Fluid Mechanics

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Lecture No. # 20

Welcome to this lecture number 20 on the NPTEL course on fluid mechanics for chemical engineering under graduate students. The topic of our discussion was integral balances of mass momentum, and energy in the last few lectures, and we want to complete this topic by discussing one final example.

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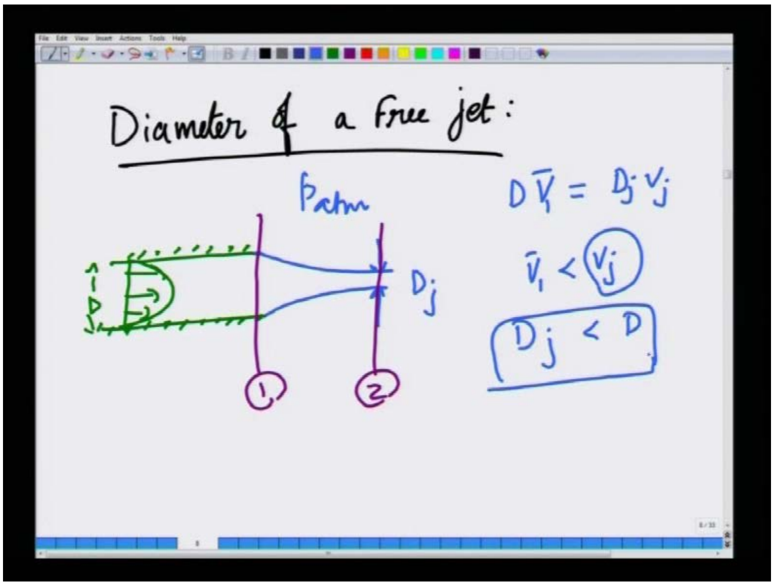

And we just briefly started doing this example in the previous lecture; essentially, it is to estimate the diameter of a free jet that emerges out of tube or a nozzle. So essentially, you have a liquid that is flowing under laminar conditions in a tube. Let the diameter of the tube be D and once a liquid exits, the diameter of the jet that emerges out of the tube is not necessarily the same as a diameter of the tube. That is because with when the fluid is flowing within the tube, there are viscous shear stresses exerted on the surface of the tube on surface of the tube that are exerted on the fluid, so which drags the fluid.

Whereas, once the fluid emerges into the atmosphere, so this is atmospheric pressure, there are no shear stresses that are acting. So the fluid tends to accelerate a little bit to reach the new steady stage. But when it tries do that, since there are lesser forces the x, let us first write down the mass balance, so it is says for an incompressible fluids D times V average is D jet. Let us call, let us try to draw the balance between stations one, which is here, and station two, which is here; D times V 1 is D j times V j.

So, V 1 will be smaller than V j, because of, so V j will be larger than V 1, because of the fact that lesser forces are acting upon it that tends to decelerate the flow which happened inside the tube. Whereas, it is does not happen. So at steady state, the mass conservation says that D V 1 times is D times V1 is equal to D j times V j. Since, V 1 is less than V j that implies that D j has to be smaller than D that is a jet diameter will become smaller than D that is from physical considerations. So how do you estimate or come up with an expression for the jet diameter? For that, we can either use the integral energy balance or the integral momentum balance. So I will, I am going to demonstrate both. First, let us use the integral energy balance, which we read in the end of the last lecture.

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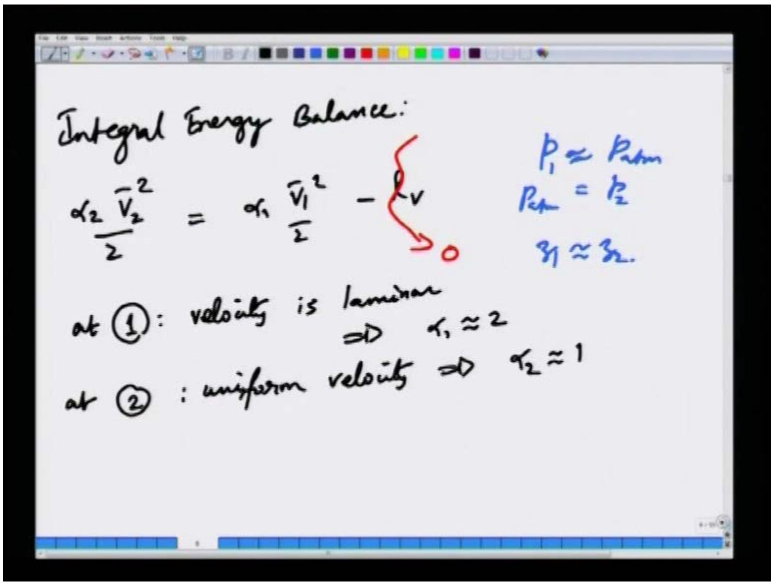

When we use the integral energy balance, we will find that alpha 2 V 2 squares by 2, the kinetic energy head is alpha 1 V 1 square by 2. I am writing the energy balance between 0.1 and 0.2, as shown here in this diagram, plus or minus viscous losses.

Now, because we have also assume that when the jet exits, p 1 is p atmosphere and that is also equal to p 2. Because in a free jet, it is of course the pressure is atmospheric as we have been mentioning. But at station one, when the just is jet is just about to exit into the atmosphere, we are assuming that so this is an assumption, whereas this is almost an exact relation. So, since p 1 is p 2, there is no p 1 pressure term and also the elevation that one is approximately z 2, so there is no gravity term in the energy balance. Now about losses, we can safely neglect them, because the losses in a free jet are negligible, because the surrounding air is not going to exert any viscous stresses. So the viscous losses are comparatively smaller, so we can afford to neglect them. Now at station one, the velocity profile is laminar, we have assumed is laminar. So alpha 1 is approximately two, at two the velocity is uniform; at station two the velocity is uniform, because it is a free jet. So uniform velocity which implies alpha 1 is 1 alpha 2 is 1.

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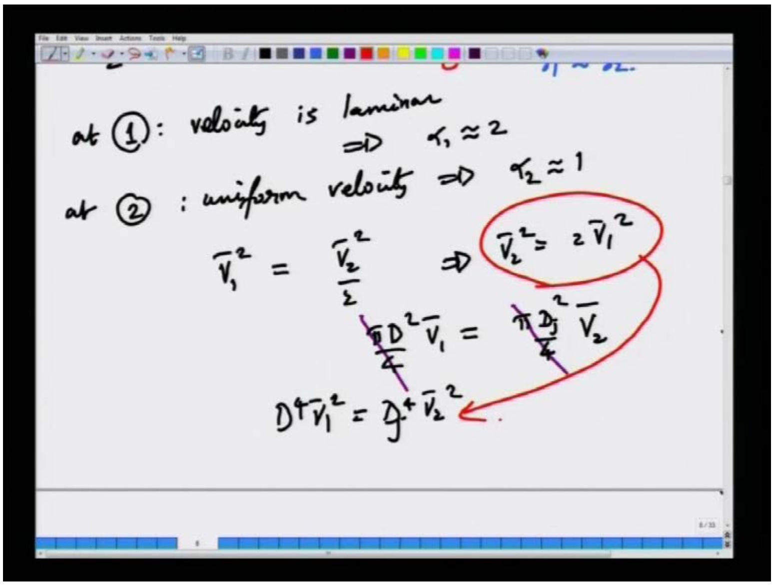

So after doing this, we will find that V 1 square is equal to V 2 square by 2 or V 2 square is 2 V 1 square. We also have the fact that pi D 1 square or pi D square by 4 times, this is the mass conservation V 1 is pi D j square by 4 times V 2. So instead of V 2, I am going to substitute, so let us cancel pi by 4 terms on both sides. So you get, D square V 1 is equal to D j square times 2 V 1 square, because V 2 square is 2 V 1 square, from here I am going to substitute.

So D j square, I am sorry, we are, let us first before I do that, let us first square the equation. So, we will get D to the 4 V 1 square is D j to the 4 V 2 square this implies that and then now I am going to substitute V 2 square here. I am going to substitute the fact that V 2 square is 2 V 1 square out here to give D to the 4 V 1 square D j to the 4 2 V 1 square.

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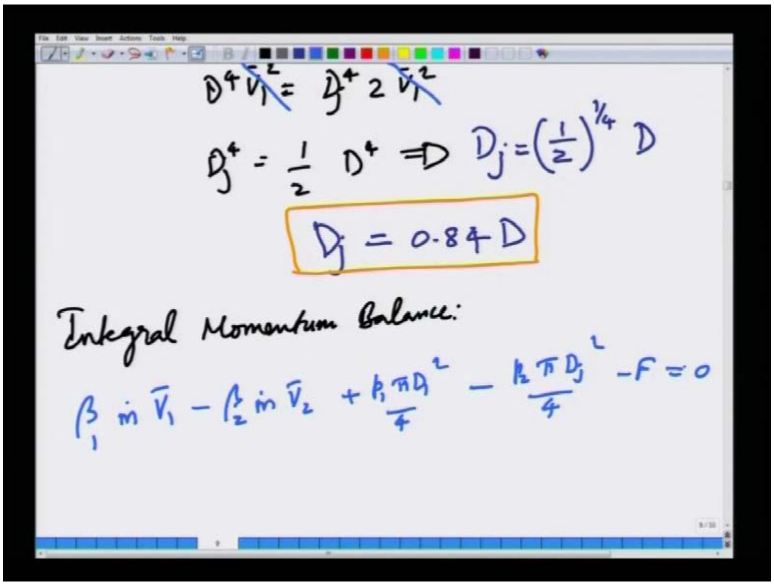

So V 1 square V 1 square will cancel out on both sides to give D j to the 4 is 1 by 2 D to the 4 or D j is 1 by 2 to the power 1 by 4th, D which gives D j is equal to 0.84 D. This is the result from energy balance. The jet diameter is 0.84 times a tube diameter from which data is emerging. Now, we can also use the integral momentum balance to solve the same problem. when we use the momentum balance, we will have beta 1 m dot times the average velocity, where beta 1 is a momentum correction factor, minus beta 2 m dot times average velocity at V 2 plus p 1 pi D 1 square by 4 minus p 2 pi D j square by 4 minus sum of all forces acting on the jet between 0.1 and 2 is 0.

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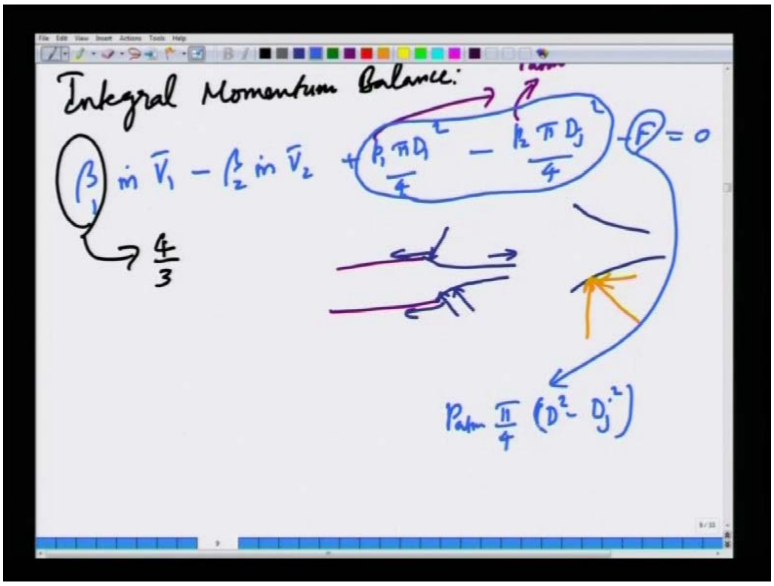

Now at the jet exit, we know that the velocity profile is laminar. So, beta 1 has to be according to whatever the momentum correction factor for laminar fluids which is 4 by 3. Now, we are going to neglect the shear stresses acting on the surface of the free jet, because it is a free jet. But this term p 1 is p 2 is equal to p atmosphere, so this is p atmosphere. Now, before I neglect the shear forces there are two contribution to the force that are emerging from a jet, from on jet that emerges from a tube. In the direction of flow, one is that tangential shear stress. The other is the fact that the atmospheric pressure, the pressure acts normally to the jet surface.

There will be a component that is acting in the direction of the flow, because the interfaces curved as the jet emerges. So there will be a contribution the atmosphere is atmospheric pressure will act like this. There will be a contribution in this direction and there will be a contribution in the horizontal direction, so we will have to take that into account. So that horizontal force due to the normal component in the force pressure is simply. So let us write this as, this is the same as before, but this force component the horizontal component of the pressure that acts is simply p atmosphere times, the projected area difference, which is d square minus d j square. So this is the change, this is the pressure net pressure force that acts on the horizontal direction in the direction of the flow.

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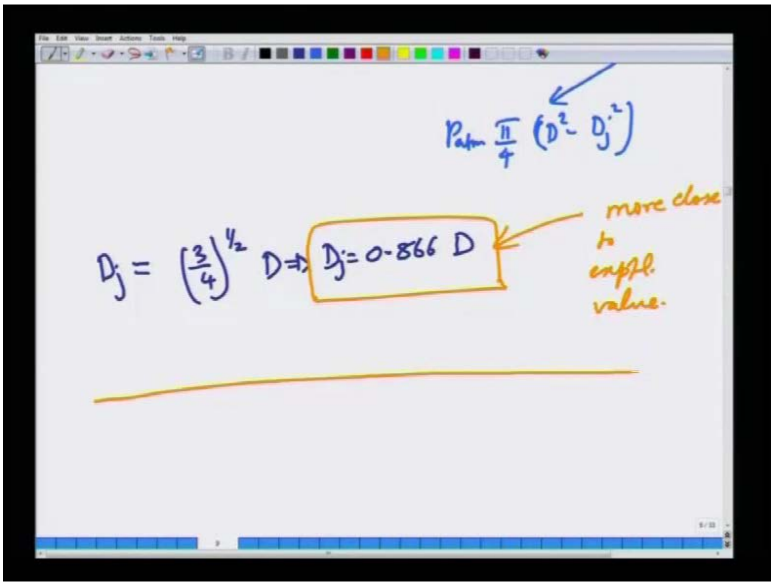

So having done that we can now simplify the above equation again, to give D j is equal to 3 by 4 to the half D, this gives you 0.866 D, this implies D j is 0.866 D, unlike the energy balance V V 0.84 D. So, it appears that the results that we obtained from differential momentum balance and differential energy balance are different, that because of the fact that the nature of assumptions that we make in this two cases are different. In the differential energy balance, we said that the viscous losses in the get jet are negligible. Whereas here, we said that the tangential components of the viscous shear stresses are negligible.

So they are not quite one and the same, so that is the reason why we get different results when we used to these two approximations. So, it so happens at this result is more close to experimental value compare to the energy balance result, because it turns out that this approximation is a better approximation that is neglecting the tangential component of the viscous stresses that exert that is exerted on the surface of the jet turns out to be a better approximation compare to neglecting the viscous losses in the jet I said exists into the atmosphere.

So this really completes our discussion on integral balances of mass momentum and energy. Now, I am going to go to differential balances of the same three quantities. But before I do that, let me also say few words repeat few things that I said on and off about this integral balances. That is they are simple fairly simple to use, because of the fact that they are they can be written across entire equipments. For example, you can write a differential integral balance about a pump or a compressor or involving pipe lines and so on.

But the price that we pay is that we need to have information about the losses, viscous losses. Which have to be either obtained from experiments or they have to be obtained from a more fundamental theory. And we will see a little later that differential balances can be used to estimate losses in simple geometries like pipes. So integral balances always come with this caveat, that we cannot use them easily that is we need to have some information about losses. Many times we may ignore losses, just as we did in the case of from the jet diameter that emerges from a pipe, the diameter of a free jet. And we found that it gives an approximate answer that is quite close to the exact answer. But it can never give the exact answer.

So, one has to always use the integral balances with caution as when one neglects losses or when one neglects tangential component to the forces. But little later, once we complete differential balances, we will see how we can obtain information for losses, for example, in flow through pipes and so on. And that will help us a great deal in completing the integral balances of energy and that can which can we then applied to various really practical applications such as pipeline networks, and including estimating pumping cost or power requirements for running a pump, to inert to make a fluid flow for a particular flow rate on and so on.

So it is possible to use integral balances with some input regarding losses and then that becomes very accurate. How thus information comes from, whether it comes from a theory such as differential balance or experiments that depends on flow regimes so on. That we will see a little later. So right now, we will stop integral balances and then we will briefly return back to integral balances when we do losses. And we will of course, use them to find various things such as pump, power requirements and so on. But we will now go on to differential balances (No audio from 13:57 to 14:06) of mass momentum and energy.

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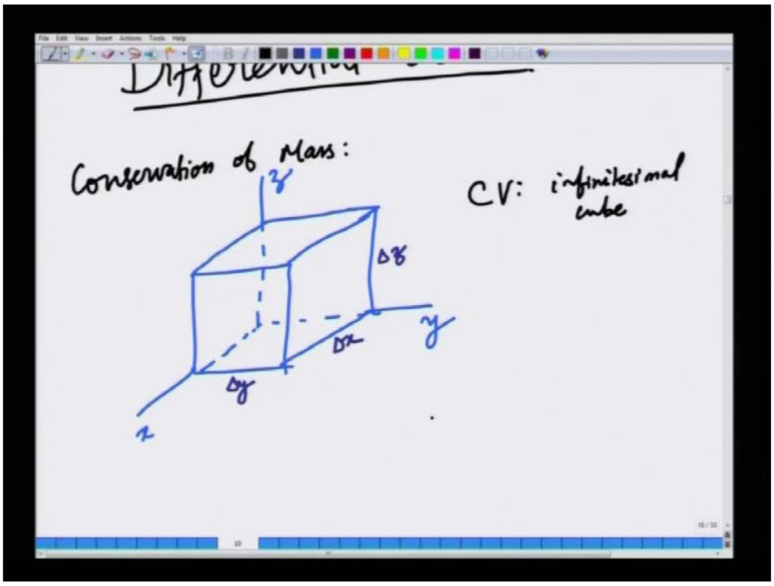

So in integral balances we wrote, we choose the control volume to be a very huge microscopic region which encompasses equipments or even tubes and you know various things like that. And then, we can actually carry out apply fundamental principles such as conservation of mass or Newton second law of motion or first law of thermodynamics, that is what gave us the equations for integral balances. Now, we are going to apply the same principles to not to microscopic regions in space, but to differential volume elements. That is very very tiny volume elements, which are so infinitesimal. That when we do appropriate simplifications using principles of infinitesimal calculus, we are going to get equations that are valid at each and every point in the flow.

So in principle therefore, one can describe after having solve the differential balances of mass momentum and energy. In principle, one can obtain point wise variations of quantity such as velocity, pressure, within a flow domain such as a pipe or a channel. So differential balances give much more detailed information about fluid flow behavior, compare to integral balances. But that also comes with a heavy price, because the equations that emerge out of the differential balances are much much more complicated, compare to the equations said that we derived for integral balance. Therefore, the solutions of these equations are not easy and often we will see that one has to go to a computer to solve the differential balances. So it is not solutions are not easy to come by when you write down differential balances. But none the less, we can the point is one can derive equation that are in principle correct.

So provided, we have the ability to solve this, then one can have the most accurate information about the flow behavior. That is the advantage of differential balances, while you are losing the simplicity of the differential balances, the while we are losing the simplicity of the integral balances the thing that we gain is the accuracy. Despite having to solve more complicated set of differential equations that is the reason why we are called differential balances, we do have now the ability to get the most accurate flow behavior for various fluids.

So, this is the motivation for doing differential balances. Having said that, a time set is not possible for as to get information such as losses by accurate solution of differential balances. So one has to in many cases especially in engineering applications, in practical applications, one has to go resort or one has to take records to doing experiments. So these are in some sense three fundamental ways of solving problems in engineering fluid mechanics. One is to do integral balances, the other is to do differential balances, on the third approach, and especially in engineering context is to use experimentation. And in the context of using experimentation, we will see that the notion of dimensional analysis helps us in organizing experimental data and also in getting better insight of experimental data.

So the sequence of our lectures will, the sequence that our lecture will follow is that integral balances which we just completed. Then we will spend some time discussing differential balances of mass momentum and energy. And finally, we will proceed to experimentation as guided by dimensional analysis. So these are the, this is the sequence of the lectures that we will go to follow in the flowing lectures. So now, when we come to differential balances, the first topic that we are going to discuss is conservation of mass. We already derived a differential version of conservation of, integral version of conservation of mass. Now, we are going to derive the differential version of conservation of mass. In order to do that, we take a very infinitesimal volume, which is in this shape of a cube.

So this, let me take a Cartesian coordinate axis of width of dimensions say delta x, delta y and delta z, the other way delta x is here and delta y is here. So it is an infinitesimal cube, our C V is an infinitesimal cube, because the sides of the cube namely delta x, delta y, delta z are very very small. So, we are going to write the conservation of law of conservation of mass as applied to this control volume.

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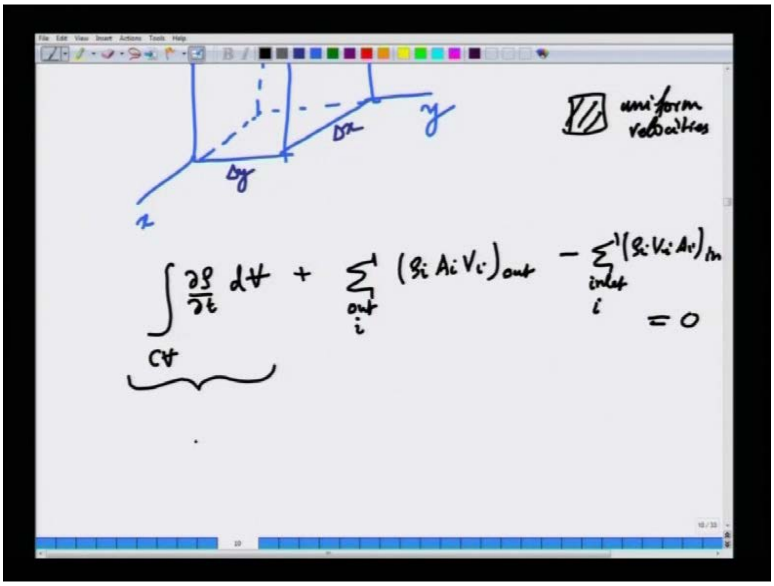

So we already have our conservation of mass relation, d rho dt dv is the rate of change of mass of present in the C V is equal to… Now, another thing we are going to assume is that since the sides of the cubes are so small where there infinitesimal. We are going to assume the uniform velocities, because the region of interest through which the fluid is going to flow so small that we can assume uniform velocity. So, I am going to write summation over all outlets, which is with an index i rho i A i V i at all outlets minus summation over all inlets rho i V i A i and all inlets is 0. This is the same conservation equation. That we wrote for integral balance, but only key difference is that now we are applying it to a very very very small control volume and infinitesimal control volume. Now, when it so happens that the control volume is infinitesimal, we can make some changes to the integrals.

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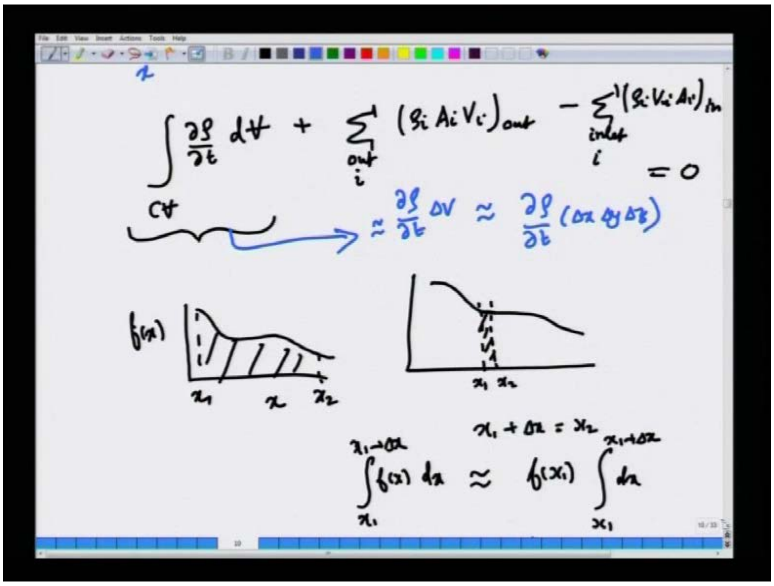

Now, when the region of interest is so small, suppose let me give you a very simple example. Suppose I have a function, f (x) and I am integrating between x 1 and x 2. Now that is really the area under this curve. Now, if I have the same functions, but I am integrating between a very very tiny difference that is x 1 plus delta x is x 2, let us say. Then, in order to do the integration, in order to find the area under the curve, I can approximate integral f (x) between x 1 and x 1 plus delta x, as f (x1) times integral x 1 to x 1 plus delta x dx. The reason is, because the integrant f (x) is not changing much over this tiny interval, so I can pull the integrant out of the integral. This is simply the value of f (x) at x 1 times the integral which is simply delta x.

So the same thing can be said not just for simple one dimensional integration, but also for a volume integral of the form written here. So, when delta V is very small, when the volume of the control volume is very very small, I can pull d rho dt outside and write it as, approximately equal to delta V; this is of course, times delta x delta y delta z. So that is a simplification that happens, when we try to use differential volumes, differential control volumes.

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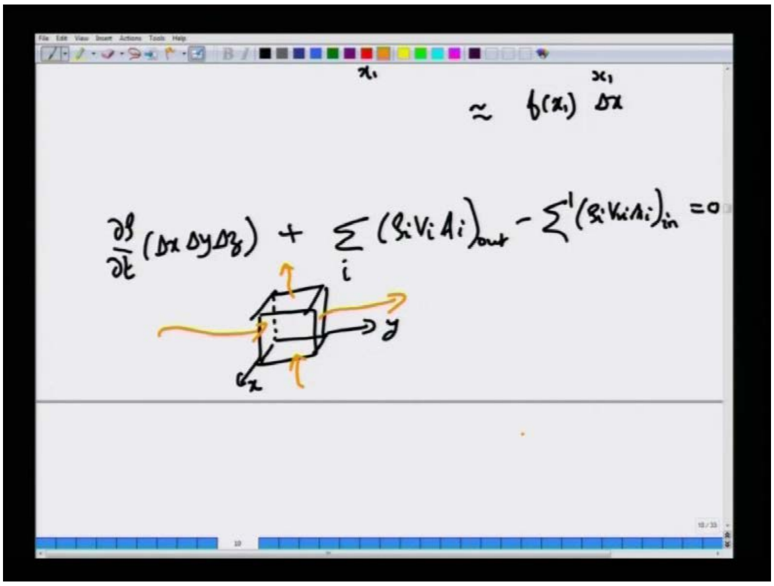

Now, we have to look at the flux terms. So essentially, let us go back lets write the equations again. so integral, so that integral has become a simple d rho dt times delta x delta y delta z plus, we have to evaluate all the flux systems rho i V i A i at the outlets minus rho i V i A i over all the inlets is 0. Now, we have to evaluate, let us go back to the C V, the C Vis a cube with six faces. That is the control volume that we're talking about. Now the fluid can come in via all the six faces, there so we can imagine for example. Since this is the x direction and this is the y direction. Imagine that the fluid can come let us say through one inlet and one outlet along the y direction and one inlet and one outlet along the x z directions and so on. So the fluid can come through any of the six faces and go through, go out of any of the six faces.

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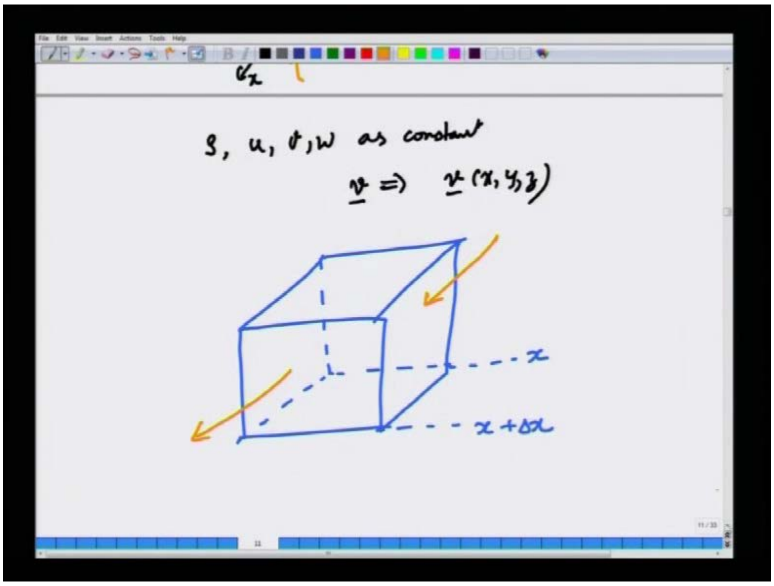

So one thing that we have to understand is that we should we cannot take rho, then the components of velocity v x, that is u, v, w, as constants, because in principle they can vary along the flow direction, so in the direction in which they are flowing. So in principle v is a function of x, y, and z, the velocity vector it is a function of the special coordinates. We have to allow for velocity variations in general. There is no need that velocity has to be constant at various points in the flow; this is as per continuum hypothesis. when we do that so when we, let us try to write the flux terms at face x. Face x is where the fluid is coming in like this, the inlet mass flow, so let us try to draw let me draw the figure again for simplicity.

So this is x, this is x plus delta x. Let us assume that fluid is coming in at x and going out at x plus delta x, that is a picture that we have. Of course, the fluid may go come this way and go that way that does not matter, because our quantities are algebraic that is they have signs associated with them. So it will take an appropriate sign negative or positive depending on the direction actual direction of flow. But right now, just for the sake of clarity we assume that the fluid is coming through x and going out at x plus the delta x.

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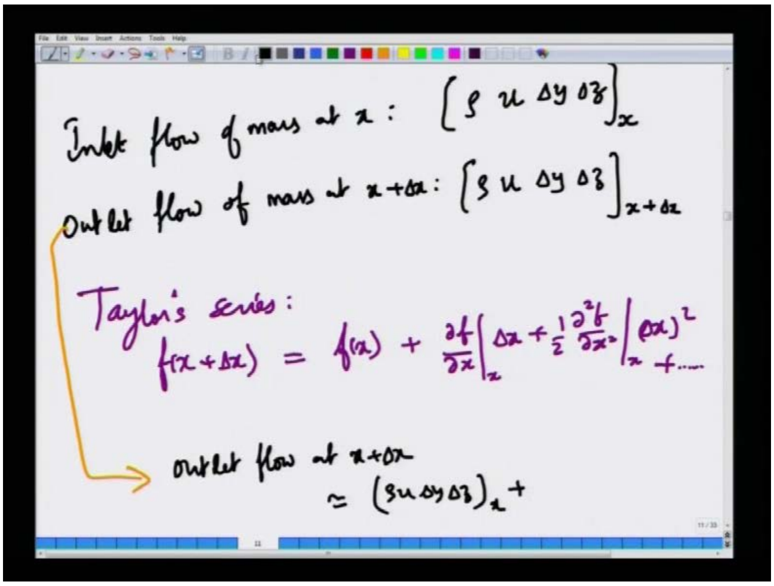

So the inlet mass flow, inlet flow of mass at x is simply the density, which is mass for unit volume, times the volumetric flow rate of fluid entering at x, which is the x velocity. Because fluid is coming in at the phase x with the x velocity, times the cross sectional area through which is flowing, which is delta y delta z, but all of which is evaluated at x. So outlet flow of mass at x plus delta x is rho u delta y delta z at x plus delta x. Now a key thing comes that we have to evaluate a quantity x plus delta x. so we have to use what is called a Taylor's series. The Taylor's series says that if you would not evaluate a function at x plus delta x, where delta x is infinitesimal that is approximately equal to the function at x plus its derivative, evaluated at x times delta x plus higher order terms.

So it is an approximation, it is called the Taylor's series approximation. wherein you can construct the nature of the value of the function at a neighboring point using the known value of the function it it's derivatives at a original point x. So we are going to apply Taylor's series expansion at face x. So this term becomes, this becomes outlet flow at x plus delta x is approximately rho u delta y delta z at x plus half partial by partial x of rho u, because delta x delta y are constant at x plus delta x, at x times delta x. So let me make some space here.

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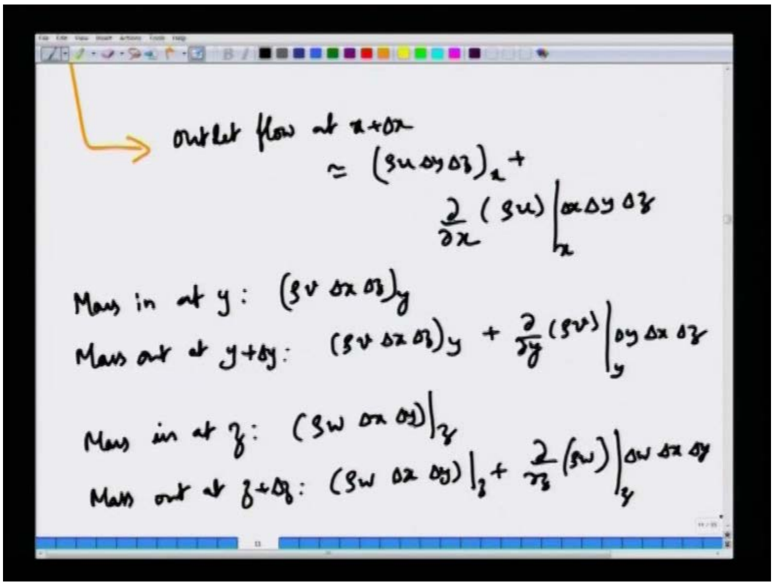

So plus partial by partial x of rho u at x times delta y delta z times delta x, because the Taylor's expansion is probably with respect to x, so we will pick up a delta x. So the net, so let us let us similarly write, mass in at y is rho u so rho v delta x delta z at y mass out at y plus delta y is rho v delta x delta z at y plus d dy of rho v evaluated at y delta x delta y delta y delta x delta z. Likewise, mass in at the other face at z is rho w delta x delta y at z and mass out at z plus delta z is rho w delta x delta y at z plus d dz of rho w evaluated at z times delta w times delta x delta y. If I substitute all this expansions in the first equation, this equation I am going to do that.

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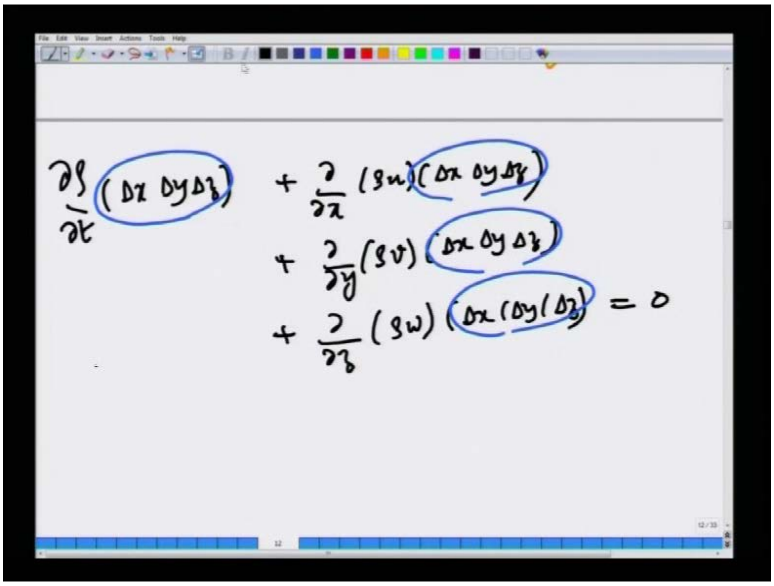

Then what I will get is the following, d rho d t times delta x delta y delta z plus… Now, let us take terms one by one, if you look at these two terms, if I have to take mass in minus mass out, so these two terms will cancel out each other. Or rather, if you look at the first equation, you have out minus n. So, if you have out minus in, then you will find that this term will cancel this term, because they are one and the same. So only term that is going to survive is this, this will be present. So likewise for the other two directions also, this term this term will cancel out minus in this will survive. So, if we write all this terms together, you will find that, this is nothing but so likewise given in the other case so here. This term will cancel, this will go away, because that will cancel this term and this term will survive. So when we do all this together, we will get d dx of rho u times delta x delta y delta z plus d dy of rho v delta x delta y delta z plus d dz of rho w delta x delta y delta z is 0.

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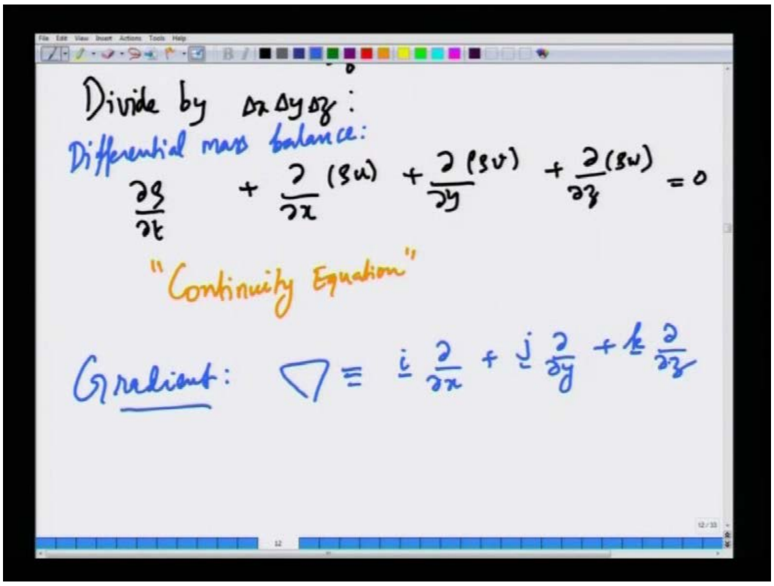

Now we find that this is common, delta x delta y delta z. So, divide the entire expression by the volume of the infinitesimal element, control volume that we have chosen. That implies partial by partial t of rho u plus partial by partial x of rho u plus partial by partial y of not rho u partial rho by partial t rho v is zero. Now, this is essentially the differential form of the mass, balance so this is the differential mass balance for any fluid. It is valid at each and every point on the flow, so that this y you have differentials.

Now we can, so this equation is valid at each and every point in the fluid, so this also called the continuity equation. Now, we can do simplification of this or make it make this equation written in a more compact form. In order to do that, we have to recall what is the notion of the gradient operator. The gradient operator grad this nothing but i times d dx plus j times d dy plus k times d dz, this is the gradient operator. It is gradient operator can act on any vector or scalar or you can also take divergence.

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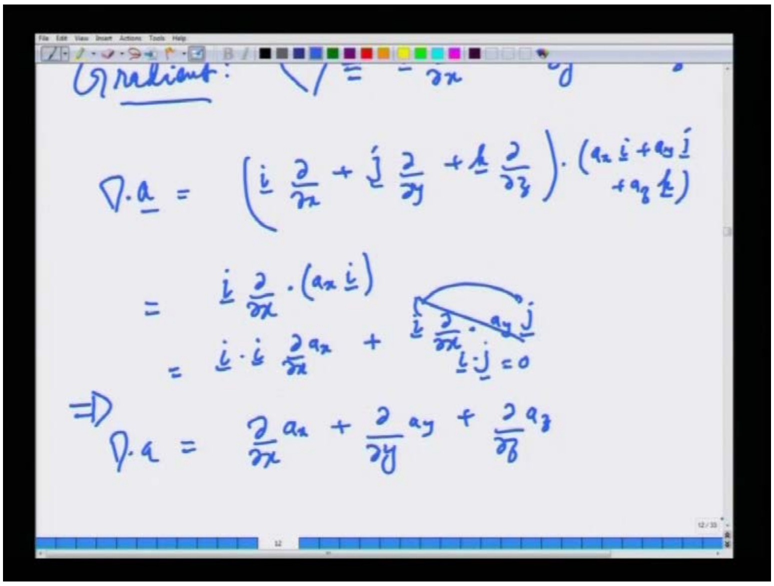

So let me explain a little bit more carefully. If you have this as gradient, then del dot any vector a can be written as i d dx plus j d d y plus k d dz dotted with a is a x i plus a y j plus a z k. So when I do the dotting, I will get i d dx dot a x i. This is nothing but now i is independent of x, the direction i is independent of x, so I can pull i out; so, i dot i d dx of a x plus, now if i take this term, i d dx dot a y j. Now, i and j are orthogonal, so i dot j is 0, so we will not get any contribution. So, only contribution that will survive in the del dot a operation, therefore is d dx of a x plus d dy of a y plus d dz of a z. because all the cross terms involving d a y dx, d a y dz, d a y d a I mean d a x dy, d a y dz and so on, they are all 0. So the only thing that will survive is this. So del dot any vector a is d a x dx d a y dy plus d a z dz, the additional of the three d a x dx plus d a y dy plus d a z dz.

Now if you look at this expression here, it appears like the divergence, this is called the divergence of a vector, any vector. Now here it appears like you have three components rho u, rho v and rho w.

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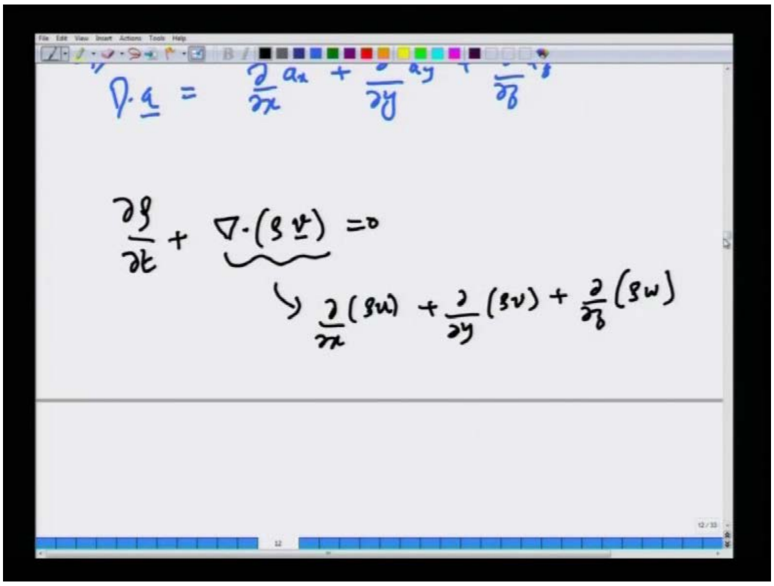

And then you have d dx of rho u plus d dy of rho v plus d dz of rho w. Therefore, we can write this as d rho dt plus del dot rho v is 0, where del dot rho v is nothing but d dx of rho u plus d dy of rho v plus d dz of rho w.

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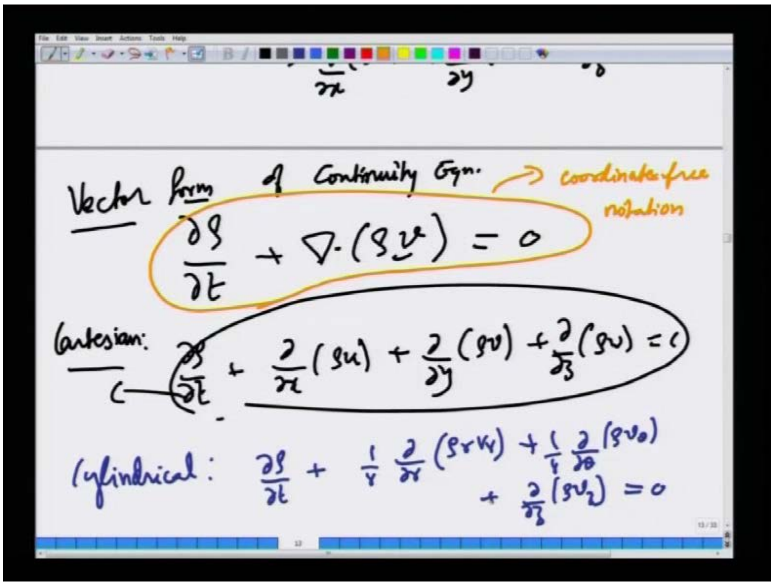

So this is a compact form of the continuity equation, d rho dt plus del dot rho v is 0, this called the vector form of continuity equation. Now, what is advantage of the vector form written like this and the component form written before. The component form for Cartesian coordinates is d rho d t plus d dx of rho u plus d dy of rho v plus d dz of rho w is 0. This form is valid only for a Cartesian coordinate system. If you want to write it for a cylindrical coordinate system, it the form will be slightly different.

For cylindrical coordinate systems, we will have d rho dt plus 1 over r d dr of rho r v r plus 1 over r d d theta of rho v theta plus d dz of rho v z, where v r, v theta and v z are the components of the velocity along the three cylindrical coordinate directions r theta and z. Now, the form of the continuity equation is very different in the cylindrical coordinates compare to the Cartesian coordinates. While the vector form of the equation remains the same, it's coordinate free. This is the coordinate free notation, whereas, the component form is a function of which coordinate you write down the continuity equation.

So the coordinate free notation is much more compact, it is a vector form. But when you actually solve a problem, you have to refer your equations or refer your problem to a given coordinate system. And you will be sort of, it will be necessary for you to write down the component form. But you must be aware that, when you write down the component form, it is the form of the equation will be very different when you choose different coordinate systems. So the form of the continuity equation looks different when you write it in different coordinate systems, as I just showed. Although, I have not derived this expression will do that little later. But right now, I am just telling you that you cannot blindly write the component form of the continuity equation in various coordinate systems in a very very similar way to Cartesian coordinates, because, while this form is independent of coordinates of systems. The component form is actually depend on coordinate systems.

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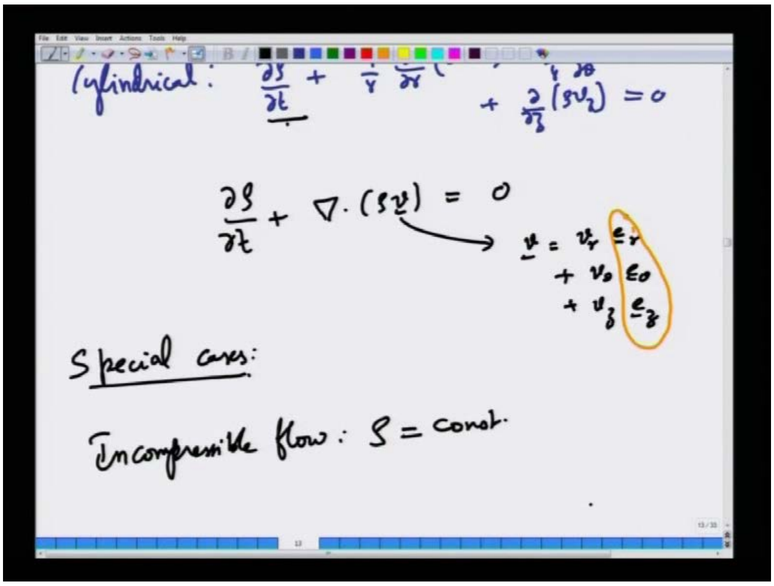

So the brief reason for that is that the when you write the velocity in cylindrical coordinate systems, you will write this as v r e r that is v theta e theta plus v phi v z e z, it is a cylindrical coordinate system. Now the unit vector themselves are now functions of coordinate directions. That is, when you go along the theta direction the unit vector e r will change its direction. So those will also contribute in your continuity equation. That is the simple region, why the continuity equation looks different and cylindrical coordinate system when you compare it with Cartesian coordinate system. Cartesian coordinate system is particularly simple, because the direction of unit vectors do not change as you go along a given coordinate direction, so that is the reason why that happens.

Now, let us also simplify the continuity equation even further. So let us consider some special cases, for an incompressible flow, an incompressible flow is defined as a flow in which the density remains a constant, it is independent of pressure. That is, whenever there is a fluid flow happening that there will be pressure differences in a fluid. But the pressure differences are not large enough to change the density, because in general, we know that pressure and, from thermodynamics we know that the density, pressure and temperature are related by an equation of state. But for incompressible flow, we can assume that the changes in density are negligible for any change in pressure that can happen in a given fluid flow.

So we will, of course derive a criterion as to when the flow can be considered incompressible shortly.

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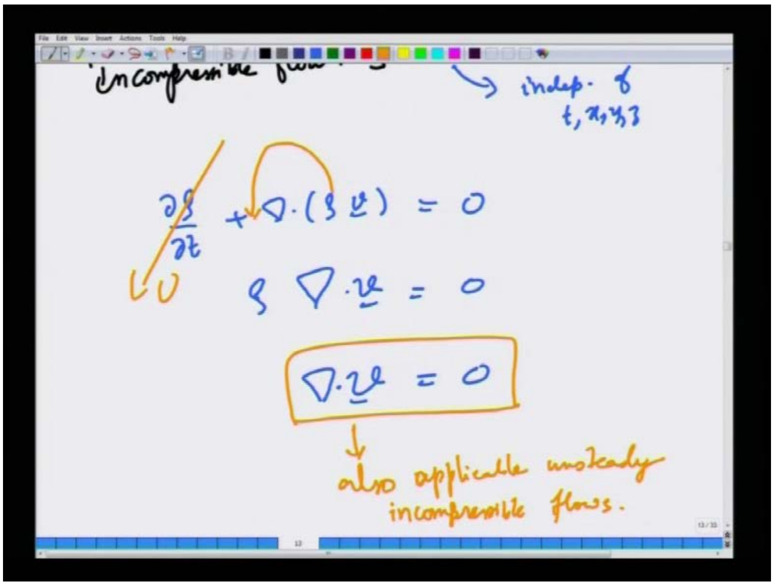

But right now, we will just write down for the simplified continuity equation for an incompressible flow, rho is a constant means that it is independent of x y z and time. So d rho dt plus del dot rho v is 0. If rho is a constant, as I have been telling d rho dt is 0, since rho is a constant you can pull out of the gradient diversion operator, so you will get a rho del dot v 0. Since rho is not 0, you will get del dot the velocity vector to be 0.

This is the simplified continuity equation for an incompressible fluid. Notice that, this is not just applicable for steady flow; this is also applicable for unsteady incompressible flows. So, we are not setting del rho del t, because the flow is steady. We are merely saying that del rho del t is 0, because the density is constant, density does not change at all. This is also applicable for unsteady incompressible flows. One should not have the notion that we are throwing away d rho dt term, because it is a steady flow. We are just throwing it away, because density is constant regardless. So, it is also applicable for unsteady incompressible flows.

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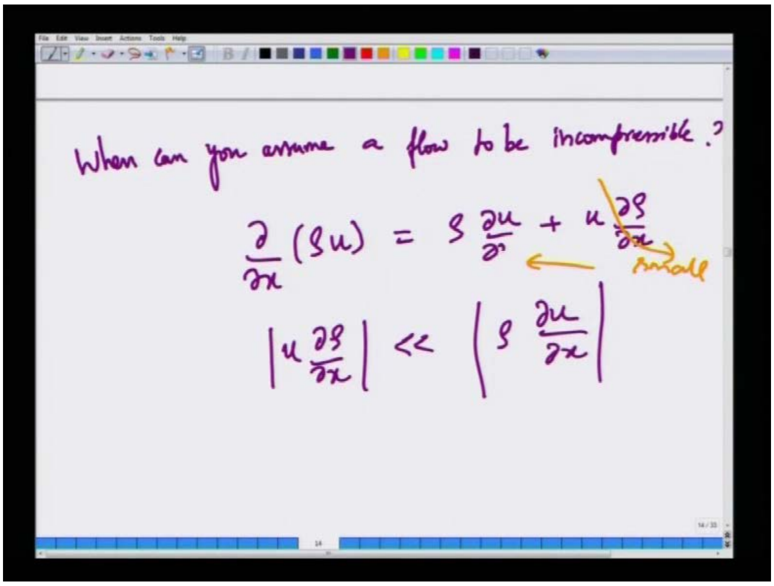

So the question is that can naturally come to mind, therefore is, when can you consider, when can you assume a flow to be incompressible? Because we just said that, in general there will be pressure variations in the flow. So, when can those pressure variations be in a small enough that density does not change. In some sense, what we are saying is that when we have a term like this in our continuity equation, this term is in principle rho du dx plus u d rho dx. What we are saying is that, this magnitude of u d rho dx is very small compare to the magnitude of rho du dx that is what we are saying. By we are essentially saying that rho is constant, so this is small compare to this term.

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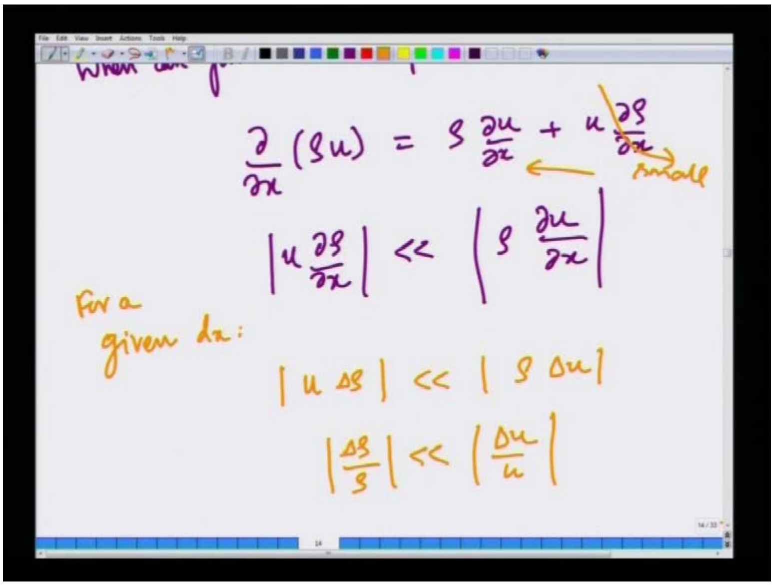

So if such as the case, we can argue when can these happen? For a given dx, we can write this as u times del rho, small compare to rho times del u or del rho by rho is very small compare to del u by u.

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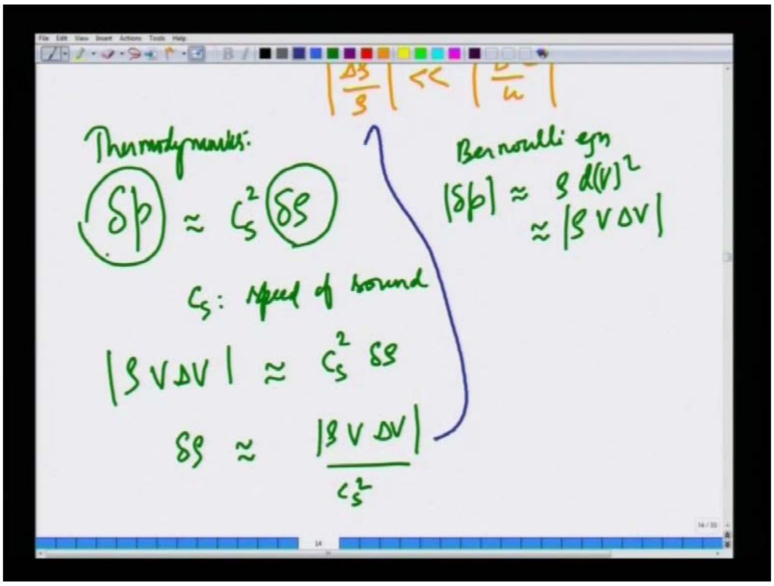

So now, we can estimate del rho by rho using thermodynamics. In thermodynamics, it turns out that delta p, the pressure difference is approximately the speed of sound square times the density difference. So, if there is a pressure difference, the corresponding density difference will be given by speed of sounds square. So for example, if you have a pressure difference, density differences given by del p by C s square, where C s is speed of sound. This is speed at which pressure wave travel in a fluid. Now, from Bernoulli equation, this is from thermodynamics. From Bernoulli equation, delta p is approximately, the magnitude of delta p is rho V del V. so delta p is approximately rho d of V square, this is rho V of del V, this is approximately. So rho V del V magnitude of that is approximately is equal to C s square del rho. So del rho is approximately rho V del V by C s square. We will substitute this out here.

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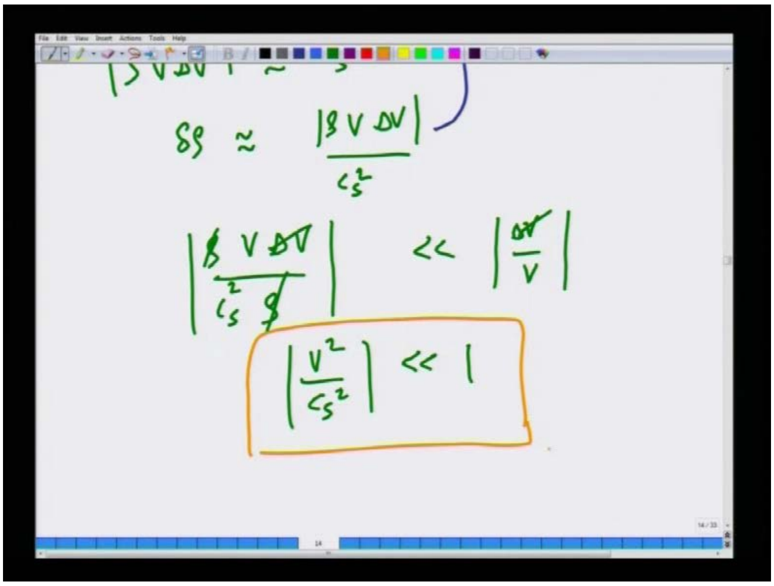

So del rho by rho is rho V del V by C s square rho is approximate is much small compare to del V by V. So rho rho cancels to give you and del V i mean, let me use the same del V del V del V cancels. So we will get that V square by C s square, the magnitude of that is very very small compare to one. When your velocities the flow velocities are very small compare to the speed of sound, then density is approximately a constant. It is independent of, the pressure changes that are there in a flow cannot cause a sufficient density changes. So when can that happen? When the velocity of the flow is very small compare to speed of sound.

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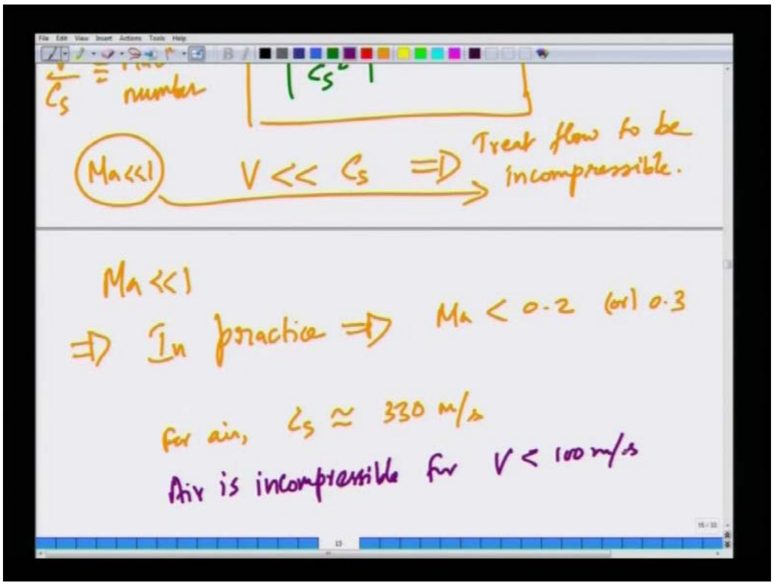

So, when V small compare to speed of sound, you can treat the flow to be incompressible. So, this V divided by C s, the velocity of the flow divided by the speed of sound is given as a special name it is called the Mach number. So when the Mach number is very small compare to one, then fluid is the flow is incompressible. So that is a very good approximation. Now we know when you can treat the flow to be incompressible. But in practice, we cannot say very small compare to one, can mean is it very small should Mach number be 10 to the power minus 8 or 10 to the power minus 3 or 10 to the power minus 1.

So in practice, this implies in practice, this means Mach number even if it is less than about 0.2 or 0.3. You can treat the flow to be effectively incompressible. It need not be very very small in the sense of 10 to the minus 8 or 10 to the minus 5; it can be even just of the order of 0.2. Since we know that for air, the speed of sound at room temperature is 330 meter per second. Then therefore, the flow can be incompressible, flow of air can be thought to be incompressible, air is incompressible for velocities less than 100 meters per second. For velocities of the order of 300 or 400 meter per seconds, of course then the Mach number for air will become 1. Therefore, we cannot treat air to be an incompressible fluid for such cases.

So but this is okay, for many many chemical engineering applications. So we will not have such cases, velocity being of the order of few hundred meters second in many chemical engineering applications. Therefore, we can treat the fluid such as air to be practically incompressible, although air can be compressed. But the density of air does not practically change much for the pressure changes that are associated with in many chemical engineering applications.

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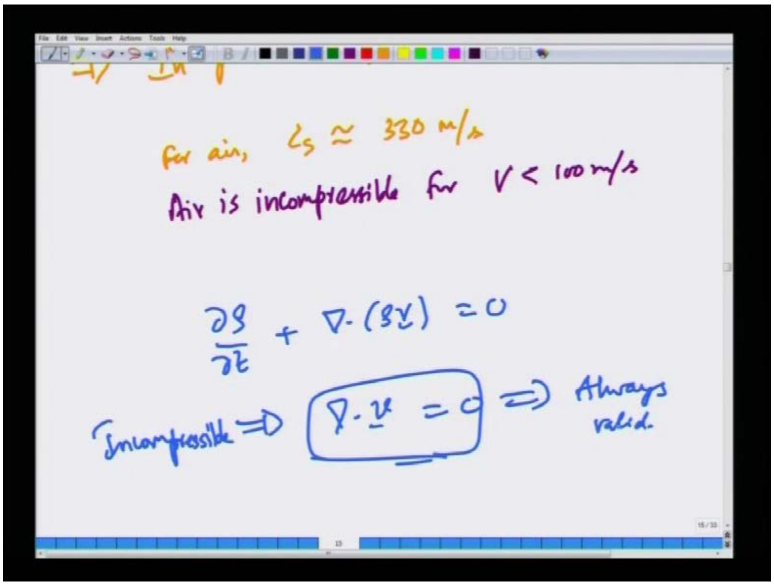

So, we will assume in this course we will largely deal only with incompressible fluids. Now, what is the importance of this continuity equation? So we have this continuity equation for a which for an incompressible fluid. This is general, for an incompressible fluid, this means del dot v 0. What is the importance of this equation? This is always valid, you cannot have a flow which violate this, because the continuity equation is merely statement of conservation of mass that is valid at each and every point in the flow.

So, we cannot imagine any fluid or any flow, where the velocity profile violets this continuity equation as long as the flow is incompressible. So, that is a very very very important statement that you have this constraint, that incompressible constraint or the equation of continuity that forces the velocity vector to be free of any divergence. That is the divergence of velocity vector is 0 for all incompressible flows. Therefore, you cannot have velocity fields that have a positive or negative divergence in any given any given real flow of an incompressible fluid. So, we will stop here at this point, and we will continue in the next lecture.