Fluid Mechanics

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Lecture No. # 14

Welcome to this lecture number 14 on the N P T E L course for fluid mechanics for undergraduate students in chemical engineering. So, let us briefly review, what we done in the previous lecture. We started doing integral momentum balance. Especially, we try to apply the linear momentum balance to a control volume. So, let me briefly outline the key steps before from which we arrive this integral momentum balance.

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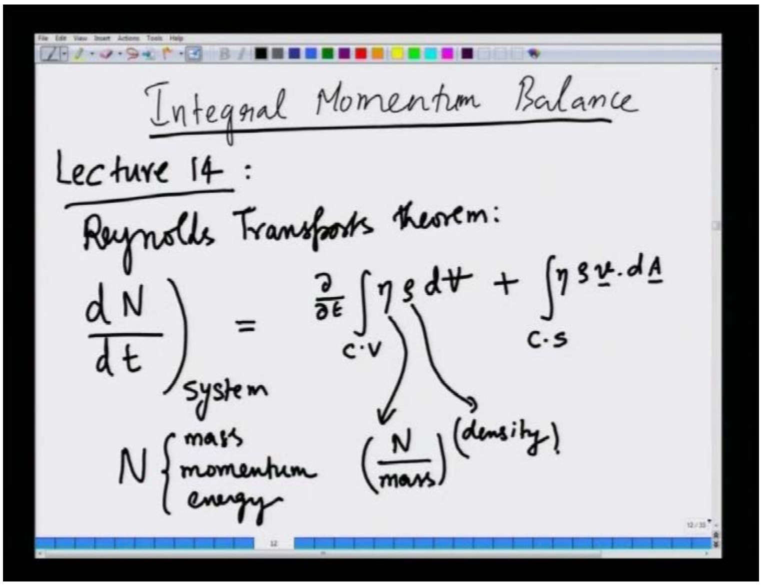

The starting point, if you remember of all this was there Reynolds transport theorem. The Reynolds transport theorem relates the time derivative of a quantity like mass, momentum or energy, for a system to the time derivative of any of these quantities that corresponds to a control volume. A system corresponds to, as you will recall a fixed or identifiable piece of mass, whereas a control volume is a stationary piece of volume that you have constructed artificially to describe or analyze a problem.

So, let me first write down the Reynolds transport theorem. So, the rate of change of quantity like mass, momentum or energy for a system, where capital N can be any one of the three; mass, momentum or energy is equal to the rate of change of the same quantity, time date of change of the same quantity over the c v, where we will use this notation, where eta is N the quantity per unit mass. So, if eta is mass, the, if N is mass then eta is 1, if n is momentum, then eta is just v. So, rho is the density of the fluid, mass per unit volume; and integrated over the whole volume in the control volume c v.

But, this is not all because, we realize that there can also be flexes at the surfaces which bring in or take out material and thereby, also bring in or take out mass momentum or energy. So, v dot dA, this is the turn that corresponds to the inlet, the entry of momentum or exit of momentum by virtual the flow into or out of the control volume through the control surfaces. So, this was the Reynolds transport theorem.

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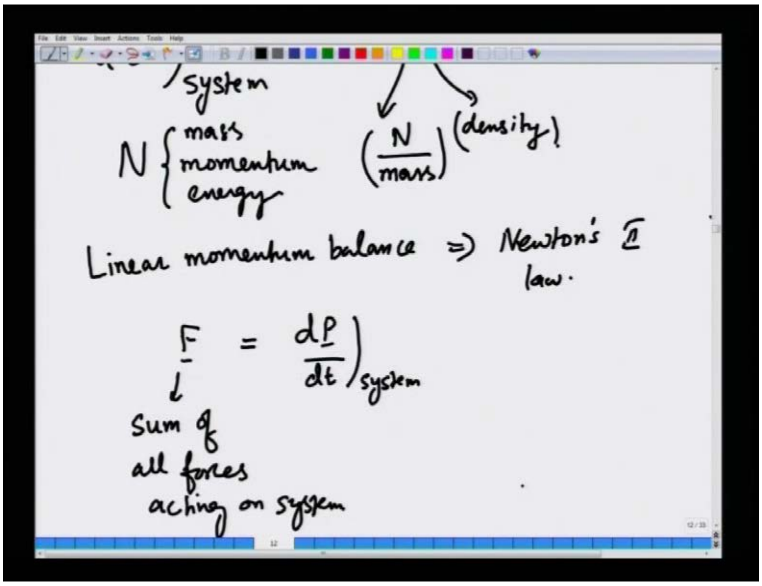

Now, the linear momentum balance is essentially a statement of, is essentially the balance, is essentially the Newton’s second law of motion applied to the system. The Newton’s second law says that, the rate of change of momentum in the system is equal to the sum of all external forces acting on the system. Now, the Newton’s second law cannot be directly apply to a control volume because, Newton’s second law is applicable only for an identifiable piece of matter, which is what the system is. Whereas a control volume is not an identifiable piece of matter because, fluid can come in and go out and therefore, it clearly cannot be called as a system.

So, Newton second law cannot be applied to the control volume and it is in fact applicable only to the system. But as I told you in the last few lectures, it is always easier for us to carry out or integral balances in the control volume frame work, because in most practical applications, we are concern only with whatever is happening with in and identifiable piece of volume.

We are not interested in following, the same set of mass, ele, mass points in a fluid, because the fluid will come and go but, all we are interested in is whatever is happening in a control volume. For example, in applications you may know all applications by involved a pump or a compressor or a turbine. In which case, we are interested in calculating the power requirement for a pump. So, our c v will essentially involve the pump and we are not clearly interested in following the same set of fluid masses as they flow through the pump.

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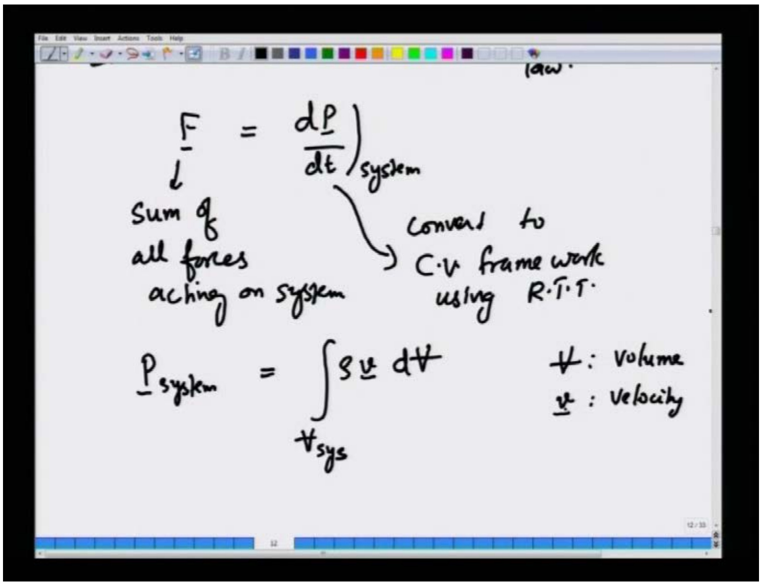

So, the Reynolds transport theorem is very critical in transforming; this time derivative to the control volume frame work. You have to convert this, to control volume frame work using Reynolds transport theorem. And in order to do that, first we have to say what is P system, the momentum present in the system, is the volume of the system, integral of the volume of the system, times rho times velocity d v. So, in our lectures v with a cross is denoted for volume, whereas v is the symbol it does not for velocity and since velocity is a vector and denoting with an underscore.

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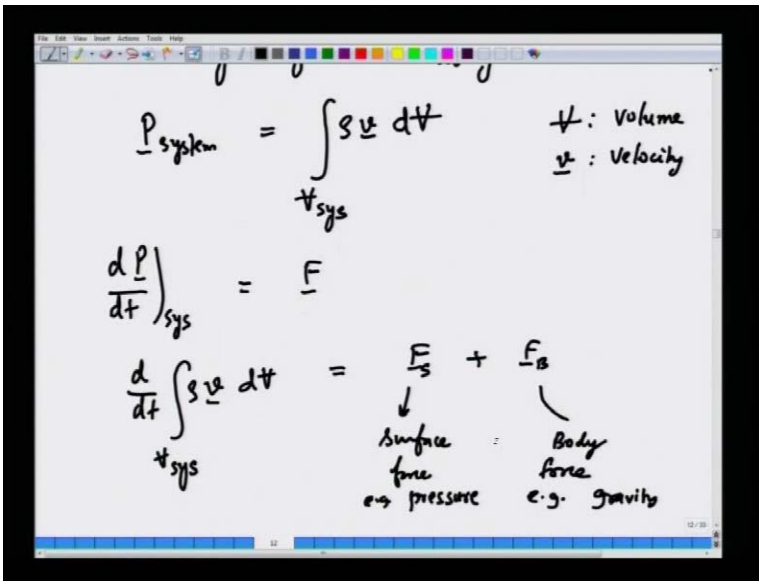

Now, so, and first of all the Newton's second law says d P d t for the system is sum of all forces and we know, what is P for a system, p is essentially rho v d v for the system, v system is equal to sum of all forces acting on the system. We discuss in the last lecture that, the forces can be divided into surface forces, which act only on the surface of the system and body force which act through the entire volume of the system. So, the body force typical example is the gravitational force. Surface force example is pressure force and other viscous stresses that act only on the surface of the control surface of the system.

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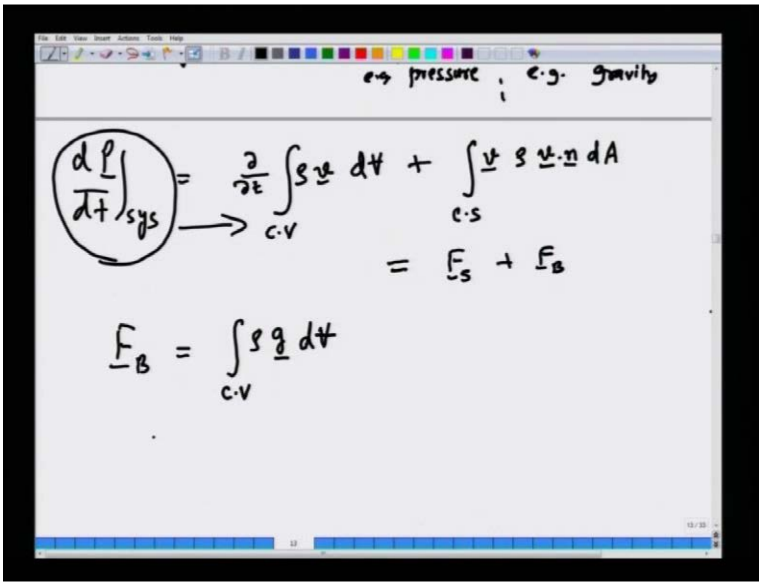

So, we now, use the Reynolds transport theorem, to write d P d t of the system to be d d t over the control volume, rho v d v this is the rate of change of momentum present in the control volume. The control volume is not changing with time, plus the flex of momentum in an out of the control volume through the control surfaces rho v dot n dA. This is the time rate of change of momentum of the system written in terms of the control volume variables. Which is what is most convenient when we try to do problems a using integral momentum balance.

Now, this is equal to therefore, the sum of forces namely the surface force and the body force. So, this is all there is to it, when you want to use the momentum balance. Now, we have to understand, what are the various forces, that are acting on that can potentially act on the control volume in various applications. The usually forces as I have been telling you the body forces typically that we will encounter is due to gravity so we have to simply use rho times the acceleration due to gravity vector g.

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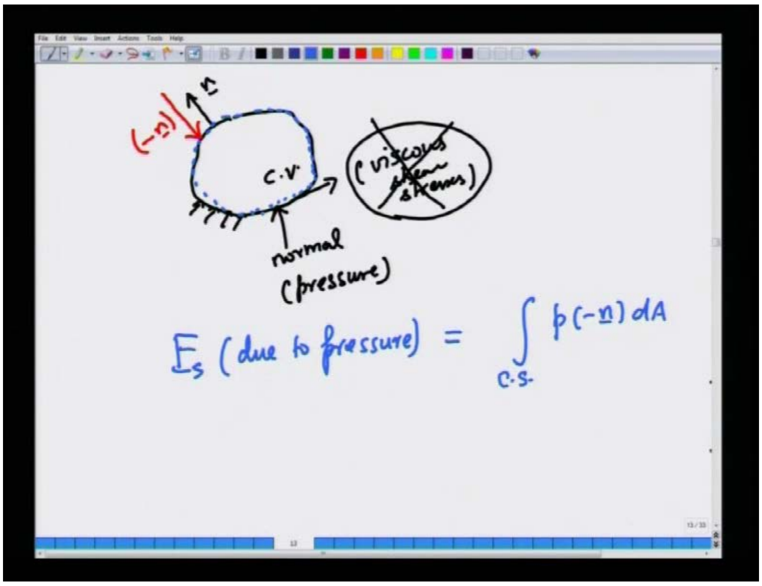

Integrated over the control volume and the surface forces are due to pressure. Suppose, you have a control volume or there can be forces that are exerted on the surfaces and these forces can have a normal component as well as a tangential component. The normal component is usually due to the pressure. Whereas, the tangential component can be due to viscous shear stresses, these both constitute the surface force. It depends on the act on the surface but the direction in which they act is different for normal forces. They act in the direction along the unit normal. Whereas, in the tangential forces, they act in the direction perpendicular to the unit normal; that is the only difference. Usually, the viscous shear stresses that act on a control surface, although, they are important in many applications, they are extremely difficult to know appropriately. And in order to have a very good knowledge of viscous shear stresses, we will see that little later in the course that we have to use differential balances in detail, to co compute or to have an idea of what are the viscous shear stresses acting or for example, on a wall of a pipe and so on.

So, usually, because it is difficult to know a priory unless you have detailed experimental data or you solved the differential balances which, we will see later. This is often neglected at this point, the viscous shear stresses so if you have a control volume usually, the only force that is taken into account is the pressure force. The only surface force that is taken into account, I am sorry, is the pressure, force and the pressure as you have seen in the first few lectures, on hydro statics, acts in the direction of minus sign.

Pressure is a scalar number, it is a number, and it is not a vector. So, pressure is just a scalar but, it acts always in the direction opposite to the unit normal, outward normal, of a surface. Pressure is compressive in nature. So, it acts, it tends to a compress the volume element. So, it acts in the direction opposite to the unit normal. So, if you want to write an expression for the pressure surface force due to pressure all you have to do is, to say that integral over the control surface p time minus n dA. Because, pressure is just a number, it is a scalar number and it is the magnitude of the force that is acting compressive to it that tends to compress a given volume element.

So, we have to put in the direction and we since it is compressive it acts in the direction of minus sign so we have to expressively put minus sign while calculating the surface force. And you have to since; the pressure can in principle vary at various points along the control surface. We have to integrate the pressure over the entire control surface to get the surface force due to the pressure forces.

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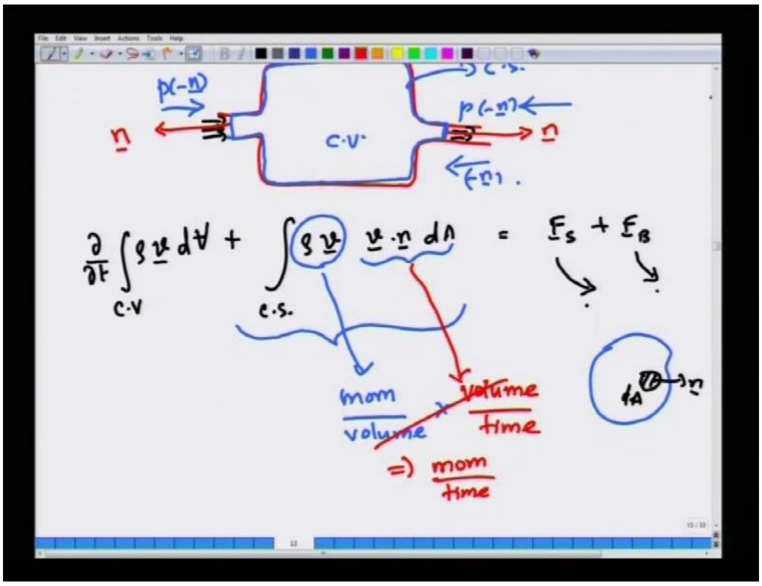

The other thing, that we reminded ourselves in the last lecture is the following that; suppose, you have a very simple problem you have a fluid in coming in like this. So, let us let the blue line be the control volume. This is for example, a container in which fluid is coming in and going out, is a very illustrative simple example. The blue line is the c s; whatever is inside the c v and let us say fluid is coming in like this and going out like this. Now, the pressure so the unit outward normal here is like this, the unit outward normal here is like this. So, here you may all agree that the pressure is in the direction opposite to the unit normal. So, you will argue that the pressure acts in this direction p minus n is in this direction at the inlet.

But, what is the important for you to remember or understand is that pressure at the exit or at controls of is of the exit, is also in the direction p minus n in the direction opposite to n. Just because of fluid is coming out it, does not mean that the force on this control surface is in the plus n direction. The key thing to understand here is that the force on this surface is exerted by the fluid outside. Which tense to a compress the fluid present inside and clearly that is obviously in the minus n direction. So, this is the common mistake that one can make based on incomplete intuition of the problem. That the pressure at the exit of a control surface is in the direction of the unit normal, it is not just as in the inlet, the pressure acts in the direction opposite to the unit normal tense to compress this control surface.

Likewise, the pressure at the exit also tense to con compress a control surface, also acts in the direction of minus n, this is a common error that one can potentially make. So, it is good to be aware of it. So, another thing to understand is the so let me just write down for your convenience the momentum balance again. So, after using the Reynolds transport theorem, c v rho v d v plus integral over c s rho v v dot n dA is f s plus f b. so, we have understood what is f b and you are said that f s is usually due to pressure. Viscous stresses although they are important in many cases at the level of integral balances at times. It is not easy to obtain detail information about the viscous shear stresses in a flow problem.

And therefore, one often neglects it a because of lack of information. Of course, what has to go back to the results and see whether the results make a physically consists and sense after neglecting the viscous stresses. So, we will have opportunity opportunities to discuss this later, when we do various examples here as well as when we discuss differential momentum balances and is to understand fluid flow. But, right now, we know at a simplistic level, what are the body forces, what is the surface forces, how to compute them?

Now, another important thing to understand is how to calculate this flex integral, the rho v is the momentum density and this is momentum by temper unit volume, times d dot n dA is the differential volumetric flow rate on; suppose, you have a control volume and you have a tiny patch dA and this is the unit outward normal. What is the volume flex, volume flowing per unit time out of this differential control volume? This is the v dot n dA. so; let me write it in red color, this is volume per time. So, this gives you the rate at which momentum in comes inside to the c v or goes outside of the c v by virtue of inflow and outflow through the control surfaces to the c v.

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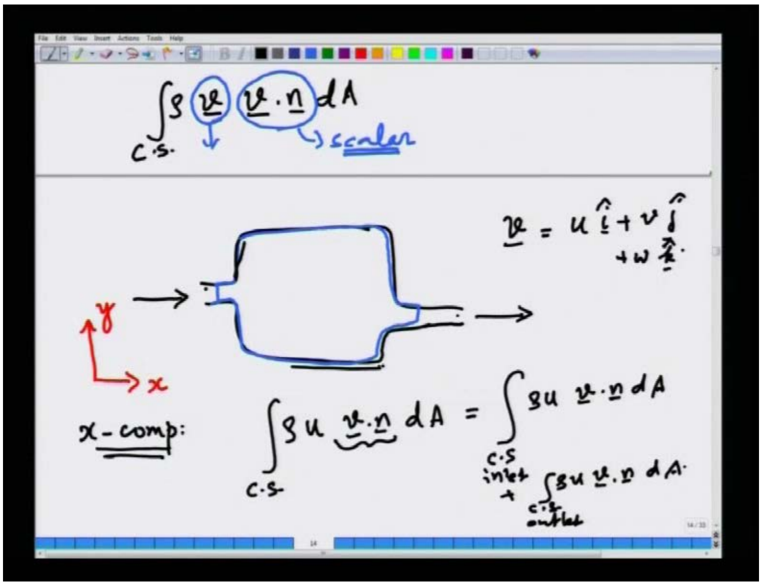

Now, what is important to understand, here is how to calculate again this v dot n integral? Remember that in the simple example that we just pointed out, let me just instead of drawn this and let me go back there. So, let me point out that the velocity inside is coming like this, the velocity vector obviously since it is coming in it is in the direction opposite to n, while going out it is in the direction of n. So, v dot n is negative for inlets while v dot n is positive by definition at an outlet. So, this is a very important to understand, that whenever you compute the surface flex term the v dot n term will always be positive at outlets and negative at inlets regardless of the co-ordinate system you choose.

Because, in many problems, you may choose a co-ordinate system, as per the convenience of as it is convenient to analyze a problem. So, for example, let me point out again, this simple example, suppose you have a container in which fluid is coming and flowing out going out. And let us choose some control surface like this, which demarcates the control volume from these surroundings. So, fluid is coming in, and going out, you may choose a co-ordinate system like this you may say this is x this is y. So, let me write down, the flex term like this rho v v dot n, this is the term that we are trying to understand carefully over the control surfaces.

Now, these are, there are, there is no flow in this part of the control surface, this part of the control surface. So, the flow is only through the entry and exit. Now, if you have the co-ordinate system like this. Now, I want to write the x component of the term, this is the vector equation remember. So, we have to actually whenever you solve a physical problem you have to actually solve it component wise. Now, if you want to write the x component, now remember the vector velocity is written as, u times i plus v times j plus w time's k. Where i, j, k are the unit vectors in the x y and z direction, u v w are the scalar components of the velocity, vector in those 3 directions.

Now we have to write the x component of this equation. In this you have first understand that this term v dot n is a scalar. So, you cannot make a component out of a scalar, the only vector available to u is v. So, the x component is obtained by just saying it is, rho integral, sorry, rho u v dot n dA, now v dot n at the inlet. So, this is over all the control surfaces and this is over the inlet rho u v dot n plus over outlet dA. Now, here u is the x component of the vector v dot n. Even though the fluid is coming like this, and you have rightly picked the x component to be like this, v dot n is still negative at the inlet, because the fluid is coming in like this. So, n is pointing in this direction, v is pointing in this direction. So, v dot n is always negative.

So, while at the outlet it is always positive. Now, in somebody else may choose to work with the problem in a different set of co-ordinate axis. Where x is pointing like, this and y is pointing like this, it just the opposite, x is pointing in the opposite direction. Even so, whether v dot n is positive or not, even so, for example, if you want analyze a problem like that, with the new co-ordinate system.

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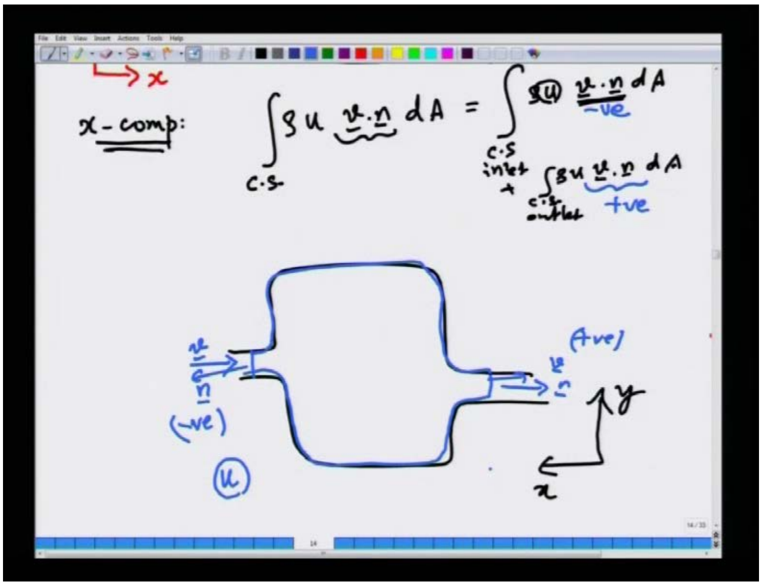

For example, this is fair. So, I am drawing the control surface again, same control surface, all you are changing is the, sorry, we are using the co-ordinate system like this. Somebody else may choose to work with this it is co-ordinate system for what ever reason. Now, what is important is velocity is coming in the minus x direction it is going out in the minus x direction, according to this problem. But, the unit, all you have to understand is that the unit outward normal and the velocity vectors are pointing in the opposite direction in the inlet. And they are pointing in the same direction in the outlet. So, v dot n is negative at the inlet and positive at the outlet.

Regardless of what co-ordinate system you choose, that is because v dot n is a dot products a dot product is a scalar and therefore, it is independent of the choice of the co-ordinate system that you choose to work it. While the component u, if you compute for this problem the numbers will come out to be negative of what you will had for the earlier problem. Because, here your excess pointing in this direction and flow is like this so obviously use in then negative direction x direction according to this co-ordinate system. Whereas in the earlier co-ordinate system which must be positive dire x direction. Now, that is of course, completely equivalent because, you are just using a different set of co-ordinate system. But, that calculation of v dot n is invariant or independent of the co-ordinate system, that is was something that we have to understand very carefully.

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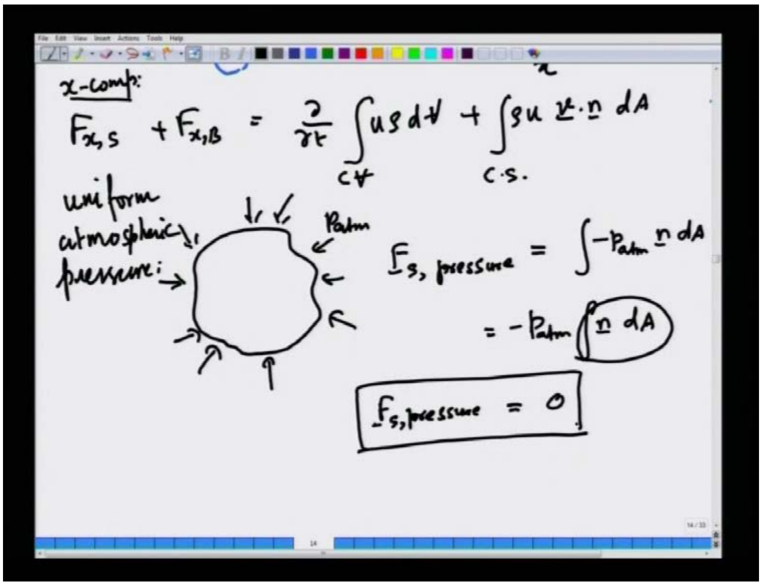

Now, another thing pointed out in the last lecture is that; so a since it is a vector equation, you may have to write the full scalar component I am just giving you an example of the x component, f x surface plus f x body is equal to d d t over the c v u rho d v plus integral over c s rho u v dot n A. This is all a that you have to do for the x component. Similarly, you can write down equation for the y and z components, which we have done in the last lecture but I not repeat that it is valise a similar. Now, another thing we pointed out is that, suppose, you have a closed surface and there is uniform pressure acting on the close surface. Everywhere on the surface there is a uniform pressure. For example, you may have a system or a control volume which is uniformly exposed to atmospheric pressure.

Now, in such a case, the contribution to surface due to pressure if it is uniform, let us say there is a uniform atmospheric pressure, f s. Pressure will then be integral minus p atmosphere n dA. Now, p is constant so, I can pull it out n dA. Now, if you have a close surface, and if you just evaluate n dA, for close surface just by symmetry this has to be 0. So, the surface force due to a uniform pressure force, acting on each and every point on the control surface is 0. In many a problems, you may not just have the atmospheric pressure; you may have something else in addition to atmospheric pressure.

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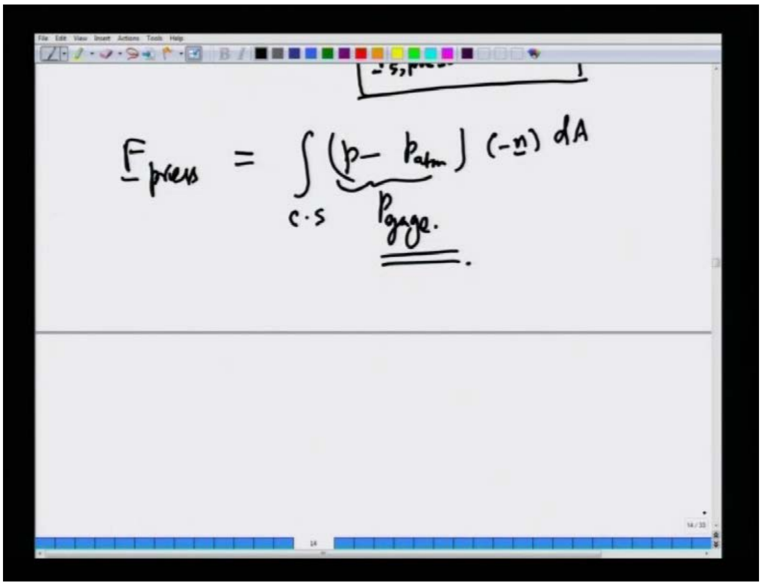

So, what will matter in such problems, if you have a let us say pump that is uniformly exposed to the atmosphere. The only thing that will matter is the difference between the actual pressures on atmospheric pressure, because the net effect of the uniform atmospheric pressure on the entire control surface is exactly 0. Because, all the components will x identically cancel off. So, when you want to calculate the pressure force, all that will usually matter is the difference between the actual pressure and the atmospheric pressure. And you have to do it with, still it is in the minus n direction, dA this is nothing but the gage pressure.

This is something that we have defined, the difference between the atmospheric pressure, the actual pressure on the atmospheric pressure if it is greater than 0 is called the gage pressure. While if the actual pressure is less than atmospheric pressure, then we define a vacuum pressure which is essentially p atmosphere minus the pressure. So, I just want to keep the number to be positive, if you calculate the gage pressure, if a p is less than p atmosphere you will get a negative number in order to keep it positive, it define the vacuum pressure as p atmosphere minus p, if p is less than p atmosphere.

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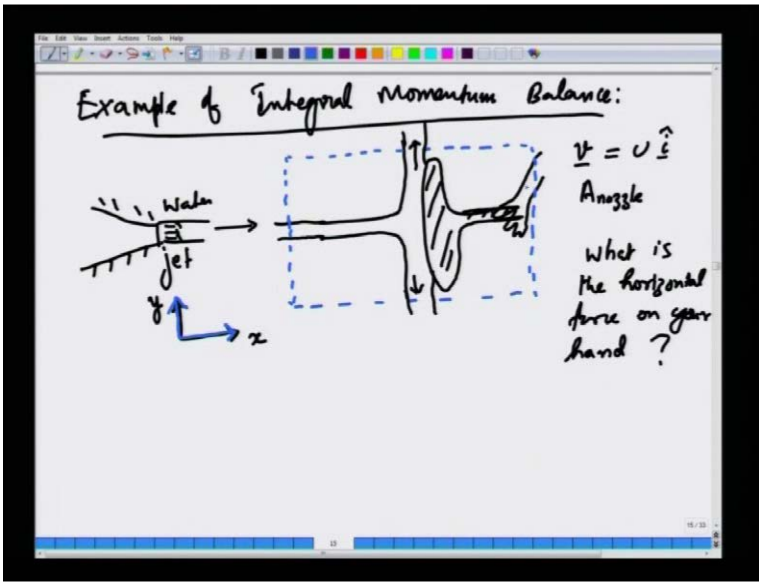

So, this is something that we get in the last case as well. Now, let us try to illustrate all of these using a simple example. And we example is the following example of integral momentum balance. Imagine, you have a solid surface, of which I am drawing just a cross section a solid surface like this, and far away there is a nozzle, through which from which fluid is ejecting. There is a eject of water, that is coming out and it impinges on the surface which is at a distance. This jet of order comes and flows like this. Now, the question that and we are given what is the velocity? The velocity is in some, we always want to put a co-ordinate system.

So, let us call direction of the flow of jet as x and the direction perpendicular as y, and we are given some number for velocity v. It is sorry u times i, and we are given what is the area of the nozzle, the cross section area of the nozzle through which water is exiting. Now the question that we want to know is that suppose you want to keep this solid surface stationary, this is a solid. Now, the fluid continues temping on it. And if there is nothing to keep the solid surface in place in stationary, then the solid surface of course, will accelerate. Because, there is a net force due to the momentum transfer conveyed by the fluid to the solid surface, because, of impingement.

But, we want the solid to be stationary. So, imagine that you are exerting a reaction force by holding the solid surface using your hand. So, you are holding the solid surface using your hand and the jet impinges on the solid surface. So, the question is what is the horizontal force on your hand? (No audio from 27:15 to 27:24) This is the question that we want to answer using the application of integral momentum balance. So, first is as I have told you is choose, in the last lecture we pointed, out some tips or useful guidelines that will help us in solving a problems, that involve a integral momentum balances.

Now, the first is the choice of a co-ordinate system and here it is pretty obvious what we have to do. We can choose a co-ordinate system, Cartesian co-ordinate system with one vector pointing along the direction of the flow, one unit vector pointing along the direction of the flow, which I will call x. And another unit vector when the direction perpendicular to the flow which I will call y. And things are ingredient in the third direction that is z direction. So, we will not worry about the third direction, to know is the first c v l use is the following. So, let us draw the jet little further up.

The first c v l use is this dotted n. (No audio from 28:26 to 28:32). So, just to make the idea clear to you, let us assume that, the jet flows like this, impinges and flows like that. Now, let us choose the c v such that it cuts across the water jet at the somewhat far away from the impingement.

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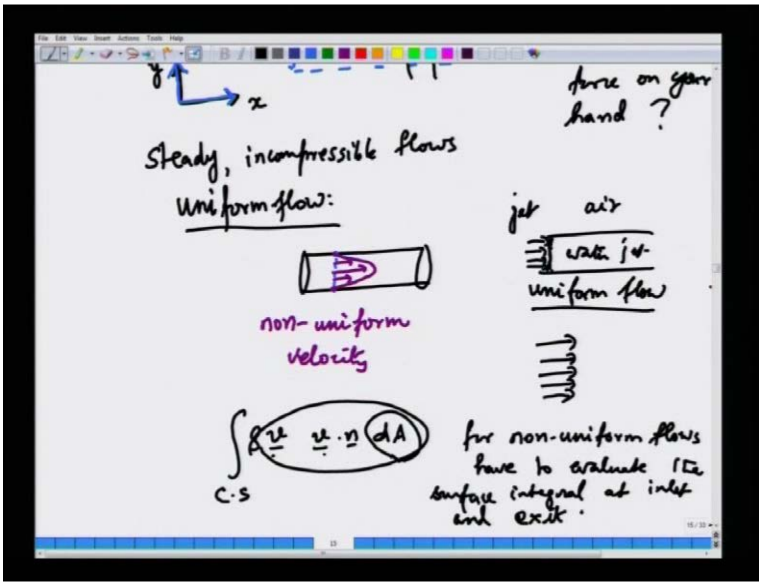

And it is also cut across cuts across, your hand which is holding the solid surface in place. Now, we are making some standard assumptions, steady flow and incompressible flow. These are all assumptions that, the flow studied at is nothing is depended on time at a given point in space as well as suppose incompressible which means a density is a constant. And we will also assume that the flow is uniform. What is an uniform flow? Whenever you have a flow, let us say inside a pipe here you have a jet afforter which is surrounded by air, this is water. Question is the velocity at each and every point or across the cross sections of such a flow same or not? If it is same then the velocity vectors will be like this such a flow is called a uniform flow. And for, what I just that exit from a nozzle to a free surface, it is a very good approximation to treat the flow to be uniform.

Because, in reality also the velocity vectors are constant, they are independent of the cross section. So, this is an example of a uniform flow, the velocity vector is independent of the cross section. Whereas, whenever, you have flow in a channel or a pipe, which are bounded by solid surfaces, we will see in detail a little later at when we discuss differential balances. That whenever, you have such a flow, the fluid velocity has to go to 0, at the walls.

And that is a pressure difference, it is driving the flow. So, the velocity is non uniform that is at (No audio from 30:54 to 31:01) the reason why, whether you have a uniform flow or non uniform flow is important; is because, when you want to evaluate the flex term rho v v dot n dA, this is an area over the inlet and outlet of the control surfaces. Now, if v is a function of cross section area then (No audio from 31:25 to 31:31) you have to evaluate the area integral. For non uniform flows have to evaluate the surface integral at inlet and exit at the outlets.

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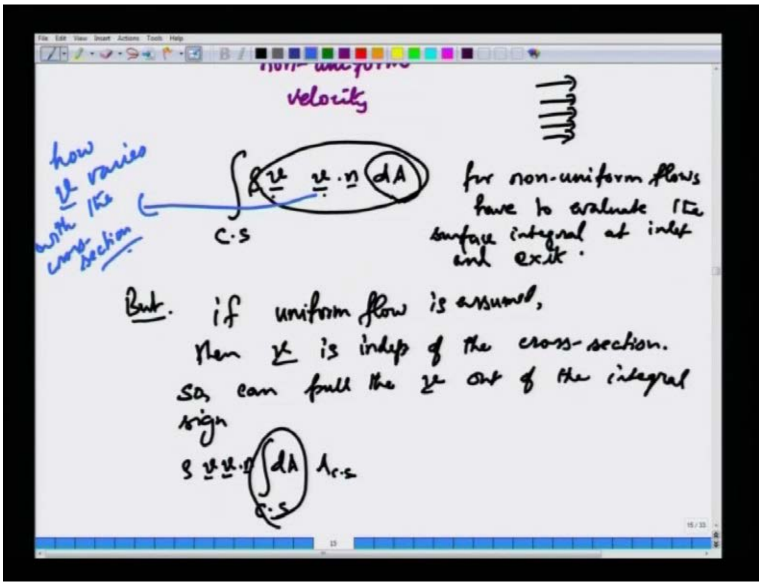

But, if uniform flow is assumed, then u at the velocity vector v is independent of the cross section, so, can pull the velocity vector out of the integral sign. So, you have rho v v dot n dA, so integral over dA is simply area of the cross section or the control surface. So, things become much simpler, when flow is uniform. With the flow is not uniform, we need to have a knowledge of how velocity varies with the cross section in order for us to be able to compute this area or surface integral. We will come to that a little while later, there are for simple cases in flow through channels and pipes.

Once you know, the cross section of the channel, there are some standard results which come from differential balances which enable us to calculate these integrals in a straight forward way. But nonetheless, it is slightly more tedious then when you have an uniform flow, because in an uniform flow, the velocity vectors are parallel to each other, and they are constant, they are independent of the cross section. Therefore, you do not have to worry about evaluating any integral. So, in this example, we will assume the flow to be uniform.

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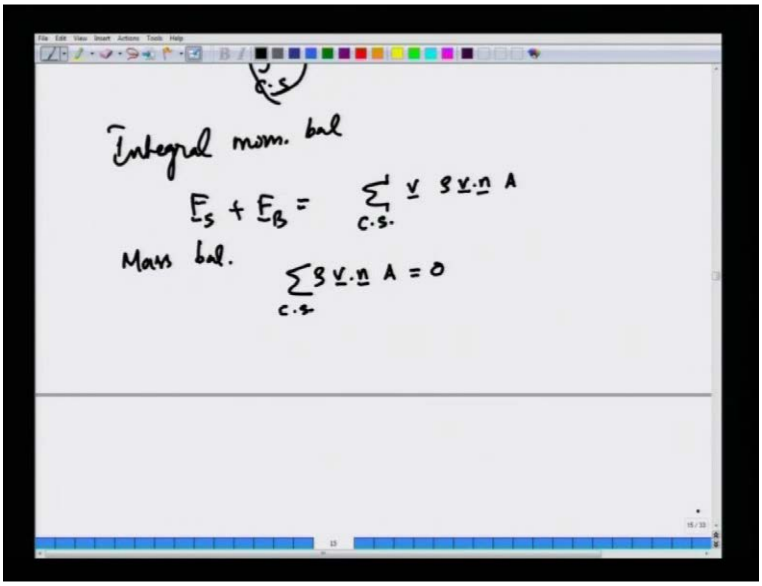

When the flow is uniform, when we use the integral momentum balance, f s plus f b is the summation over all the cross sections, v rho v dot n times A, this is the momentum balance.

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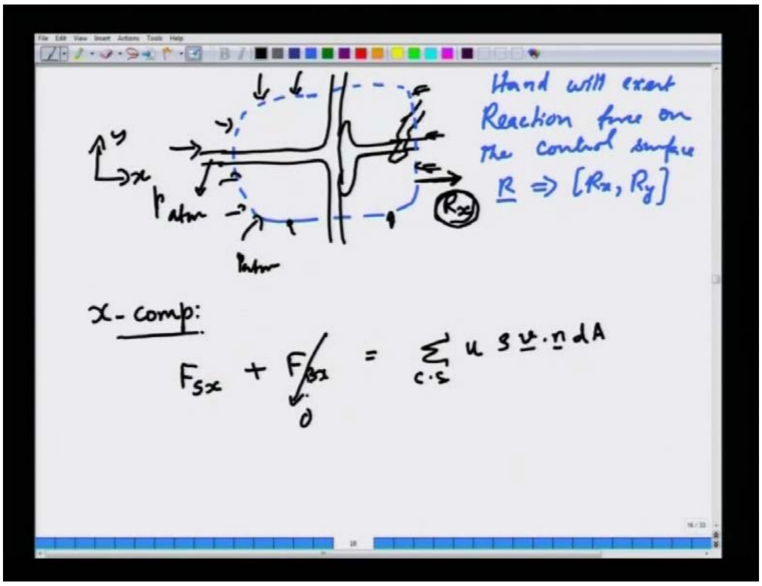

The mass balance will simply reduce to summation over c s, rho v dot n A is 0. Now, the thing that is important for us to understand, is that suppose you are, let us go back to the problem you have the surface it is a trying to fold by your hand and you have the c v is cutting across your hand. Now, since the c v cuts across your hand, your hand will have to exert a reaction force, hand will exert a reaction force on the control surface. In order to keep the solid surface from moving your hand is of course, trying to resist it by exerting the force. And how is that force felt by the c v? It is it is felt via the control surface because the control surface is cutting across your hand.

On the control surface and let us call that reaction force r with components r x and r y. Now, the question that is being asked that the question that was asked to us to compute is the force that is exerted by the c v on your hand which is exactly the equal and opposite of the reaction force exerted by the hand on the c v by Newton’s third law of motion. So, what we have to compute eventually is r x and r y and the negative of that will be the force exerted by the c v on your hand is they adjust equal and opposite. Now, let us go further into the problem, now you have a free jet, it is coming and exiting in into an out of the c v.

Now, whenever you have a free jet the pressure of the jet is atmospheric and the pressure outside is also atmospheric. So, there is uniform atmospheric pressure everywhere acting on your c v. So, if you have uniform atmospheric pressure then of course, it will not contribute anything. Now, let us call the force, the component x component of the force exerted by the hand on the c v as r x. Because, whether r x is in the direction along the plus x and let us put a co-ordinate system x and y like this, and flow is in the plus x direction. So, whether r x is positive or negative come out of the answer.

So, we have to understand that the reaction force, that is exerted by the hand on the surface is r x, But, when we when and we want to compute r x and the negative of that is a force exerted by the c v on the hand. But, right now let us just keep r x to be acting in the r x is a component along the x direction. Whether it is acting in the direction of x or minus x, will come out of the problem that is all I am saying is that r x is an algebraic quantity. It has a sign, and it will come out naturally from our calculation and it will agree with the physically expected result also as we will see shortly. Now, let us ester do the x component of the momentum balance. So, the sum of surface and body forces along the x direction is equal to summation over c s, u rho v dot n dA. Now, the body force is not, there is no body force in the x direction that is 0.

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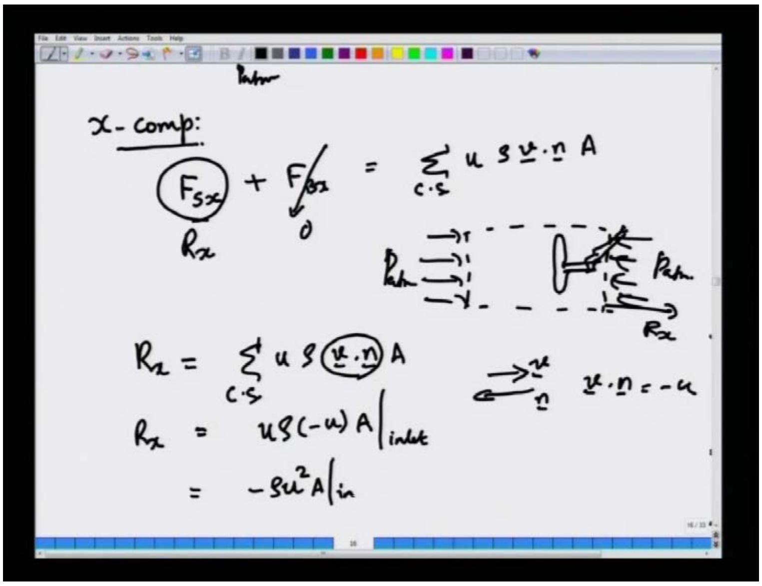

Now, what is f x s x, what are the surface force is acting on the control surface, in the x direction? We have already said that the pressure force, uniform pressure force that acts on the entire controls surface that will act to 0 because it is uniform everywhere. So, one way to see it is very simple, that suppose you have the control surface to be like this there is p atmosphere here and there is p atmosphere here in both sides. And of course, you have this surface and your hand is holding the surface, but there is uniform p atmosphere on both sides, which will balance out and that will not contribute to the surface force. The only thing that will contribute is the reaction force exerted by your hand on the c v through the control surface.

Because the reaction force is that there is a reaction force that is exerted by your hand on the c v is felt only through the control surface. Because it is a surface force and since, the control surface cuts across your hand, that force will be transmitted as r x in the momentum balance. So, r x is equal to summation over the control surface u rho v dot n dA, sorry, times A, because you have already integrated it down it is uniform flex approximation. Now, along the x direction there is only inlet of momentum there is no exit of momentum. So, along the inlet you have r x is u remains u v dot n v is in the direction of plus x, whereas unit outward normal is in the direction of minus x v dot n is minus u, u times rho times minus u times A at the inlet.

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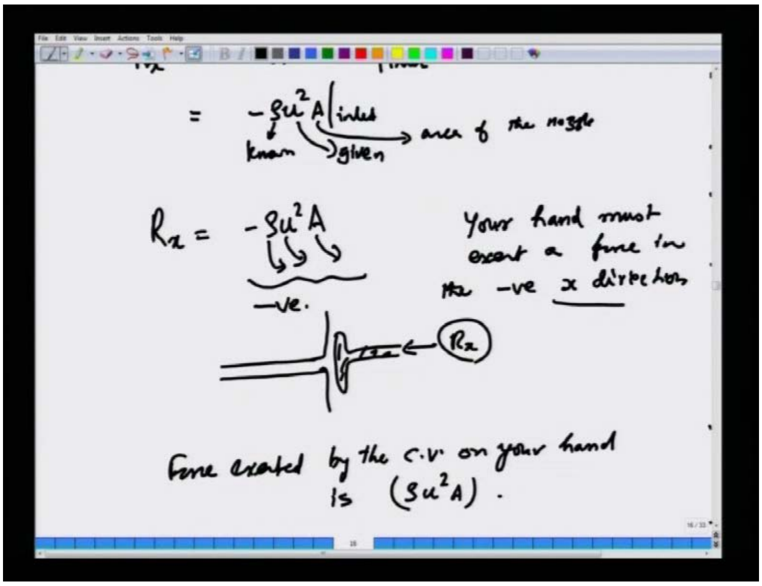

So, this is equal to minus rho u square A at the inlet. So, r x since we know the density of the fluid is known velocity is given the average velocity, that the jet is impinging is given. And the area of the jet is the area of the nozzle, this is an assumption that the, jet is not pinning down as it impinges, which is an assumption so that is a reasonably good assumption to make. So, r x is minus rho u square A, u square is positive quantity, use the x component of velocity, use u squared is positive, rho positive, A is positive. So, r x is negative that is your hand must exert a force in the negative x direction.

(No audio from 41:20 to 41:26) Which completely agrees with our intuition because, the fluid is, jet is impinging on the surface in the plus x direction. In order for you to keep, the solid surface stationary you have should of course, resisted with the force in the minus x direction so that is clear. So, the force exerted by the c v on your hand is negative of r x which would be rho u square, that is the answer. So, this is a very simple example of the application of integral momentum balance.

We learnt couple of things here, that whenever you have a free jet, it is important, it is very good assumption to make a it is to consider the jet to have uniform velocity. And the pressure inside the jet is also atmospheric, whenever the jet is a free that is it is exposed to the atmosphere outside. The pressure inside the jet also becomes atmospheric it is a good assumption. So, once you make these assumptions it is very easy to use the integral momentum balance to calculate the force.

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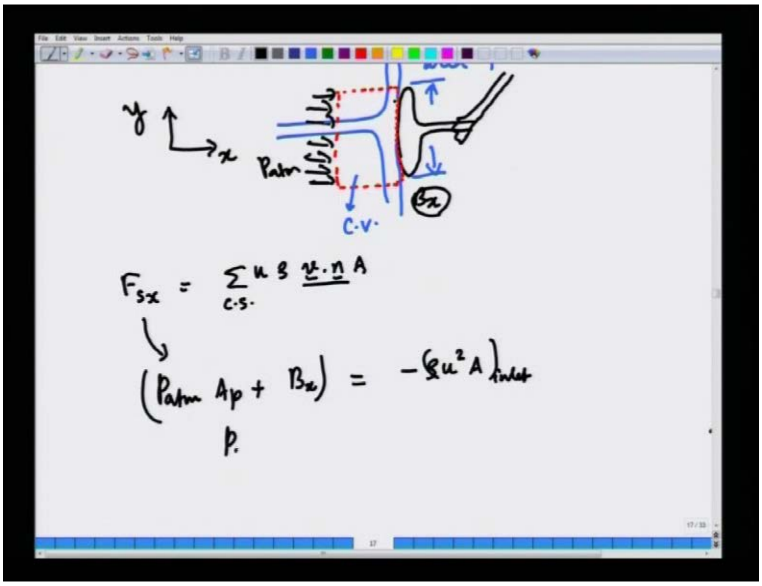

Now, I want to illustrate this with a slightly different choice of c v, because the choice of c v is completely in the hands of the person who is solving the problem. Therefore, we can always choose some other c v. So, I am going to illustrate this like this, this is the solid surface and your hand is holding it. Now, let me draw the jet with blue color, it is going like that. Now, the c v which I am going to draw with red color is exactly, it does not include the solid surface. It just adjoins the solid surface and it in closes the fluid that is trying to impinge on the solid surface but it does not include the solid surface.

Now, let us call this area as A p, and this is our new c v, we are going to apply the momentum balance now, f s x everything remains same as before. Uniform flow assumption, pressure being atmospheric, everything remains same A. Now, what is, what are, what is the f s x here? Now, that is clearly atmospheric pressure acting and there is a force exerted by the solid surface on the fluid, Let us call that force b x, if this is the force exerted by the solid surface on the fluid.

Again let us not worry about this sign, it will come out naturally from the problem b x is an algebraic quantity. It is merely a component of the force exerted by the solid surface on fluid, on the control surface of the fluid. So f x x is therefore, p atmosphere times A p, plus b x is equal to; now, this is same as before minus rho u square A, that inlet. This because u dot n is, v dot n is again negative for the inlet; therefore, you get this answer.

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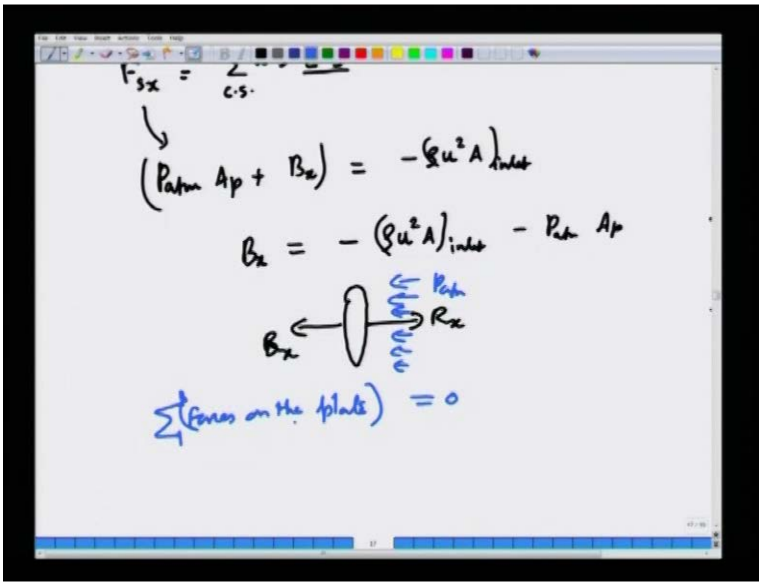

So, p atmosphere, so we have this expression. Now, the question that we want is, what is the force exerted by the c, by the what, is the force felt by your hand? So, let us first eliminate B x from here, it is rho u square A inlet minus p atmosphere times A p. Now, we do a free body diagram on the plate the force exerted by your hand on the plate is r x. The force exerted by the plate on the fluid is b x. But, there is also uniform atmospheric pressure acting on the plate from the right side. So, we will have to include that in the free body diagram on the plate.

So, these are the only 3 forces acting on the plate some of since the plate is not accelerating sum of all forces on the plate must be 0 so that simply means 0 is equal to minus b x. So, we have taken b x to act in this direction, sum of all forces on the plate in the x direction is 0.

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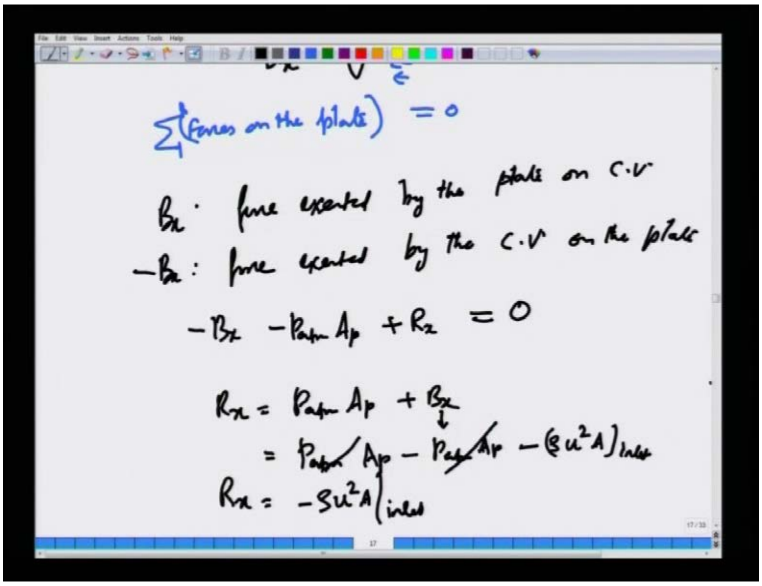

So, let us let me explain this carefully, b x is the force exerted by the plate on the c v. So, minus b x is the force exerted by the c v on the plate by Newton's third law of motion. So, minus b x minus p atmosphere A p because, that is acting in the minus x direction plus r x, we do not know what is us, what is the sign of r x whether it is positive or negative. It will come out naturally from the problem. So, r x is nothing but p atmosphere A p plus p x but we already have an expression for b x. So, it is p atmosphere times, A p minus p atmosphere A p, minus rho u square A at the inlet. Now, the 2 p atmosphere a p will cancel to give Rx is minus rho u square A at the inlet and remember that Rx is the force exerted by the hand on the c v.

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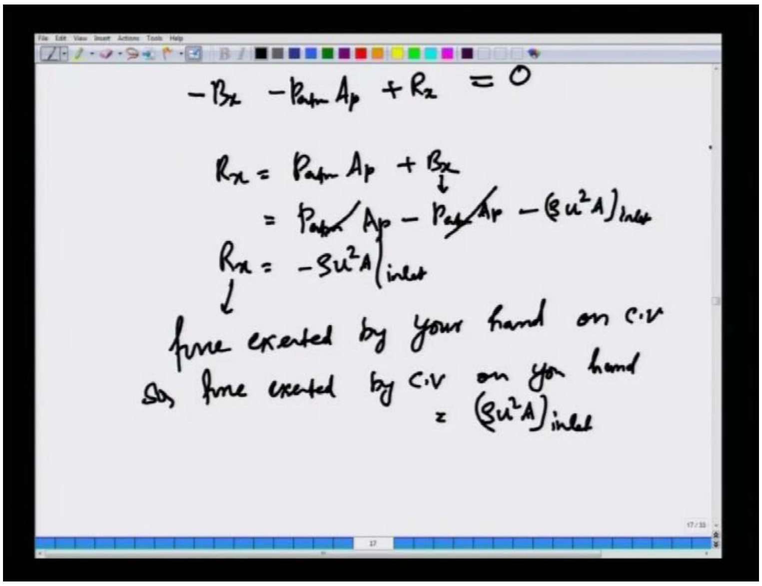

So, the force exerted this is the force exerted by your hand on c v. So, force exerted by c v on your hand is simply rho u squared A at inlet, it is clearly in the plus x direction as you would expect physically. So, this clearly tells you, that you can choose to work with a c v of your choice and the answer if you do every a step correctly the answer will be immediate to the c v that you choose to work with. But, some c v are easier to apply for a given problem then other c v. For example, in our example here, we had to do some extra work in the second c v because; we have to again draw a separate free body diagram on the plate.

In order to take the contribution of the atmospheric pressure out whereas, in the first c v we immediately concluded that the atmospheric pressure bal cancels out balances out. So, we have to merely focus on the force exerted by the hand on the c v, the r x force that is a reaction force. Some c v is our more convenient time the other but if you do all the steps correctly can carefully all the c v should give raise to the same answer. And clearly the physical answer, the c v is actually our constructions the problem does not care for c v.

So, if you do the, if the actual physical process is done a care for c v, whereas if you want to analyze it you choose a c v of your convenience. But, the answer should come out to be the same regardless of the way in which the way in which the c v is chosen. So, long as all the steps are done correctly so that is the key thing that we have to remember, when we do momentum balance. I think, we will stop here this completes an example and we will continue from this point onwards in the next lecture. Where we will finish our discussion on momentum balance and we will start energy balance. Thank for your attention and we will see you in the next lecture.