

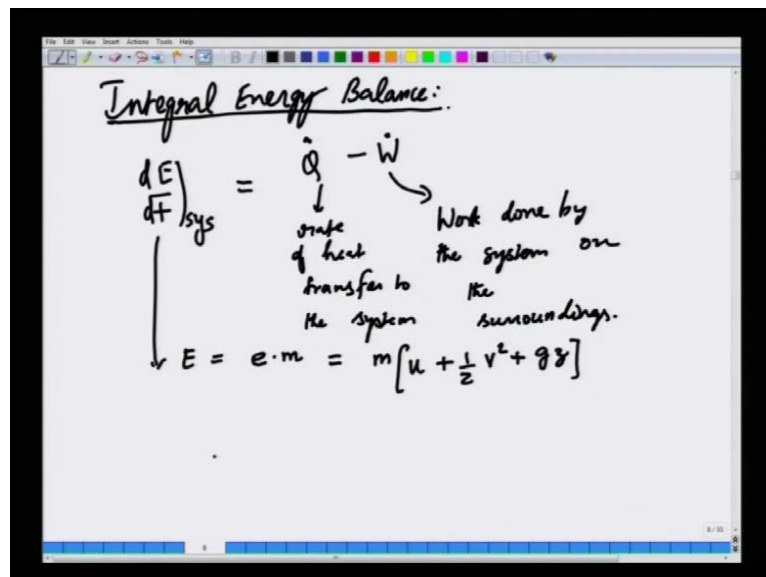
Fluid Mechanics
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Lecture No. # 18

Welcome to this lecture number 18 on this NPTEL course on fluid mechanics for undergraduate chemical engineering students. We have been discussing the ideas and integral balances of mass momentum and energy. And in the last lecture we completed the derivation of integral energy balance; and we just started out doing an application of the integral energy balance.

But before completing that application, I thought I will say a few words more about the integral energy balance and more specifically I want to make a few additional comments on the nature of losses and their connection to the loss of thermodynamics. So let me very quickly do that, before I proceed towards application of integral balances. So, after essentially the integral energy balance is a statement of the first law of thermodynamics applied to a continuously flowing system.

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Integral Energy Balance:

$$\frac{dE}{dt}\Big|_{\text{sys}} = \dot{Q} - \dot{W}$$

↓
rate of heat transfer to the system

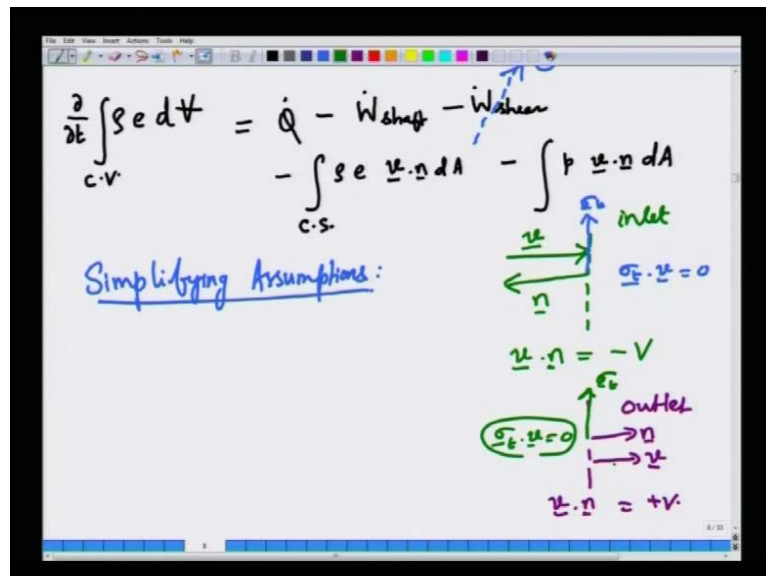
↘
Work done by the system on the surroundings.

$$\downarrow E = e \cdot m = m \left[u + \frac{1}{2} v^2 + gz \right]$$

And that merely says that the rate of change of the total energy of the system is equal to the rate at which heat is transferred to the system minus the rate at which work is done by the system. The dot above Q and W refer to the rate of heat transfer to the system. The sign convention is very critical, because heat transferred to the system is considered positive while heat transferred away from the system is considered negative. In the case of work, the convention that we are going to follow is work done by the system on the surroundings is positive while work done by the surroundings on the system is negative.

This is typically the convention used in engineering thermodynamics and fluid mechanics, while there may be in other context such as physical chemistry and physics, both heat transferred to the system, and work done on the system is considered positive. But that is not the convention that we are going to use in this course; so, work done by the system on the surroundings. Now we can convert this to the where E is essentially e times mass - the mass of the system and that is equal to mass times u - the specific internal energy plus half V square specific microscopic kinetic energy of flow and the potential energy g z times m..

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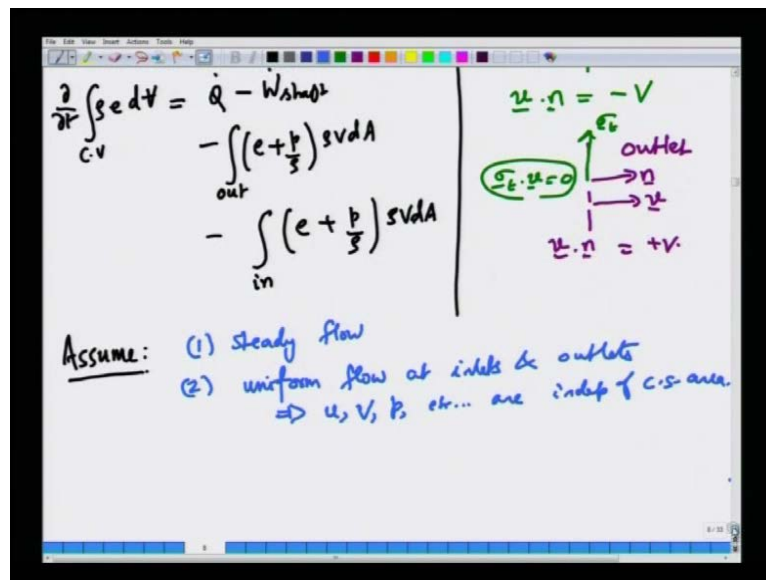
And when we apply the Reynolds transport theorem, we said that this must be equal to $\frac{d}{dt} \int_{c.v} \rho e dV$ integrated over the control volume. That is equal to the rate at which heat is transferred to the system minus the rate at which shaft work is done by the system. Then we also had in general there is a shear work done

at the entry and exit through the control surfaces. But we also said that that can be normally said to 0 by choosing the control surface perpendicular to the flow velocities. But this is not all, when you convert from the system approach to the control volume approach the rate of the change of the energy of the control volume will not just change because of the heat transferred and the work interaction between the system and surroundings, but also because of the fact that that energy can flow in and out the system by virtue of flow.

So that term is there, $\rho e \mathbf{v} \cdot \mathbf{n} dA$. But we also had one more term that is due to the pressure work done by the fluid that is entering and pressure work that is done at the inlet and exit control surfaces by virtue of fluid flow in and out of the system. This is also sometimes termed as the flow work. It is $p \mathbf{v} \cdot \mathbf{n} dA$. So, this is all the work and heat interactions that causes the rate of change of total energy present in the control volume. Now we will make some simplifying assumptions and then make the connection to thermodynamics, where we can concretely say something about the losses. In the last lecture, I sort of physically motivated it, now I am going to rigorously show that.

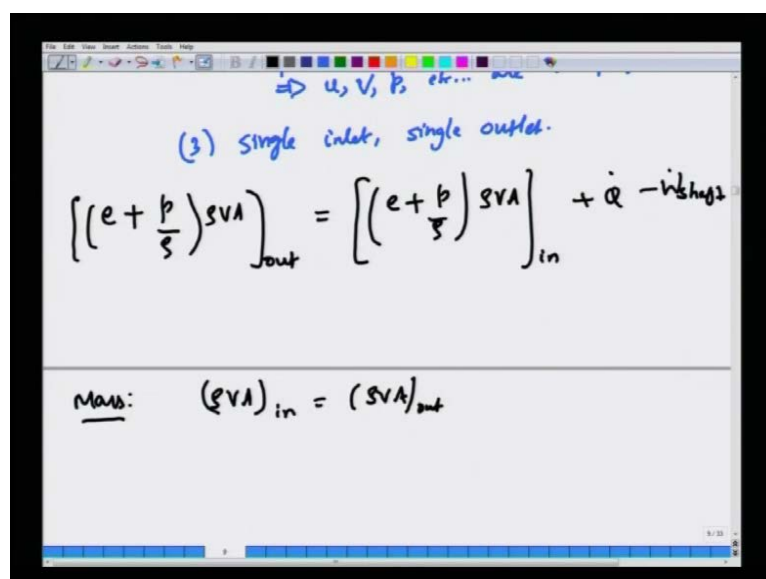
So when you choose the control surface like this, suppose this is the unit outward normal, this is the inlet and the velocity is coming in exactly perpendicular to the control surface then $\mathbf{v} \cdot \mathbf{n}$ is minus V . If it is an outlet, \mathbf{n} and \mathbf{v} are pointing in the same direction. So, $\mathbf{v} \cdot \mathbf{n}$ is plus V . But whenever you choose your control surface like this the shear stress act on the control surface or flow surface in this direction. So $\boldsymbol{\sigma} \cdot \mathbf{v}$ is 0. Same for the outlet, this shear stress act perpendicular to the direction of the velocity. So the rate at which shear work is done, at the control surface is zero if you chose the velocity exactly perpendicular to the control surface. So that is that is often possible. So we will just keep it like that so we will neglect shear work from now on. Once we do that, we will also find that.

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So let me now write this integral of the total energy present in the C V is equal to Q dot minus W shaft minus integral e plus p by rho times rho v d A at outlets minus e plus p by rho, rho v d A at inlets. Now I am going to make further simplifications. Now I am going to assume steady flow, now uniform flow at inlets and outlets. This means that quantity such as, whichever is happening within this integral such as u V p etc. are independent of the cross section area and their constants. This is an assumption because we know how to correct it by using the kinetic energy correction factor little later. But right now let us keep thing simple by assuming uniform flow.

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So once we do that and then we further assume single inlet single outlet. For simplicity there is only one inlet and one outlet, in which case this equation becomes $e + p$ by ρ times $\rho V A$ at the outlet is $e + p$ by ρ times $\rho V A$ at the inlet plus $Q \dot{m}$ minus $W \dot{m}$. If you use some mass balance for a single inlet single outlet system for steady flow, you will find that $\rho V A$ at inlet is the same as $\rho V A$ at outlet. This is mass in the rate at which mass is flowing is equal to rate at mass is flowing out, at a steady state for a single inlet single outlet system if there is no generation of mass or consumption of mass within the control volume and this is essentially the mass flow rate that is occurring through the system.

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Mass: $(\rho VA)_{in} = (\rho VA)_{out} = \dot{m}$

Divide by \dot{m} :

$$\left(e + \frac{p}{\rho} \right)_{out} = \left(e + \frac{p}{\rho} \right)_{in} + q - w_{shaft}$$

$$\left[u + \frac{p}{\rho} + \frac{1}{2} v^2 + gz \right]_{out} = \left[u + \frac{p}{\rho} + \frac{1}{2} v^2 + gz \right]_{in} + q - w_{shaft}$$

So let me divide this **this** equation by \dot{m} throughout, to give $e + p$ by ρ at outlet is $e + p$ by ρ at inlet plus small q which is essentially $Q \dot{m}$ by \dot{m} , minus small w shaft which is essentially $W \dot{m}$ by \dot{m} . Small q is the heat transferred to the control volume per unit mass of the fluid and small w is the shaft work done by the control volume on the surroundings per **unit volume of the fluid sorry per** unit mass of the fluid because now we have divided the rates with the mass flow rates. So the resulting quantity will be independent of time and it will be on the basis per mass of the fluid.

So once we have done that, I am going to write e as $u + p$ by ρ plus half V square plus $g z$. Outlet is $u + p$ by ρ plus half V square plus $g z$ at inlet plus q minus **this is**

heat into the system sorry volume per unit mass this is the work done by unit volume per unit mass. There is no rate involved..

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The image shows a whiteboard with handwritten mathematical equations. The top equation is:

$$\left[u + \frac{p}{\rho} + \frac{1}{2}v^2 + gz \right]_{out} = \left[u + \frac{p}{\rho} + \frac{1}{2}v^2 + gz \right]_{in}$$

Below this, the change in the quantity is defined as:

$$\Delta () = ()_{out} - ()_{in}$$

Then, the change in the specific energy quantity is given as:

$$\Delta \left[u + \frac{p}{\rho} + \frac{1}{2}v^2 + gz \right] = q - w_{shaft}$$

A note indicates that this is the differential form:

↔ differential form

$$du + d\left(\frac{p}{\rho}\right) + \frac{1}{2}d(v^2) + g dz = \delta q - \delta w$$

Now I can use a new symbol delta as that quantity evaluated at the outlet minus quantity evaluated at inlet. So delta of u plus p by rho plus half V square plus g z this q minus w shaft. Now I want to make connections with thermodynamics specially the second law of thermodynamics. So I want to write this in differential form that is I am going to evaluate this so that the inlets and outlets are very **very** close to each other, so that we can convert this delta to a normal differential. This is du plus d of p by rho plus half dV square plus g dz is del q minus del w shaft.

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$$\Delta \left[\frac{1}{\rho} \quad \frac{1}{2} \quad 0 \right]$$

differential form \leftrightarrow

$$du + d\left(\frac{p}{\rho}\right) + \frac{1}{2} d(v^2) + g dz = \delta q - \delta w_{shaft}$$

From Thermodynamics:

$$du = T ds - p dv$$

$$du = T ds - p d\left(\frac{1}{\rho}\right)$$

$$T ds - p d\left(\frac{1}{\rho}\right) + p d\left(\frac{1}{\rho}\right) + \frac{1}{\rho} dp + \frac{1}{2} d(v^2) + g dz = \delta q - \delta w_{shaft}$$

Definitions:

$$v = \frac{\text{Vol}}{\text{mass}}$$

$$\rho = \frac{\text{mass}}{\text{Vol}}$$

$$v = \frac{1}{\rho}$$

So now, I am going to use relations from thermodynamics. Thermodynamics tell us that du is $T ds$ minus $p dv$ v is the specific volume for unit mass, well ρ is mass per unit volume. So v is 1 over ρ in our notation. So this is well known relation in thermodynamics, the combination of first and second law of thermodynamics. So we will write this as du is $T ds$ minus $p d$ of 1 over ρ . So I am going to substitute du out here, to get $T ds$ plus, now let me first write minus $p d$ of 1 over ρ . This term out here, becomes plus $p d$ of 1 over ρ , I am going to differentiate over parts, plus 1 over ρdp plus half dV^2 square plus $g dz$ is equal to δq minus δW_{shaft} .

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$$(T ds - \delta q) + \frac{dp}{\rho} + \frac{d(v^2)}{2} + g dz = -\delta w_{shaft}$$

$$\geq 0$$

$$T ds - \delta q = \delta w_{shaft} \quad (\text{viscous losses})$$

$$\frac{dp}{\rho} + \frac{d(v^2)}{2} + g dz = -\delta w_{shaft} - \delta w_l$$

Single inlet, single outlet

$$\left[\frac{p}{\rho} + \frac{1}{2} v^2 + g z \right]_{out} - \left[\frac{p}{\rho} + \frac{1}{2} v^2 + g z \right]_{in} = -w_{shaft} - w_l$$

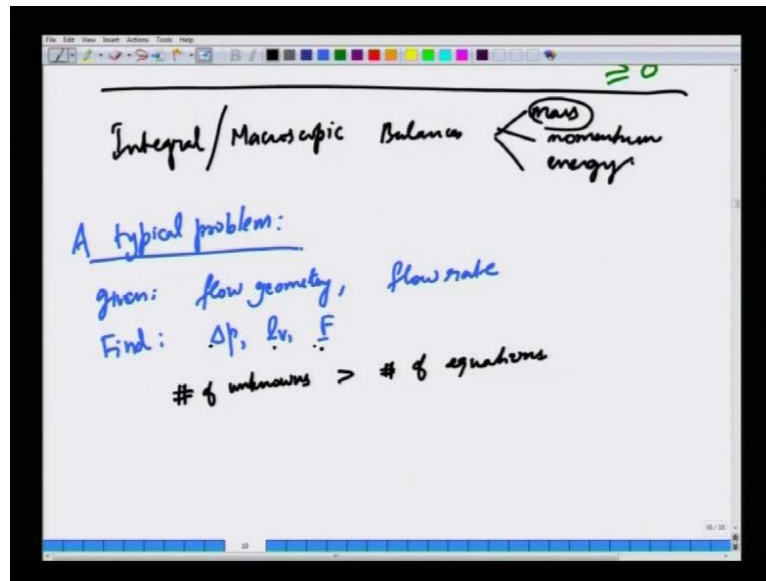
Now these two terms knock off each other and all you are left with is $T ds$, now I am going to bring this $d q$ out here, minus $d q$ plus dp by ρ plus d of V square by 2 plus $g dz$ is equal to minus W_{shaft} . Now the second of thermodynamics tells us that this quantity is always greater than or equal to 0. $T ds$ minus $d q$ is always $d q$ is always greater than or equal to zero and the equality is valid only for a reversible process. For any process that is not reversible for example, any real flow process is not a reversible process, there is uni directional conversion of mechanical energy to internal energy. As we mention in the last in the previous lecture.

So this is always greater than 0. So this is given the symbol ΔW_{loss} , viscous losses which cannot be retrieved as useful work because of irreversibility of the process there is always a systematic conversion of microscopic energy to internal energy. So mechanical energy to internal energy. So we cannot retrieve this back as useful work. So these are forever loss to internal energy of the internal degrees of freedom which will eventually manifest as an increase in temperature of the fluid.

So these are the viscous losses. So we will go to write this as dp by ρ plus d of V square by 2 plus $g dz$ is equal to minus ΔW_{shaft} minus ΔW_{loss} - the viscous loss. Now, I can now change the differential form to the input minus output form. So I am going to change it back to single input single output kind of the system inlet single outlet to give p by ρ plus half V square plus $g z$ at outlet minus p by ρ plus half V square plus $g z$ at inlet is minus w_{shaft} minus w_{loss} viscous losses. And this was the equation that we derived in the last lecture as well.

Last lecture I motivated this viscous loss term more physically, but now I have also told you that by just using a fundamental relation from thermodynamics you can actually show that this $T ds$ minus $d q$ is what eventually becomes a viscous loss and that cannot be retrieved as useful work. So this viscous loss is something that has to be either calculator or it has to be obtained from experiments. So, we will come to how to evaluate viscous losses a little later, but this is quantity that is always greater than 0. It is 0 only in the case of an idealized liquid with 0 viscosity, the losses of 0, but no real fluid such as air or water has 0 viscosity. They always have, even if it is a small number they have a finite viscosity. So there will always be loss of microscopic mechanical energy to irreversibly, to internal energy which will eventually manifest as an increase in temperature of the fluid.

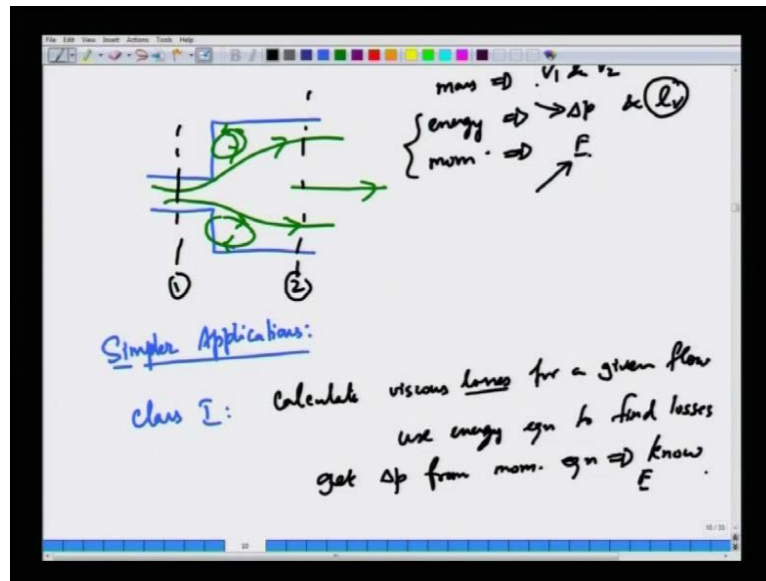
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Now before I go to applications, I want to make some general comments on microscopic balances. So we have three equations, integral these are also called as microscopic balances because they pertain to entire cross sections of equipments, we are not going to worry about detailed variation of flow quantity such as velocity or pressure at each and every points in the fluid, but, we are essentially doing a balance about entire equipment; so, sometimes called also as microscopic balances, which we refer to as integral balances. We have three equations for mass momentum and energy. So the question is how do we solve problems? A typical problem in real application might involve the following. This is rough statement of the problem, we will not give you concrete, I am not telling you a concrete statement. Suppose you may be given a flow geometry and you may be given a flow rate that is happening through the geometry - flow rate.

And we may be asked to find for example, what is a pressure drop required to make the fluid flow? What are the losses that are involved? What are the forces that are exerted on various parts of the control surfaces because of the fact that fluid is flowing through this geometry. Now it turns out that such a problem is not well specified because we will eventually find that this number of unknowns that we have is greater than number of equations; essentially this, because mass balance will trivially relate inlet and outlet velocities. So we will be left with three unknown Δp , loss and force but we have only the two balances moment and energy. So you have only two balances while we have three unknowns. So that leads to a problem.

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So for example, I am going to illustrate this situation in the case of an expansion, you may have channel which is expanding suddenly and fluid is flowing like this through, at steady state. Now, because of the fact that there is a sudden expansion there are these recirculating zones of fluid. And this leads to loss because some of the energy that you are trying to supply to make the fluid flow in this direction, is lost in making the fluid go round and round in this sort of dead pocket. So this leads to loss. So this is reason why expansion always has losses. Now if you construct a CV like this at point 1 and at point 2.

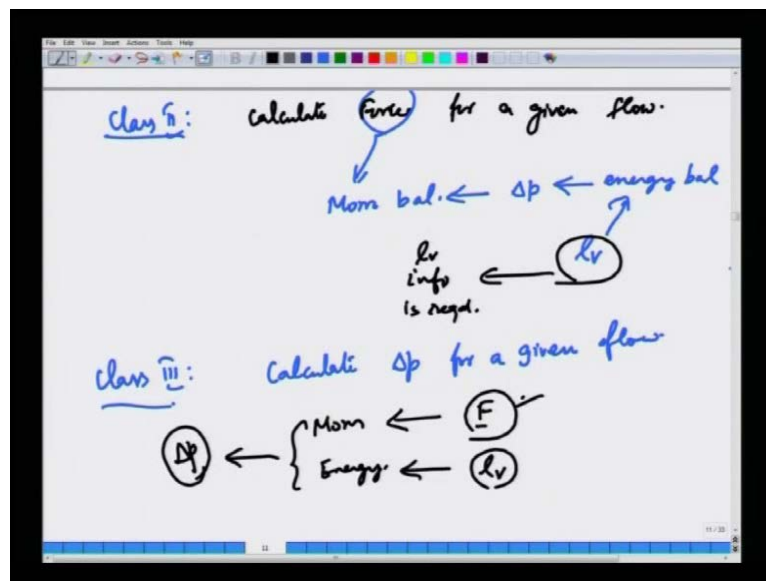
The mass balance will simply relate V_1 and V_2 , it simply tells you $V_1 = V_2$. The energy equation as two unknowns Δp and viscous loss, but, the momentum balance has an unknown the force - that is also an unknown. So we have only these two mass balance, simply relates V_1 and V_2 . So given V_1 , we can find V_2 . So that is the only job of mass balance. You have two equations, but three unknowns. So in order to calculate losses you should either have an idea of what the force is or what the pressure differences are? Either through experiments or some way we have to, otherwise this problem cannot be solved using the integral balances.

So we can roughly categorize the application, typically simpler applications of integral balances will fall in one of the following classes. Class 1 problems, let us say we will be asked to calculate viscous losses for a given flow. So here the idea is to use energy

equation to find losses. But in order to do that you need to know the pressure drop and you should get pressure drop from the momentum balance. So in order to find losses you need pressure drop in the energy equation, but the momentum equation relates the pressure drop to the forces. So if you want to get pressure drop from momentum equation, the momentum balance. Then you need to have we need to know the forces acting on the CV, otherwise we cannot.

So some information about the forces required in class 1 problems, if you want to calculate viscous losses, for a given flow and if you have no idea about the pressure gradients or pressure drops, then we need to appeal to the momentum balance, but they are the process relate to the forces acting through the control surfaces. Now we need to have some information on what the forces are acting on a control surface. Then we can proceed to calculate the pressure drop from the momentum balance and then substitute that back to the energy balance to estimate the losses given in the given application.

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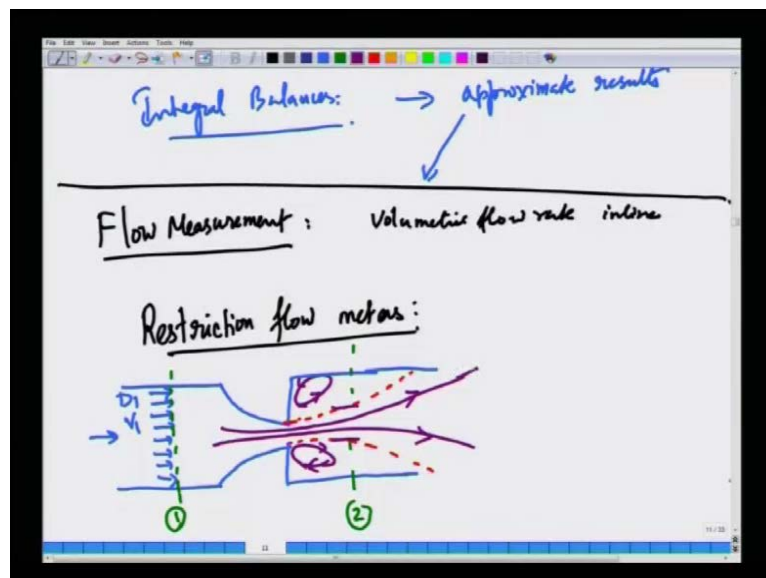


Now class 2 problems, second class of problems will involve, force must be calculated from the momentum balance. So essentially the question that will be asked or post is calculate force for a given flow force for a given flow. If you want to do that the force must be calculated from momentum balance, now in order to do that you need delta p. In order to get delta p you have to go to the energy balance. So, for that you need to know the viscous losses.

So you need to know you need to have some idea of this, information is required. Now to compute force in class 2 problems the second class of problems. The third class of problems involved. So here we may be asked to calculate Δp for a given flow. So you will be given a flow the velocity, what is a Δp that is required. So you have two options you could either use the momentum balance or the energy balance. So in order to get Δp from the momentum balance, you need to know the forces in order to calculate Δp from the energy balance, you need to know the losses.

So depending on whichever information is available, you may be able to calculate Δp from either of the approaches. So the answers may not always be the same because some of this information, the knowledge of either forces or losses might itself be not so accurate. So, some of the information about forces or losses of themselves are not accurate. So the resultant answer for Δp in the this third class of problems will be different depending on the approach you take. And of course, one has to choose whichever is approximation or is more accurate. You will have to get a better agreement with experimental data. So you can have different answers for Δp .

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So the point I am trying to drive across is that in general, integral balances are simple because you do not need much detail information about the velocities, distribution of velocities across various points in the flow, they are much simpler but they yield only approximate information as we just pointed out, because often our knowledge of the

forces exerted through control volume or the losses they are not perfect and many a times they have to be inferred from doing additional experiments or existing experimental data. So, you do not have a rigorous estimate of these, we solve these quantities such as forces or losses. So many a times we may have to end up making some severe approximations regarding forces or losses. In such a case the integral balances will give you approximate results. But the estimates, the approximate estimates that one obtains from integral balance, many a times can be good enough for a certain engineering applications.

So, we may we may actually a be very successful in getting some fairly good estimates for quantity such as flow rates, by making some clever approximations. But in some other application we may not be able to use the integral balances to great effect because the information that is required is simply not there, such as forces or losses. So the approximation cannot be good enough. So we need to have a more detail understanding of a the flow, which will come through what we will see a little later called differential balance approach or the microscopic balances approach. But so far as the integral balances are concerned, you have only three equation mass, momentum and energy.

The fairly simple and we indicated the three kinds of problems that one has. One has namely class 1 problems where for example, you are asked to, in class 1 problem you are asked to calculate the viscous losses for a given flow. So if you are given a flow then you can use the energy equation to find the losses. But the energy equation requires Δp . Now if you want to calculate Δp from momentum balances then you need the force information. So that is 1 class of problem and the second class of problem, you may be asked to calculate forces for a given flow. If you are asked to calculate the forces for the given flow, then you may have to use momentum balance, but the momentum balance will again need the information on Δp . In such a case you have to use it energy balance to calculate Δp , for which you need the knowledge on losses viscous losses.

So for class 2 problems, you need to know the losses information, while for class 1 problem you need to know the forces information. Now class 3 problems, you may ask to calculate, you may be asked to calculate Δp for a given flow and that will depend on whether you have to use momentum balance or energy balance. Suppose the force information is there for you, then you could use the momentum balance and if the losses information are there, you could use energy balance

Now the answers from either approach would of course, not exactly match because that it depends on the nature of approximations you have made in getting an idea of what the forces are or what the losses are. So in general the answers you will get will not be the same for the same quantities such as the pressure for a given flow. And we just pointed out that integral balances also give you only approximate results, they will not give you exact results because many of the unknown such as losses are known only approximately. So but in many engineering application these are fairly successful primarily in getting a trend on a various quantities such as flow rates or pressure drops and with additional experimentation we may be able to understand the given engineering problem variable.

So that is the philosophy through which or that is the a view point will take on the application of integral balances that they yields simple, but approximate the yields they allow for some simple solutions, but they give only approximate answers. So that is the key thing that we have to understand. Now, we will get back to the problems that we started doing in last lecture that is an application of integral balances to flow measurements. By flow measurement I mean measuring volumetric flow rates inline that is if you have a piping network through which fluid is flowing continuously in an application, in an industrial application you need to have a device that is fixed in within the pipe line system and we need to be able to measure the volumetric flow rate.

For example, in many applications in chemical engineering, in a chemical plant we may want to know, what is a flow rate that is entering a reactive? So the performance of the reactor will a function of flow rate and if the flow rate is not equal to the desired flow rate, then you may have to do some corrective measures using process control. So it is very critical for us to be able to tell what is a flow rate that is entering a given reactor or a given equipment in many chemical engineering application. The question is how are we going to measure a flow rate, as it is flowing through a pipeline network, through an equipments such as the chemical reactive.

So one simple way to measure flow rate is to go to the end of the pipeline system collect amount of liquid that flows, a fluid that flows to a particular, for a particular amount of time and if everything is in steady state you will get some mixture of the amount of volume that has flown per unit time just by dividing the amount of volume collected divided by the time for which you collected the volume. But those are indirect methods

and that relies on the fact that you can actually go to the exit of the pipeline network and its sort of collect the liquid.

But suppose you are interested in finding what is the fluid flow rate into a reactive which is completely closed to and there is not accessible for you to collect the volume. So you needs some indirect methods, where and you can infer the velocity or flow rate through maximum of the pressure drop. So such flow meters are called restriction flow meters. These are flow measuring devices which are fixed within a pipeline network in which fluid steadily flowing and by measuring an pressure drop between 2 appropriate points within this device, then we can correlate the pressure drop to the volumetric flow rate. That is the philosophy of flow restriction flow measurement devices or restriction flow meters.

So a typical representatives sketch it is not a detailed geometry of given flow meter is that you may have a pipeline through which fluid is flowing and you may have a contraction and a sudden expansion back to the original pipe diameter. And fluid is flowing upstream with some let us say the pipe diameter is D_1 and let us say that the normal sorry the velocity magnitude of the velocity that is flowing normal to this control surface is, so let us call this station as 1. There is an average velocity that is entering the way fluid is going to flow is the following; this going to flow like this.

So I am typically drawing, it is going to expands slowly and there will be recirculating zones. Now we can put the control surface somewhere sorry the station to somewhere here where the fluid velocity is uniform in this direction and it is flowing through. So let me also draw the streamline that separates in red. So you may have a streamline like this. So let me draw this streamline first. So then you can draw the flow velocity.

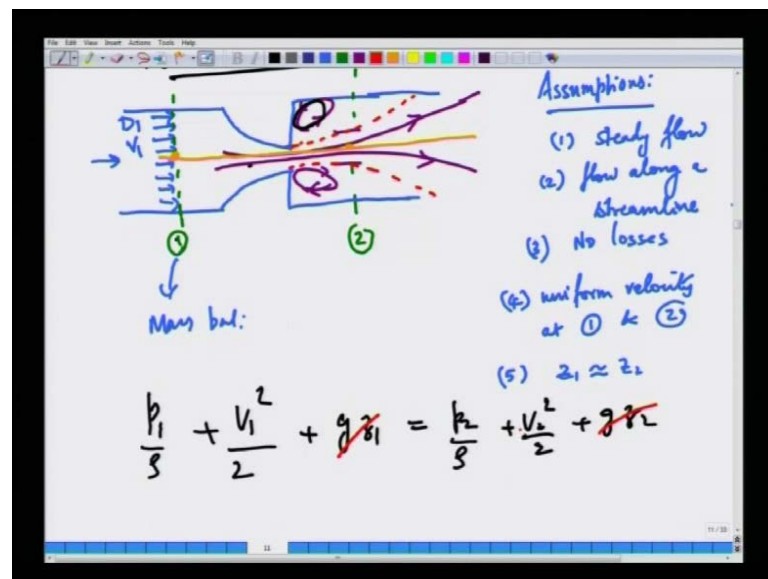
So this is to where there is, the cross section through which fluid is uniformly flowing is roughly a constant. Eventually it diverges and then it is sort of diverges then approaches the tube diameter itself. Now we want to apply the energy balance, but there are too many losses here, first of all the major contribution to the loss is due the fact there is an expansion and due to the expansion there are ades that are formed which are recirculating within the dead pocket through which fluid can either enter or leave.

So the fluid just mainly flows through this minimum area, whereas this area is not available flow because of recirculation. So it leads to the lots of losses. Now we do not

have any idea about what these losses are because these are such a complex flow process that we do not know exactly how the losses are related to the average velocity we want, that is flowing into the tube.

So that information is not available to us. So in lack of such information which is required to solve the problem rigorously, we will rather take a simpler approach whereby we say that let us just use the Bernoulli equation to begin with between points 1 and 2. The Bernoulli equation as I told you, remember the last lecture is applicable only for an **inviscid** fluid. A fluid with a hypothetical fluid with zero viscosity. **It is applicable along a streamline** sorry It is applicable only along the streamline. So the Bernoulli equation is applicable only along the streamline. So we will take a streamline that flows between point 1 and 2 like follows.

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So let me draw this streamline with an orange. So you may have streamline that goes, so we are going to apply Bernoulli equation between these point 1 and point 2 along the streamline. So this is admittedly an approximation, we will correct for this approximation later because remember that the Bernoulli equation is valid for a hypothetical inviscid fluid while no real fluid is inviscid that is no real fluid has zero viscosity, so there will be losses. Now losses are neglected to begin with and they will be corrected for later using an approximate method. So this is one of the simplest illustration of the application of integral energy balances or integral balances in general.

So the mass conservation equation will simply tell you, if I apply mass conservation between point 1 and point 2. Let us let us make the assumptions first, the assumptions are steady flow. Flow along a streamline because we want to be able to apply Bernoulli equation, no losses which will be corrected for later and you will have uniform velocity at stations 1 and 2. And we will further assume that the elevation of points 1 and 2 is same, which is a very reasonable assumption. So the Bernoulli equation will simply become for this problem p_1 by ρ plus V_1 square by 2 plus $g z_1$ this p_2 by ρ plus V_2 square by 2 plus $g z_2$ minus W . So we are neglecting shaft work, we are neglecting losses because it is Bernoulli. Now z_1 is the same as z_2 so that is 0.

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The image shows a whiteboard with the following handwritten equations:

$$(p_1 - p_2) = \frac{\rho}{2} (v_2^2 - v_1^2)$$

$$\rho v_1 A_1 = v_2 A_2 \rho$$

$$v_1 = v_2 \frac{A_2}{A_1}$$

$$\rightarrow (p_1 - p_2) = \frac{\rho}{2} \left[v_2^2 - v_2^2 \frac{A_2^2}{A_1^2} \right]$$

$$(p_1 - p_2) = \frac{\rho v_2^2}{2} \left[1 - \frac{A_2^2}{A_1^2} \right]$$

So essentially you will get a relation between p_1 minus p_2 , the pressure drop between point 1 and 2 which that is equal to ρ by 2 v_2 square minus v_1 square. Now the mass conservation equation tells us, that $v_1 A_1$ is $v_2 A_2$. Of course, ρ is there, but ρ is cancelling out because it is an incompressible fluid we are assuming incompressible fluids ρ is a constant - does not change. So you can write v_1 as v_2 times A_2 by A_1 . So, that this equation becomes p_1 minus p_2 is ρ by 2 times v_2 square minus v_2 square A_2 square by A_1 square or p_1 minus p_2 is ρv_2 square by 2 times 1 minus A_2 square by A_1 square..

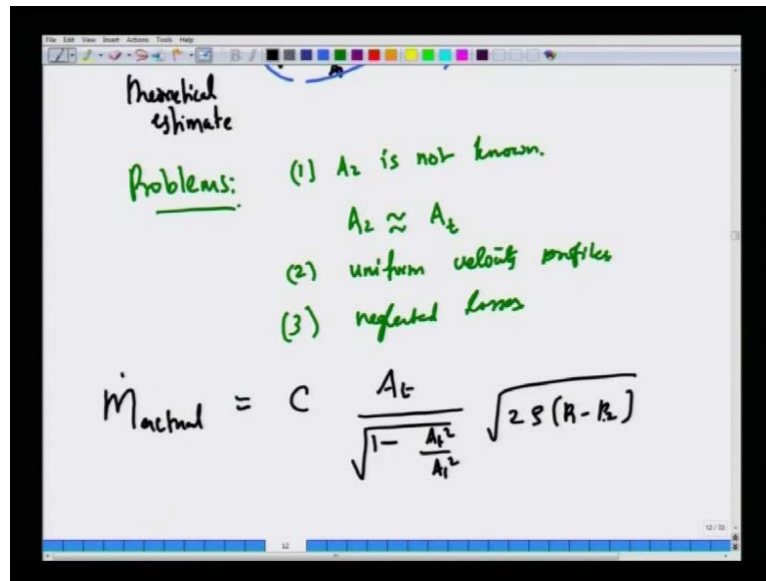
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The image shows a whiteboard with handwritten mathematical equations. The first equation is $V_2 = \left[\frac{2}{\rho \left[1 - \frac{A_2^2}{A_1^2} \right]} \right]$. Below it, the mass flow rate is given as $\dot{m}_{th} = \rho A_2 V_2$. The final equation is $\dot{m}_{th} = \frac{A_2}{\sqrt{1 - \frac{A_2^2}{A_1^2}}} \sqrt{2 \rho (P_1 - P_2)}$. A blue circle highlights the fraction $\frac{A_2}{\sqrt{1 - \frac{A_2^2}{A_1^2}}}$, and an arrow points from this circle to the text "theoretical estimate". Another blue arrow points from the term $\sqrt{2 \rho (P_1 - P_2)}$ to the label Δp .

So according to Bernoulli equation you can find what is the velocity at point V2 in terms of the pressure torque. If you measure the pressure between points 1 and 2 and if you know, what are the areas A1 and A2? Then we can find, what is the average velocity at point 2? So the mass flow rate of which is related to the volumetric flow rate quite simply cause it is of fluid incompressible, simply the density times the area times velocity V2.

So this will now become A2 by square root of 1 minus A2 square by A1 square times square root of 2 rho p1 minus p2. That is a this th represents for theoretical estimate for mass flow rate because in reality this may not be exactly true as we have neglected losses, but now we are just estimating based on the application of the Bernoulli equation. So what are the things that we need to know to compute the mass flow rate? We need to know rho, which can be typically found from handbooks. We need to know these differences and pressure delta p and we need to know the geometric factor A2 and A1.

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So what **is the what** are the problems with this equation? What are the issues? The problematic issues are the area A_2 is not known, it is not the conduit area or tube area. It is essentially if you recall the picture, this is the point at which the fluid is flowing uniformly and we choose arbitrary the point 2 to be this, we do not what is this area through which the fluid is flowing appearing without measurements.

But this area is roughly taken to be this area of the throat, this point at which the cross section is the smallest called the throat and this is called the diameter of the throat, the **throat** diameter. So typically the assumptions that is made, of course is that A_2 is a throat. This is an assumption which has to be checked. Now we also assumed uniform velocity profiles while applying the Bernoulli equation, but in reality that is not the case. So there will be some errors that accrue because of the fact that we have made the velocity profile to be uniform because the most important is we have neglected losses - the expansion losses. So the actual flow rate, the mass flow rate will be not the same. But it will be some other constant which has to be empirically or experimentally determined divided by $1 - \frac{A_t^2}{A_1^2}$ times square root of $2 \rho (P_1 - P_2)$.

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$$\beta = \frac{D_t}{D_1}$$

$$\dot{m}_{\text{actual}} = \frac{C A_t}{\sqrt{1 - \beta^4}} \sqrt{2 \rho (P_1 - P_2)}$$

$$\dot{m}_{\text{actual}} = K A_t \sqrt{2 \rho (P_1 - P_2)}$$

K (discharge coefficient)
 Calibrated (K)
 K (V)

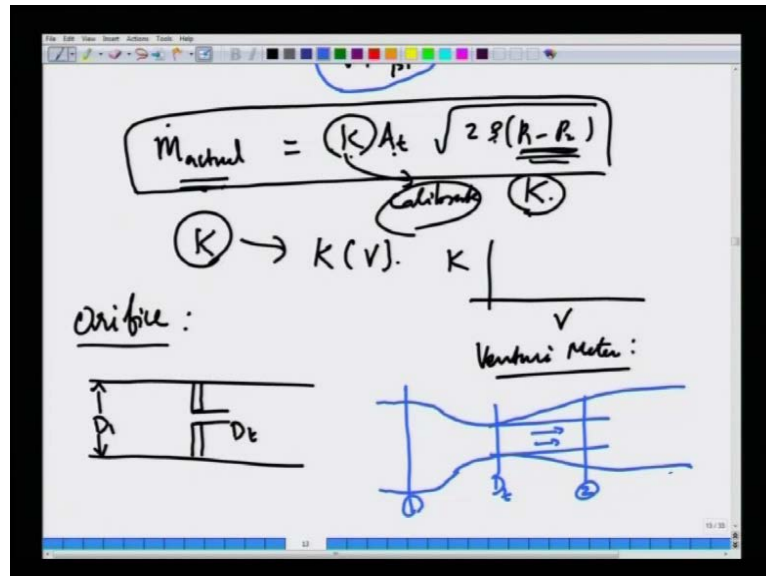
Now it is conventional to use beta as the ratio of throat, the two diameters D_t and D_1 . Beta is D_t divided by D_1 ratio of the throat diameter to the inlet diameter. So if I do that then the actual flow rate will be some C times A_t divided by square root of $1 - \beta^4$ times $\sqrt{2 \rho (P_1 - P_2)}$. Now this is called the discharge coefficient it is completely non dimensional, is called the discharge coefficient. So the actual mass flow rate becomes $K A_t \sqrt{2 \rho (P_1 - P_2)}$. Now this is. So, how do we do this for a given flow meter and for non-flow rates we measure pressure and calibrate the flow meter by finding K for known values of pressure drops and for known values of flow rates and by measuring pressure.

Once you have calibrated K for a given flow meter for an unknown value of flow rate you can use this equation by just simply measuring the pressure between the 2 points 1 and 2. So essentially what is normally done is, to put a manometer at this between these two points and to measure the pressure drop and that will give us an accurate estimate of what is the actual mass flow rate that is happening but for that we need to know what is K this discharge coefficient and in principle this K can be function of the velocity itself.

So you will find that in calibration charts the K is actually a function of velocity in some way. So that comes from the experimental fitting of the equation. So essentially this is the expected equation and you try to fit it with known values of flow rate and pressure since A_t and ρ are known you can find out what is K and if you do that generally find

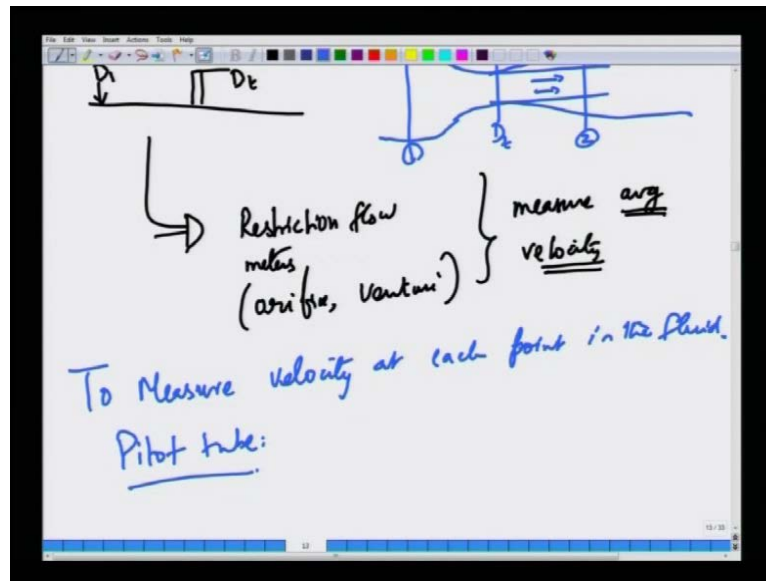
the K is not a constant it is really functional velocity, at which fluid is flowing through this flow meters..

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So, what are the types of such flow meters 1 is called the orifice meter. **Orifice meter** is generally you have a circular disk with a hole that is an orifice which acts as a restriction device D_1 and you also have venturi meter. So here you have this, let me draw with some other color, you have this upstream tube but it gradually converges and gradually diverges and again fluid is flowing in this region like this. So this is point 1, this is point 2, this is the throat diameter. So these are the two common configurations of the restriction flow meters that are often used. So essentially, the basic idea is the same; you are trying to estimate rather the mass flow rate or volumetric flow rate through a pipe by merely measuring the pressure drop before and after or at the point of minimum cross section that is a throat and by measuring this pressure drop we are able to relate that pressure difference, shows the actual flow rate that is happening in the system. Now these devices is measure average.

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So what is the major idea behind restriction flow meters? The restriction flow meters such as orifice and venturi meters; they measure average velocity. They do not tell you, what is the velocity at each and every point across the cross section of a pipe? They measure only the average velocity. So if you want to measure, what is the point wise velocity? You cannot use restriction flow devices, restriction flow meters; to measure velocity at each point in the fluid, you need what is called the Pitot tube. It is essentially a very small tube that is inserted into the flow, then which is used to infer, what is the velocity at a given point in the fluid? By varying the location of this tiny tube at various points across the cross section of a pipe for example, then we can infer the velocity distribution or the variation in velocity at each and every point across the cross section of the pipe. So, I we will stop here at this point, and we will start in the next lecture.