Fluid Mechanics

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Lecture No. # 19

Welcome to this lecture number 19 on this NPTEL course on fluid mechanics for chemical engineering under graduate students. In the last few lectures our topic of discussion was the derivational application of integral or microscopic energy balance and just to quickly remind you the microscopic energy balance is derived using the first law of thermodynamics. The first law of thermodynamics applies to a system which contains a particular mass of a fluid while in many engineering flow applications we deal with flowing systems.

So therefore, we have to convert the first law of thermodynamics to a control volume which is basically a fixed region of space of interest to us in an engineering operation.

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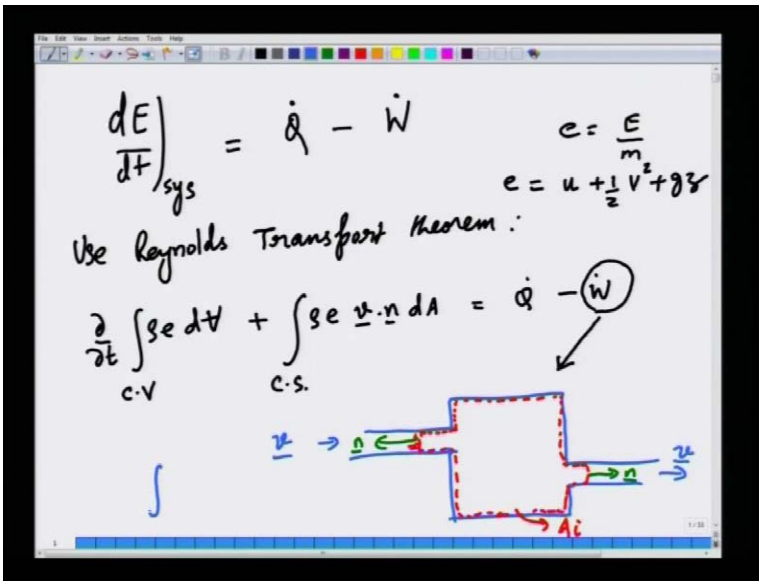

So we have to use the Reynolds transport theorem to convert the rate form of the first law of thermodynamics which simply says that d E d t for a system is rate at which heat is transferred minus to the system minus rate at which work is done by the system just to reiterate again the work transferred between the system and surroundings involves a sign convention and we are going to follow the sign convention that the work done by the system or the C V on the surrounding is positive while the work done on the system or the C V is negative.

So, that why you see a negative sign in this expression for first law of thermodynamics, while if you use the other convention that work done on the system is positive then you will have a plus sign. So that is the sign convention that can also be adopted but we are going to follow this sign convention which is typically in many engineering context.

So, then use the Reynolds transport theorem to convert this into a C V formulation where in you get rho e d V where e is the total energy per unit mass and e is written as the internal energy plus unit mass plus the microscopic potential energy plus unit mass plus the gravitational potential energy plus unit mass this is the rate of the change of total energy present in the control volume but this is not all you also have the flux term over the control surface rho e V dot n d A is integral sorry(Refer Slide No 01:10) is equal to rate at which heat is transferred into the system minus rate at which work is done by the system.

Now, there are various contributions to work suppose you have a control volume which looks like this there. So this is a system in which fluid is entering through a pipe and exiting through a pipe let us say. So now, we are going to draw the control volume like this using the red dotted lines now that is the red dotted lines therefore, becomes the control surface that demarcates whatever inside which is the control volume from the surroundings, now fluid is entering like this here. So the unit output normal here is like this only or normal output like this the velocity vector is pointing against the unit outward normal at the inlet and along the unit outward normal at the outlet.

Now, there are various types of work that that are possible but what is important for us understand is that work is done only if there is a force that acts and there is motion in the line of action of force. So in this control surface suppose you construct this internal areas where there is no flow because a fluid is stationary at the wall that is called the no slip condition, which we will see in detail little later. So all though a stress acts on these internal surface is there is no work done on those surfaces because the surface is itself stationary.

So, here there is a pressure force that acts there is a normal force that acts and fluid is also flowing. So therefore, there is a work contribution at the inlet and outlet that will be given by we later saw it to be equal to the inlet and outlet.

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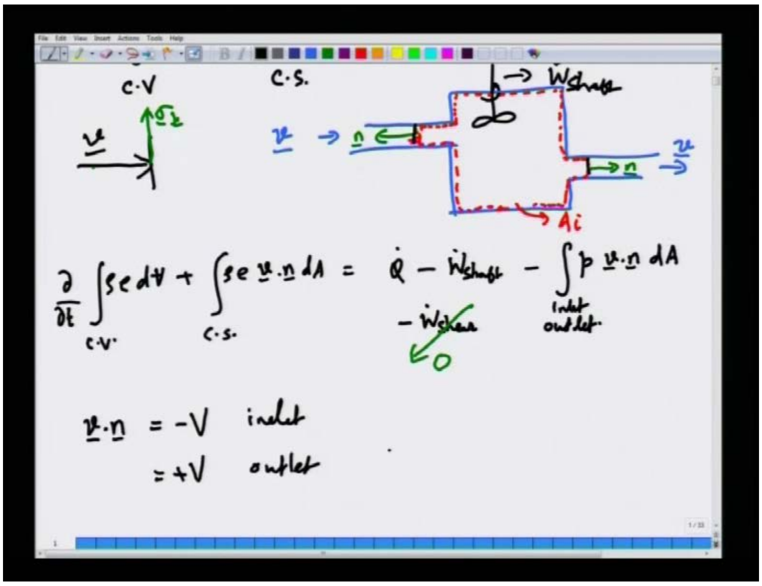

So, there are these normal contributions to the force that comes from the work done by the normal stresses at the inlet and outlet which I will write shortly and there are also share contributions because of the fact that you can have share work done by an impeller at the control surface. So having included both these terms we saw that the left side remains as such as rho e d Vplus integral C S rho e v dot n d A is Q dot now this is denoted as the shaft work okthat isassociated with palm compresses turbines. and so on. So either you could do work on the C V or the C V you can extract work out of the C V.

Now the term that arises due to the fact that you have a normal force due to pressure is given by rho sorry (Refer Slide No4:53) integral p v dot n d A integral over the inlets and outlets ok and in general there is no shear work at the entrance and exit because if you choose the control volume that is normal sorry(Refer Slide No 4:53) if you choose the control surface to be normal to the inlet velocity the shear stresses act in the direction perpendicular to the inlet velocity. So the dot product of these two is orthogonal vectors is, obviously zero. So the shear work can be set to zero by choosing the control surface carefully. So after having done all this you therefore, get and then we will let us simplify v dot n as minus v for inlets is plus V for outlets.

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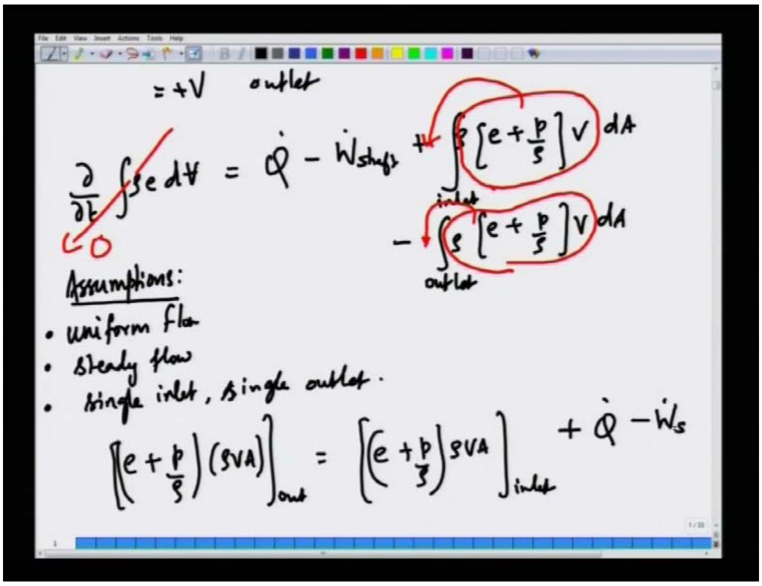

So once you do this you get a very simple d V is equal to Q dot minus w dot shaft minus integral rho times e plus p by rho plus V for inlets minus rho e plus V by rho v d A over outlets because of this fact that v dot n is minus V therefore, minus of minus becomes plus here that is plus v for inlets well it is minus it just plus v for outlets. So this minus sign stays here so this is the integral energy balance.

Now, we also said that we can simplify this further by making assumptions of just for the sake of obtaining clarity and so you can use the assumption of uniform flow we of course, saw that we can correct this assumption by using the kinetic energy correction factor for the energy balance ok the alpha term, but, right now if you assume uniform flow and steady flow and single inlet single outlet when the flow is steady this becomes zero when the flow is uniform all these quantities are independent of the cross section.

So you can pull it out at the integral same here. So, the area integral becomes just the area of inlet and outlet.Ok so having done that you will get e plus p by rho times rho v a at at the outlet is e plus v by rho rho v A at the inlet plus Q dot minus W dot shaft.

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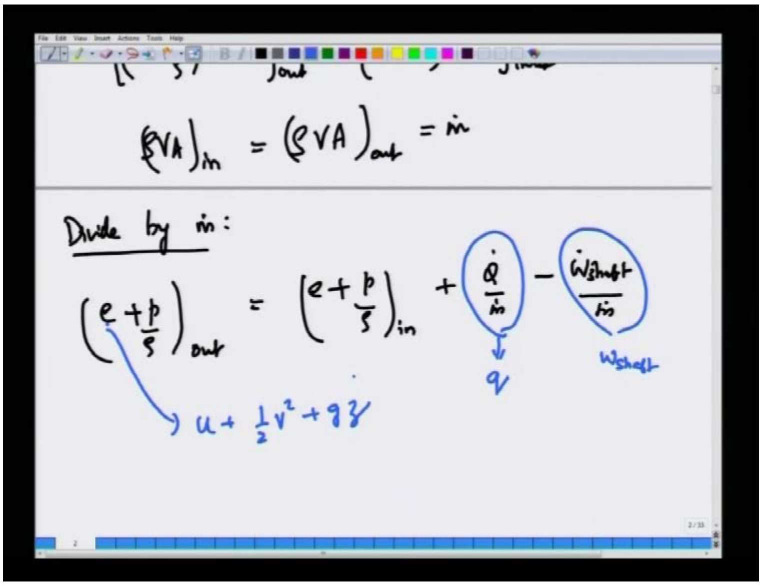

Now the mass balance will save for a single steady system at with single inlet and single outlet that rho v A at inlet is the same as rho v A at outlet because at steady state there is no accumulation of mass in the C V. So that is equal to m dot the mass flow rate. So if we divide by m dot we get e plus p by rho at outlet is e plus p by rho at the inlet plus Q dot by m dot minus w dot shaft by m dot now this is nothingbut, the amount of heat transferred to the C V per unit mass there is no rate involved because you are rate dividing one rate by another rate. So they cancel out and this is denoted as w shafts there is no rate involved anymore. So now as per second law of thermodynamics.

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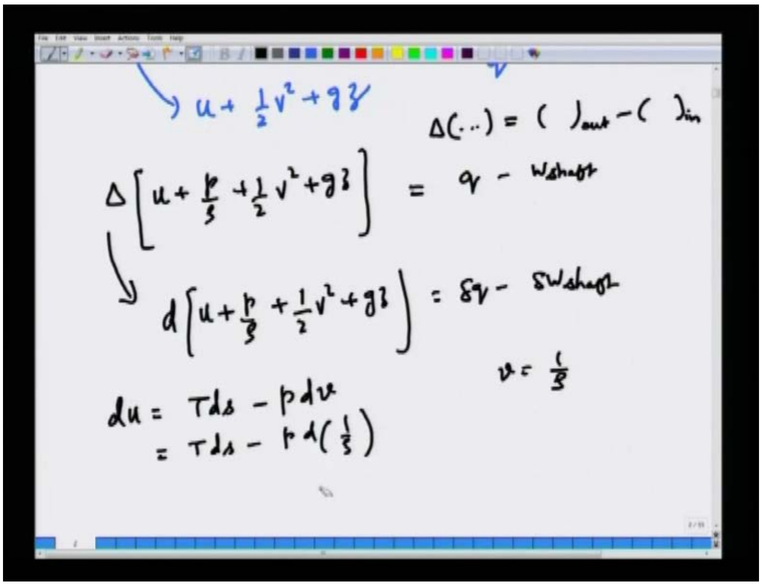

So once you expand e to be u plus half V square plus g z you get change where delta means change in out minus in u plus half V square plus V by rho lets write V by rho first plus half V square plus g z in minus out where delta of any quantity is that quantity evaluated at out minus in delta symbol implies that you are evaluating quantity at the outlet minus the quantity at the inlet. So this is equal to q minus W shaft if you want use thermodynamics I want to change it to a differential where the inlet and outlet are just separated by small distance so you can convert it to a different cell d of u plus p by rho plus half V square plus g z is del q minus del w shaft.

Now you can borrow or adopt a relation from thermodynamics d u is Td s minus p d V, V is a specific volume which is 1 over density is T d s minus p d of 1 over rho when you substitute this back out here then you get d u instead of d u.

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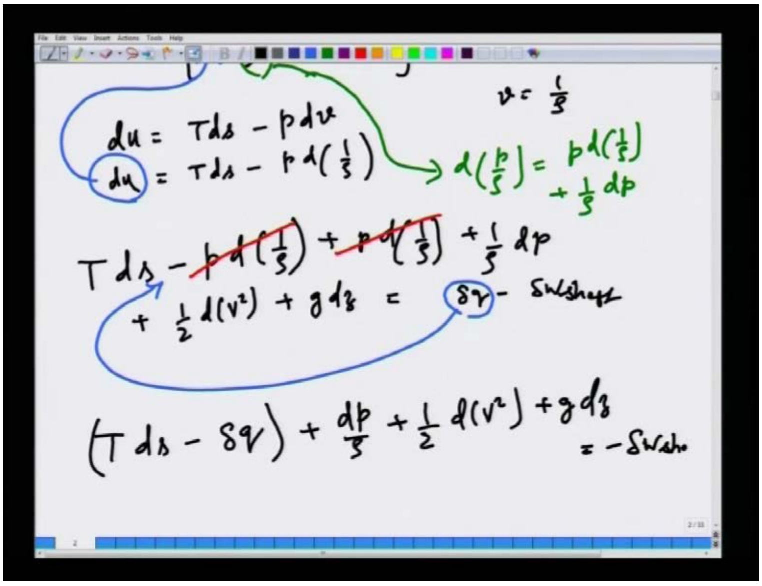

So I am going to substitute this in the d u here. So instead of d u I will get T d s minus p d 1 over rho but d of p over rho will give me d of p over rho will give me, p d 1 over rho plus 1 over rho d p. So I am going to write this as plus p d 1 over rho plus 1 over rho d p plus half d V square plus g d z is del q minus del w shaft now these two terms will cancel each other to give and I am going to bring the del q to the left side to get T d s minus del q plus d p by rho plus half d V square plus g d z is minus del W shaft.

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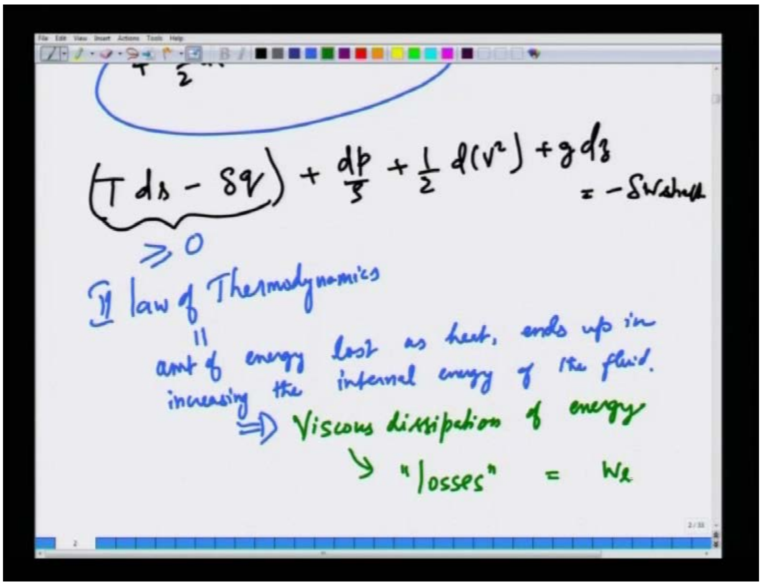

Second law of thermodynamics tells you that this term is always greater than equal to zero this is second law of thermodynamics therefore, and this is basically the amount of work that is lost irreversibility due to heat and this equality is valid only for reversible process but engineering applications involving flow are irreversible process you do not do them infinitesimally slowly over a sequence of equilibrium states. So they are irreversible processes therefore, second law of thermodynamics tells us that this is greater than zero for flow applications this is the amount of heat sorry (Refer Slide No12:49) amount of energy that is lost as heat and ends up in increasing the internal energy of the fluid of the fluid this is also called as the viscous dissipation of energy viscous dissipation of energy.Now, from the point of view of fluid mechanics this is the amount that is lost. So these are termed as losses. Ok so this is written as W l amount of energy or work that is lost.

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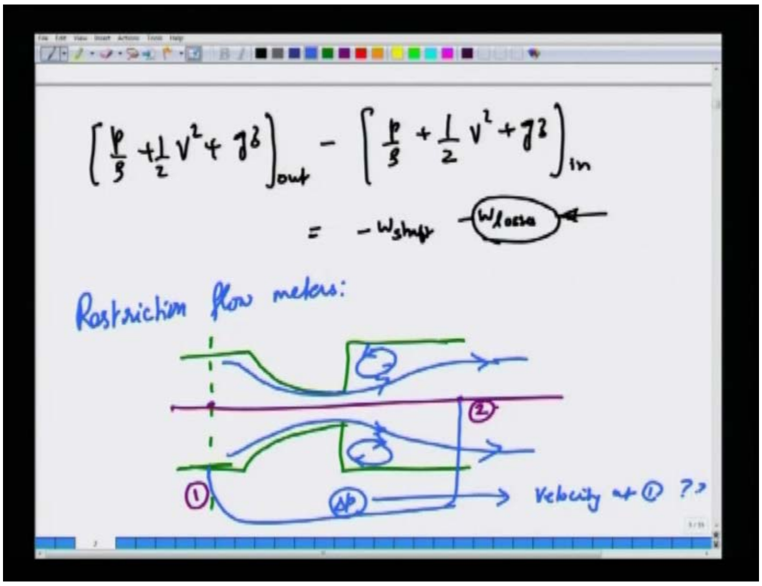

So you rewrite your energy equations simplified energy equation as p by rho plus half V square plus g z at outlet minus p by rho plus half V square plus g z at the inlet is minus W shaft minus W loss losses this are the viscous this is the viscous dissipation of energy that implies that energy is transformed from microscopic forms like work two internal energy. Ok now, how to compute this losses will come to a little later how we do differential balances but right now this you have two sort of know the losses if you want to find the pressure difference and so on. So, either the losses come as an input to the calculation to experimental data or they come out as an output of the calculation.

So this is the energy balance after taking into account the losses. Now, after finishing this we also looked at the application of integral energy balance or through flow measurement where we try to understand restriction flow meters wherein essentially you had a configuration like this you had a gradual contraction on an expansion. So essentially fluid is flowing like this eventually occupying the full region.

So, there are recirculating zones like this both sides. So idea is here to have a knowledge suppose you take streamline that goes from here point 1 to point 2 now if I measure the pressure difference between these two points there is a delta p can I relate it to the velocity at 1. That is idea.

So you measure pressure difference between 2 points 1 upstream of the contraction and another downstream of the contraction and see whether you can relate the pressure to the velocity we found that after making simplifying assumptions that there are no losses and by applying the Bernoulli equation between point 1 and 2 remember that the Bernoulli equation is essentially cross simplification of the energy balance by taking the C V to be stream cube and by assuming that there is no viscosity in the surrounding fluid.

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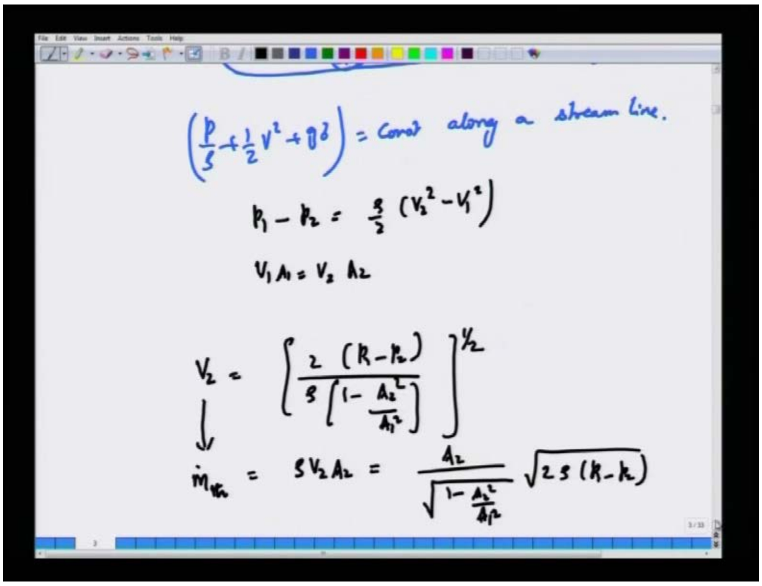

So there is no work then and by shrinking the stream tube to a stream line you find that p by rho plus half V square plus g z is a constant along stream line. So, this is the Bernoulli equations. So by applying this. To this 2 points along this stream line we found that we can say that p 1 minus p 2 is rho by 2 V 2 square minus V 1 square and by using mass conservation we say that V 1 A 1 is V 2 A 2 therefore, we were able to write this as V 2 is 2 V 1 minus p 2 by rho 1 minus A 2 square by A 1 square whole to the power half.

So this was the theoretically expected V 2 from here we can find the theoretical expected mass flow rate as rho V 2 A 2 is equal to A 2 divided by root of 1 minus A 2 square by A 1 square root of 2 rho p 1 minus p 2.

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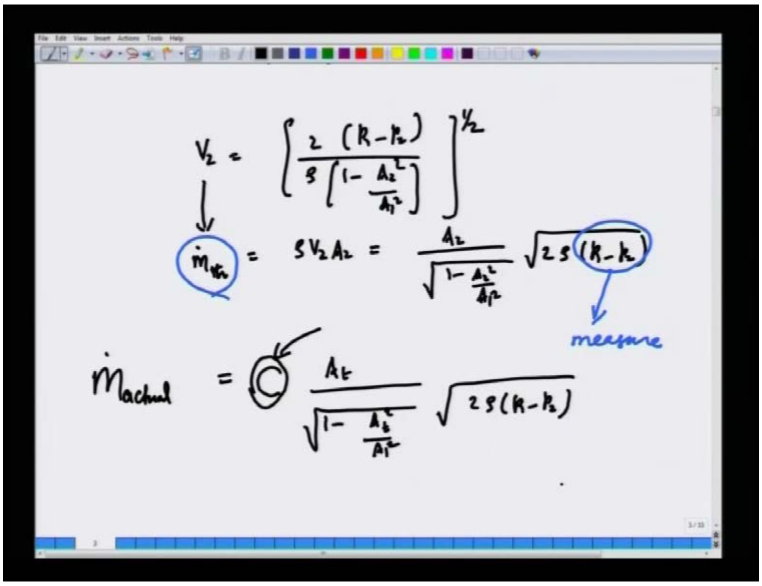

So what is idea? the idea here is that by measuring this pressure difference between points 1 and 2 and by knowing the area A 2 and A 1 and by knowing the density we can find the mass flow rate, but, this is a theoretically estimated mass flow rate but it has a lot of assumptions. So in reality the observed mass flow rate will not be the same as this because we have assumed that there are no losses which is a gross over simplifications and another thing is that we do not know what is the flow area at point A 2 but point 2 we do not know what is the area because the flow area is only this not the entire because of the recirculation zone occupies the zone like this.

Until this streamline occupies the entire the cross section. So this area is something that we do not know. So usually what is done is to use the throat area this region is called throat area of the minimum cross-section of the flow meter so instead of A 2. Now therefore, you write m dot observed or actual is A t instead of A 2 you write A t because that is something that we know from the construction of the device a t square by a 1 square root 2 rho p 1 minus p 2, but, since there are.

So many approximation that are involved there will be the actual mass flow rate will be some constant which are we fitted experimentally or empirically by doing the following experiment by knowing the given mass flow rate and by knowing the pressure by measuring the pressure drop we can fit the constant.

So that this equation is satisfied and then this can be used to create calibration charts for a given meter that is how the constant C changes as a function of flow velocity and So on that will give us way to measure or way to measure way to compute mass flow rate in a real application by simply measuring the pressure difference at an upstream point of the flow meter and at the downstream point of the flow meter. So that is the idea of restriction flow meters.

We also mentioned that this is generic discussion and in practice there are orifice meters and venturi meters which all fall in to this restriction flow meter class of flow measuring devices. But one drawback of restriction flow meters is that you will be able to measure only the average velocity cross section average velocity or the entire cross section of the pipe. But suppose for some in some applications if you need the local velocity at a given point in the cross section of the pipe then what do we do. So this is where we left of in the last lecture we just started this discussion on what are called pitot tubes.

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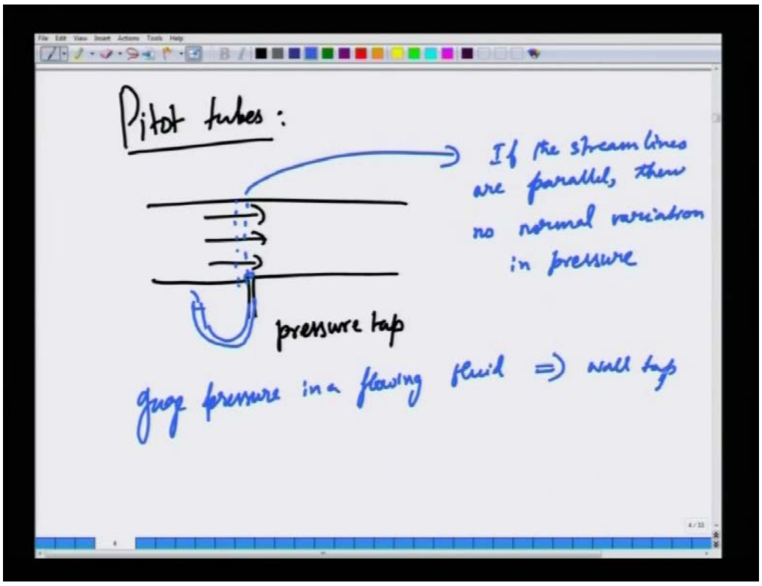

Now, one way to measure the pressure in a flowing liquid is the following suppose you have a pipe or a channel in which fluid is flowing you can make a small hole a pressure tap this is called a pressure tap or a wall tap and let us say fluid is flowing on the average in this direction then it turns over that there is no normal variation of the pressure with the stream lines are straight which we see later if the stream lines or parallel then no normal variation in pressure.

So we can just measure the pressure here by having a small tap and then connecting it to the manometer which is exposed to the atmosphere. So you can measure the gaze pressure at this point of a flowing fluid by simply having a wall tap. So the gage pressure in a flowing fluid can be measured by a wall pressure tap and a manometer is called also a wall tap. But that is not sufficient to calculate the local velocity at any point in the fluid in order to do that what we have to do is what is called use what is called a static pressure probe.

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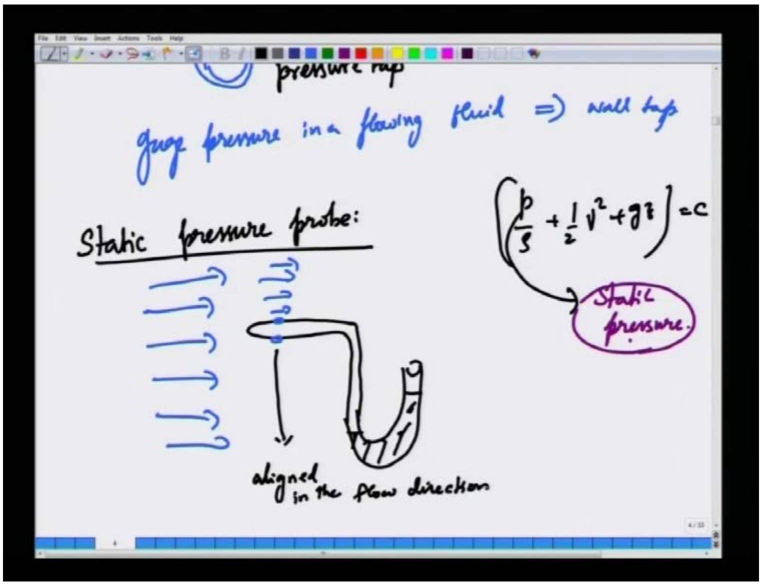

Essentially what this static pressure probe is it has a small tube and there are two holes at the top and bottom. So this 1 is another way of measuring the pressure in a flowing fluid. So the pressure that happens in the Bernoulli equation is also called this is a Bernoulli equation this is also called the static pressure 1 way to measure the static pressure that is the pressure in a flowing fluid is to use a wall tap another way is to introduce this static pressure probe which is very thin cube shape like this now fluid is flowing like this and this region will therefore, will fill the pressure of the flowing fluid and if it is connecting to a manometer then you will be able to measure the pressure that is present inside a flowing fluid which is also called the static pressure or which is also called the static pressure the key thing is to align this in the flow direction this much be aligned in the flow direction.

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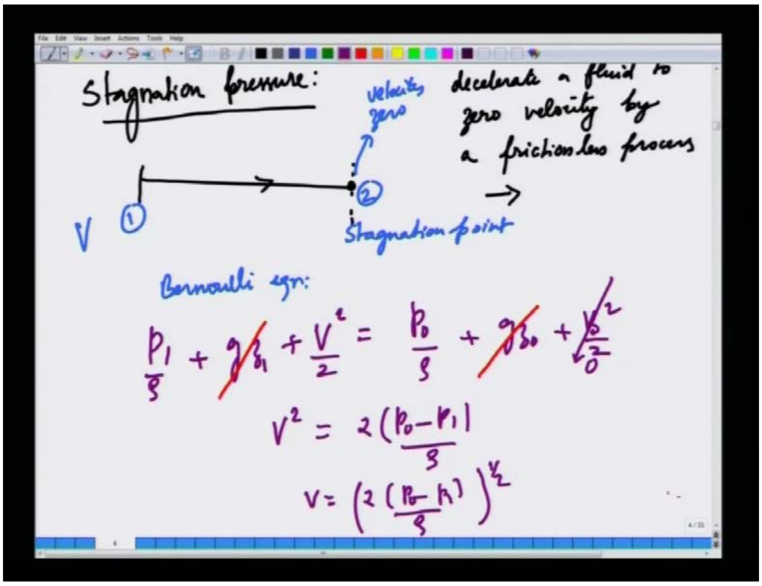

Now, another quantity of interest is called the stagnation pressure let me explain what this means the stagnation pressure is that pressure hypothetically suppose a fluid is flowing along the stream line suppose you bring it to rest you decelerate to a zero velocity by a frictionless process.

So you decelerate a fluid to zero velocity this is a conceptual idea by a frictionless process in reality of course, it is not possible to you know in general to have a frictionless d k of deceleration of a fluid to zero velocity therefore, but it is a conceptual idea it is a concept that if you are able to decelerate a fluid to a zero velocity by a frictionless process then what will happen is that if you take the stream line and apply Bernoulli equation between point 1 and 2 here the fluid is velocity of the fluid is zero.

If the velocity is zero here and it is V 0 t here if you apply Bernoulli between point 1 and 2 Bernoulli equation between point 1 and 2 what will happen is that p 1 by rho plus g 1 z plus V naught square which is the upstream let us call all these terms as let us not call this V naught let us call it V let just call the velocity in the fluid as V plus V square is at the point this is called the stagnation point where the fluid velocity is zero is called the stagnation point at the point where the fluid velocity is zero this becomes equal to let us denote the subscript this point is p 0 plus g 1 sorry (Refer Slide No 25:04) g z 1 and similarly, g z 0 plus V 0 square but V0 t square is zero because the fluid is static it has gone to zero velocity.

Now, if you assume that the elevations are the same then these two terms cancel out. So, then that gives you an expression for V square this is not V 2 square this simply let us call this V square. So, V square is p 0 minus p 1 by rho or V since you have V square by 2 we should have V square by 2 here. So, V is 2 p 0 minus p 1 by a rho whole to the half.

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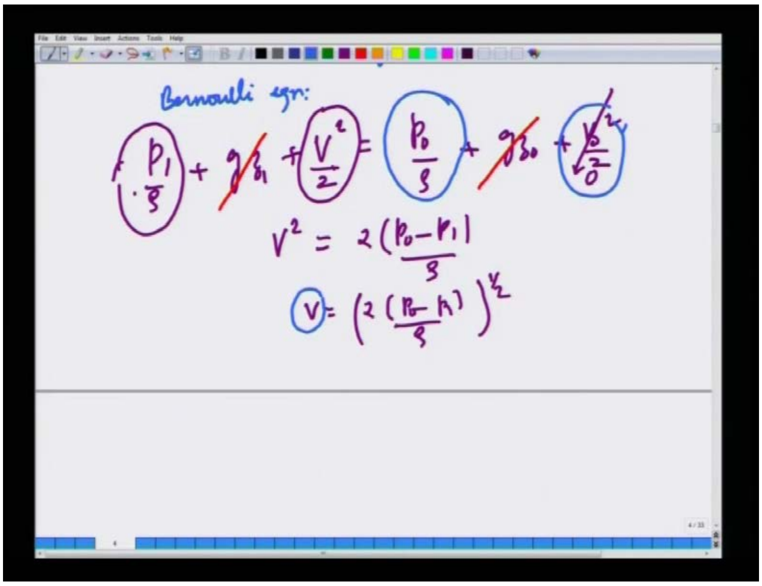

So this is essentially says that the entire pressure had a there is in on the upstream at point 1 there is a pressure head as well as the kinetic energy head. So both the pressure head in kinetic energy head are at the stagnation point are converted to just pressure because the kinetic energy head is zero there.

So that gives you an estimate for what is the velocity up stream. So if you measure the pressure at a stagnation point and if you measure the static pressure which is the pressure in a flowing fluid then that gives you the velocity at which you want to you are measuring the static pressure suppose you are able to measure this pressure difference between a stagnation point and another point where the fluid is flowing then you can estimate the what is the local velocity of the fluid at that point where you have measured the static pressure now how do we do this experimentally.

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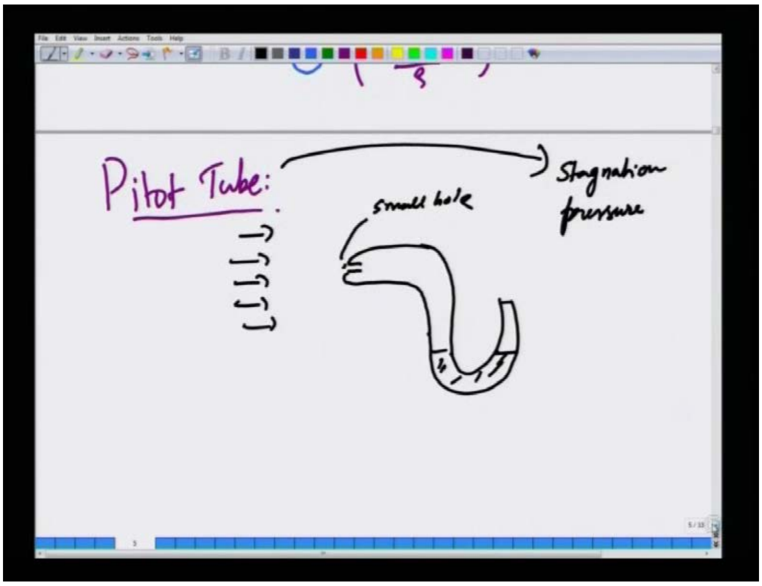

So this is conceptually how it comes about but we also have to find a way out experimentally and it is done by what is called the pitot tube. The pitot tube is a way to measure stagnation pressure. So how it works is the following. So you have a tiny cube it is a small hole and it is aligned in the direction of flow and the fluid comes to rest at this point approximately and therefore, if you connect this to a manometer you will get the stagnation pressure with respect to the gaze pressure with respect to atmospheric pressure that is the gaze stagnation pressure. So the pitot tube can measure stagnation pressure.

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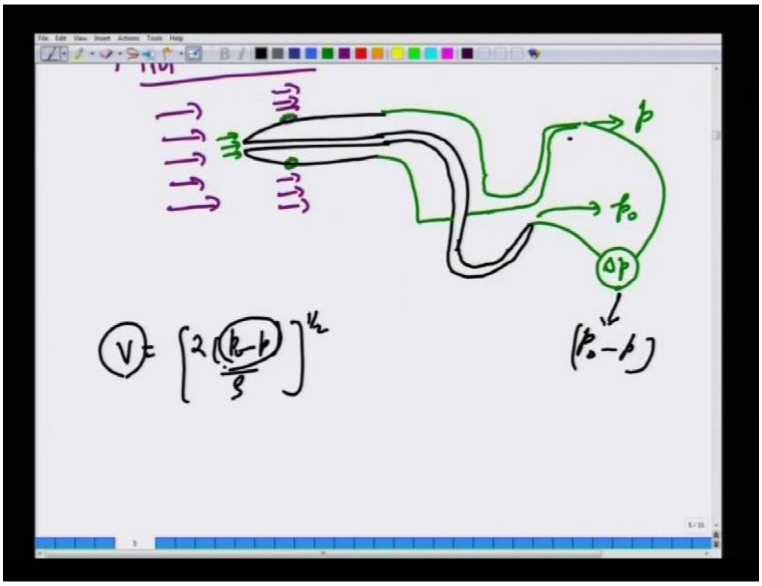

So the pitot tube can measure stagnation pressure but there is also another variation that called the pitot static tube which simplifies the two measurements in a single device which makes possible to measure both stagnation pressure and static pressure and the difference between them in a single device this is called the pitot static tube which goes like this which looks like this.

So you have the tiny hole and there is the inner one the inner tube is one tap which measures the stagnation pressure because fluid is coming like this and then there are holes which measure the static pressure.

So here is this. So let us show it some other color. So the inner tube this gives you the static pressure p where as the outer as the the sorry (Refer Slide No 30:11) this the tubes where this holes are on the surface gives a measure of the static pressure p whereas, this tube where the fluid comes to a halt it gives you a measure of stagnation pressure p 0 if you connect these two ends to a manometer this change in pressure will directly give you what is a local velocity using the formula we just derived because we derived this formula which says that V is 2 square root of 2 p 0 minus p divide by rho whole to the half.

Since we know p 0 minus p through the measurement by connecting a manometer between this inner tube and outer tube then we can directly substitute that value out here and you will get a measure of the local velocity. So you can place you can imagine placing the pitot static tube about various points cross section of a pipe or a channel and you can therefore, measure the local velocity at each and every point .

As I told you sometime back the velocity that of a fluid that flows within a inside a tube or a channel varies at various points in cross section of the channel and that can be measured by using the pitot static tube because it gives you a handle to measure the local fluid velocity now this will completes our discussion on flow measurement.

So and I will what I will do next is to apply the engineering balances that is both and that is all the three balances mass, momentum and energy to couple of problems to illustrate the certain points that comes out upon applying this mass by integral balances because in general what happens a while using integral balances is that we do not have all the data to completely solve a problem.

Therefore we are forced to make assumptions. Assumpations such as that there is no viscous shear a stress existing on a wall when we use the momentum balance for the simple reason that we do not know in a complex engineering situation what those forces are. So we are forced to make the assumptions that those forces are negligible but then such assumptions may work in some context and they may not work in some other context.

So this is always an issue while applying engineering sorry (Refer Slide No 30:11) while applying microscopic or integral balances to engineering applications because the because of lack of complete information to solve the integral balances. So this I will try to show you in the context of two problems.

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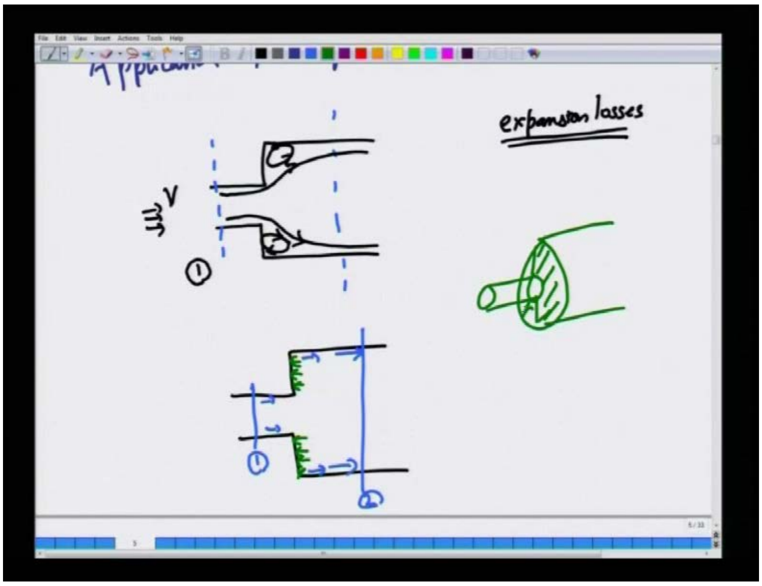

First let us imagine applying integral balances. So imagine you have a sudden expansion, of a flow. So we have fluid flowing and you have 1 station 1 upstream. So let us draw the C V and another stream let me draw it again.

So, you have fluid flowing there is a sudden expansion you have station 1 up stream and station 2 downstream of the expansion whenever you have a expansion what happens is there are losses because not all the energy that you put in by virtue of let us say a pressure drop goes in pushing the fluid in the flow direction there are recirculating zones which are just there, because of the sudden expansion which cause additional losses such losses are called expansion losses.

So let us say if fluid is flowing with a given velocity and we want to be able to find the amount of viscous losses that happen in a sudden expansion. So we are now going to calculate the expansion losses the viscous losses that occur in the energy balance that is our aim when you have a sudden expansion now the equations that we have are the three balances mass momentum and energy. So we wont to be able to find the losses viscous losses due to the sudden expansion. So, using the three balances now how do we how are we going to do that.

Now, if you want to do if you want to use the energy balance to calculate the viscous losses as I have told you we need to know what is the pressure drop between point 1 and 2. If you want to know if you want to know what is the pressure between point 1 and 2 then you have to use the momentum balance between these two points between those two control surfaces points in the control surface you apply them momentum balance across these points to estimate delta p for that we need to know what are the forces.

So let me just draw this diagram once again. So, as a fluid is flowing there are two types of forces 1 is a tangential force acting between point 1 and 2 on this surface other is the normal force acting on this suppose you imagine this to be a pipe this pipe is going like this and it suddenly expanding to a bigger pipe. So, there is an annular region where you have this normal force due to pressure. So, there are two types of forces that act on the surfaces of this C V. So, we have to keep that in mind now let us first use.

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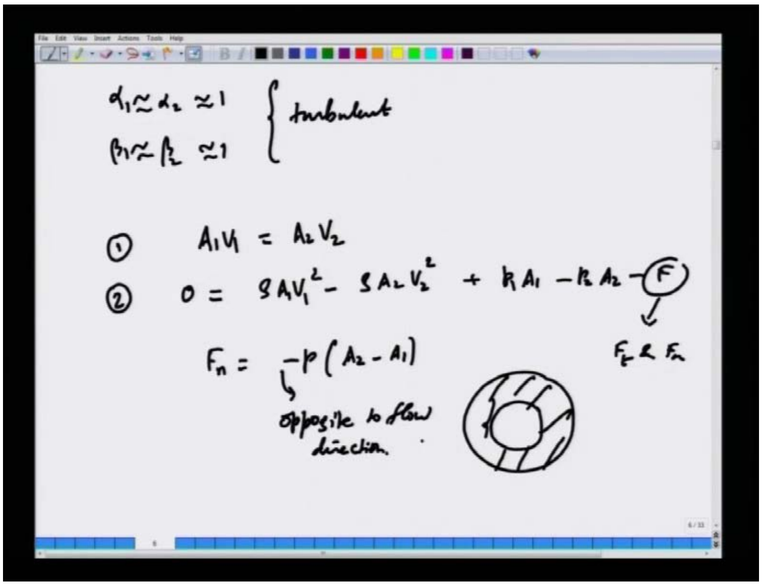

So, let us say use assumptions that we will use alpha 1 is alpha 2 is approximately 1 beta 1 is beta 2 is approximately 1 that is essentially we are saying that let us assume that the flow is in the turbulent region just to keep the flow to be reasonably uniform.

So, you have A 1 V 1 is A 2 V 2 this is mass conservation for an incompressible fluid this is equation number 1 the momentum balance tells you that at steady state zero is rho A V 1 square minus rho A 1 V 1 square rho A 2 V 2 square plus p 1 A 1 minus p 2 A 2 minus F where p 1 A 1 are the forces p 1 A 1 and p 2 A 2 are the forces acting when the fluid is entering and leaving the control surface and F has both the tangential and normal components so I just say as I just told this is the normal component due to pressure these are the tangential components in the direction of flow.

So but we are going to say that the normal component is simply the pressure times A 2 minus A 1 because this is the annular area remember always the pressure is acting and the negative sign is because of forces acting in the direction opposite to the flow direction. So you have to put a negative sign.

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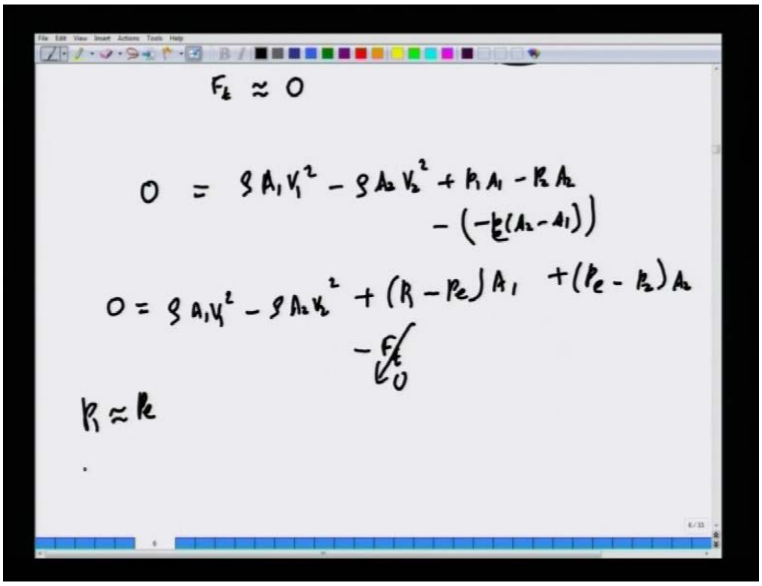

So that is one thing that is a normal force the shear stresses exerted by the fluid on the solid surfaces are very difficult to calculate or estimate because we need a more detail model such as a microscopic or differential momentum balance to get that information. So, due to the lack of the information either from A more fundamental model or from experiments we merely set it to zero at this point. So, this is an assumption that we have to live with because otherwise it is very difficult to find what the tangential forces are due to shear stresses.

So, the momentum balance becomes rho A 1 V 1 square minus rho A 2 V 2 square plus p 1 A 1 minus p 2 A 2 minus, minus p A 2 minus A 1. So, this is what we have at. So now the point is this p is the pressure at the point where the sudden expansion occurs. So let us call that plane as plane e because that is where the pressure is being exerted. So you can also rewrite this as zero is rho A 1 V 1 square and as rho A 2 V 2 square plus p 1 minus p e (Refer Slide No 38:58) there is there are two negative signs. So, let us remove 1 plus p 1 minus I am sorry, there are two negative signs that is plus p 1 minus p e times A 1 plus or minus we can even write plus p e minus p 2 times A 2 and then minus F tangential which is neglected in our approximation.

So now we have to know what is the difference between p e and p 1 because p e is the pressure at the expansion whereas, p 1 is the pressure at the upstream of the expansion where the fluid is flowing with the some uniform velocity now this the pressure between point 1 and point e differs because the factor fluid is flowing in the short segment but by choosing the station 1 is sufficiently close to the expansion we can assume that p 1 will be very close to p e because the only thing we are now neglecting is the viscous losses due to the flow in that short segment.

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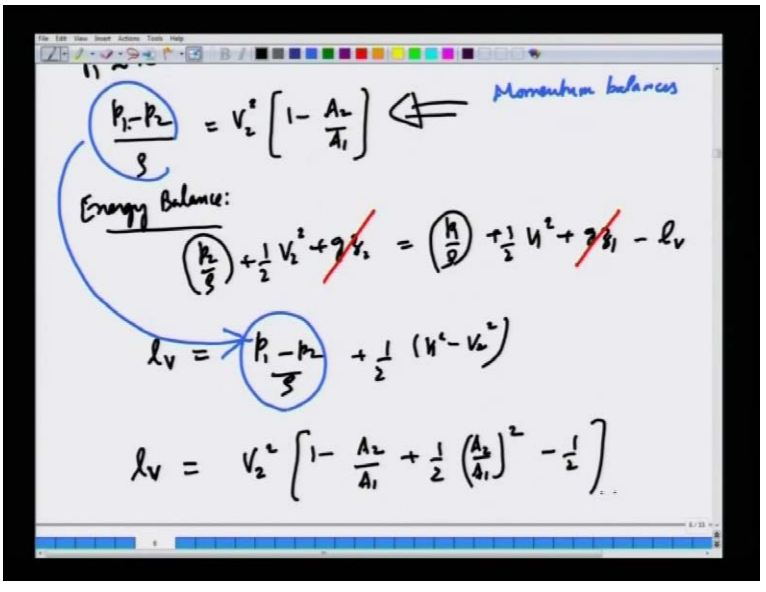

So, we are going to assume that p 1 is approximately p e. So we have then by rewriting this equation p 1 minus p 2 by rho is V 2 square times 1 minus A 2 by A 1 this is from the momentum balance this is what the momentum balance gives us the integral momentum balance after making the assumption two assumptions that the share stresses are zero and the pressure at the a sudden expansion is same as the pressure at the upstream now if you use the energy balance integral energy balance you will get that p 2 by rho plus half V 2 square plus g z 2 is p 1 by rho plus half V 1 square plus g z 1 sorry (Refer Slide No 41:52) g z 1 minus viscous losses there is no share there is no shaft work done because there are no shaft that are cutting across the control surface through this simple expansion.

Now, one thing that we can say safely is that the elevations of the upstream and downstream roughly the same. So there is not much different potential difference due to gravity head now then we have to substitute for p 2 minus p 1 from the momentum balance to get an expression for viscous loss.

So you have p 2 minus- rho by rho minus p 1 by rho which is obtain from the momentum balance after substituting that here we will get an expression for viscous loss which is p 1 minus p 2 by rho plus half V 1 square minus V 2 square here I have not substituted p 1 minus p 2. So, instead of this p 1 minus p 2 we have to essentially substitute that. So you will get the viscous loss l viscous the viscous losses due to expansion as V 2 square times 1 minus A 2 by A 1 plus half A 2 by A 1 whole square minus half.

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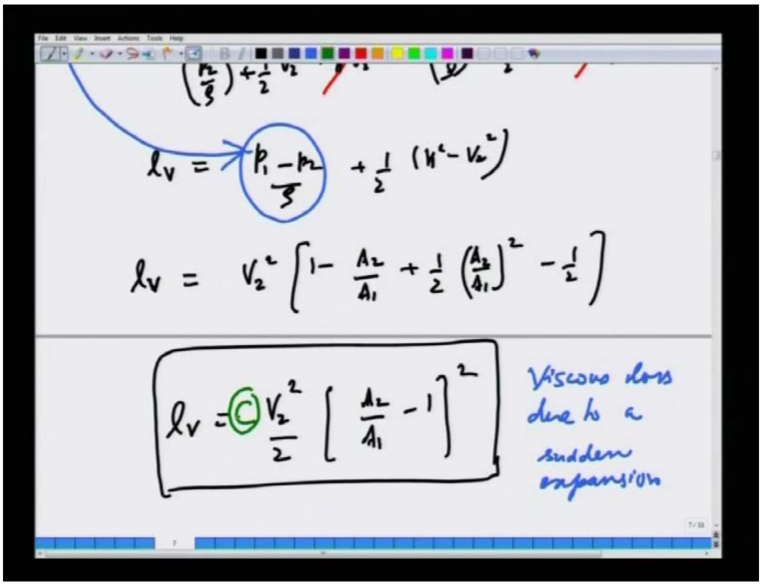

So the viscous loss is equal to V 2 square by 2 A 2 by A 1 minus 1 whole square. So this is the viscous loss that happens due to a sudden expansion of course.In reality this will not be exactly true because of the assumption we make. So, we may have to do the measurement and up and then fit a constant with that but the form of the dependence of the loss on the velocity is exactly like this and the areas of the sudden contraction but of course, this relation will not be exactly true because of the fact that we have made these assumption that there are no share stresses and the pressure of the expansion is exactly equal to the pressure up stream and so on.

So certain things are neglected in our analysis that makes our analysis approximates but, none the less what is powerful about the integral balances that despite being making such grossly simplifying assumptions the functional form of the viscous dependence of the viscous loss on velocity and area is exactly the same as you obtain from the simplifying analysis simple analysis but, it says that the pre factor which turns out to be 1 our analysis is not exactly one but it will be some number which can be fitted by using experimental data.

So, this is one example of an application of integral energy balances where we have not just applied one balance but we have applied a combination of mass, momentum and energy balance towards obtaining the viscous losses that there in a sudden expansion.

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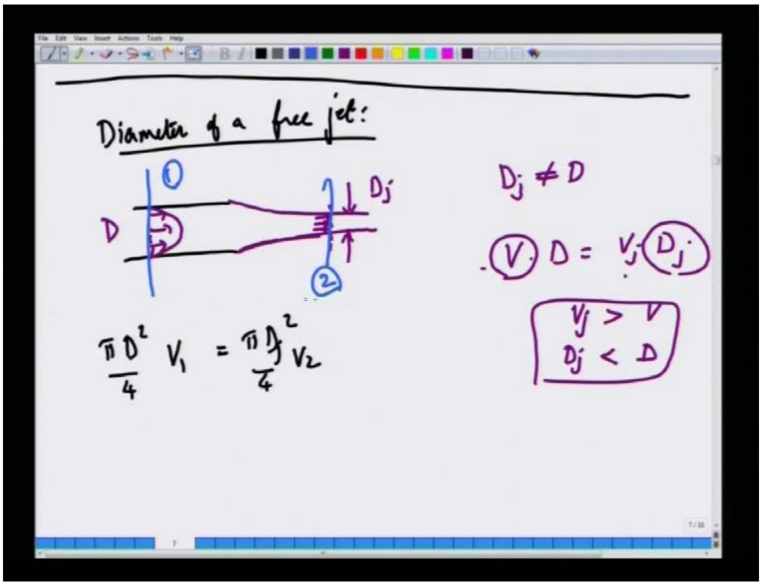

Now another example of an application of the integral energy balance is to obtain the diameter of the free jet, that exist from a tube. So imagine you have a tube in which fluid is flowing under fully developed condition let us say it is a laminar for simplicity.

Now, this fluid is existing now the moment it exist it goes into that atmosphere and there are no share stresses exerted by the surrounding atmosphere on the fluid because the atmosphere is static and it is just an. So shear stresses exerted by the atmosphere on the fluid is negligible compare to the amount of share stress the fluid faces when it flows through the wall when it flows through the tube a rigid tube with tube with a rigid wall.

So in general the diameter of the tube d and the diameter of the jet d j that emerges a sufficiently downstream from the exit of the tube they are not the same because this is simply because of the lack of resistance to flow when the fluid exist into the atmosphere. So what happens is it generally thins a little bit the reason for that is because at constant flow rate the amount of volume that flows inside and outside is the same so we have V times D is V jet times D jet.

So this is from mass conservation since the fluid will accelerate that is V will be less than V jet because of the fact that there is no there is no resisting force here the fluid is somewhat accelerate here that means, that d jet has to be. So let see. So V in general so the diameter is in general different at the downstream because of the fact that there is no resisting shear forces here.

So V jet will be greater than V therefore, d jet will be smaller than D took satisfy this mass conservation conditions. So D jet is in general smaller than d and usually as I have told you in several occasion before that whenever you have a free jet velocity is uniform across the cross section of the jet. So having set these let us use the mass balance pi D j square by 4 V 1 sorry (Refer Slide No 46:16) pi D square by four V 1 is equal to pi D j square by 4 V 2 let us just call this station as one and this station as two in our analysis.

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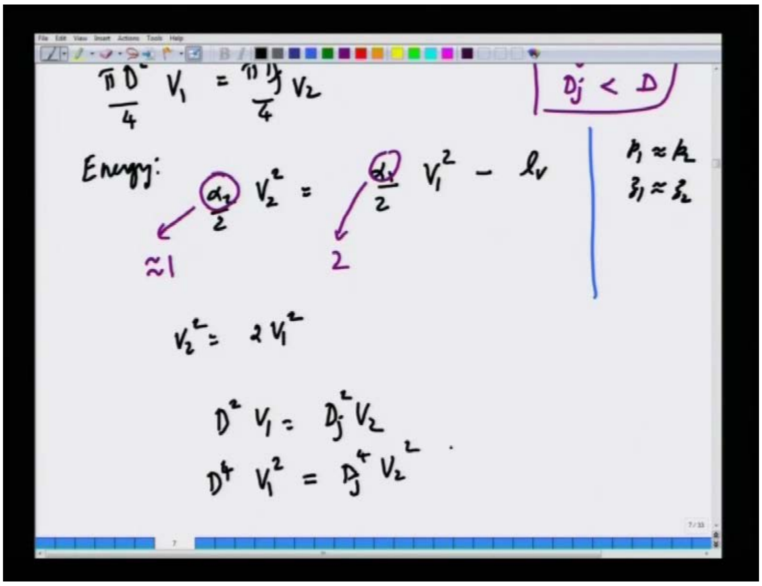

Now, this is mass integral mass balance applied between points 1 and 2 of the C V this is a mass balance the energy balance becomes alpha 2 by 2 times the average velocity square at 2 is alpha 1 by 2 the average velocity square is at 1 minus the viscous loss now we have assume that p 1 is approximately equals to p 2 and z 1 is approximately equal to z 2 because…

So now, I am going to make a simple change in the C V that my C V is right at the exit the point 1 the plane is at the exit of the tube.

So the velocity profile is just parabolic as it is just exiting in the tube, but, the pressure is approximately the atmospheric pressure that is a assumption that we make although it is not rigorously correct this is an assumption that pressure as soon as it exist the tube is the same as the pressure far away in the free jet which is the atmospheric pressure but the velocity profile is still parabolic.

So if you do that then alpha 2 is approximately one because the flow is uniform here whereas, alpha 1 is 2 because the flow is parabolic there. So use these two relations then I get a simple relation for you cancel let say alpha 1 is alpha 2 is 1. So V 2 square is simply 2. V 1 square that is one relation that we get and therefore, substitute this in continuity equation we get d square times V 1 is d j square times V 2.

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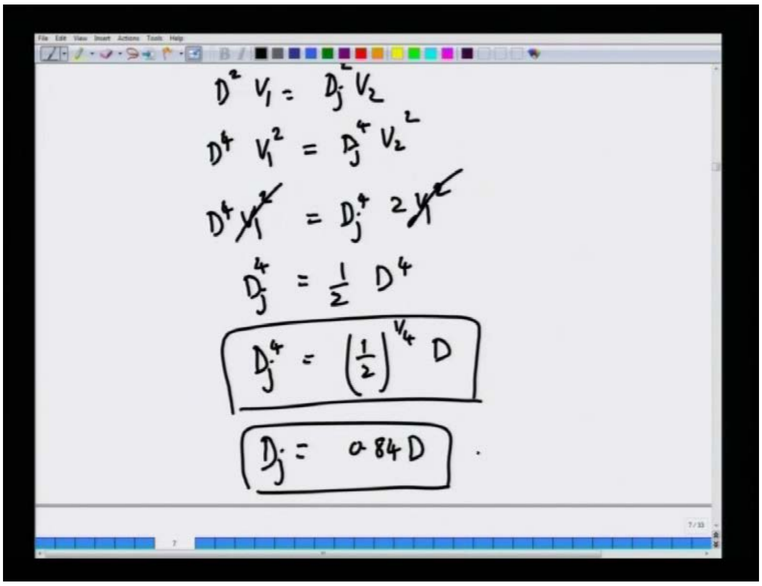

So, if I square equation D square time d to the 4 times V 1 square is D j to the 4 times V 2 square but, V 2 square is simply 2 V 1 square8d j to the 4 times 2 V 1 square V 1 ,V 1 cancels off. So D jet is to the 4 is half d to the 4. So, d jet to the 4 is 1 by 2 to the power 1fourth times D or D jet is point 84D this is the equation that we get while using the integral energy balance between points 1 and 2 where the point 1 is just exiting the tube and the velocity profile is parabolic while the pressure assumed to be atmospheric point 2 is sufficiently downstream the jet where the jet velocity is uniform and the jet pressure is atmospheric now the reason why jet things is because as soon as the jet exits the liquid exits the tube then it does not have any shear stresses also.

So, it accelerates a little bit. So, its velocity will be more then what it was at the tube but continuity of mass implies that therefore, V 1 A 1 is V 2 A 2 therefore, the jet area will be smaller than the tube area therefore, the jet diameter is smaller than the tube diameter and it is small by factor of point 84 now we will stop here and in the next lecture we will use the momentum balance integral momentum balance to solve the same problem and we will see that the answers are not exactly the same. So, we will meet in the next lecture.