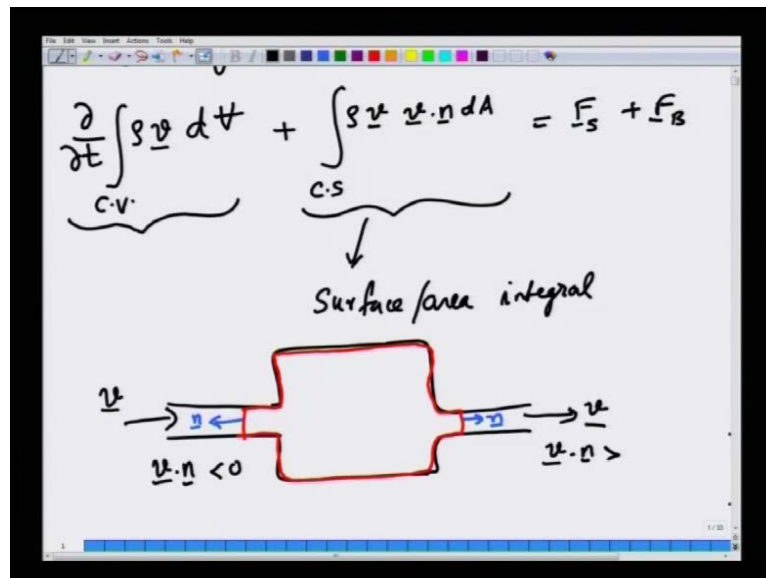


Fluid Mechanics
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Lecture No. # 15

Welcome to this lecture number 15, on fluid mechanics for undergraduates students in chemical engineering. Let me begin by briefly recapitulating what we were doing in the last lecture. In the last lecture, we completed nearly the discussions on integral momentum balance and we also illustrated the application of integral momentum balance through an example, whereby we calculated the force due to a jet that impinges on solid surface and we showed how the integral momentum balance can be used evaluate force and there is one further thing that we have to discuss before we complete this topic and before we do that let me first write down the integral momentum balance for your convenience.

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So, the integral momentum balance for a control volume takes the following form. This is the time rate of change of momentum present in the control volume plus integral $\rho \mathbf{v} \cdot \mathbf{n} dA$ over the control surface is equal to the sum of all the surface forces and body forces present in the control volume.

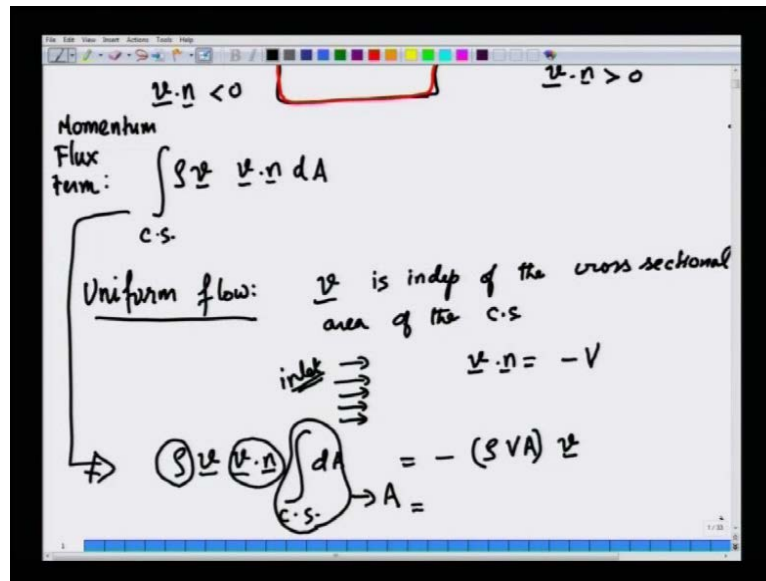
So, this term is the term that tells you that how momentum changes within a fixed region of the space, that is control volume and this is the term that denotes how momentum enters and exits the control volume by virtual flow. So, this the momentum flux term that enters and exits the control volume through the control surfaces and these are surfaces and body forces and we gave several hints or tips has to how to use this for a given control volume for a given problem and we also saw in the last lecture number 14, that for the same problem we could choose different control volumes and as long as you use the correct as long as you do the steps correctly the answer will be the same regardless the choice of the control volume.

Haven said that of course, sudden control volumes are easier or more convenient for problems sudden problems compare to others. So, it comes with experience and some judgment as to which is the most appropriate control volume for a given problem, but even if we do not hit the right or the most appropriate control volume as long as the steps are done correctly the answer will remain same and it regardless of the choice of the control volume.

Now, there is one thing that you should remember that whenever we have to evaluate the momentum flux stem, it is surface or area integral. Now, for example, you could imagine that fluid is entering a simple control volume, let us demarked that show the control volume using this red line, lets imagine that fluid comes through here and exits through here now as I have told u several times in the last two lectures.

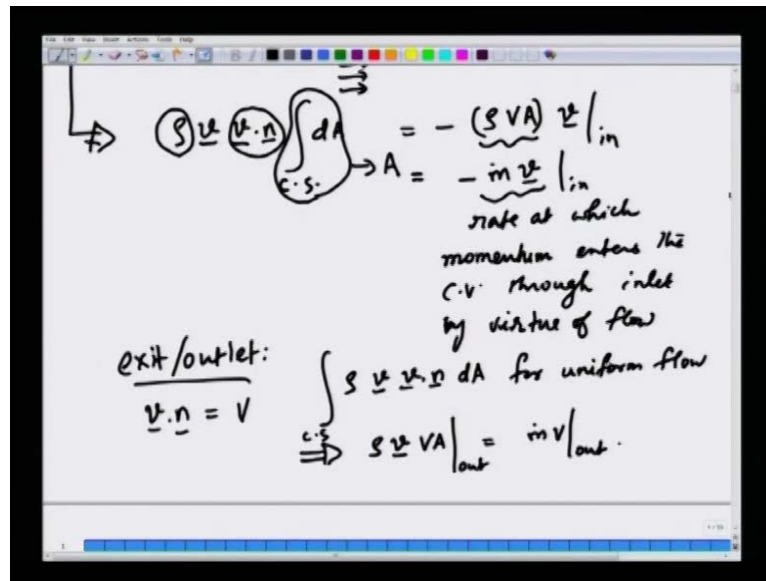
The term, the velocity enters and exits like this. So, $\mathbf{v} \cdot \mathbf{n}$ is always negative in the entrance and it is positive in the exits, regardless of the choice of the coordinate system Because $\mathbf{v} \cdot \mathbf{n}$ is a scalar quantity, it is a dot product of the velocity and the normal to the control surface and the velocity and regardless of how you choose the coordinate system velocity $\mathbf{v} \cdot \mathbf{n}$ is always negative in the entrance and $\mathbf{v} \cdot \mathbf{n}$ is always positive in the exits.

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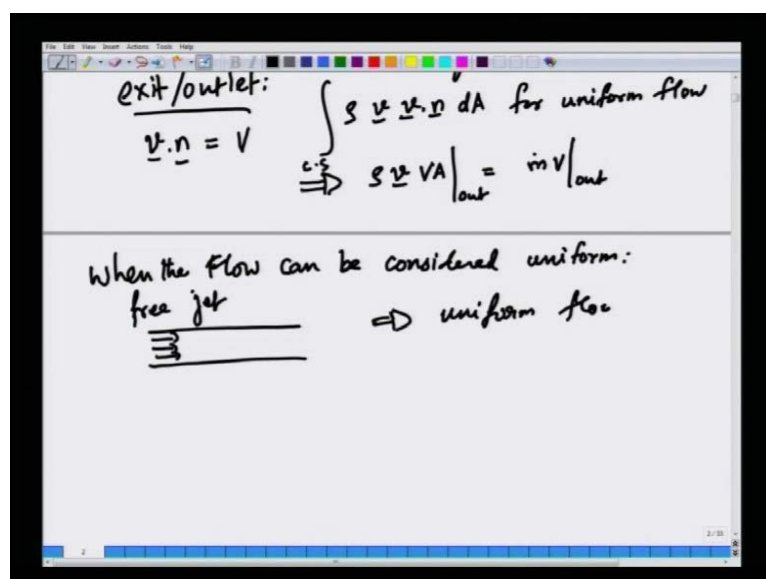
So, once you do that you have to typically. So, let me focus on the momentum flux term in the integral momentum balance that appears like this $\rho \underline{v} \underline{u} \cdot \underline{n} dA$ over the control surface. Now suppose we assume uniform flow, if the flow is uniform then \underline{v} is independent of the cross sectional area of the control surface of the c.s, this implies that the velocity is a constant. So, you can pull this equation to simplify to the following $\rho \underline{v} \underline{u} \cdot \underline{n} \int_{c.s.} dA$ and this is simply now ρ times $\underline{v} \cdot \underline{n}$ is essentially let me just complete this is simply equal to summation and integral over dA is simply A . Let us focus on let us the inlet part of the control surface. So, $\underline{v} \cdot \underline{n}$ is minus it is a V and so, this gives $\rho V A$ times the velocity vector the negative sign is because, we are let us a focusing on the inlet.

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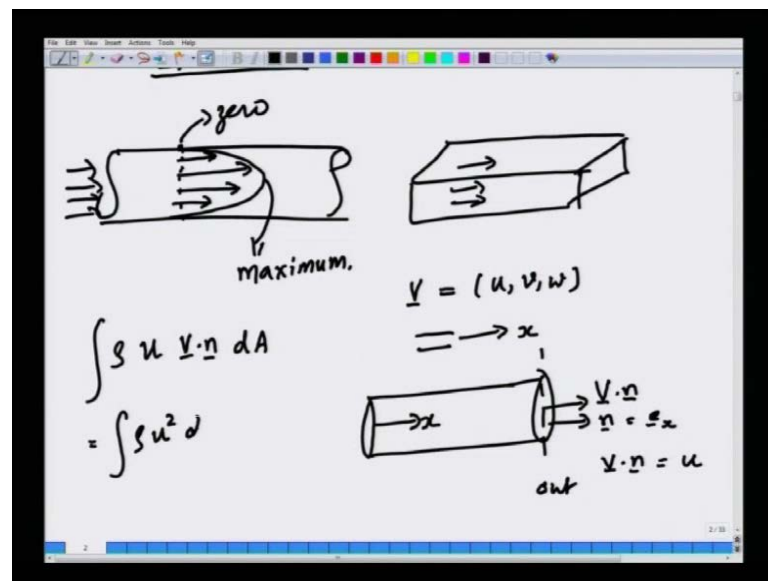
So, this is also equal to $\rho V A$ is nothing, but a mass flow rate \dot{m} times v the velocity vector this. So, mass and velocity is momentum mass flow rate times the velocity that is the rate of the mass entry times; the velocity vector is the rate at which momentum enters the C.V. through the inlet by virtue of flow. So, that is the inlet flux term. So, likewise for the exit or outlet you will find that $v \cdot n$. So, this all evaluated inlet. So, let us put in $v \cdot n$ is plus v . So, the integral $\rho v v \cdot n dA$ for uniform flow will become $\rho v v A$ this is simply equal to evaluated at the outlet $\dot{m} v$ evaluated at the outlet.

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So, when the flow is uniform then we get very simple answers, when the flow is uniform we get very simple answers for the influx or outflux term, but having said that we also have to worry about more realistic cases and let me before I do that let me also say that when can when the flow can be considered uniform well, as I saw as we saw in the last lecture, whenever we have a free jet a jet of water that exits into the atmosphere is called a free jet.

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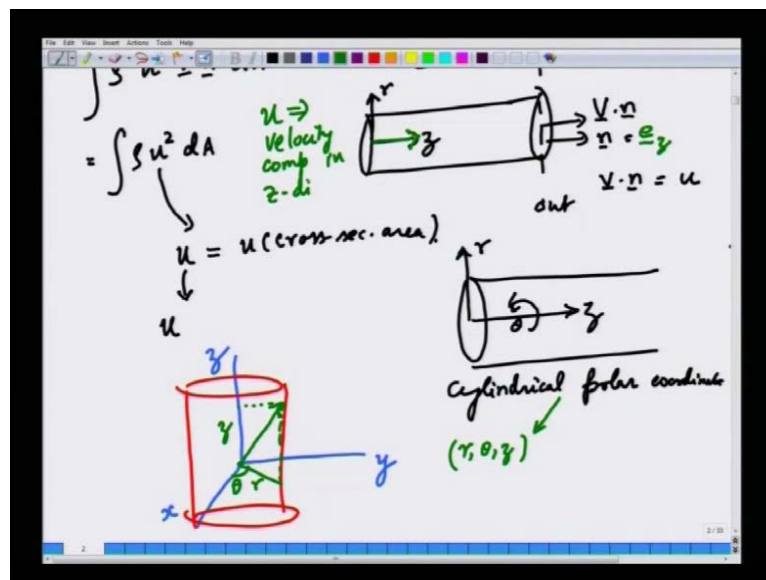
Then uniform flow is a very good approximation, but in many engineering applications we will see as we go further in the course, that you have flow within a pipe. So, fluid is flowing within a pipe or within a channel rectangular channel bounded by rigid boundaries surrounded by rigid boundaries. Whenever, we have flow through a duct a pipe or a channel regardless of the shape of the duct it is experimentally observed, that the velocity is not a constant it varies with the cross-sectional area. So, this is an experimental fact and also when we do differential momentum balances, we will show this rigorously by deriving the equations of motion the linear momentum balance for a linear differential linear momentum balance for a very very tiny control volume, we will show this can be derived exactly or rigorously.

So, the fact is flow is flow through ducts and channels are not always uniform in the sense that the velocity is not a constant with respect to the cross-sectional area through which fluid is flowing and in fact, the velocity varies as we go from center towards the

wall and typically the velocity is zero at the wall and maximum at the center. So, there is clearly a variation of fluid velocity across the cross-section of tubes and channels are conduit in general. Now we have to solve problems involving integral momentum balance even with such conduits, then we are left with evaluating terms of this type where let say velocity v is given by components u v w .

Let say the flow is in the x direction now, let say we are looking at the x component of the momentum balance. So, let us imagine that let us say we focus on the outlet the flow is in the x direction. Fluid is flowing in the plus x direction n is e_x . So, $v \cdot n$ is simply equal to plus u . So, that is what we are looking at we have to evaluate terms like this in flux term, where u is not a constant u is actually a function of the cross-sectional area.

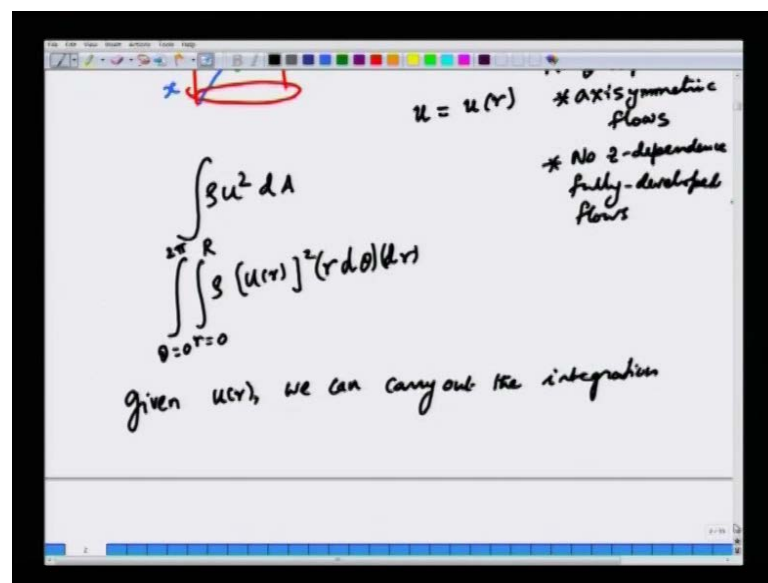
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Now, but in principle you have to evaluate first of all you need to know what is u , how is u a function of let say we call, let say a flow in a pipe. If we have a flow in a pipe, it is useful to use polar coordinates where to be on the safer side to be correct we should call z . So, to be the flow direction z and then r and then theta goes this is called cylindrical polar coordinates. So, theta goes around the access of the tube. So, let me illustrate this with respect to the Cartesian coordinates which we are familiar with suppose we have x y and z and let us put a cylindrical tube whose axis is along the z axis. Now any point let us keep it u this is cylindrical coordinates system any point is denoted by, this is the point this is the position vector of the point it is denoted by projection.

First, take the projection of the position vector with respect to x y plane the distance from the origin to the projection is r, the angle made with respect to x axis is theta and the vertical z the vertical projection is z which remains the same. So, any point is denoted by r theta z in cylindrical coordinates and whenever we have flow through tubes flow through pipe it is convenient to work with cylindrical polar coordinates, because it simplifies the problem by grade d. Now, the flow is fluid is flowing in the instead of n calling as e x, n is in the direction e z because we are using polar coordinates, but let us call the velocity component in the z direction as u in the z direction.

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So, we want to evaluate and integral of this type rho u square dA. Now u is a function typically only of r we will see little later when we do differentially momentum balance we will precisely say when this is valid, but let us now take it as a fact u is not a function of theta, because the flow symmetric about the theta the z axis is called axis symmetric flow. So, the flow is symmetric about the theta axis is not varying along the theta direction such flows are called axis symmetric flows and the flow is independent of axis symmetric flows means no theta dependence.

The flow is also independence of independent of z direction. So, no z dependence that is the velocity that the z direction is independent of the z coordinate itself such flows are called fully developed flows. We will of course, discuss these assumptions a little later when we do differential balances now I am just treating it as more like a fact.

So, essentially we have to do integral rho u square and where u is now a function only of r. So, let us keep it as u r whole square, now dA is nothing, but in polar coordinate so r d theta dr. This is the area element in the polar coordinates. So, you are integrating r from center to the radius and theta from 0 to 2 point. So, provided given u r we can carry out the integration to calculate the momentum flux term in the integral momentum balances, because in general the flow is not uniform.

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$$\int \rho u^2 dA \equiv \beta \rho A V_{av}^2$$

$$V_{av} \equiv \frac{1}{A} \int u dA$$

$$V_{av} = \text{c.s. average velocity.}$$

Momentum correction factor

Laminar flow in pipes:

$$u = U_0 \left(1 - \frac{r^2}{R^2} \right)$$

Turbulent flow:

So, we have to do this in order to calculate the flux terms. Now it will be nice however, if we can write the integral in terms of a very simple algebraic quantity beta rho A v average square where, v average is the cross sectional average or velocity. So, v average is defined as integral over it is a u dA follow over A where this beta is called the momentum correction factor. So, instead of evaluating this entire integral if we can evaluate this once and for all and write it in terms of an algebraic quantity and like the average velocity times the factor that includes for long uniform that account for long uniformity of the flow then that will be very very handy when we do integral momentum balances.

So, in order to this first we have to calculate what beta is, now let me just do this for two cases, one is called laminar flow in pipes in tubes or pipes and the other is for turbulent flow in pipes. Now, first of all I have to tell you what are laminar and turbulent flows.

Now, it turns out that fluid flows in general exists in two types of regimes one is the slow and orderly motion of flow, where fluid elements are nicely flowing passed to each other and the flow is usually in one direction. The flow is steady in the sense that at a given point in space the velocity is not a function of time such flows are called laminar flows and typical example of laminar flow is suppose, when we try to open tap in your house. When the tap is open very little you see a very smooth jet of fluid that exits a tap, but when we open the tap fully then you see that the smooth jet is no longer a smooth jet and its breaks and it's very very complicated the motion is very complicated.

So, the smooth jet regime is called laminar regime in the flow from a tap and the more when you close the when you open the tap completely full then the more complex flow that exits from the tap is indicative of the turbulent nature of the flow. So, it turns out that the velocity how the velocity varies with the cross sectional area is very very different for laminar and turbulent flows. For laminar flows, for which I can give you a full illustration of how to calculate of beta, the velocity in a pipe is parabolic in nature.

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The image shows a handwritten diagram of a pipe with a parabolic velocity profile. The maximum velocity is labeled u_0 (maxm. Velocity). The area is given as $A = \pi R^2$. The equations are:

$$\int_{\theta=0}^{2\pi} \int_{r=0}^R r dr d\theta (u(r))^2 \equiv \beta \int V_{av} A$$

$$V_{av} \equiv \frac{1}{A} \int_{\theta=0}^{2\pi} \int_{r=0}^R u_0 \left(1 - \frac{r^2}{R^2}\right) r dr d\theta$$

That is, if I plot the velocity as a function of the radial coordinate it is a parabola velocity goes to zero at a r equal capital R and it is a maximum u naught, it is the maximum velocity at the center at r equal zero at the center. So, this is both an experimental fact as well as we will illustrate this how to derive this theoretically or exactly by doing the differential momentum balance a little later in the course.

But we right now we will just take it as experimental fact. While the turbulent flows the velocity profiles looks very different. So, u is approximately $1 - r/R$ to the power one-seventh, this is an experimental fact we cannot derive this rigorously from first principles. So, this we will take it as an experimental fact this now although we take it is as an experimental fact, this will derived later in the course when we do differential balances. So, once you have let me once you have the form of the velocity you all you need to do is integrate u^2 theta is 0 to 2π r is 0 to R $r dr d\theta$ $\rho u r^2$.

Now once you have come up with this expression you equate it to ρv_{average} times A , this all A is essentially πr^2 for cross-section of the pipe. Now, in order to do that we have to tell what v_{average} is we have to first compute what is v_{average} ? v_{average} is nothing, but one over an integral theta is 0 to 2π r is 0 to R u naught times $1 - r/R$ square by capital R square.

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$$\begin{aligned}
 V_{av} &\equiv \frac{1}{A} \int_{\theta=0}^{2\pi} \int_{r=0}^R \underbrace{U_0 \left(1 - \frac{r^2}{R^2}\right)}_{u} r dr d\theta \\
 &= \frac{2\pi U_0}{A} \int_{r=0}^R \left(1 - \frac{r^2}{R^2}\right) r dr \\
 &= \frac{2\pi U_0}{A} \int \left[r - \frac{r^3}{R^2}\right] dr
 \end{aligned}$$

So, first notice that and of course, you have to do that $r dr d\theta$, first notice that the theta the velocity expression the integral and the independent of theta. So, you can readily do the theta integral 2π by A times integral r equals 0 to R and as seen u naught is constant you can pull it out u naught is a maximum velocity at r equal to 0.

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$$\begin{aligned}
 &= \frac{2\pi U_0}{A} \int_{r=0}^R \left(r - \frac{r^3}{R^3} \right) dr \\
 &= \frac{2\pi U_0}{A} \left[\frac{r^2}{2} - \frac{r^4}{4R^3} \right]_0^R \\
 &= \frac{2\pi U_0}{A} \left[\frac{R^2}{2} - \frac{R^4}{4R^3} \right] \\
 V_{av} &= \frac{\pi U_0 R^2}{2}
 \end{aligned}$$

So, 1 minus r square by r square r d r. So, this is 2 pi u naught by A integral r minus r cube by R square d r this is 2 pi u naught divided by A r 0 to r. So, this is r square by 2 minus r to the 4 by 4 r square r is 0 to capital R. So, when you evaluate this 2 pi u naught by A r square by 2 minus R to the 4 by 4 r square this is nothing, but pi u naught by A this is v average pi u naught by A times r square by 4.

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$$\begin{aligned}
 V_{av} &= \frac{\pi R^2 U_0}{2 A} & A &= \pi R^2 \\
 \boxed{V_{av} = \frac{U_0}{2}} & \Rightarrow V_{av} A = \frac{\pi R^2 U_0}{2} \\
 \int_{\theta=0}^{2\pi} \int_{r=0}^R u^2 r dr d\theta & \equiv \int \int (V_{av} A) \\
 &= \int \int \frac{\pi R^2 U_0}{2} \\
 \int_{r=0}^R \frac{2\pi U_0}{2} \left(\frac{1-r^2}{R^2} \right)^2 r dr & \left| \begin{array}{l} u = U_0 \left(\frac{1-r^2}{R^2} \right) \\ u^2 = U_0^2 \left(\frac{1-r^2}{R^2} \right)^2 \end{array} \right.
 \end{aligned}$$

So, v average is nothing, but pi r square u naught, there is a factor of two. So, that will give you pi or this two will cancel factor of two here. So, that will give you pi r square

times u_{naught} divided by $2A$, but A is the area of the cross section of the pipe that is simply πr^2 . So, v_{average} is u_{naught} by 2. So, we have to find from the knowledge of the velocity profile for laminar flow in a pipe. So, we have to do another integral you have r is 0 to R θ is 0 to 2π $\rho u^2 r dr d\theta$ is equal to remember, where this is definition of the momentum correction factor and we want to find what β is we want to essentially find an expression for β .

So, to that end we are doing all these calculations $v r A$. So, v_{average} times from this expression are nothing, but $\pi r^2 u_{\text{naught}}$ divided by 2. So, you write this as $\beta \rho \pi r^2 u_{\text{naught}}$ and now we have to do this integral again the θ integral is straight forward because integral independent of θ . So, r is 0 to R ρ is a constant, so I pull this out. Now u is u_{naught} times $1 - \frac{r^2}{R^2}$ by r^2 . So, this is nothing, but. So, u^2 is u_{naught}^2 times $1 - \frac{r^2}{R^2}$ by r^2 whole square. So, you have u_{naught}^2 times $1 - \frac{r^2}{R^2}$ by r^2 whole square dr this is the integral we have to calculate.

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$$\begin{aligned}
 & 2\pi \rho u_0^2 \int_0^R \left[1 - \frac{2r^2}{R^2} + \frac{r^4}{R^4} \right] r \, dr \\
 &= 2\pi \rho u_0^2 \int_0^R \left[r - \frac{2r^3}{R^2} + \frac{r^5}{R^4} \right] dr \\
 &= 2\pi \rho u_0^2 \left[\frac{r^2}{2} - \frac{2r^4}{4R^2} + \frac{r^6}{6R^4} \right]_0^R \\
 &\Rightarrow 2\pi \rho u_0^2 \left[\frac{R^2}{2} - \frac{R^4}{2R^2} + \frac{R^6}{6R^4} \right] = \frac{\beta \rho \pi R^2 u_0^2}{2} \\
 & \quad 2\pi \rho \frac{R^2}{6} = \beta \rho \pi R^2
 \end{aligned}$$

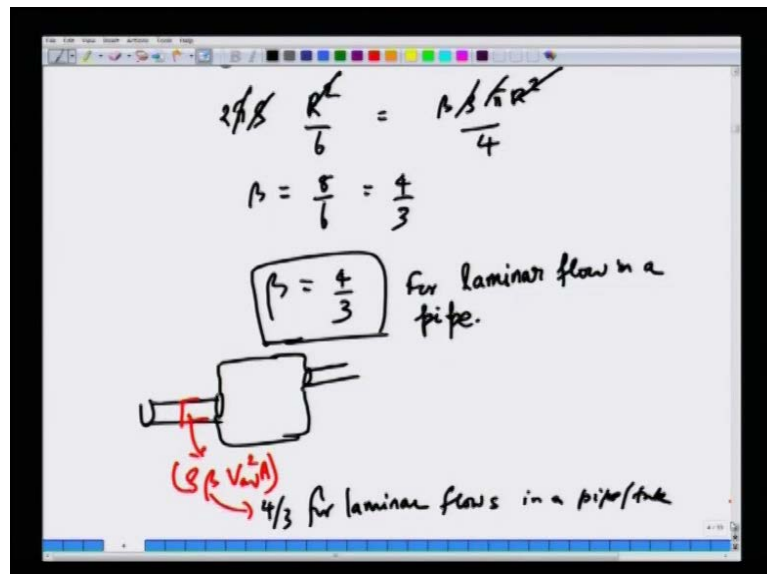
So, $2\pi \rho u_{\text{naught}}^2$ r is 0 to R capital R . So, then let's explain this square out. So, it is $1 - 2r^2/R^2 + r^4/R^4$ by r^2 plus r^2 to the 4 by capital R to the 4 times $r dr$. So, $2\pi \rho u_{\text{naught}}^2$. So, let me multiply through by r by r^2 plus r^2 to the 5 by r to the 4 dr . So, if I integrate this I get r^2 by 2 minus $2r^4$ by 4 r^2 plus r^6 by 6

the 6 by 6 r to the 4, r is going to 0 to capital R if I do this two pi rho u naught square r square by 2 minus r square by 2 again plus r square by 6. So, you get these 2 r square by 6 will cancel out and if you see the right side, you have this is this much equal beta times rho beta times rho by this expression r square u naught by 2.

So, go back once to see. So, this should be v average square because that is the definition of beta. So, that is why we are missing a factor of. So, if you remember the definition of beta. So, this is v average square. So, I missed out v average factor. So, we have to write this is v average square. So, that is why we were missing factor of v average here. So, let me go back. So v average squared here. So, it is u naught square by 2. So, this implies you have to u naught square lets cancel out things u naught square will cancel.

So, the left side will have r square by 6 times 2 pi rho the right square we will have beta rho pi r square by 2. So, beta should become I am still missing a factor of 2 here, lets us case go through the algebra a little I repeat once again. So, v average is u naught by 2 which just find. So, lets us look at this term. So, there is a factor of 4 because we squaring v average. So, its u naught square by 4 which is why we were missing a factor of 2.

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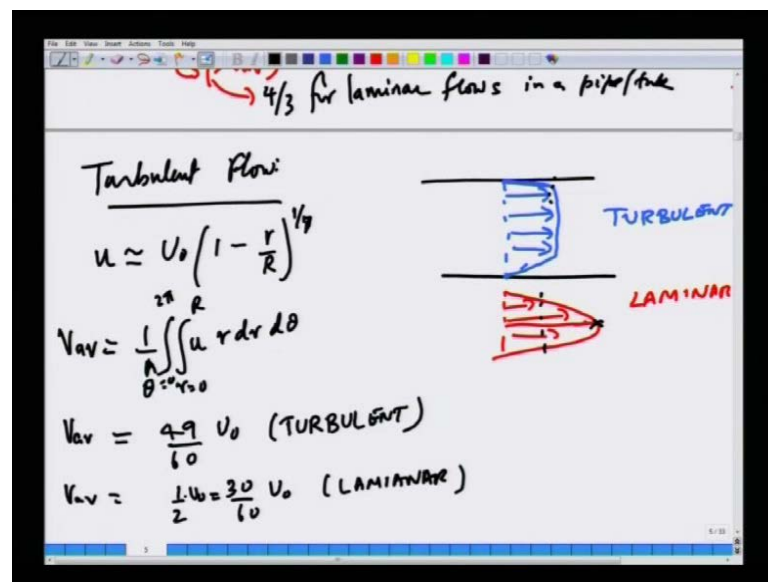


So, let us go back here and replace this by 4 and then by 4. So, beta will become simply. So, everything will cancel pi will cancel with pi r square r square rho rho. So, beta will become simply 8 by 6 or 4 by 3. So, beta is simply 4 by 3 for laminar flow in a pipe.

Likewise, we can also this is the momentum correction factor what is the advantage. So, whenever we have a problem in which the laminar flow in a pipe that connect various equipment and if your control surface cuts across the cross section of the pipe, whenever you want to calculate the momentum flux integral all you can do is you can write this as rho beta v average square.

You can simply times area you can simply replace it by this were beta is two by 3 4 by 3 for laminar flow beta simply 4 by 3 for laminar flow in a pipe. So, that is the simplicity that the momentum correction factor gives us pipe or tube because, otherwise we have to evaluate the integral each and every time when we do the problem.

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So, I will quickly tell you what is beta for turbulent flow in a pipe. So, it turns out that for turbulent flow u is approximately u naught as I told you this is more like an experimental fact, it is not rigorously derivable from first principles like the laminar flow expression for the velocity none the less the velocity is a function of the radial coordinate, but if you qualitatively sketch this velocity profile, you will find that the velocity is more plug like in compare to this is for turbulent flow. If you compare and contrast this side by side for laminar flow the velocity profile for laminar flow was like this. So, there is already a huge difference in terms of the nature of the velocity profile there is there is a large variation in the laminar flow, while in the turbulent flow you see that except very close to the walls the velocity is almost uniform.

So, you would expect even before calculating the momentum correction factor that the momentum the uniform flow assumption is more appropriate for turbulent flow were as compare to laminar flow. So, let us first calculate the average velocity the velocity is one over a integral $u \, dA$ is simply r is 0 to r theta is 0 to 2π , u which is this expression times $r \, r \, d\theta$. So, if you do this which I want do the algebra fully the answer is answer you will get is $\frac{49}{60} u_{\text{max}}$. So, the average velocity the first from the maximum velocity only by a factor of $\frac{49}{60}$ where as if you were comparing it to this is for turbulent, where well for laminar flow if you sees it is half.

So, it is $\frac{30}{60}$. So, you can see that the variation in the turbulent flow is much more the in laminar flow is much more because the average velocity is only the half the maximum velocity, while in a turbulent flow the average velocity is pretty much flow to the maximum velocity that is, because the flow is already uniform in the core of the pipe except very close to the walls, where the velocity is varying rapidly to 0 at the walls.

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The image shows handwritten notes on a whiteboard. On the left, a box contains $\beta = 1.020$ with the text "(TURBULENT FLOW)" next to it. An arrow points from the box to the text " β more close to 1". On the right, $\beta = \frac{4}{3}$ is written. Below these, the equation $\int \rho u \cdot \eta \, dA$ is shown with an arrow pointing to $\rho \beta V_{\text{av}}^2 A$. A note next to this equation says " ≈ 1 TURBULENT" and " $= \frac{4}{3}$ LAMINAR FLOW".

So, likewise you can calculate momentum correction factor and you will find that this is just using the procedure above and changing the velocity profile to this you will find. So, this I will leave it as an exercise for you. So, this is the momentum correction factor for turbulent flow. So, beta is very close to 1 it is more close to 1 the turbulent flow, when compare to laminar flow where beta was essentially $\frac{4}{3}$. So, in many practical problems if the flow is turbulent then it is good approximation to keep the momentum

correction factor as 1, because the errors incurred by replacing 1.02021 is not much whereas, for laminar flows it is always good to use the actual momentum correction factor β by $\frac{4}{3}$.

When you compute various forces using the integral momentum balance because indeed the differences are huge when you consider the laminar flow. So, whenever you have flow through pipes or channels it is always good to in order to compute the momentum flux term which is essentially this you want to be able to write this as let us take one component you want to be able to write this as ρv average square β times the area.

So, we can use β to be approximately 1 for turbulent flow and β is exactly equal to $\frac{4}{3}$ of laminar flow. So, this is the use of or this is the advantage of using momentum correction factor whenever you have flow through internal flow through conduits such as pipes and so on. So, there is one more, this nearly brings to the end of integral momentum balance we will of course, use momentum balance little later in the course as in when we need them in order to basically compare and contrast the results, that we get from integral momentum balance with a lets a energy balance which we do next.

So, we will of course, use this idea and concepts frequently, but this is the at this point I would like to end, but before I end I will also like to say that the choice of control volume is very very critical in the in the solution of many integral momentum balance problems. The control surface which is essentially the boundary that separates the control volume from the surroundings is an imaginary surface. So, the control surface in the no way tends to abstract the flow or anything it is just in your mind you are essentially trying to do a balance of various agents by through which momentum is coming and going out and it is changing.

So, the control surface is essentially an imaginary surface it can cut across rigid, you know things like flanges and so on. Like for an example in the previous problem, that we did in the last lecture we you choice the control surface which cut across the hand which is trying to resists the motion of the body. So, control surface is clearly an imaginary construct that helps us in the solution of various integral momentum balance problems. So, that is something that we have to keep in mind and whenever you have flow of jet that comes out of a nozzle, remember that we have to treat the pressure in the jet to be atmospheric to a very good approximation and if the jet is free and it expose to

atmosphere, then the pressure inside the jet is the same as the atmospheric pressure and also that the velocity is pretty much uniform in whenever you have flow of a jet that is a very very good assumption now.

Now, next comes the force part of the momentum balance, there are two components to forces one is the two contributions to the forces, one is the body force and there is the surface force. In this course we will usually treat gravity as the only body force now surfaces, as I told you can be due to pressure forces and viscous shear forces or due to reaction forces that come through the fact that your control surface is cutting across various solid surfaces.

So, you have to take a careful accounting of various reaction forces also like we saw in the previous example. Now viscous forces are usually neglected in a first approximation because, we do not have a very good knowledge of what the viscous force is are. So, typically the surface forces are thought to be a pressure force alone. Now, another important fact that we saw was that when we want compute the pressure it is only the gage pressure that usually matters because if your cv is completely surrounded by atmosphere and if there exists a uniform atmospheric pressure across all the boundaries of the c v that is across the control surface that uniform contribution to the pressure will not contribute to any net force on this cv through the control surface.

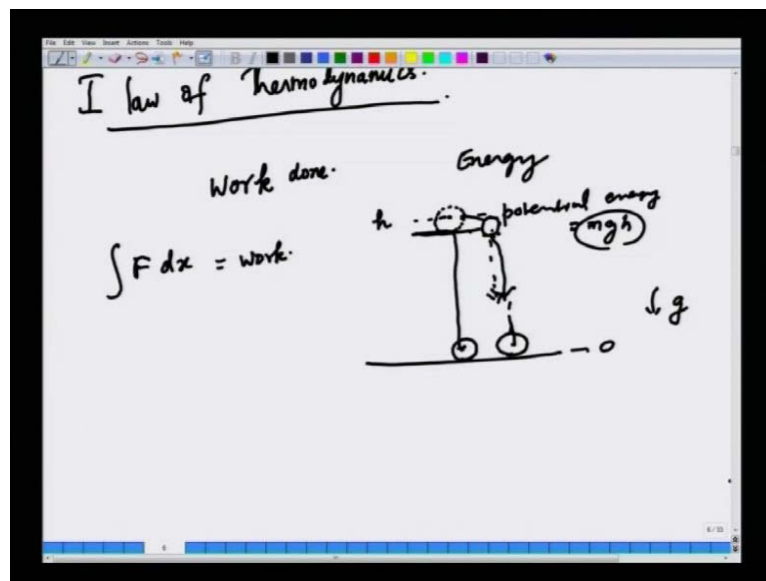
So, only the difference between the actual pressure and the atmospheric pressure will matter in any momentum balance problem and that is of course, the difference is of course, the gage pressure. So, usually the gage pressure is what is important in actually calculating physically important relevant variables such as forces. So, this nearly completes the integral momentum balance and we will soon start the discussion on energy balance.

So, now we will start our discussion on new topic we going to do a integral balance of energy and this topic is very very important, because the use of energy equation is very very frequent in many engineering fluid mechanic problems as we will see little as we see little later after we discuss the basics of the integral energy balance, but first is thing is first. First, we have to understand that what is the fundamental principle which gives us the integral energy balance, just to recapitulate we started doing integral balances using Reynolds transport theorem as applicable to a cv control volume.

So, so far we have done two types of integral balances, integral mass balance which is essentially a statement of law conservation of mass as applied to control volume and then we moved on to do the integral momentum balance which is essentially a restatement of Newtons second law of motion as applied to a control volume we said that Newtons second law of motion is typically is a valid only for a system not for a control volume. So, we had to use the vehicle of the Reynolds transport theorem to transform the time derivative with respect to system to time derivative with respect to control volume and this transformation entitle it result in a fact that there is momentum flux term.

So, the principles there are underlying principles which lead to the integral balances the law of conservation of mass or principle of conservation of mass which relate to the integral mass balance and the Newtons second law of motion as applied to a system which leads to integral momentum balance.

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Now, we have to first understand what is the underlying principle that we have given us integral energy balance? And the answer is it is the first law of thermo dynamics. So, many almost all branches of engineering will have a separate course in thermo dynamics where you learn the principles and applications of thermo dynamics in greater details. So, I will take only very few, I will do a very I will do a brief discussion on the meaning of the first law of thermo dynamics, as it is applicable for our course which is essentially

a course in fluid mechanics. But the principle of conservation of energy essentially comes from first law of dynamics.

So, it is an important that we understand, what is the meaning of first law of thermo dynamics? Now, there are two important principles or two important quantities, that play a big role in the thermo dynamics that is one is work; another is energy. So, work done the principle or the motion of the work is familiar to us from mechanics whenever a force acts over a distance that gives rise to work. So, we know this from mechanics for example, we could have a ball at the ground level let say gravity is acting below down.

So, suppose I apply a force over a distance and push this ball up. So, we have we have done work on this ball by moving this ball over a distance. So, that is work done on the ball, but what happens to the work done well, we have the work done has gone into increasing its gravitation potential energy, because it is now at a higher position h compare to its initial position which is 0. So, this potential energy gained is essentially $m g h$ and that must be work done in moving the ball from the ground level to a particular height h .

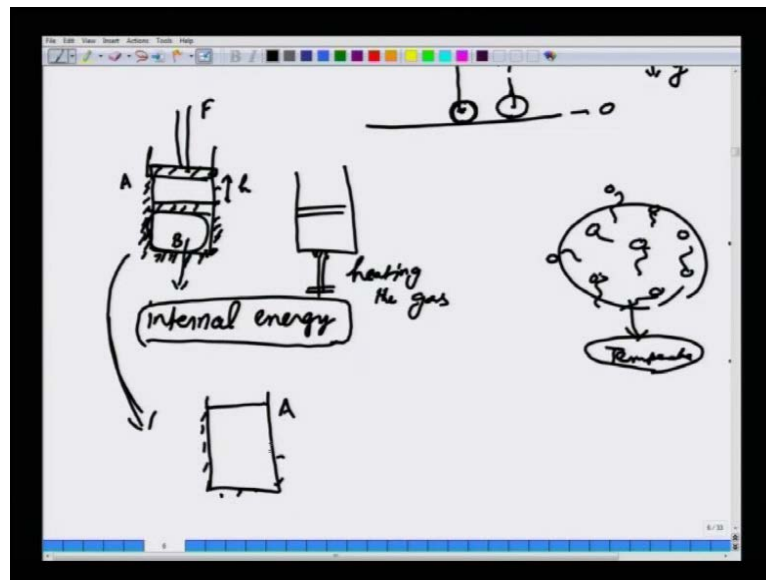
So, this motion of work done and equivalence of work and energy is very very familiar in mechanics. So, we can change the energy of the ball by doing work done by doing work on it. We can also take out work out of the ball and we can change energy for example, if I drop this ball suppose you place this ball on a plate. So, your moving this ball from ground level to up by doing work on it. So, it is increasing its potential energy. Now, if you if you drop the ball from this height of course, it is going to fall freely now this potential energy which was stored in the ball by the virtue of its height.

Now the moment you tip it up a table let us say from edge of the table it will convert to kinetic energy motion of the ball. So, there is energy kinetic energy of course, is energy by virtue of the motion of the ball. So, that is conversion of the potential energy which was there in the ball to kinetic energy and so when the ball comes and this kinetic energy can be converted to work by some mechanical device. So when the ball again comes to the ground its potential energy again becomes zero.

So, you can imagine transferring this kinetic energy, when the ball collides to some piston cylinder assembly by so that you can move a piston or something like that. So, that constitutes that amounts to extracting work from the motion of the ball and finally,

the ball again comes to rest at the ground level going back to its original state of zero potential energy. So, this is the motion of work energy in mechanics. Now the question is doing work only is it the only way to change energy of the system, the answer is no it is not the only way.

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Suppose, you have a fluid now I initially insulate it thermally, I have gas in it and I do work by compressing the gas by moving the piston over a distance h . I apply a force over a distance h . So, I am doing work explicitly. Now, what will happen to that work? Well so, this piston here originally has come here and you have done work by compressing the gas. Now let us call this state B and this state as state A the original state as A and this state as B. So, the gas is now in a compressed state and work has been done on the gas because clearly the piston as moved over a distance h .

So, work has been done by u under so that as resulted in a compress state of the gas. Now for a moment now I can also now you can ask the question, what happen to the work done in terms of energy. So, here I macroscopically if I take a ball of course, if you do work by lifting it up and placing it on the table, you are changing its potential energy. But, here it is not clear what energy we are changing because macroscopically the gas is stationary the on the average it is not flowing, it is in a static container it is in a container it is in static macroscopically speaking.

Because of course, now in the state B also once you have done work and once you have done given some time for the ball to settle in the state B also the ball for the gas to settle in the state b also the gas will be static. So, what as change from state one state A to state B? Now what has changed is the internal energy of the system? This is the new concept. It is not there in mechanics, it is the concept that comes on thermo dynamics.

Internal energy of a gas is fluid in general is that part of energy that stored in the molecular degrees of freedom of the fluid. So, remember that when we started out teaching fluid mechanics, when we started out discussing fluid mechanic in the very first few lecture as we said that we are going to take the continuum approach, where we are going to completely discard any molecular interpretation or any molecular description of fluid flow. So, but none the less you cannot in order to take into account in order to do a proper balance of energy.

The first law of thermo dynamics says that if you do a work on a completely thermally insulated system and the gas is initially stationary and final stationary. So, there is no kinetic energy of motion macroscopic of motion of the gas. So, what has happen to work done it must go into the molecular degree of freedom of the gas. So, essentially if you think of a very simplistic picture of the gas as a dilute system of spheres, which are randomly moving. Now by doing by compressing the gas you have increased the kinetic energy of motion of the molecules and from elementary kinetic theory, which we have must been familiar from physical chemistry courses, physics courses.

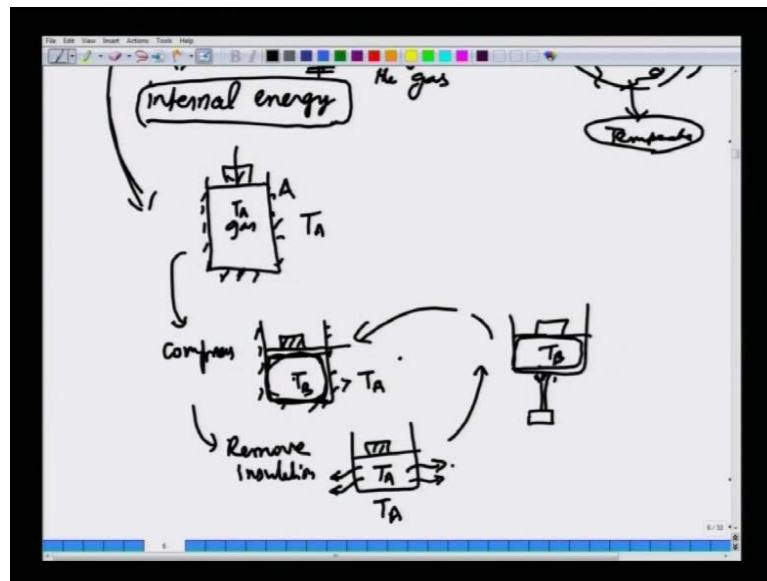
You know that this translation of kinetic energy of the gas is related to the temperature of the gas. So, essentially by compressing a gas and completely insulating the container while you compress only of done is to increase the internal energy the work that you have done, as gone on increase internal energy of the gas which minifies as increasing temperature of the gas. Because the temperature is directly related to the kinetic energy of molecules starts less kinetic energy of molecules. So, this is the new concept by doing work you cannot just change the macroscopic kinetic energy or potential energy of an object, but you can also change the internal energy of an object.

Now, the first of thermo dynamics tells you that this is also not very different from what mechanics is doing because, mechanics saying that by doing work you can change microscopic energy like microscopic kinetic energy of the ball or its potential energy.

But here we are merely saying that the work that you done are going to do something else that is change the internal energy of the gas. Now what is very important is that first law of thermo dynamic tells us that that is not the only way to change internal energy.

So, mechanics tells us that the change to in order to change the energy of the system we have to do or take out work from the system. But, now the first law of thermo dynamics tells you that the internal energy of the system can be change by merely heating the gas.

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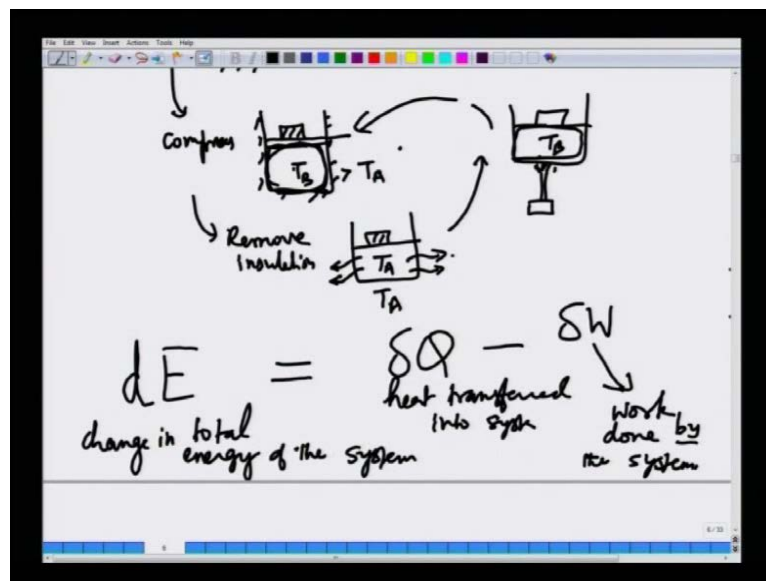
By heating that gas imagine, so let us do the following experiment. Initially you had state A and you have an insulated container state A gas, now you compress the gas to state B. I argued that the work done in the compression has gone on to increase the internal energy of the gas and therefore, its temperature now remove insulation and expose, but maintain. So, let say we are compressing a using maintain the position of the wait. So, that you are not allowing the piston to expand to. So, just remove the insulation. So, what will happen if you remove the insulation the since the gas in the cylinder inside is piston cylinder assembly is greater than has a higher temperature.

Then T_B is greater than, initially let us say the gas is at temperature T_A same as the ambient outside, but T_B is greater than T_A . So, there will be if once you remove the insulation then since, this is at higher temperature than this then energy will flow from inside the gas to outside. Now I will take the same system I can bring it back to state B by merely applying heat to this system. So, I take this system once I have allowed this

gas to reach by virtue of heat transfer it will lose heat. So, it will eventually reach the same temperature T_A . Now I can bring it back to the same system with temperature T_B by a completely different route.

I am just supplying heat by virtue of let us say Bunsen burner or something like that or an electric heater whatever it is. I can bring the system back to the same state state B by an entirely different route that is through heat transfer. So, the we have we said that by you can change the internal energy of the gas by compressing it and not allowing the heat escape the gas by insulating it, but we can also change internal energy of the gas by heating it. So, there are two difference different modes of changing the internal energy of a system one is by doing macroscopic work on the system that like compression and the other is by supplying heat to the system.

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So, this is the sum and substance of the first law thermo dynamics and we can mathematically express this as follows. The change in total energy of the system like gas in a piston cylinder assembly is equal to the amount of heat transferred into the system minus the amount of work done by the system. So, in thermo dynamics in engineering thermo dynamics at least this work always involves in sign convention. So, one has to understand very clearly whether the sign convention is work done by the system on the surroundings is positive or whether it is a vice versa.

In engineering thermo dynamics it is always taken that the work done by the system on the outside that surroundings is positive while work done on the system by the surroundings is negative. So, that is why we have a negative sign because if the system is doing work on the surroundings it is work is done by the system on the surroundings. That means that will lead to a decrease in energy of the system. So, this is the mathematical representation of the first law of thermo dynamics. The change in total energy of the system is equal to the amount of the heat that is transferred to the system minus the amount of work that is done by the system. We will stop here at this point and we will continue from here in the next lecture.