Fluid Mechanics

Prof. Vishwanathan Shankar
Department of Chemical Engineering
Indian Institute of Technology, Kanpur

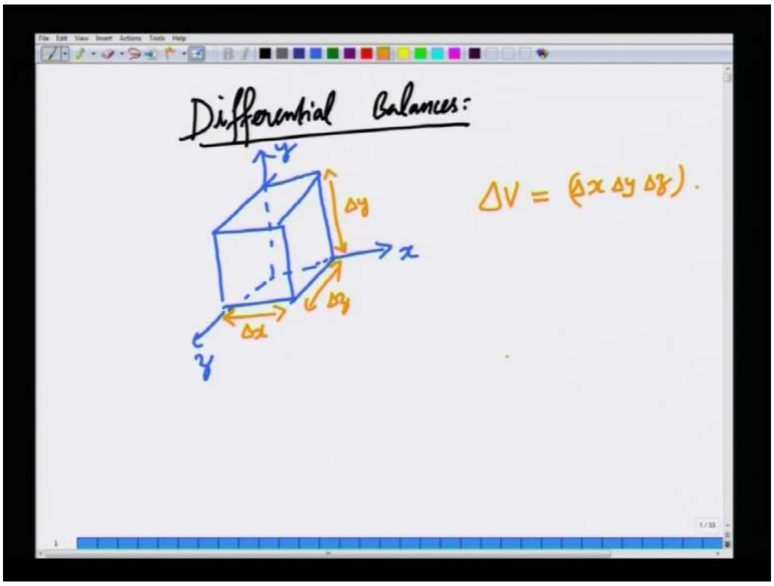
Lecture No. # 21

Welcome to this lecture number 21 on this NPTEL course of fluid mechanics for chemical engineering undergraduate students. The topic of our discussion in the last lecture, lecture number 20 was differential balances. Differential balances are balances of mass momentum and energy that are written down for a fluid, and these are valid at each and every point in the domain of the fluid. See, so the advantage of using differential balances is that one can after solving the equations that come out of differential balances. One can get a detailed description of the velocity and pressure distributions in a flow field, such as flow in a pipe or flow passed a body like a sphere or an airplane wing.

So, while it has the most exhaustive or accurate information that one can get for a fluid flow, but the down side is that the solution of these differential balances are not easy. And often one has to use computer, a very powerful computer to solve the equations that govern the mass, momentum, and energy for (( )) at each and every point in the fluid. But these are the most rigorous equation that one has for describing fluid flow. So, it is useful to know, and have a good feel for how these equations are derived, and how they are simplified to get some solutions that are often used in many applications. The other approaches that we already discussed, the other approach that we already discussed is that of integral balances. In integral balances, we do not write down equations that are valid at each and every point in the flow domain. Rather, we write down we choose a very huge control volume which encompasses many equipment, such as pumps, and turbines, and so on.

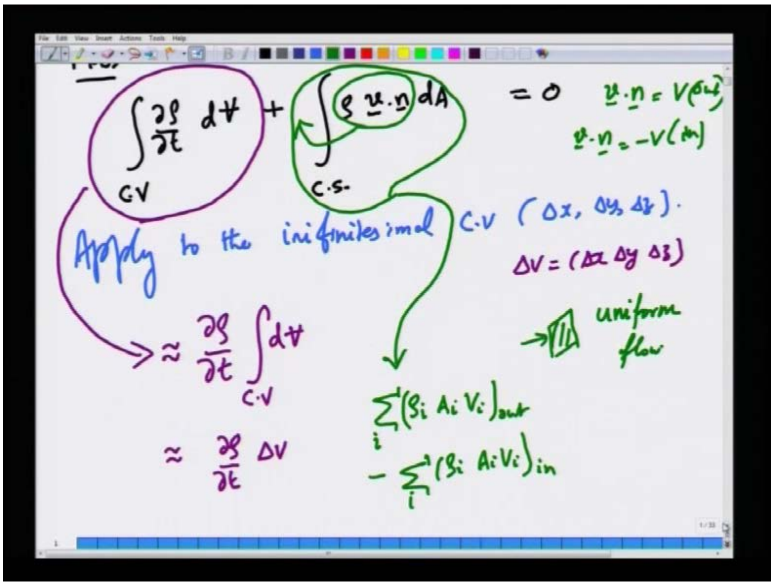
And one writes the overall balance of mass momentum energy, but that so the integral balances are fairly simple. But the point is the use of the integral balances is will involve quantities such as viscous losses or forces on solid surfaces which are not known a priory. So one is left with rather incomplete set of equations, because one does not know in general what are the losses. So, one is often left with the choice of neglecting certain important feature like losses, in order that one can solve the integral balances. At times experiment data is available to estimate these losses, so one can use them or as we will see a little later. By solving the differential balances in simple systems, one can actually find out the losses. So, the basic idea in differential balances like the basic idea in differential balances is that one takes a very very tiny control volume of infinitesimal size, so let us try to draw for simplicity cubic control volume.

(Refer Slide Time: 02:51)



So you have three cartesian directions. Lets draw it lightly short, you have x, y and z, three orthogonal directions. And the distance is along the three directions are simply, so this is simply delta y and this is delta s this is delta z. So, the volume of this infinitesimal control volume is delta x delta y times delta z. And we are going to take an infinitesimal control volume in the sense that, we are going to eventually take the limit of delta x delta y and delta z going to zero. So, essentially this control volume will shrink in the limit to a point. And thereby, we are we going to derive equations that are valid at each and every point in the fluid. Since, we are taking the limiting process of delta x delta y delta z going to zero; we can anticipate that we can get differential of a various quantities.

(Refer Slide Time: 04:39)

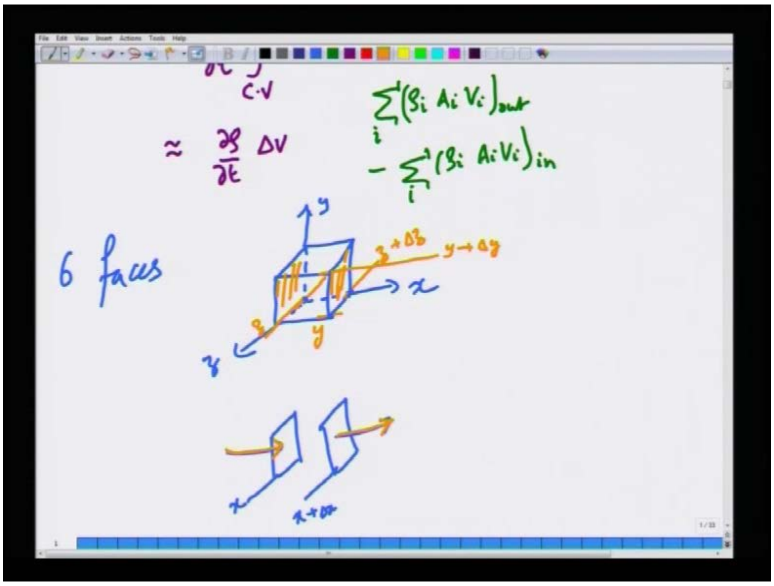


So that is why we use it is these balances are called as differential balances. What one does is to derive the differential balances is, to use the integral balance for c v and take the limit of an infinitesimal c v. So, we look at mass conservation of mass as a plate with c v. We already derived that rate of change of mass present in the c v plus integral rho v dot n dA over the control surface is zero. This is the most general equation that governs mass conversation for c v. Now, we are going to apply this to this infinitesimal c v. So apply to the infinitesimal c v with sides delta x delta y delta z, the cubic c v that we just drew here. Now, few simplifications arise, when we apply a general conservation equation such as this integral balance to c v. First of all, since let us look at term by term, so if you look at this term. Look at this term, now since the c v so tiny, the variation of density within this c v can be neglected to a first approximation.

So we can pull this out of the c v, this is the approximately this, in the limit of this c v volume tending to zero. We can write this very easily and this is nothing but, if you integrate this volume we will simply get times delta v, where delta v is delta x times delta y times delta z, the volume of the c v. So, that is one simplification that comes in the first term. Even if you look at the second term, we can do some simplification. Now, since the c v cross sectionals areas are so small, we can assume uniform flow.

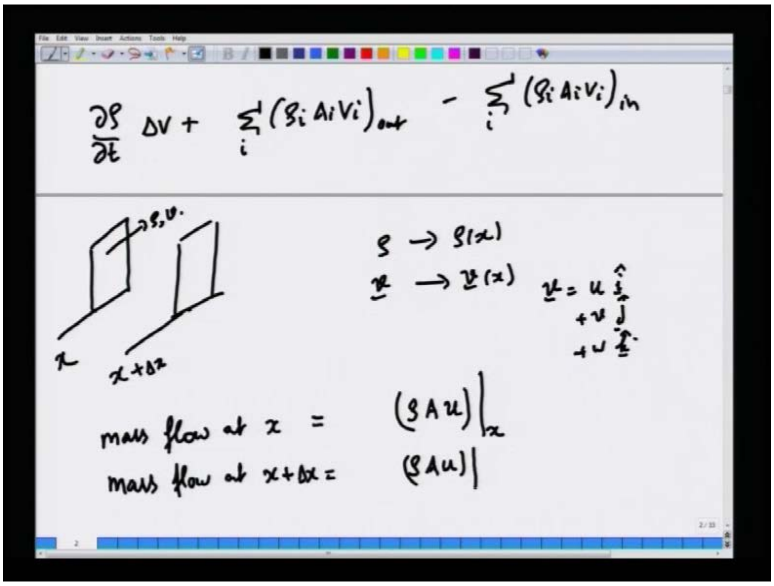
We can simply write this as, summation over all outlets, for outlets v dot n is positive remember is v for outlets and is minus v four inlets, this is something that we had seen before. So, i times rho. So let us, now the key thing is that if you pull, if things are uniform we can pull this outside the integral. And integral over d will simply give you A, the area of cross section true which fluid is flowing, so we will give rho a rho i A i V i out minus rho i A i V i through all the all the inlets.

(Refer Slide Time: 07:41)



Now we do, we have c v like this, a simple cubic c v and x, y, z. This cubic c v is has six faces, one at x one at x plus delta x another at z plus delta z delta l z one at y one at y plus delta y, so there are three six faces at various stations. Therefore, fluid can come in and go out of any of the six faces. Just for the fake of clarity, we going to assume to that fluid is going to enter for example, the face at y this is the face at y, face at let see say the face at x and its going to leave the face at x plus delta x. That is fluid is entering the face at x and fluid is leaving the face at x plus delta x. So this is, so we made a small mistake here, the face at x, so this is the coordinate x, the face at x is here, the face at x plus delta x is here. So that is what I have drawn here. The face at x is the left side of the c v, the face at rise at x plus delta x here is here the right side. So fluid can enter through this and leave through this. Well, you can also have that the fluid entering through this and leaving through this or both sides fluid entering through both sides and leaving through some other face. That does not alter any of our derivations, because we are going to assume the velocities to be algebra quantities that is they have sign associated with them.

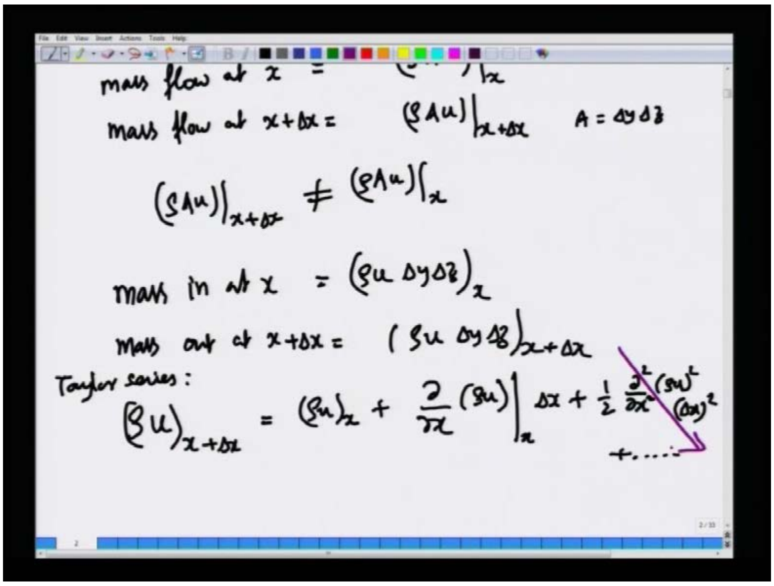
(Refer Slide Time: 09:55)



So, accordingly in a given problem they will take a plus or minus signs accordingly. But just for the sake of clarity in derivation, we are going to assume that fluid is coming in through there and leaving through that. So we have had this equation d rho dt times delta v plus summation i rho i A i V i over the three outlets minus rho i A i V i over the three inlets, this is what we are going to use to derive the equation. Now to do that, first of all we have to account for the amount of mass is that enters at x and then we have to find out the amount of mass that leaves at x plus delta.

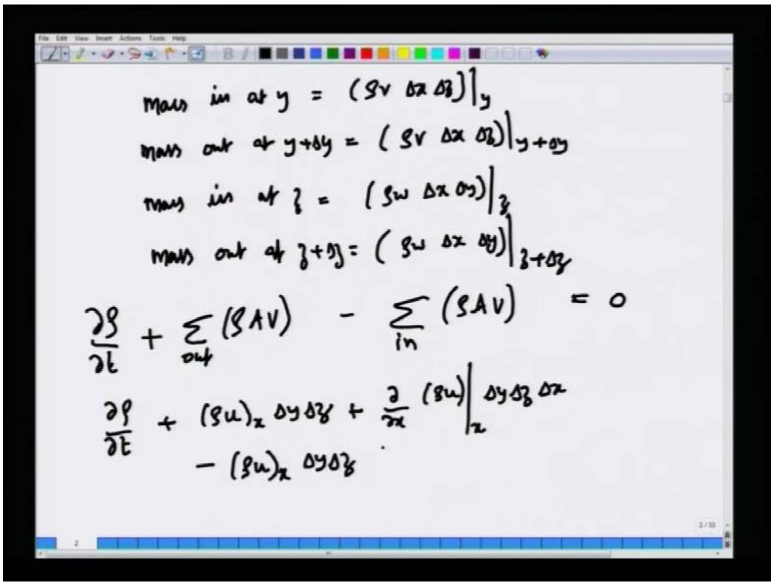
So, in general the quantity such as rho v will be a functions of x, the velocity vector will also be a function of x, because we have use the continue hypothesis. And many all fluid quantities such as rho, v, p, (( )) pressure rho density, velocity, pressure they all vary smoothly and continuously with special coordinates. So, we will have to evaluate in order to find out the mass flow, at x is equal to rho a times the x, so velocity is a vector its u times i plus v times j plus w times k. So, this is the x component of the velocity evaluated at face x, mass flow at x plus delta x is rho A u at x plus delta x.

(Refer Slide Time: 11:46)



Now, in general rho A u at x plus delta x is not the same as rho A u at x. All though A is same, if you assume that that the cube area is not changing, but rho and u in general are not same. So, first of all, what is A? A is equal to in this face delta y times delta z. Here also, A is delta y delta z. So, the mass flow, mass in at x is rho A sorry rho u times delta y delta z evaluated at x. Mass out at is rho u delta y delta z at plus delta x, this in general not the same as the quantity rho u at x.

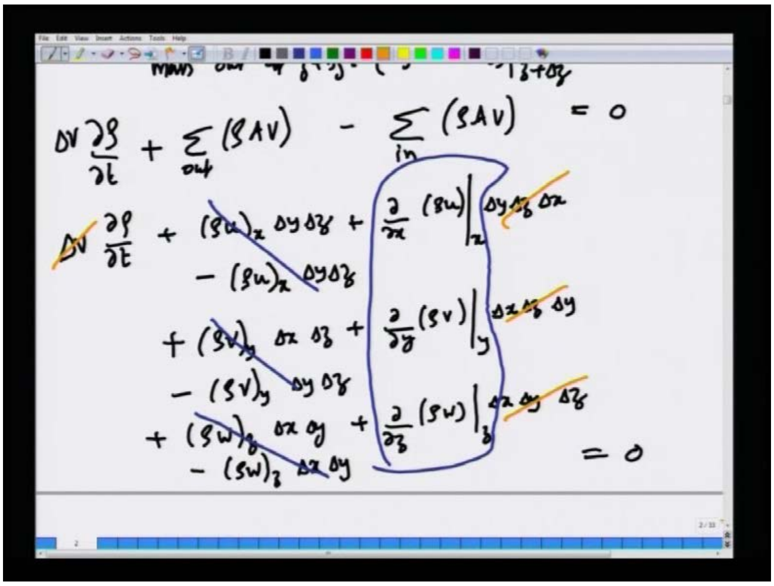
(Refer Slide Time: 13:39)



But you can use, what is called Taylor expansion or Taylor series expansion. To find out what is rho u at x plus delta x, this is rho u evaluated at x plus d d of rho u evaluated at x time delta x plus half d dx square d square x dx square times delta x square plus so on. Now, we are going to drop all the higher order terms. So, we are going to restrict only to this order. So, if you want write down and likewise you can write down at the directions, mass in at y is rho v delta x delta z at face y, mask out at y plus delta y is rho v delta x delta z at y plus delta y. And then mass in at face z is rho double w delta x delta y, mass out at rho w.

Now, we going to use this equation d rho d t plus summation over all outlets rho A v minus summation over inlets rho A v is zero. So, d rho dt plus. Now, let us look at all the three faces separately. For the face at x, the outlet well mass flow rate is rho u at x times delta y delta z plus d dx of rho u at x, I am sorry its delta y delta z times delta x that is the Taylor series expansion. We are neglecting higher order terms, minus this inlet flow rate in the x face which is rho u at x times y delta y z.

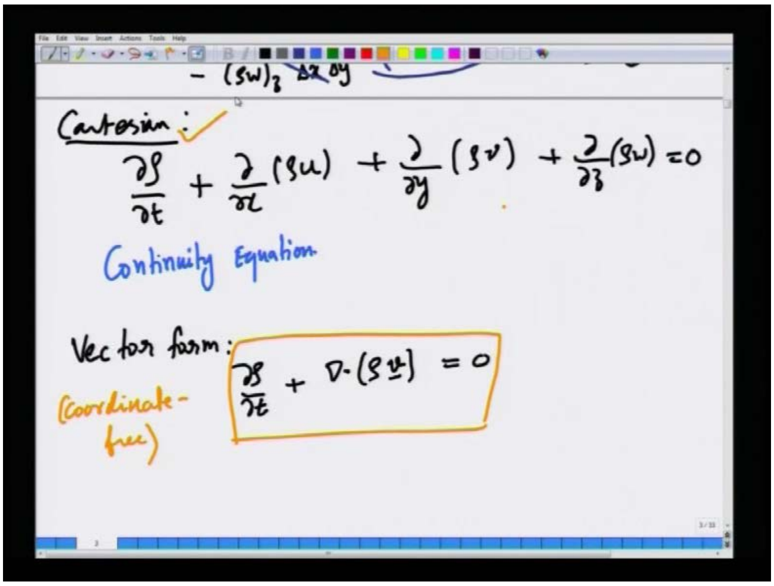
(Refer Slide Time: 15:35)



Similarly, you can write down for other two faces, plus rho v at y delta x delta z plus d dy of, after having Taylor expanded, rho v at y delta times delta x delta z times delta y. This delta y comes from the Taylor expansion, minus the inlet which is rho v at y times delta y delta z plus the z face rho w at z times delta z, delta x delta y plus d dz of rho w valuated at z times delta y delta x delta y k times delta z, which comes from the trailer expansion, minus rho w z times delta x delta y is equal to zero.

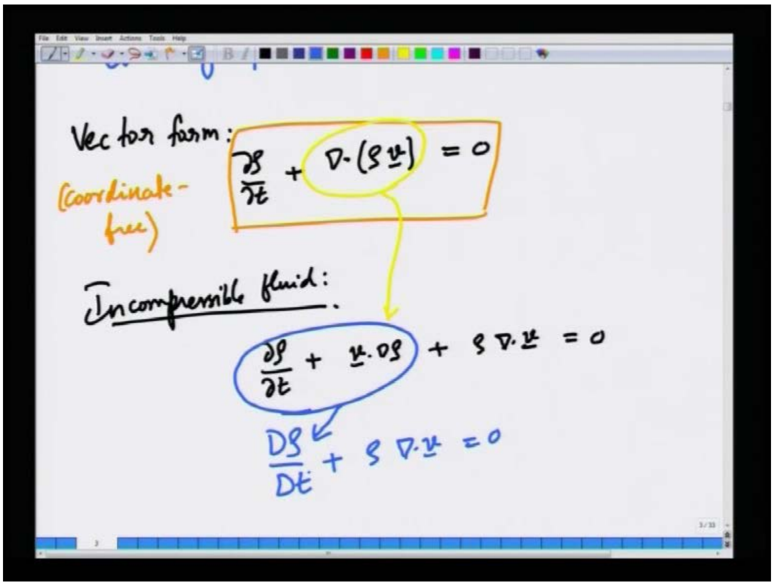
Now we can see that several terms are going to cancel away. For example, these two terms will cancel, these two terms will cancel, and these two terms will cancel leaving us with only this term. Now, if you see all these terms are multiplied by the volume and this term is also multiplied by the volume, if you remember which we wrote earlier, is delta v is nothing but, delta y delta z and del times delta x. So, you can divide the entire equation by delta x, delta y, delta z, to give simple expression for the mass conservation d dx of rho u plus d dy rho v plus d dz of rho w is zero.

(Refer Slide Time: 17:08)



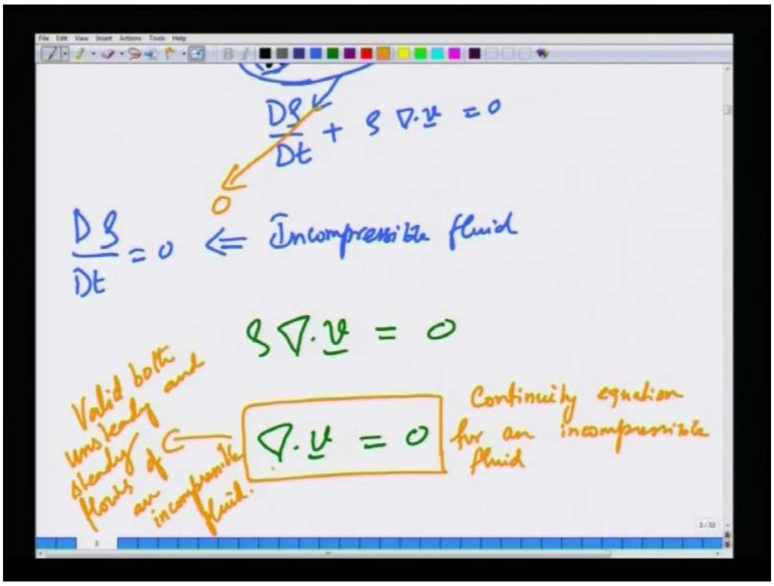
Now, this equation is valid at each and every point in the fluid, this is also called the continuity equation or equation of continuity. Now, we also wrote down a coordinate free vector form of this equation. This form of the equation is valid only in cartesian coordinates, cartesian reference frame. Now, if you want to write it in general other coordinates, in other coordinate frames, we will use the quantity the propriety of a gradient or a divergence to write this as follows. And this is a vector form which is coordinate free form.

(Refer Slide Time: 18:35)



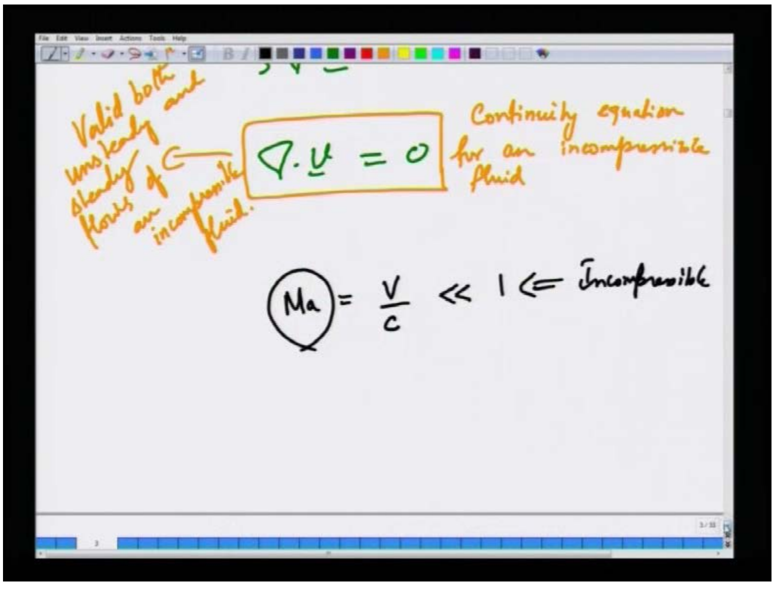
So, once you now what are the proprieties of divergence in other coordinate systems, you can use this form and write down the equation of continuity in other coordinate systems. While you cannot use this form in other coordinate system that is only valid for cartesian coordinate system. So, having said that we proceeded further to use this continuity equation to simplify it for an incompressible fluid. Before I do that, I can write this equation as d rho dt plus v dot del rho plus rho del dot v is zero. If we expand this term, you get two terms by using differentiation by parts. Now, this expression combined together is the substantial derivative of density plus rho from this term del dot v is zero is this.

(Refer Slide Time: 19:35)



The substantial derivative of any quantity is the rate of change of that quantity with time, as you follow a given fluid particle. Whereas, this term is simply the rate of change evaluated at a fix point in space. Now for incompressible fluid, d rho dt is zero. Suppose you follow a given fluid parcel, its density will not change with time, for an incompressible flow. So, if you knock this term of for an incompressible fluid, we get the continuity equation for an incompressible fluid to simplify, as rho times del dot v is zero. But since rho is a non-zero constant you have, this is the equation of continuity for an incompressible fluid, continuity equations for an incompressible fluid. It is important to understand that the time derivative goes not, because the flow is steady. It is because of the fact that the substantial derivative of density is zero for an incompressible fluid.

(Refer Slide Time: 21:12)

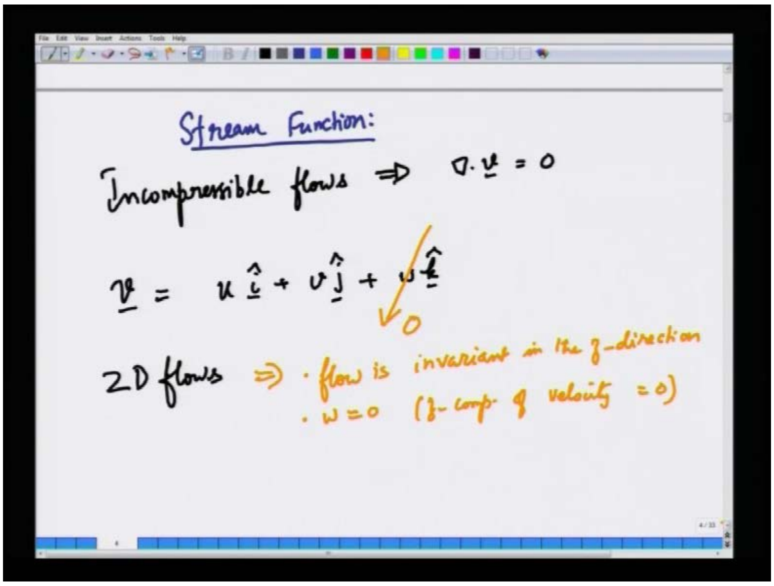


So this is valid for both steady and unsteady flows of an incompressible fluid, both unsteady and steady flows of an incompressible fluid. So that is something important for us to remember that this equation is in general true for both steady and unsteady flows, but it is applicable only for an incompressible fluid. We also saw that incompressible condition is valid when, in the last lecture, I would not go through this again. When the velocity typical magnitude of the velocity relative to the speed of sound is very very small, that is ratio of c is very very small compare to 1.

That is the Mach number is small compare to 1, then the flow is incompressible. If you want know that derivation, you please refer to lecture number 20, the previous lecture, I would not derive this here again. So, for this course will be predominantly focusing only on incompressible flows, wherein the density of a fluid parcel does not change as you follow the fluid parcel. So our continuity equation will always take the form of a del dot v equal to zero. That is the velocity field, the velocity is a vector quantity and it varies at each and every point in space, in general in a flow. But what this means is that del dot v equals zero means is that the velocity v field vector field has no diverges in an incompressible flow. The velocity can vary at each and every point in space, but in such a manner that the divergence of the velocity vector always has to be zero.

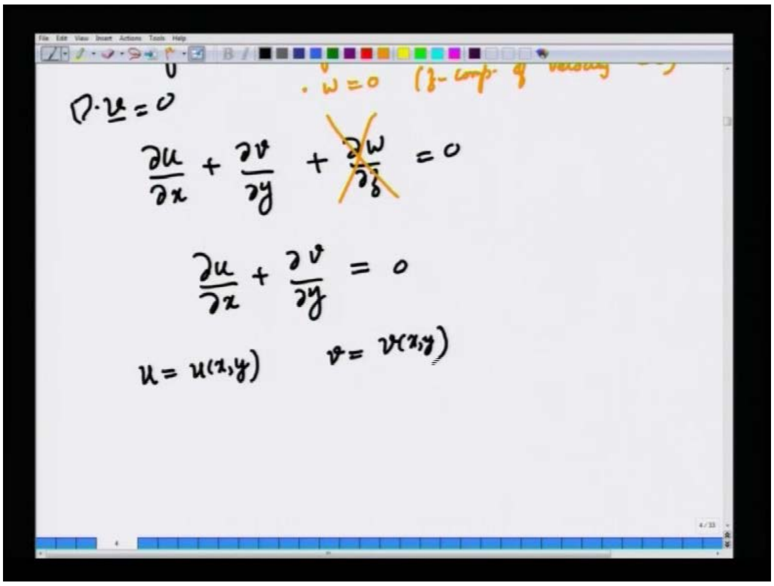
This is a very very rigorous and important constraint on the possible values which the velocity vector can take, because it cannot take any arbitrary values. It always has to satisfy the continuity equation del dot v equal to zero for an incompressible fluid, because it is essentially a statements of conservation of mass. So it is a very fundamental principle that no flow can violate. So although, we cannot show individual applications of continuity equation right now. What is important for us to understand is that the continuity equation must always be satisfied by any fluid flow. And for an incompressible fluid, it takes a simple form that the divergences of velocity filed is zero.

(Refer Slide Time: 23:15)



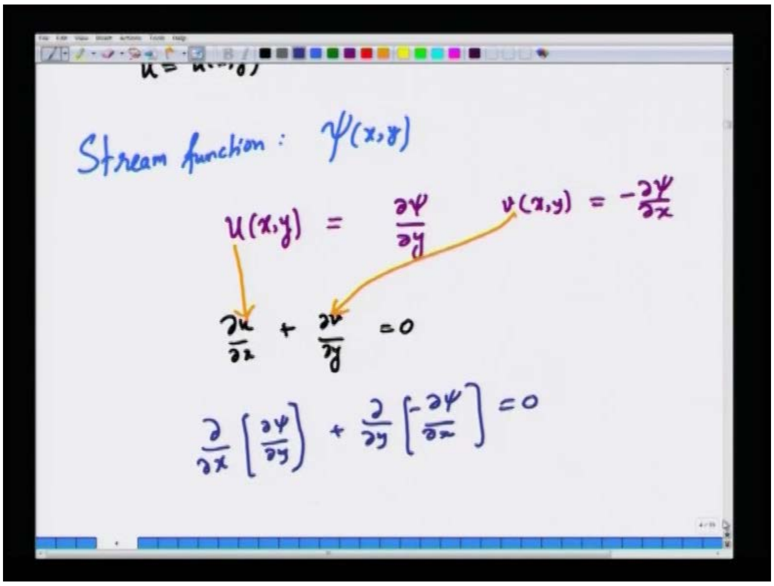
Now, we are going to go further, proceed further and we are going to introduce a notion of a stream function. Now, I am going to assume again incompressible flows. For incompressible flows, we always have del dot v is zero. In general fluid flows, the velocity vector velocity is a vector can have component in the i direction plus component in the j direction plus a component in the k direction that is the z direction. But we will restrict our attention now to what are called 2 D flows that is only these two components are non-zero. We can choose the coordinate frame in such a manner, that the z component of the velocity is zero. So the flow is invariant, 2 D flows means flow is invariant in the z direction and this is first point and the z component of velocity is zero. If you do this, so you have only in these two velocity components u and v.

(Refer Slide Time: 25:06)



The continuity equation for cartesian coordinates for an incompressible fluid del dot v equal to zero means partial u partial x plus partial v partial y plus partial w partial z is equal to zero. So this is del dot v is zero, but since we are said that the w is not there, it is a 2 D flow, it is an incompressible 2 D flow. So the in the continuity equation simplifies to or the mass conservation simplifies equation simplifies to partial u partial x plus partial v partial y zero.

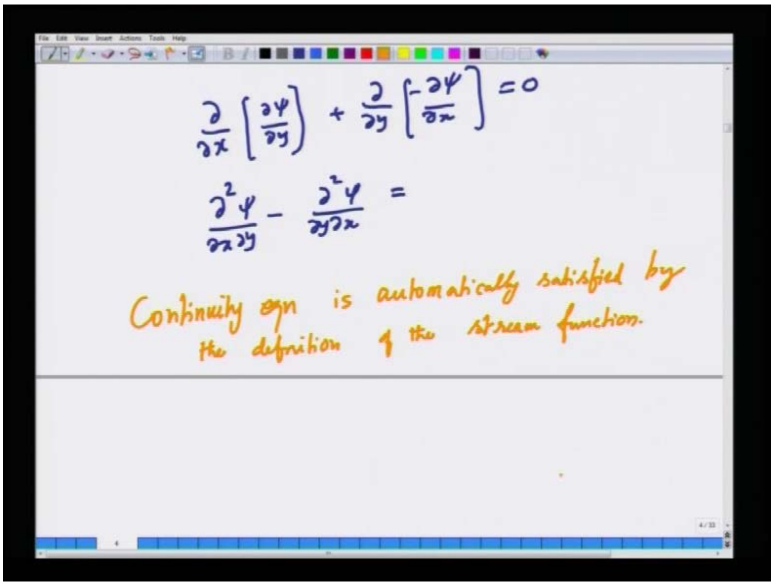
(Refer Slide Time: 26:02)



So and both the components of velocity u and v are functions of only x and y. Because, we have saying that assuming that the flow is invariant in the third direction. Now, we are going to introduce a concept, new concept called a stream function. It is a function of the two independent variables, its denoted by the Greek letter psi is a function of both x and y, and its related to the fluid velocities in the following manner. So, its related as follows u of x y is partial psi by partial y v of x y is minus partial psi by partial x.

That is this new function is constricted is defined in such a manner. That if you take and it is a function of the both the independent variables x and y in a two dimensional flow. If you take the partial derivative of the new function with the respect to y, you get the x component of the velocity u. And if you take the partial derivative of the new function psi with respect to x that is the negative of that is basically the y component of the velocity v, so this is the definition of string function.

(Refer Slide Time: 27:51)

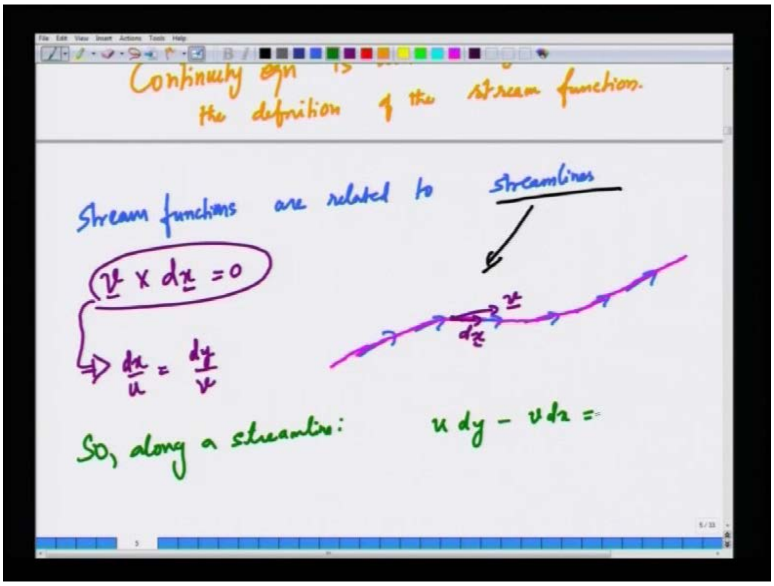


What is the advantage of this definition? Now, let us look at the continuity equation, you have partial u partial x plus partial v partial y is zero that is the continuity equation. Now, I am going to substitute u here and v here through this definition. So you get partial partial x of partial psi partial y plus partial partial y of minus partial psi partial x is zero that is the continuity equation. Now, I can pull this minus sign outs, you get partial square by partial x partial y psi minus partial square by partial y partial x psi, this is the continuity equation. But partial square psi by partial x partial y is actually partial square psi by partial y partial x, because the order on which you take partial derivative should not matter for a continues function of two variables. So this means that an important conclusion can be drawn that the continuity equation is automatically satisfied by the definition of this stream function, where the definition of the stream function.

So this is a very important conclusion, because now we are finding the, by constructing the stream function in this faction by defining the stream function in this fashion, your automatically satisfying the continuity equation for two dimensional in compressible flows. That is a great simplification, because when you want to proceed further to momentum balance, you will find that you will have two sets of equation. The mass balance and momentum balance which are linked with each other or they are coupled to each other.

But by defining this stream function, you are doing away with one equation, your we have to solve one equation less. And it is always nice to solve a lesser number of equations and more number equations mathematically speaking. So, in that sense the stream function is very very helpful, in the solution of two dimensional incompressible flow problems. So, it is a purely mathematical reason has to why this I mean this motivation of stream function is purely mathematical. That of course, it simplifies the solution of the equations by reducing the number of equations by one.

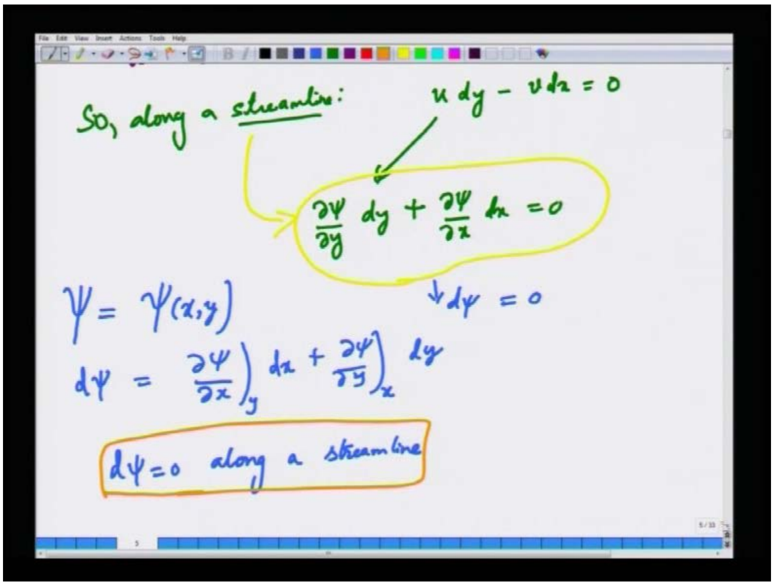
(Refer Slide Time: 29:58)



So that is one motivation for introducing this stream function. Another motivation for the stream function is that, we have to first understand that stream functions are related to; we are going to show functions are related to stream lines. We are discussed stream lines, stream lines long back when we discussed fluid kinematics. Kinematics, if you remember is a description of flow without worrying about what are the underlying causes of the flow that is forces acting on the fluid. So there we define stream function, stream lines as lines which are always parallel to the fluid velocity vector. Suppose you have fluid, let us just first draw the fluid velocity vector, suppose you have at each and every point the velocity vector is pointing in this direction. We can connect each and every point a line that is tangential to the fluid velocity vector and such a line is a stream line.

Stream line is tangential to the, it is tangent to the fluid velocity vector at each and every point in the flow. So if two, if you have a local velocity vector v and the displacement along the stream function is dx. And if these are parallel, then the cross product of v cross dx is zero. This is something that we did it last time; we did few lectures back when we did kinematics. Now this gave rise to the condition, if v cross dx is zero, then we found that du by u dx, we found that dx by u is dy by v. This is something that we derived by using the fact that they are two vectors with the velocity vector and the displacement vector along the stream line they are parallel, so the cross point of these two vector should be zero, that give rise to this equation.

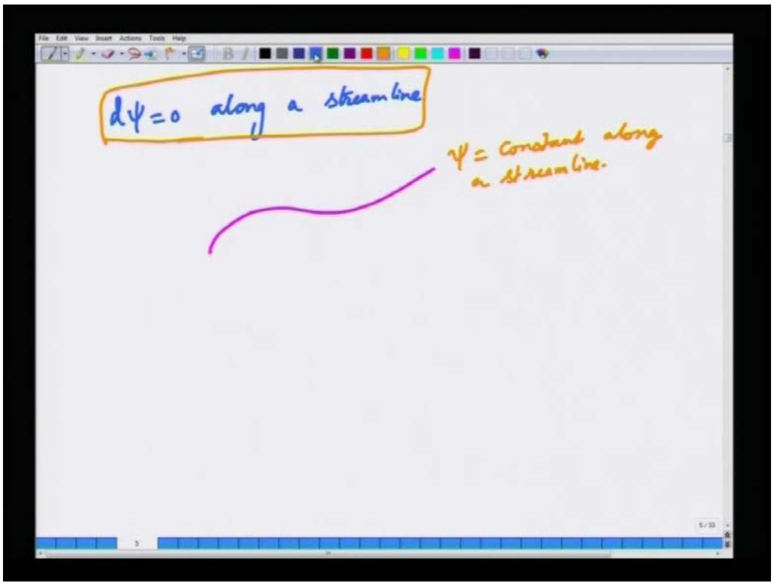
(Refer Slide Time: 32:24)



So along us stream line along the stream line, we can say that u dy minus v dx is zero. This is along the stream line from the definition of the stream line. Now, I am going to use the definition of the stream function that u is partial psi by partial y dy v is minus partial psi by partial x, there is also one more negative sign. The two negatives will make a positive partial psi by partial x dx is zero. So along the stream line, we are finding this relation. Now, if you know little bit of calculus, you will realize that if psi is function of two variable x and y. Then the variation in psi a small variation in psi that occurs because of small variations in x and y can be written as partial psi by partial x at constant y times dx plus partial psi by partial x, partial psi by partial y at constant x times dy is zero.

This is I am not saying it zero, this is the variation in psi because of variations in small variations in x and y. But we find from a definition of stream function that along the stream line, from the definition of a stream line this quantity identically zero, so d psi is zero along the stream line. This something that we just found, because this is evaluated along the stream line. And this is nothing but, d psi the total change in psi that is incurred, if you change so d psi, if you change x by a small value and y by a small value. So, if d psi zero along a stream line what it means is, what this means is that if you go along a stream line psi has to be a constant. Because along a stream line, if you change the variation in the stream function, it turns out to be zero rigorously, so psi is a constant along a stream line. This is a very major result, because if you want to plot stream lines, all you have to do is to flat lines of constant psi.

(Refer Slide Time: 34:27)

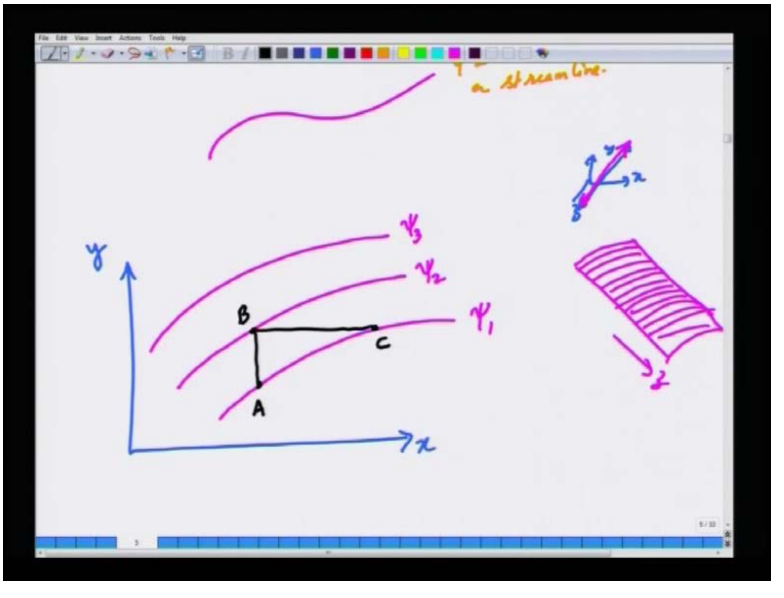


Once you have the information of how psi varies as a function of x and y, psi some of now two variables for a two dimensional flow. Then we can simply plot constant values of psi and those will be stream lines. So this is a very very useful result, because if you want to plot fluid flow behavior, if you want to visualize fluid flow behavior, it is often useful to use stream line stream lines to do that both experimentally as well as from a computational point of view. In that context, it is much much easier to find out the stream lines by just drawing lines of constant psi.

So this is one another use of stream function. So we found two uses so far of a stream function. Firstly, the stream function simplifies the solution of problems for two dimensional incompressible flow situations. Because by defining the free stream function like the way we defined just few minutes before, we found that the incompressibility condition is identically satisfied. So you have you are left one equation less in your solution scheme, so it is always helpful form a mathematical point of view. The second advantage is more physical that if you find the stream function, then lines of constant stream functions are stream lines.

Now stream lines are special lines in a flow domain, where the velocity vector the stream lines are always parallel or tangential to the velocity vector each and every point in the fluid. So, by knowing by plotting the stream lines in a flow domain, it is easier to have a visual understanding of what the flow is a going to be like. Now, there is another important third major implication of using stream functions and that relates to the volumetric flow rate that flows between two stream lines. So, let me explain this bit slowly. Suppose you have flow domain and we are any way restricting ourselves 2 D flows.

(Refer Slide Time: 36:49)

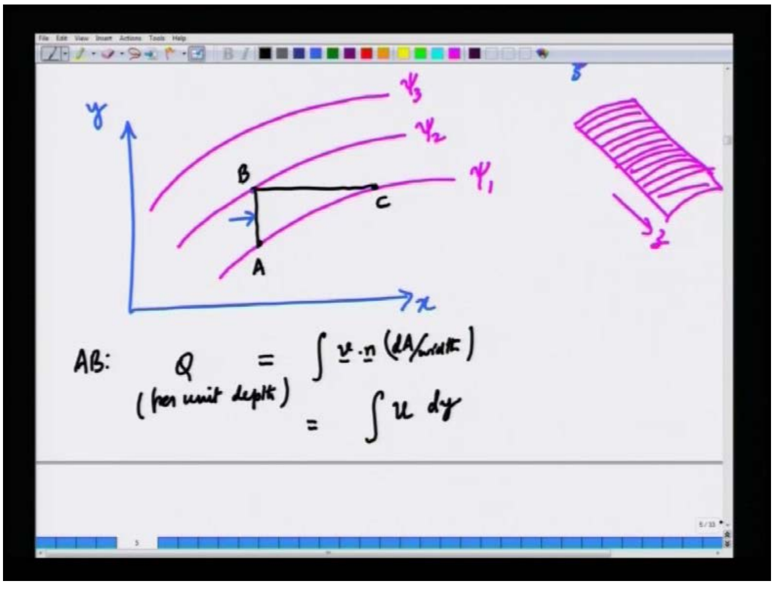


So you have a 2 D flow domain in the x y plain, what we mean by say, what we mean by 2 D flow is that the variation long suppose we have x, y and z. The variation along the third dimension z, this is the x is less very very less, so there is no variation in the third direction. So that is the meaning of saying that flow is two dimensional.

So, the another use of stream function is the fact that this values the difference between values of stream functions between any two points in a flow domain has some relation to the volumetric flow rate that flows in between the two stream lines. So, we first saw that lines of constant values of stream function denote a stream line. So, let us imagine a two dimensional flow, in any case our restriction our discussions are now restricted 2 D flows. So we have, let us say three stream functions, three stream lines one, two and three. And each stream line is denoted by a constant value of stream function psi 1, psi 2 and psi 3. Now, the claim is that and remember imagine remember that the flows are three–dimensional, two-dimensional in the sense there is no variation in the third direction z.

So these stream lines are almost like sheets, so in the third direction and there is no variation. Because you can always construct many such stream lines in the third z direction and there is no variation of any quantity in the third direction. So, you can cut at any cross section and they will look like that lines. Now suppose, if I look at any two points A, B, you can ask the question what is the volume of fluid that flows between these two points per unit width of the third direction z. And you can likewise consider some other orientation of the area, what is the amount of fluid that is flowing between volume of fluids that is flowing between this area per unit width in the third direction per unit time. So, that is the question that we want to answer.

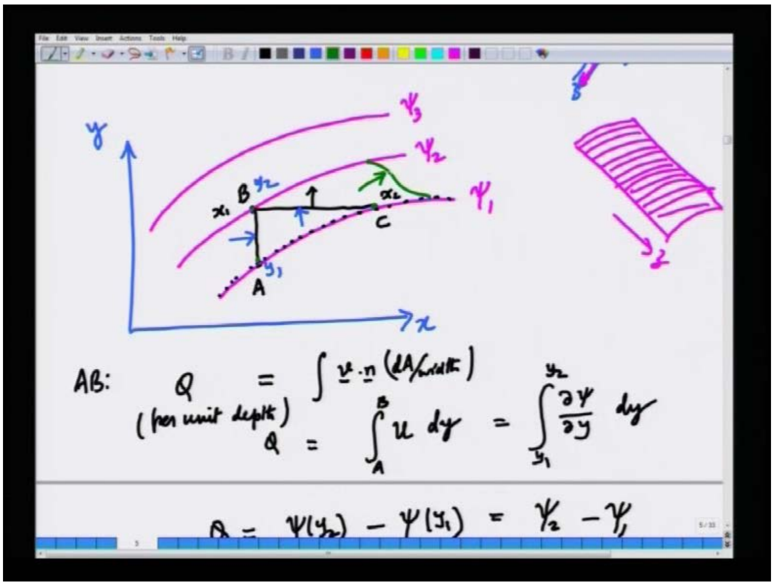
(Refer Slide Time: 39:18)



So, let us find out the volumetric flow rate. So the volumetric flow rate let us use the line A B. the line A, B is actually a surface, because you have to extend it in the third direction. And since things are invariant in the third direction z direction, you can calculate everything per unit width of the third direction. For the volumetric flow rate per unit depth, let us say the third direction across this surface A B. Now, I am choosing this surface A B, because in this surface along this surface A B, this surface is directly perpendicular to its along its perpendicular to the x axis.

So the only component to the velocity vector which is going to a cause flow it towards to the surface is x components. Because the y component is parallel to this surface so it cannot cause any flow. So the volumetric flow rate is nothing but, integral it, in principle it is v dot n times dA. now, in principle but, since it is unit width we cannot write dA. So, we have to write it as some write it as dA per width. And for this simple system, since v is purely along the x direction v dot n becomes u. And the coordinate for integration is simply dy, but per unit width of the third direction, so you do not have to worry about the third direction.

(Refer Slide Time: 40:56)

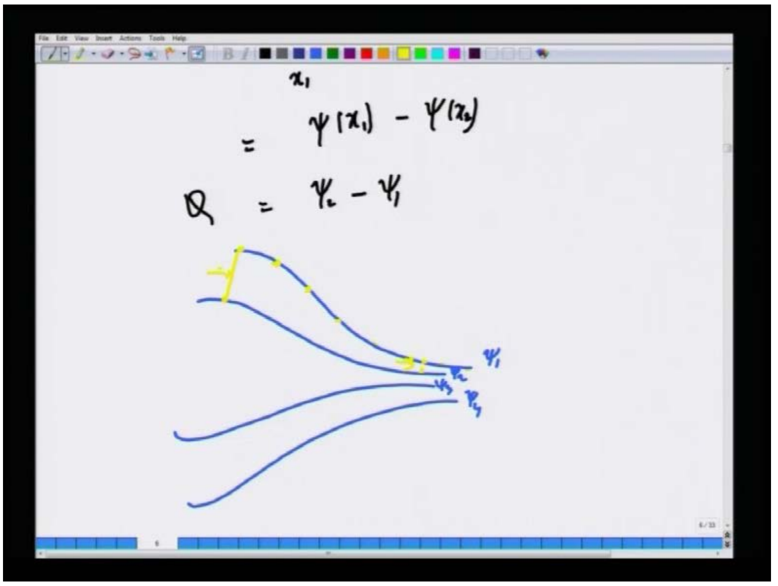


So, q is simply integral A to B u dy. and what is the differentiating thing between points A and B. it is the y coordinates, this is y 1, this is y 2. So this is simply y 1, simply equal to y 1 to y 2. Instead of u, I can put partial psi by partial y t y. so q is nothing but, psi at y 2 minus psi at y 1. So and psi at y 2 is simply psi 2 and psi at y 1 is psi 1, because along each and every point in this line the stream function is a constant psi 1.

I can do this for the psi surface or lines B C also, let us do that for the line B C q is integral, so if you look at this diagram. Now, this is completely perpendicular to the x, it is completely perpendicular the unit normal is perpendicular to the y direction; it is along the y direction. And the surface is completely normal to the y direction, so the only component of velocity that is going to contribute is the y component of the velocity. So the normal component is just the y component. So, q will be written as per unit width will be written as, now the only thing that differentiates between these two points between B and C is their x positions, here its x 2 here its x 1, so integral x 1 x 2. So we are now computing the point x 1 here, x 2 here. Volumetric between these two points u v dy v dx, because the area of the its going to be integrated along the x directions, so dx.

V is nothing but, minus partial psi by partial x dx, so this is psi of x 1 minus psi of x 2. Now psi of x 1 is psi 2, because x 1 lies on the psi 2 line, psi of x 2 psi 1, so this is nothing but, psi 2 minus psi 1. So, what we have shown is that by choosing two different completely different areas. We have shown that the amount of fluid that flows between these two points is the same as the amount of fluid between flows at between these two points.

(Refer Slide Time: 44:06)



So if you take any two orientation, the amount of fluid that flows between these two points per unit width of the third direction per unit time is always given by the difference in the value of the stream functions between those two stream lines. So, this the difference in values of stream function between any two stream lines directly gives you a measure of how much volume of fluid that is going to flow between those two points between those two points.

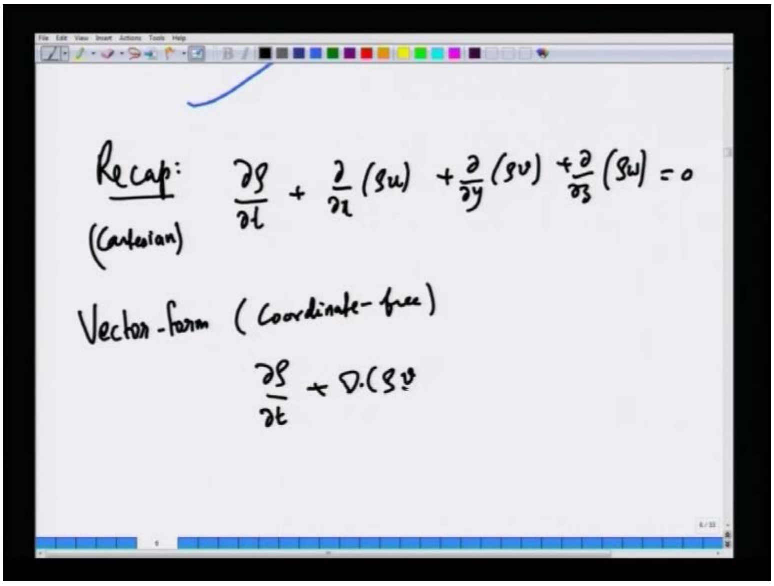
So, what is the big advantage of this description? The big advantage of this description is that, suppose you have a stream flow pattern like this. These are stream lines, so these are values of stream lines psi 1, psi 2, psi 3, and psi 4. So, along at each along at each and every point along this stream line time, psi is constant psi 1. So if I take psi 1 minus psi 2 the difference, it is constant from here to here, because it is the same stream line, the same two stream lines. But here the width, the distance between those stream lines is much more compare to a point like here.

So, if the difference between two points the (( )) stream function values is related to the volumetric flow rate. More amount of volume, the same amount of value has to flow between these two points as well as these two points, because psi two minus psi is constant. That means at the velocity here will be less compare to the velocity here, because of the fact that this two stream lines are converging.

So by immediately having, by having quick look at the stream line distribution present in a fluid flow, you can immediately say where the fluid velocity is going to larger and smaller. Because, the difference between values of stream functions between two, any two stream lines is always equal to the amount of volume that is going to flow across any line drawn between these two stream lines. Since, psi 2 minus psi minus constant along the stream lines at along any two points along the stream lines.

So when the whenever the two stream lines converge, we can immediately conclude that along at the point of converge the velocity vectors will be much much greater compare to the point where the stream lines are divergent. So, this gives rise to a way very quick aid for flow visualization, because once you have a huge data, let us say from a computer solution or an experiment in fact.

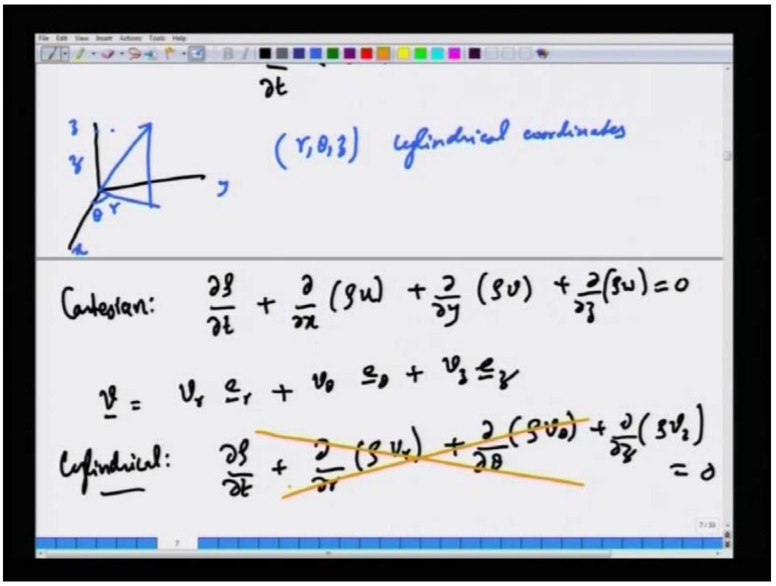
(Refer Slide Time: 46:38)



By plotting all the data in terms of stream lines, one can immediately get a qualitative feel as to where the flow velocities are large and where the flow velocities are small. So this is the big advantage of differential stream function. So to just to recapitulate what we did in today’s lecture. We first derived the most general form of continuity equation, for cartesian coordinate system that becomes the following, this is for cartesian coordinates. Now, we can also write this in vector form which is coordinate free. That is the nature of the equation that we are going to write in the vector form is coordinate free is independent of coordinate we choose to work with.

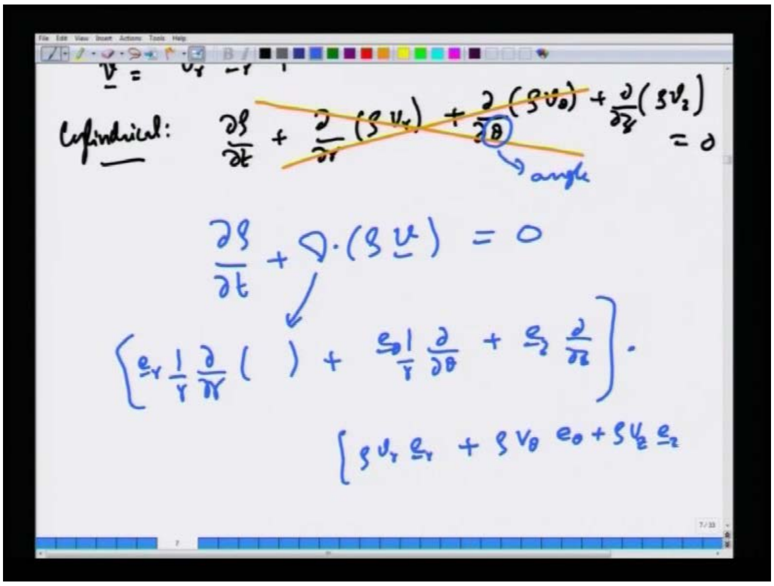
It is always the same, it is also called the abstract form, d rho dt plus del dot rho v is zero. Now, in many applications you will see little later that it is often convenient to use coordinate system that corresponds to the symmetry of the particular system. We may be interested in flow through pipes or flow past a sphere. In which case, you may not it may not be convenient to work with cartesian coordinate systems and it may be beneficial for us to use a cylindrical coordinate system or a spherical coordinate system.

(Refer Slide Time: 47:48)



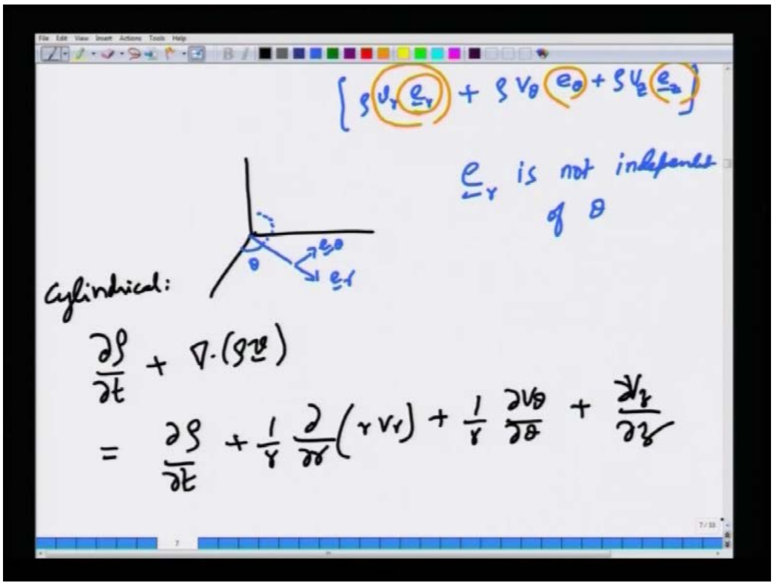
In a cylindrical coordinate system any point is denoted by r, which is the distance from the origin to the projection of the vector in the x y plane, theta which is the angle the projection makes with the x axis and z which is the normal z coordinate in the cartesian coordinate direction, so you have r theta is z in cylindrical coordinates. Now just, so in cartesian coordinates, the continuity equation or the mass conservation equation in the differential form becomes this. But, suppose you in a cylindrical coordinate, the velocity vector is written as v r e r plus v theta e theta plus v z e z.

(Refer Slide Time: 49:24)



So, if you want to write the continuity equation, we cannot write by steady by just extending the continuity equation in the cartesian coordinates, this is wrong. Because for the first reason that this equation is such a straight forward extension of the cartesian continuity equation in cylindrical coordinate, it is not even dimensionally consistent. Because this is an angle variable, theta is an angle it has no dimensions does not have dimensions of length, whereas all r and z have dimensions of length. So, clearly you cannot readily extend this two more other coordinate system such as cylindrical and spherical coordinate systems. A more fundamental reason is that, if I look at the abstract form, it becomes divergences of divergence of rho v. Now, divergence in the cylindrical coordinates system is one over r plus e theta one over r d d theta plus e z d dz.

(Refer Slide Time: 50:50)



This is a gradient operator dotted with rho v r e r plus rho v theta. So, plus rho v phi, rho v z e z. if you do this operation, key thing for us to remember is that in cylindrical coordinates. If I look at the r direction and theta direction, this is e r, e theta is along this direction. If I move along the theta direction, you can see that e r is going to change its direction. So e r is not independent, it is not always pointing in the constant direction; if you change theta e r varies. So, this is a very generic feature in curvilinear coordinates wherein the unit vectors will change the directions as you go along a given coordinate direction. So unlike in cartesian coordinates where the unit vectors i, j, k, are always pointing in the same direction, regardless of no matter where how you very x, y, and z, that is not the same. So essentially the crux of the matter is that, when you take the divergence, you not only must take the divergence of v r, but also e r, e theta and e z and this will give rise to a different kind of equation.

So, I would not to do the details, but it is important to know that in cylindrical coordinate systems the continuity equation will become the following. So, one cannot readily extend the cartesian continuity equation like here to cylindrical coordinate systems. Because of the fact that unit vectors themselves will vary with special coordinate in curvilinear coordinate systems. So that is something that we must keep in our mind, and the same thing applies even for spherical coordinate systems. Having done that, we introduce the notion of stream function which is very very useful, especially when you consider two dimensional flows. Because, you need not satisfy the in compressibility equation or continuity equation separately, because the stream function by definition satisfies del dot v equal to zero. So that is one big advantage of using a stream function for two dimensional incompressible flows.

The second advantage is that by, it this the stream functions, if you plot lines of constant values of stream function, they are exactly the stream lines. Remember that, the stream lines are always locally tangential to the fluid velocity vector. So, it is a very very useful way of visualizing fluid flows. The third important result that we derived is that, the different between two stream function values between any two stream lines gives you a measure of the amount of volumetric flow rate per unit width in the unit depth the other direction, for two dimensional flows.

So this also helps in flow visualization, because when the stream lines are far apart; and if they converge at some point. That also means at the velocity vectors are velocity magnitude of the velocity will be increasing, wherever the stream lines are converging. So, these are the very, so these three are very important implications of the definition of stream function. So, this really completes our discussion on a differential mass balance, which gave rise to the continuity equation. The continuity equation is the most extremely fundamental, because it always must be satisfying, but it cannot be solved in isolation. We have to combine it with linear momentum balance as well as energy balance. So, we have to go to the next step of deriving the differential momentum balance for fluid flows, which we will do in the next lecture.