forward methodologies to solve non-linear partial differential equations.

So, it is often decided that one has to approximate the navier stoke equations into suitable forms, then which can be solved hopefully in a in a systematic analytical way. Otherwise, one has to take recourse to computational methods. And that branch of fluid mechanics that deals with the computational solution or numerical salutation of navier stokes equations is called c f d, computational flow dynamics. Of course, in this course we will restrict ourselves to merely analytical solutions of the navier stokes equations in simplified settings by making suitable approximations physically motivated approximations. But, it is also important to keep in the back of once mind that these are extremely difficult equations to solve in general without making in simplifications.

So, if you want to solve these equations without making any simplifications, the only recourse for us to use powerful computing computers and to solve these numerically. So, that is an important thing to keep in mind, one when considers navier stokes equation. Now, just as a aside, you also have what are called Euler equation. The Euler equations are the navier stokes equations without any viscous effects.

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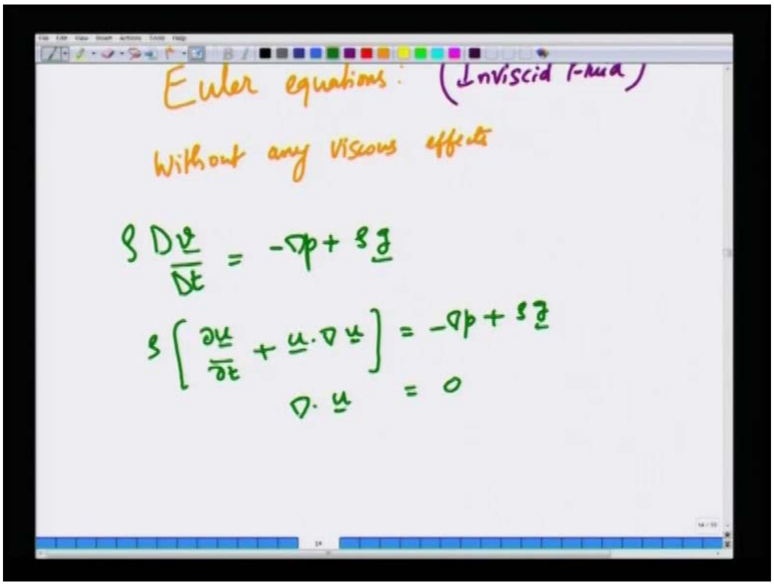

That is, if you set the viscosity to zero which you cannot do in reality, because no there is no fluid that has zero viscosity. All fluids have some viscosity some could some may be higher, some may be lower, but they do have some non-zero viscosity. But if you consider hypothetical fluid of zero viscosity, then we say we get what is called the Euler equation. So, this is applicable for an inviscid fluid by that we mean a viscous fluid with zero viscosity that is a non-viscous fluid. So, what are the equations for such a fluid? Well, it is rho Dv by Dt is minus grand p plus rho g, these are the Euler equations. But, if you want to of course, expand out the substantial derivative in order to clearly see the non-linearity.

So this is the Euler equation in conjunction with, in association with the continuity equation. So, this is for an inviscid or frictionless flow or a non-viscous hypothetical non viscous fluid. Of course, there is no fluid that has in reality has zero viscosity. But we will see a little later that in some regimes, we can assume that the viscous effects negligible compare to other effect such as inertial forces or pressure forces. So, it is possible for us to model the fluid as though it is inviscid, so will come to that later. So this is just to tell you that the navier stokes equations are the most general equations, but if you set the viscosity to zero we get what are called the Euler equations.

So the navier stokes equation are the most general set of equations that describe the balance of linear momentum at each and every point in the fluid. And these are restricted of course, to special class of fluids called Newtonian fluids, where the stress tensor the stress is directly proportional to the rate of deformation and the constant of proportionality is the viscosity fluid. The navier stoke equations are in their most general form are extremely difficult to solve, because they are partial differential equations, they are non-linear and they are coupled. So in what follows, we are going to make some very simplifying approximations to get to be able to solve the navier stokes equation, since some simplify settings. But one must keep in mind that, in general one has to use a computational technique to solve the navier stokes equation using numerical methods.

But having said that we still have to supply some more information, in order to able to solve navier stokes equation. As we know, in order to solve any differential equation completely, we have to if you integrate a deferential equation, you will get constants of integration. And those constants can be fixed only by applying conditions at the boundary, so there are called boundary conditions. The boundary conditions also are physical statements that are not part of the conservation principle that is a linear momentum balance, which essentially came from a Newton second law of motion.

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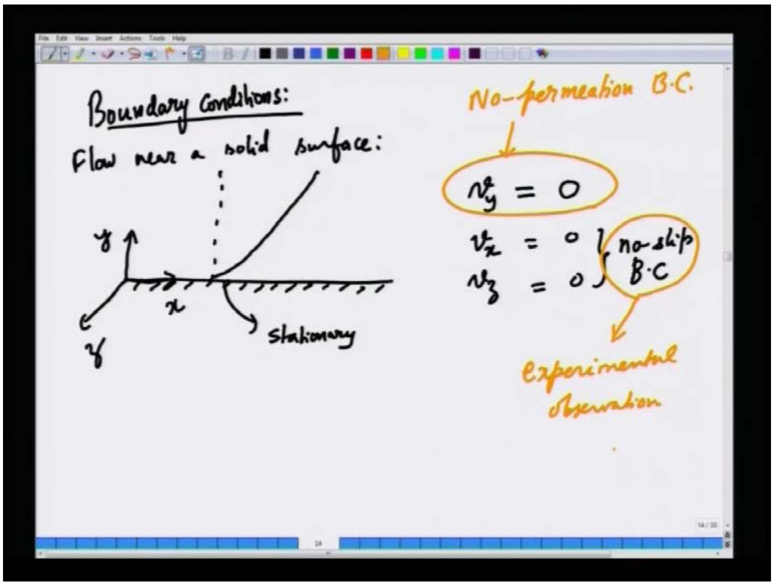

So the linear momentum balance is a fundamental principle. While the constitute relation such as navier stokes equation, the Newtonian constitute relation is not as fundamental. the boundary conditions are some statement of physical fact that when, let us say you have fluid flowing pass a solid surface or when you have two fluid flowing pass to each other. So, these are the physical nature of the conditions that happen at the boundary of the flow. Typically, you will find that you will have a fluid flowing past a solid wall, so flow near a solid surface; you have flow near a solid surface. Let us say the solid surface, the boundary is stationary. If the boundary stationary, then one of the most important conditions is that, first of all if the boundary is stationary, let us assume the flow in the x y plane, the third direction is z. The boundary is stationary first thing we can say is that, if the boundary rigid first thing we can say is that v y equal to zero.

Because you cannot have any fluid flow in to a rigid boundary, because it is impermeable, so this is the normal velocity condition. But it also turns out that whenever a fluid flows past a solid surface, the tangential components of the velocity v x and v a v z are also zero, this is called the no-slip boundary condition. Boundary conditions are all often abbreviated with the let us B C. Where is this condition is more fundamental in the sense that it merely tells you the fact that, if you if the fluid is the wall is impermeable to the fluid, then you cannot have flow into the wall, so this is called the no permeation boundary condition.

While this is these two the tangential components of that velocity vector v x and v z that they are zero is a consequence of what is called the no-slip boundary condition. This is merely an experimentally observation. It turns out that most fluids when they flow past a solid surface obey this condition. It cannot be provide rigorously, whereas this is merely a statement of mass conservation. That if there is no fluid flow into the solid boundary then the normal component of the velocity is zero.

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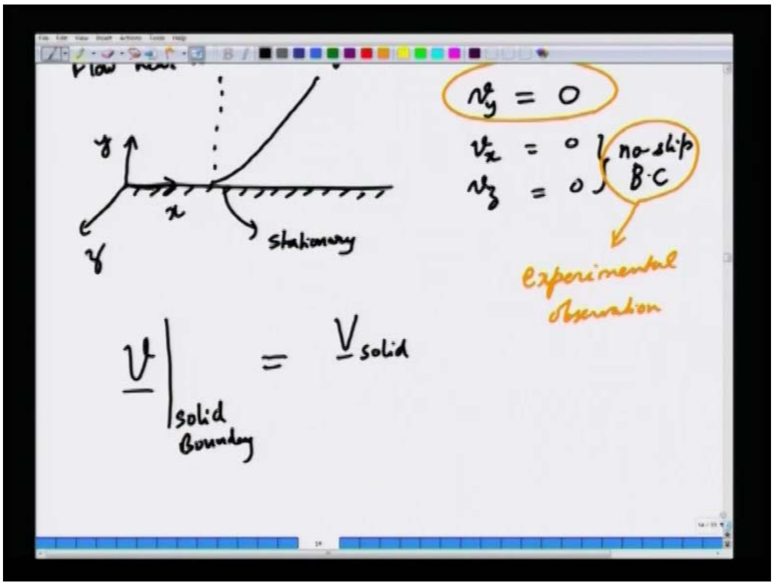

So essentially what we are saying, therefore are that when you have flow past a wall, the velocity vector of the fluid at the boundary, and at a solid boundary are equal to the velocity of the solid. If the velocity solid is stationary then the velocity of the fluid is zero at the solid boundary. The solid is moving with velocity, the fluid will take on the same velocity as the solid. So this is one important set of conditions that we will use while solving problems. Now, it is not that you will always have only fluid flow past a solid surface, you may often have flow of two liquids, two invisible liquid passed to each other. So, let say x then z, it is a y and the third direction is z, this is liquid one, and this is liquid two.

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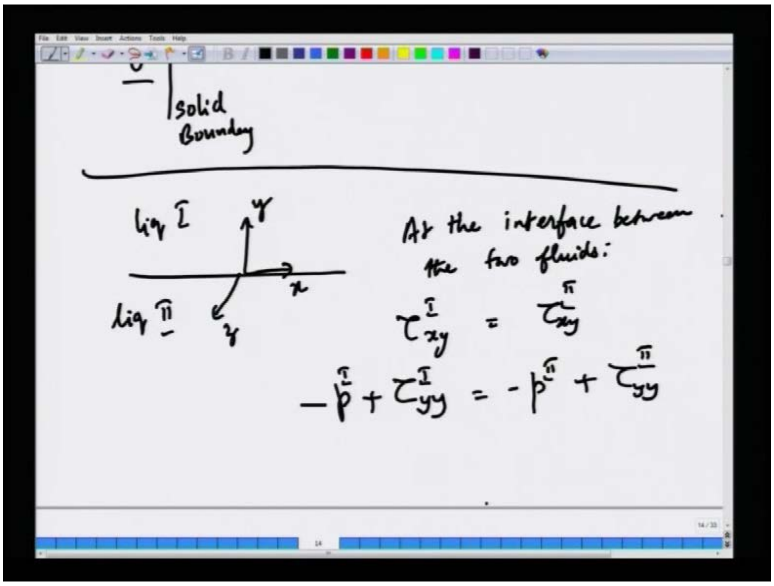

Then the condition are that the stress must be continues at the interface, this is at the interface between the two fluids, tau x y in fluid one is equal to time tau x y in fluid two. Then the normal component of the stress is also continues tau y y in fluid one is also equal to tau y y and fluid two. And in order to do this, we have to the normal component has pressure also, we have to include that as well pressure in fluid two plus the normal component of the viscous stress in fluid, this is the condition at the interface between two immiscible liquids

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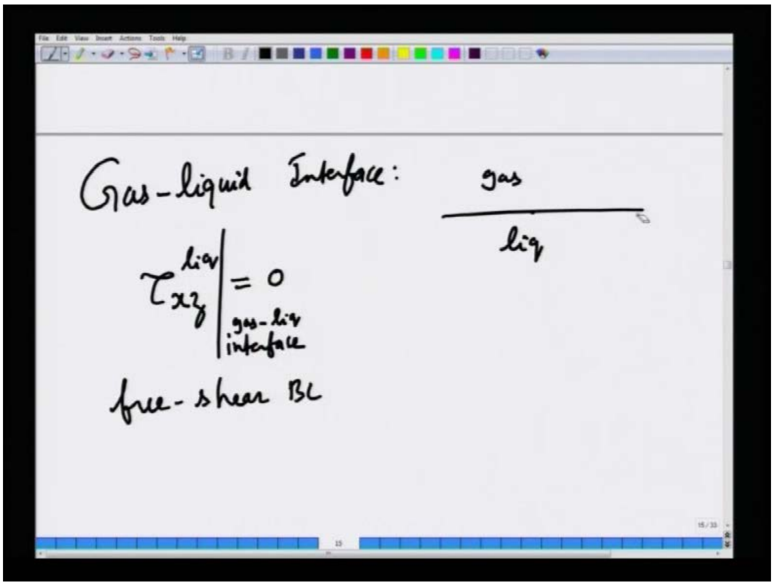

One of them could be a gas also. But in the specific case of Gas-liquid interface, since the viscosity the gas is very small, it is you can often approximate that the shear stress exerted by gas on the liquid is zero. So, what is often for found is that, if let us say you have a liquid and a gas, then the shear stress at the interface the stress exerted by the gas on the liquid is negligible. So, we say that the shear stress on the, at the of the liquid at the gas liquid interface is zero, this is called the free shear boundary condition.

At a gas liquid interface, the shear stress of the liquid is zero, because the gas exerts negligible shear stress on the liquid. In principle, the shear is continues, but since the magnitude of shear stress exerted by the gas on the liquid is small, you can as well neglected, you can assume that it is merely zero. So, this is another condition that one often uses, when you have either flow of a film that is exposed on one side to the atmosphere which is comprised of air.

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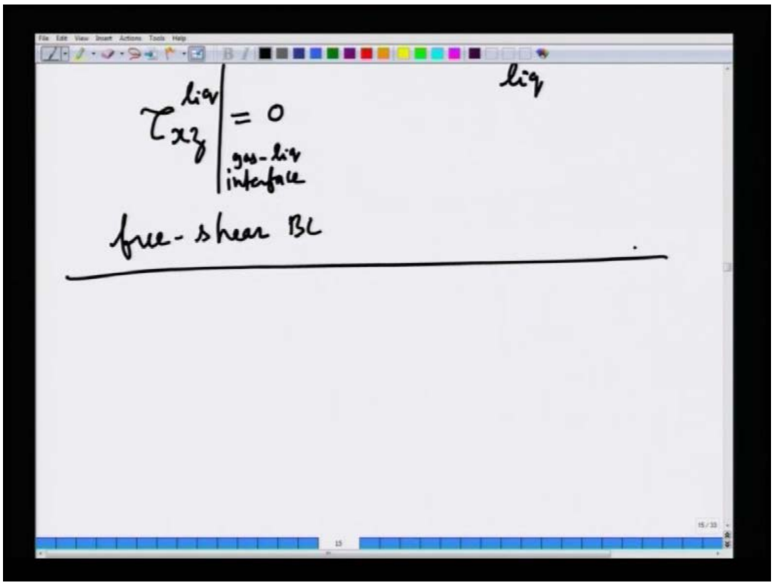

So, we can neglect the shear stress exerted by the air on the liquid. So, to recap what we did today, we understood clearly what the different, the constitutive difference between a fluid a viscous fluid and the elastic solid is. In an elastic solid, the stress is directly proportional to the amount of deformation, that is the strain and the constant of proportionality is shear modulus or the elastic modulus of the solid.

And if the elastic modulus is more, then we will find that it is very very difficult for us to deform the solid. While, if the elastic modulus is less it is easy for you to deform the solid. Where as in a fluid the stress not proportional to the amount of deformation or extent of deformation, it is proportional to the rate at which the fluid is going to deform. Or the rate of deformation is essentially a measure of the flow of the fluid. So in a fluid, the stress is proportional to the rate of deformation, which is measured by the gradient of velocity and the constant proportionality is the viscosity. Now, higher fluid of higher viscosity is difficult to flow when compare to fluid of lower viscosity. By the same argument, because the stress is directly proportional to the rate of deformation and the constant of proportionality is the viscosity.

Now, when you substitute the constitutive relations back into the momentum balance, the differential momentum balance, we obtain for a Newtonian fluid, what are called the navier stokes equations, which are non-linear coupled partial differential equations in general very difficult to solve. But, we will solve the navier stokes equations by making simplifying approximations. In order to solve navier stokes equations, you must supplement or you must provide additional conditions at the boundary, because any solution of a differential equation will have some constant of integration. And when you have constant of integration, they can be fixed only by specifying, let us say what is a velocity or the stress at the bounding surfaces. By doing so, we can finally, fix the constant of integration. Now, we are ready to apply the navier stokes equations to very very simple flow settings by making suitable approximations. We will continue this in the next lecture.