Fluid Mechanics

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Lecture No. # 13

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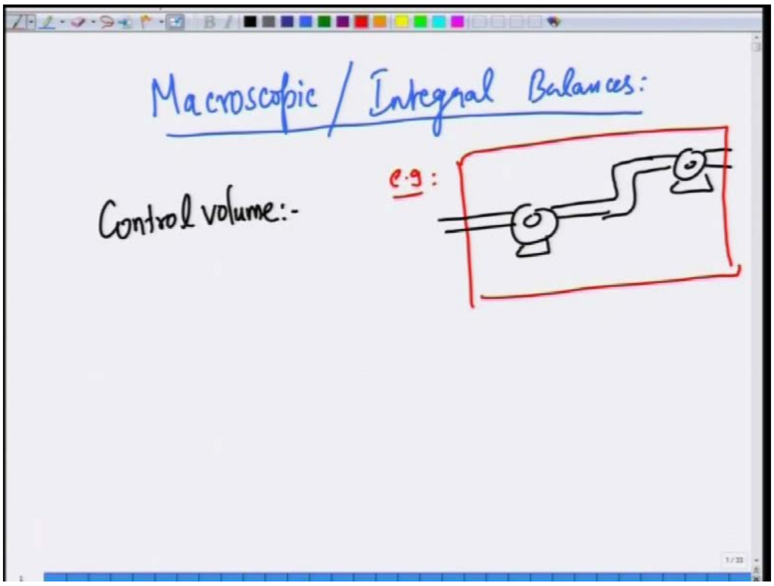

Welcome to this lecture number 13, on N P T E L course on fluid mechanics for undergraduate students in chemical engineering. The topic that we are currently discussing is under the title of macroscopic balances or they are also referred to as integral balances, just to remind you of what we were discussing in the last lecture. Macroscopic balances are concerned with entire equipment, process equipment like pumps, compressors or tubes networks of tubes and so on.

So, we are going to apply the principles of conservation of mass, momentum and energy to control volume what is called a control volume. A control volume is any fixed region in space that is of interest to us. So, you could have for example, a pipe leading to a pump and then there could be several other pumps also, all this could come under a single control volume, depending on the nature of the application. This is just an example of course; the context in which the problem arises will suggest control volume by itself. And it comes from some experience by solving various problems in fluid mechanics; one learns how to choose appropriate control volumes that is convenient given problem.

But, none the less in general a control volume is a region in space that is of interest to us in which we want to compute certain quantities such as forces or power requirement in pumps or and so on, similar reason similar quantities.

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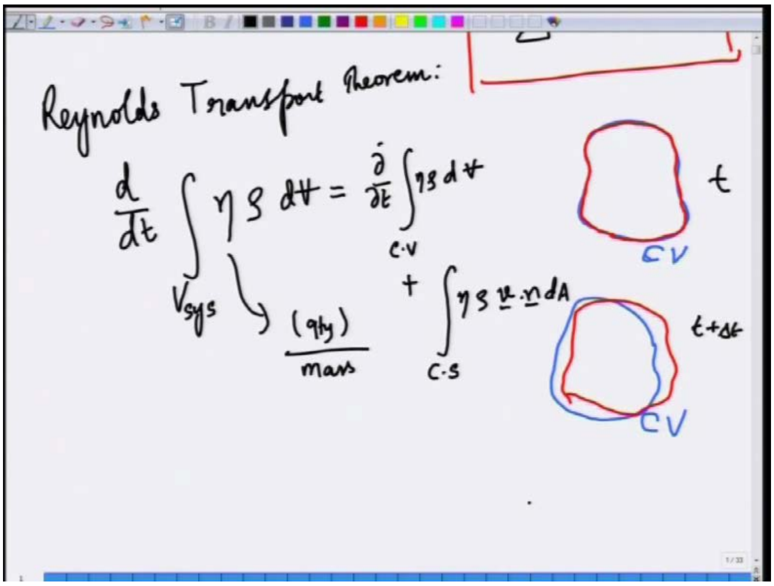

So, whenever you have a control volume. We have this instrument or we have this tool called the Reynolds transport theorem, (No Audio Time: 02:04 to 02:10) transport theorem which relates the rate of change of quantity in a system. So, just remind you a control volume is a fixed region in space. Here I am denoting the control volume by blue line a system is that fluid that occupies the C V at a given time t but, at a time immediately later t plus delta t the system has moved slightly away from the C V. A system or a material volume contains a same set of material points; there is a same set of fluid particles.

So, you are following the same set of fluid particles while a C V is a fixed region in space. So, the Reynolds transport theorem tells you that suppose you have a C V and at a time t plus delta t, a system has moved slightly away. So, this is time t, this t plus delta t. What is the rate of change of various quantities at a present in this system, at time t as a function of, in terms of the variables that are expressed in terms of the control volume?

So, the system rate of change of any quantity. So we introduced a general quantity called eta which is per quantity per unit mass. This quantity could be mass momentum or energy integrated over volume is d dt over C V eta rho d V. The time rate of change of that quantity present in the C V plus flux contribution which is over the various inlets and exits of the C V eta rho V dot n d A, where n is the unit outward normal to the control surface.

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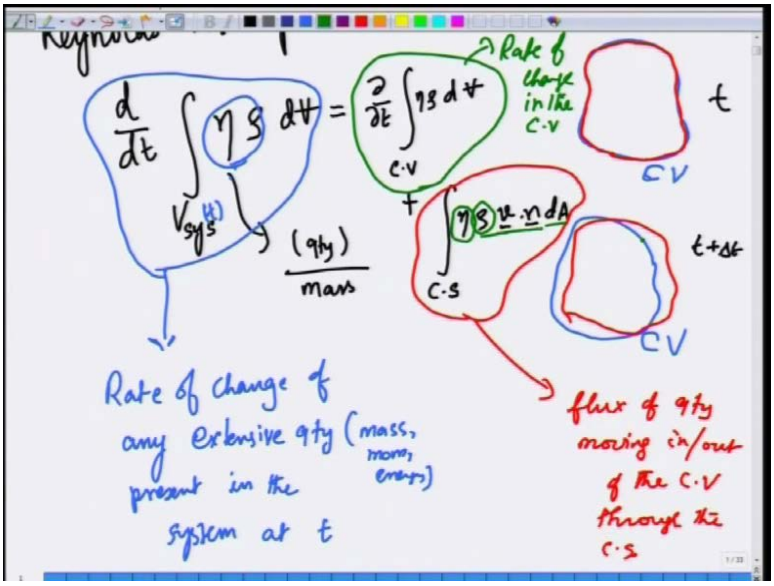

And this is the Reynolds transport theorem. This is the left side of the Reynolds transport theorem. Let me just write in words, is the rate of change of any extensive quantity such as any extensive quantity such as mass momentum or energy, momentum or energy present in the control mass or system at time t.

The system and control volume coincide at time t but, at a slightly later time t plus delta t the control mass would have moved away from the C V because if you follow fluid particles by virtual flow, all these particles will be moving slightly away. So, that is the idea and the right side, the first term on the right side contains the rate of change of that quantity present in the C V itself. So, there are two contributions to the rate of change of this quantity extensive quantity as you follow this system.

One is because of the fact that there is an inherent rate of change present in the C V itself. This is a rate of change in the C V and things could also change because of the fact that fluid is moving in and out by virtue of this flow in and out of the C V. There are fluxes of quantities inside and outside the C V and that is this quantity.

So, this is the flux term, this is the flux of quantity moving in or out of the C V. A C V is visible to the outer surface through the control surface so through the surface, the control surface. So, the interpretation of various terms as follows. So, just to remind you eta is any quantity per unit mass time’s rho which is density, which is mass per unit volume. This product will give you any quantity per unit volume in a tiny volume in the C, in a system integrated over a entire volume will give the total amount of mass momentum or energy that is present in the system and this is d dt of that.

The time rate of change of any quantity, the same interpretation but, here instead of integrating over the system which is a function of time, we are integrating over C V which is fixed region in space and this is the flux term. The interpretation as follows V dot n is a normal component of the velocity at the control surface times, d A is the normal volumetric flow rate of the fluid times. Density is the mass flow rate times the quantity per unit mass will give the rate at which any quantity such as mass, momentum or energy is entering or leaving the C V through the control surface.

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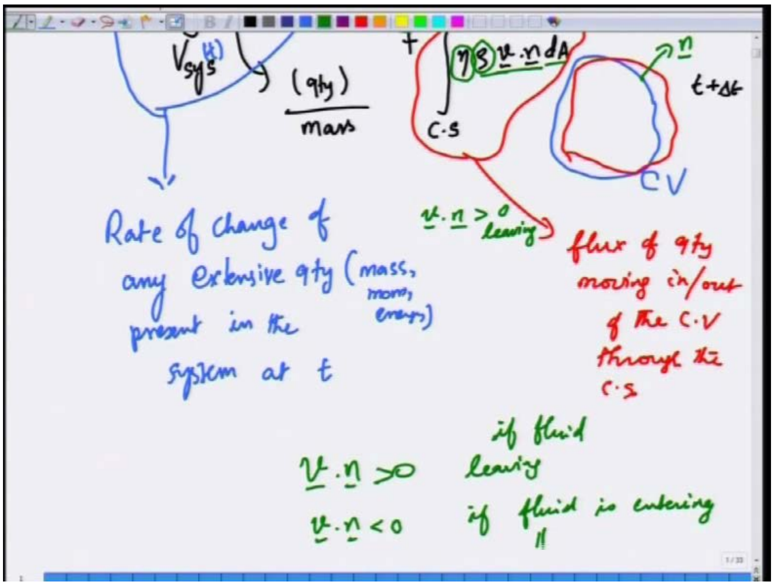

V dot n is positive, if there is, if the fluid is leaving because n is a unit outward normal to the C V. So, V dot n is positive if fluid is leaving, and it is negative, if fluid is entering the C V.

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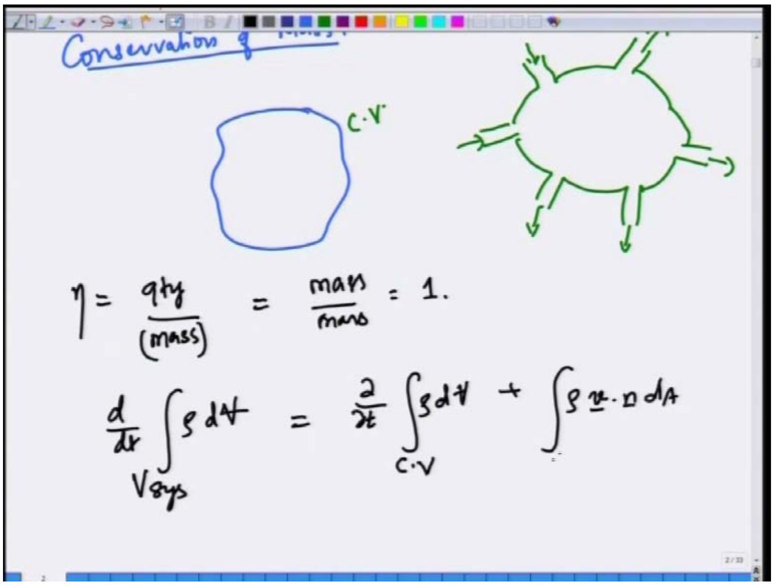

So, that is the general Reynolds transport theorem. We next applied it to the case of conservation of mass, the principle of conservation of mass. So, this is the integral balance for mass conservation. Suppose, you have an arbitrary C V and in reality you may think of this as a container with several inlets and exits that is how it will appear in a real problem.

So, fluid may enter through certain entries and exit through certain exits. So, this is the C V, so you identify the C V. How does the conservation of mass principle work for this? So, eta is quantity per unit mass, now the stuff that we are interested in mass itself. So, it is mass per unit mass is unity. So, you have to substitute 1 in the Reynolds transport theorem. So, d dt of rho d V of the system is d dt of V, control volume over the C V rho d V plus integral rho V dot n d A over the control surface. This is by applying Reynolds transport theorem with eta equals 1.

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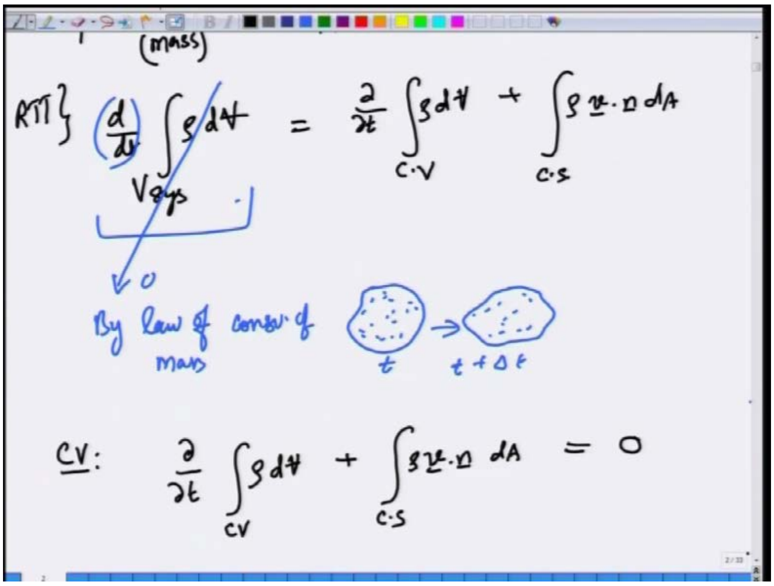

Now, the conservation of mass principle says that if you follow the same set of mass points, masses neither created nor destroyed. So, this is 0 by law of conservation of mass. If you follow the same set of mass points or fluid particles, since mass can neither be created nor be destroyed. If you follow the same set of mass particles, fluid particles their mass will remain a constant. So, that is a meaning of this derivative where you are following a system, that is by system we mean that you have system is like this at time t is equal to t at any time t. A little time later the system will deform but, it will have the same set of mass points has it had a t.

So, if you follow the system at a later time t plus delta t. The mass will remain the same so the time rate of change of mass of the system is 0. So, the mass conservation principle as applied to a C V reduces to d dt of integral over C V rho d V plus integral C S rho V dot n d A is 0. Because this term is 0 so this right side must be equal to must be equal to 0.

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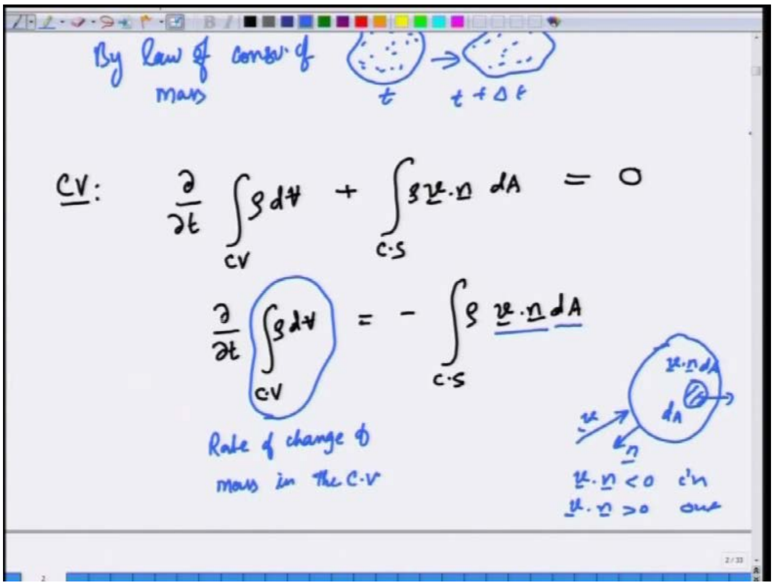

Now, I can interpret this in the following manner. I can take the flux down to the other side C s rho V dot n d A. Now, the physical interpretation for this is, this is the rho times d V is the mass that is present is a tiny pocket, tiny volume in a C V integrated over the entire C V will give you the total mass. This is the rate of change of mass present in this C V.

Now, why should the mass in the C V change? Well it changes because there are entry and exits to the C V. And through the entry and exit fluid is flowing in and out and if there is a net flow in then there is an increase of mass. If there is a net flow out then there is an decrease of mass. So, what is the right side, V dot n is a normal compare to the velocity times d a is the volumetric flow rate over a tiny part of the control surface. So, if you look at a control Volume like this then if you take a tiny area d A, the net volumetric flow rate is V dot n d A.

Now, times rho will be the net mass flow rate. Integrate over the entire surface will give you the total net mass flow rate in or out of the system. Now, V dot n is positive if there is a net flow out. If there is net flow out everywhere on the control surface then this integral will be positive, negative of a positive quantity is negative. That means that there will be decrease of mass respect to time in the C V. If V dot n is negative through every part of the control surface then there is a net inflow because n is the unit outward normal.

If V is pointing coming into the C V, V dot n will be negative, if there is inflow and V dot n is positive if there is outflow.

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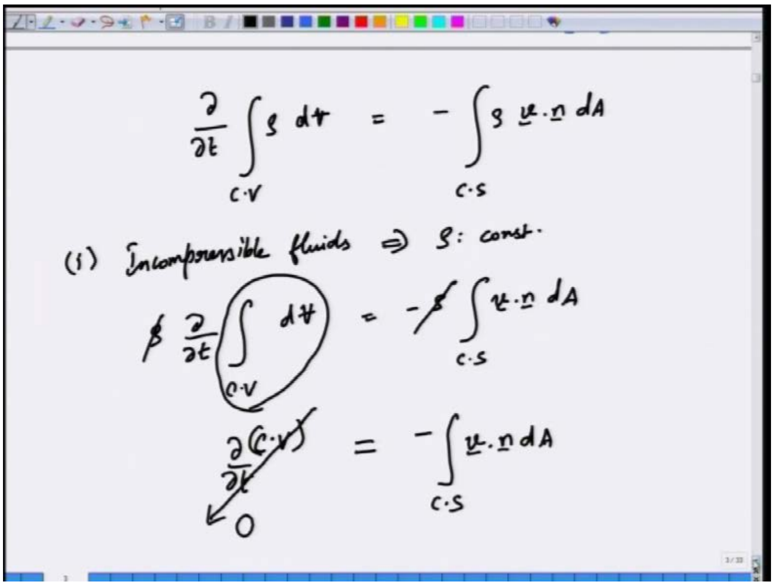

If V dot n is negative, then negative of negative is positive. So, this leads to a buildup of mass that is d dt mass in the C V is positive. So, this is the most general form of mass conservation principle the d dt of integral over C V rho d V is minus integral over C S rho V dot n d A. Now, we can also simplify this, to many special cases. This is the most general form of mass conservation that is Valid for any system in fluid mechanics any problem in fluid mechanics. Now, for some special cases, the first special case we did is for the case of incompressible fluids. When the fluid is incompressible, rho is a constant, rho is independent, and rho does not change with pressure.

Whenever there is a fluid flow there is pressure changes associated with any flow but, if the density changes are small or negligible then we can treat the density to be constant with respect to variation in pressure. Such fluids are called incompressible fluids.

If rho is a constant then you can pull rho outside the integral. So, d dt integral, this is C V rho d V is again rho is constant, V dot n d A. Now, since rho is there in both sides of the equation it cancels out. Now integral over d V of the C V is nothing but, the d dt of the C V itself, the volume of the control volume. Now, if the control volume is fixed region in space it is constant so it is 0, it does not vary with time. So, minus integral C S V dot n d A is 0.

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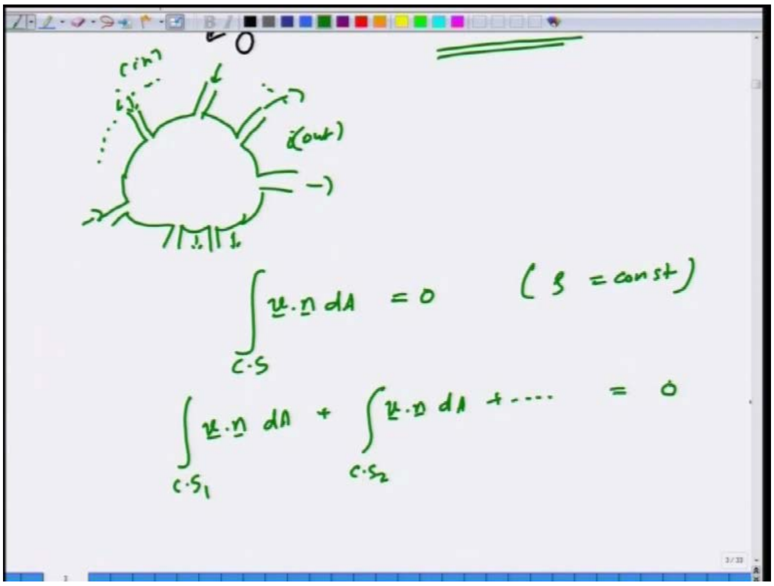

Now, let us apply to a case where you have typically C V can be a container with region of interest with several inlets and several outlets. There can be several inlets i and several outlets, just schematically fluid can come in and go out through several outlets. Now, I can therefore, split this flux term. So, for an incompressible fluid V dot n d A is 0. This is the conservation of mass principle; this is simplified form for incompressible fluids rho is constant.

Now, the control surface is split into various control surfaces, C S 1 plus C S 2 plus so on is 0. Because there are typically several countable number of inputs and outputs in generally in realistic problem in fluid mechanics. In such a case, you can write like this.

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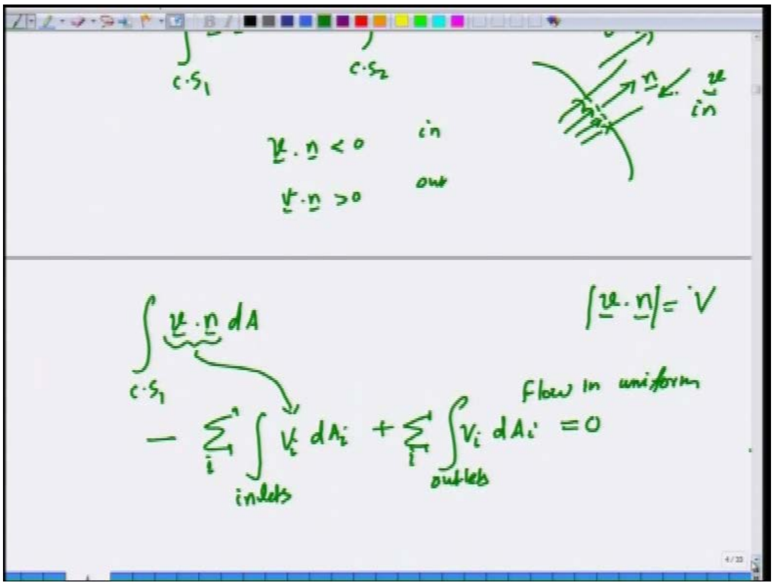

Now, you can also demarked inlets and outlets. So, for an inlet V dot n is negative inlet and V dot n is greater than 0 for an outlet because whenever you have a control surface. Suppose, you have a conduit if this is the n is the unit outward normal if V is opposite then V is entering in V dot n is negative, if V is leaving in out. If V is leaving fluid is leaving the control Volume through the control surface V dot n is positive.

So, we can split this into various inlets. So, since V dot n is negative, let us call V dot n as V and the magnitude of V dot n is V and the sign of V dot n is negative for inlets and positive for outlets. Now, usually what happens is that, let us just write it like vector summation over various inlets and outlets. Now, if the flow is uniform, that is the velocity we have in general terms like this C S 1 V dot n d A. Now, we can choose velocity to be completely normal to the C V either it is entering or leaving we can choose our control surface such that the velocity is normal.

So, V dot n is essentially sum V, V dot n is the normal component of the velocity that is V and then there is a sign associated with it. So, if it is an inlet we will put a negative sign. And this is for all the inlets minus summation over all the inlets plus summation over so let i denote all the indices for inlets and similarly, for summation over all outlets is 0.

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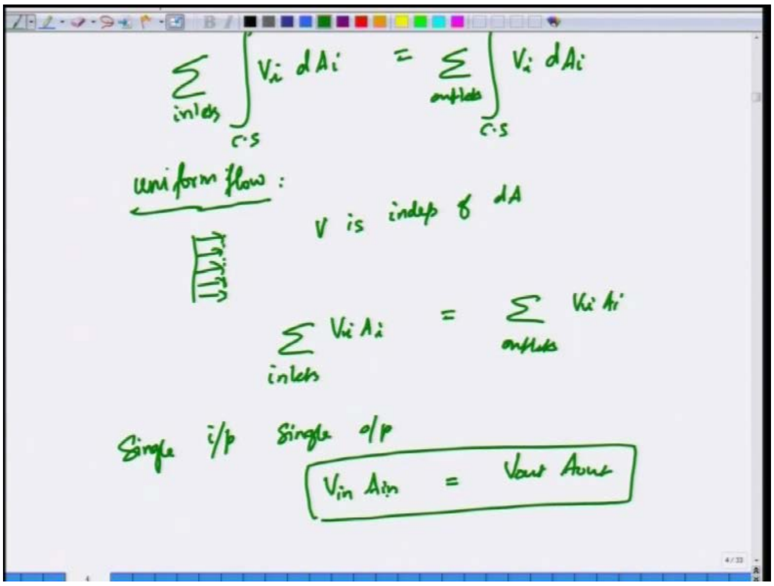

So, in general we have V i d A i summation over all inlets is integral summation over all outlets V i d A i. Now, if the velocity is uniform, if the flow is uniform then the velocity vector is independent of the cross section of the control surface, the velocity vectors is a constant, uniform flow. Usually flow is not uniform in flow through tubes and channels as we will see later but, if the flow is like a free jet that is going in atmosphere that is moving in atmosphere, in air then can reasonably think of it as a uniform flow.

So, as a matter of convince in many simplified settings one may think of the flow to be uniform. In such cases V is independent of d A. So, you can pull it out and integral over d A over the control surface is simply A. So, summation over all inlets V i A i is summation over all outlets V i A i.

If there is only single inlet, single outlet, single output then V in A in is V out A out. This is a very simple form of conservation of mass applied to incompressible flows whether it is steady or not it is not a problem because as C V it is a fixed region in space. This is valid for both steady and steady and unsteady flows as long as the flows incompressible.

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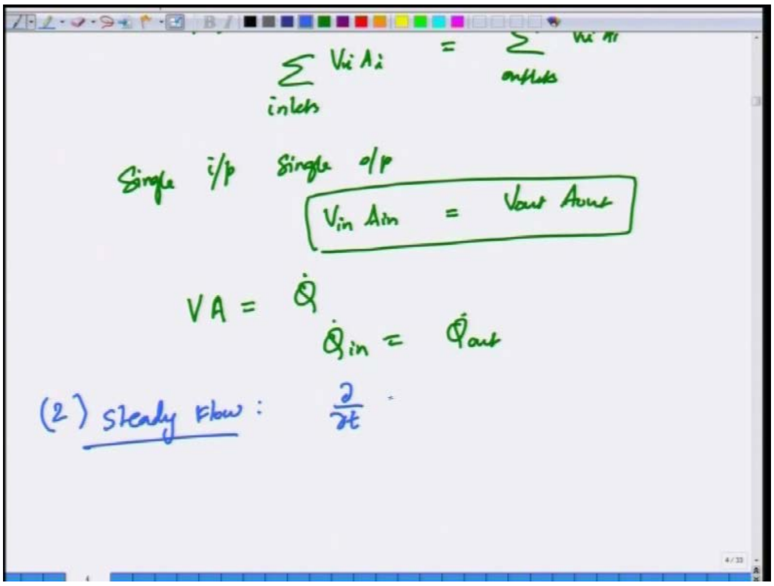

So, if the flow is uniform then it that is a velocity vector is independent of cross sectional area. Then this is the simplified form of mass conservation equation V times A is the volumetric flow rate Q. So, Q in is Q out. So, generally if I was saying single input single output problem mass in must be equal to mass out because the fluid since the fluid density is constant. That means that the volume rate at which volume is flowing in is equal to rate at which mass is volume is flowing. This is denoted by volumetric flow rate is denoted by the symbol Q dot. So, this is the simplified form of mass conservation equation that is useful in many contexts in practical applications.

Now, you can also consider another approximation that is the steady flow approximation. When the flow is steady d dt is 0, whether the fluid is incompressible or not is not a matter. We are merely considering steady flow that is d dt is 0.

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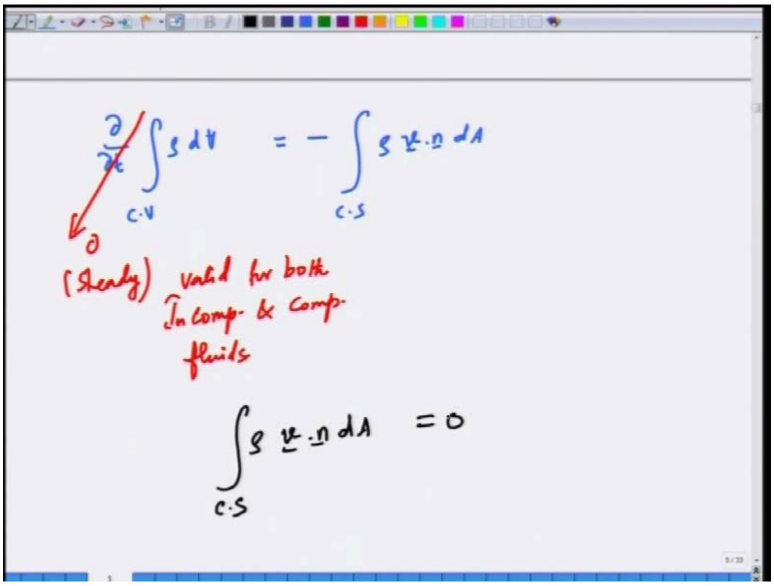

When d dt of any quantity is 0, so we have the general mass conservation equation rho d V over the C V is minus integral over C S rho V dot n d A. Now, for steady flow approximation d dt is 0, valid for both incompressible and compressible fluids.

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So, this is independent of whether rho is constant or not because we are assuming the flow to be steady. So, here integral over C S, now since rho is not a constant we have to keep rho inside the integral is 0.

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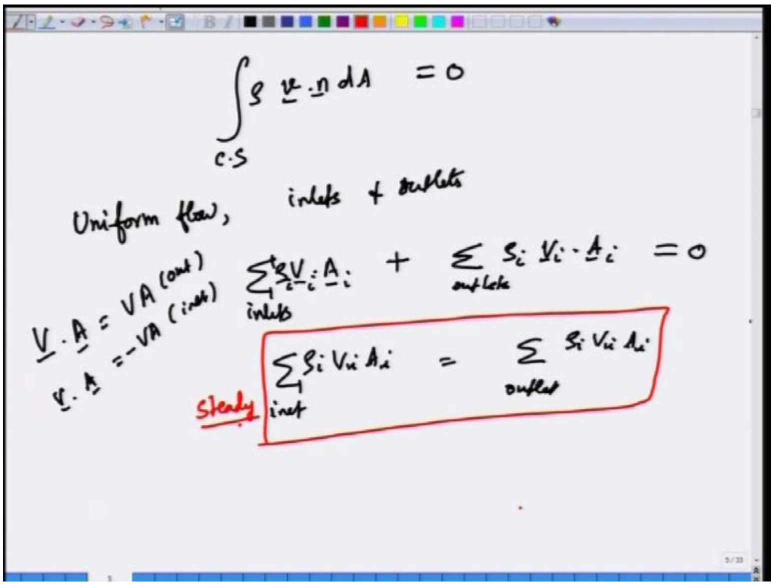

And if the flow is uniform and you assume inlets and outlets that are discreet in number. Then you can write this as summation over V dot A i over all inlets is equal to summation over rho i V i dot A i over all outlets. So, this is the simplified form of mass conservation equation for steady flows and if the flow is uniform then V dot A will be simply.

So, this is still without taking into account, the sign convention for inlets and outlets. So, V dot A is V A for outlets and V dot A is minus V A for inlets. Then we can take one of these out and write V rho i V i A i over inlets is rho i V i A i. So, this is a simplified form of mass conserve equation valid only for steady flows and density can vary from inlet to outlet. Although the form appears fairly similar to what we had before for incompressible flows, there the equation that we derived was valid whether the flow is steady or not, here it is valid only for steady flows.

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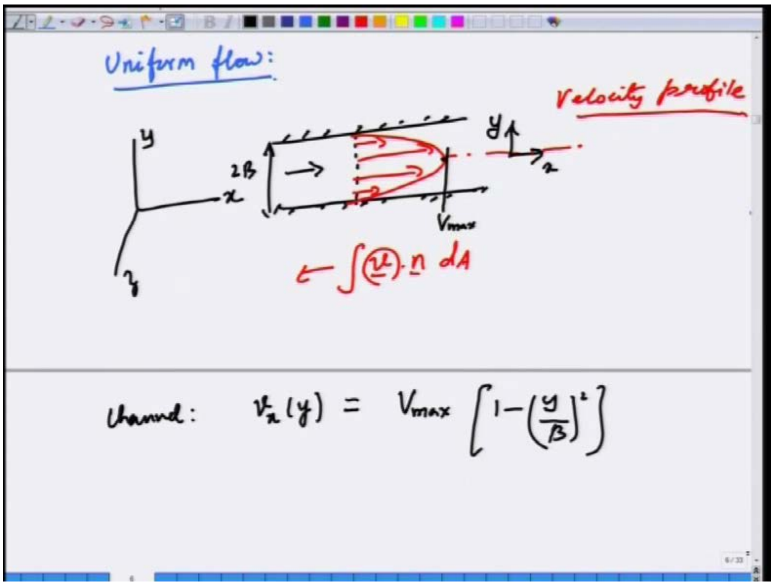

Now, I told you a thing about uniform flow approximation, we will come to this little later and we do differential balances but, let me just tell you what will happen in reality. Suppose, you have a flow in the x direction, let us say in a rectangular channel. That is extending in the z direction that is coming out of the board in the z direction the flow is in the x direction. Now, if you have flow between two rigid plates. Then the velocity in the fluid well in general be non-uniform. That is the velocity will be 0 at the vase and it will be maximum at the center.

So, and this is called velocity profile, it is called a velocity profile and in general therefore, one cannot pull the velocity if whenever you have V dot n d A. You cannot pull V out because velocity is a function of the coordinate with which you are integrating. So, one has to do this integration in general.

So, for example, in a channel flow, flow in a rectangular channel the velocity in the x direction is a function of the y coordinates in the following manner. It is the maximum velocity which is at the center this is called V max times 1 minus y by B. Suppose, this is the 2 B and the y coordinate is set like this then whole square.

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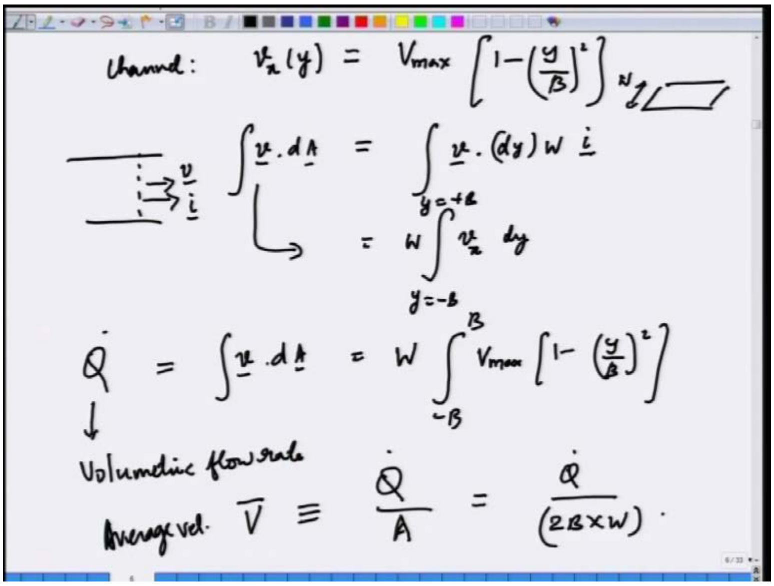

Then if you want to integrate integral V dot d A what this would mean. Suppose, fluid is entering in the x direction the unit outward normal is or let us keep the exit for simplicity. Suppose, fluid is exiting the channel in the x direction and the unit outward normal is also in the i direction that is along the x direction. Then V dot V A will be simply d y times W i where W is the width of the channel in the third direction.

The channel is also wider in the direction perpendicular to the board. So, this will simply give you V dot i is simply and W is a constant can be pulled over. It is V x times d y and integrated from y equals minus B to y equals plus B. This is the meaning of this term V dot d A, this is W times y equals minus B to plus B times. V x of y is V max times 1 minus y by B square. So, this is the meaning of integral V dot d A in general for a non-uniform flow and this is nothing but, the volumetric flow rate in the channel.

So, if you want to calculate the volumetric flow rate, this is how one calculates. And to get definition of an average velocity in the channel, V bar. It is defined as the volumetric flow rate divided by the cross sectional area. So, this is Q dot divided by the cross sectional area of the channel is nothing but, 2 B times W.

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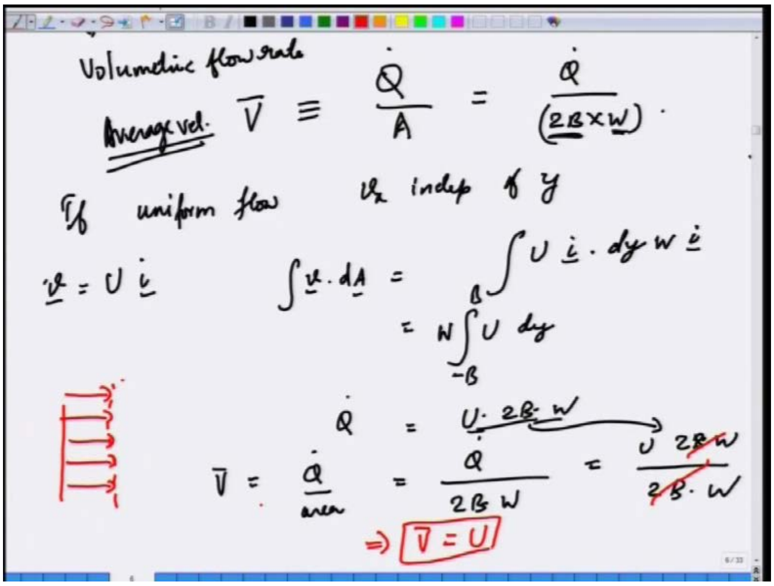

The whole width of the channel is the gap of the channel is 2 B and the width out of the paper is W. So, two times W is the width of the channel. This one calculates or defines average velocity by taking computing the volumetric flow rate for a non-uniform flow and dividing with the area. Suppose, the flow is uniform, if you have uniform flow then V x is independent of y the normal coordinate.

So, integral V dot d A will be some constant U times. So, let us just do it. If V is simply a constant in the x direction so, you will have U i dot d A is nothing but, dy W i. So, this is nothing but, integral U dy and y going from minus B to B. This is nothing but, U times 2 B times W, this is the volumetric flow rate. The average velocity is Q dot divided by area cross section area. That is nothing but, Q dot divided by 2 B times W. That is equal to U times 2 B times W, from this expression divided by 2 B times W.

So, the 2 B W 2 B W cancels implying that the average velocity is the uniform velocity. That makes physical sense because if the velocity is constant independent of the cross sectional coordinate. Then independent of the coordinate perpendicular to the direction of the flow then obviously of you even if you average it will give you constant answer. So, this is the meaning of this particular quantity called Average velocity. So, this completes our discussion on mass conservations.

So, that that really completes the discussion on mass conservation and we will apply mass conservation is so important. That in any problem that involves other balances such as momentum balance or energy balance we will see little later when we apply these principles. That mass conservation will give you one input to the problem. That is it will give you one equation to relate various unknowns with. So, mass conservation is always satisfied. So, it must be always satisfied in any problem in fluid mechanics.

So, the integral form of mass balance we will see a little later is also always applied in some very rare cases. It gives you a very trivial consequence but, mostly it gives you useful information in terms of relating the unknown variables that we want to solve for. The next in our topic in momentum, in integral balances is the integral momentum balance.

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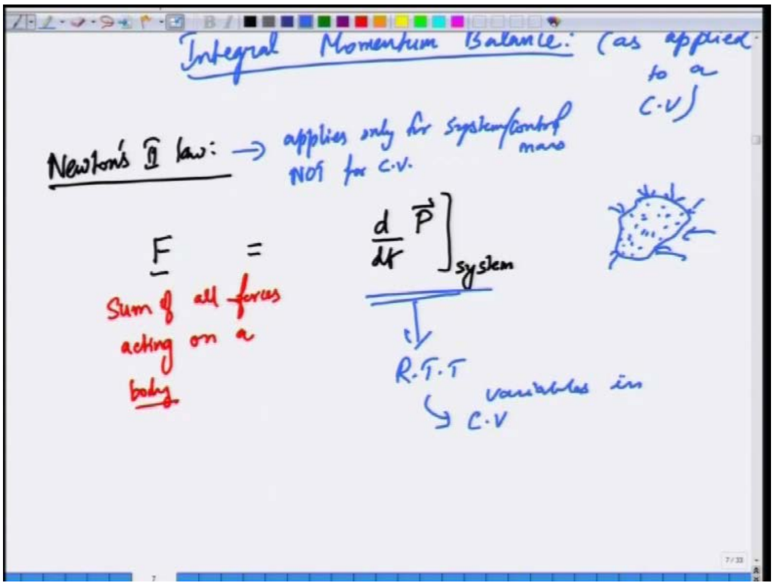

Integral momentum balance applied to a C V, as applied to a C V. We want to derive an integral balance for momentum as applied to a control volume but, we have to go back to the control mass approach because really speaking, the momentum balance rests on Newton's second law of motion.

Law which says that the sum of all forces acting on a body, a body is an identifiable piece of matter in space. Sum of all forces acting on a body is equal to the rate of change of the time, rate of change of momentum that is present in the body which is the system. That is you follow the same set of mass points. The time rate of change of momentum of all these mass points which found this system or a control mass is equal to sum of all the forces that are exerted by the surroundings on the control mass.

So, the Newton's second law is really applied to a control mass, it applies only for a control mass or a system control mass and not directly applicable for C V. So, we have to use Reynolds transport theorem to simplify the time rate of change of momentum that is present in the system and relate it to variables in a C V. This has been our strategy even in the conservation of mass principle but, there it was very straight forward because the time rate of change of mass of a system was 0. If you follow the same set of mass points then it is mass will not change as per conservation of mass principle. Therefore, that was simple but, here it is slightly more involved as we will see.

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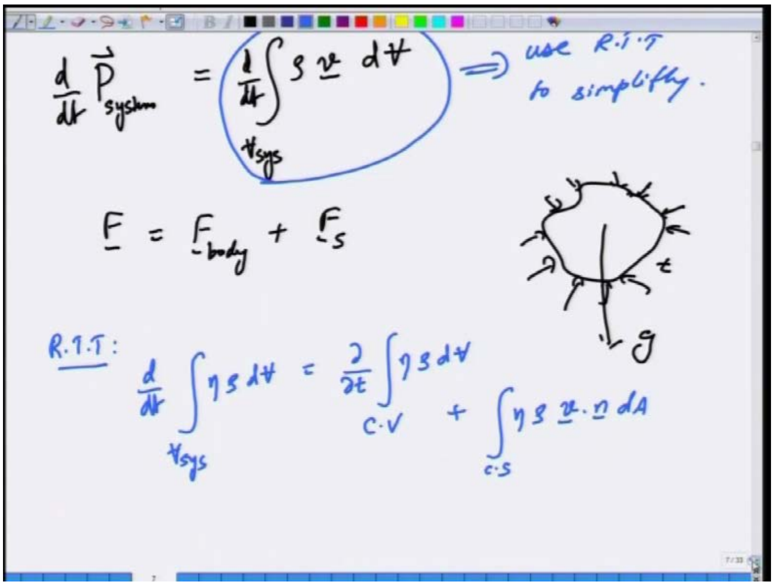

So, the total momentum that is present in a system which is identical to the C V at time t is equal to integrated with the volume of the system rho times velocity times d V. Mass times velocity vector is momentum vector, momentum is a vector mass times velocity vector is momentum vector.

In a fluid density can general change from point to point? So, we take a tiny volume then density times velocity is the momentum per unit volume times the tiny volume will give you the differential volume will give you the momentum present in that volume. Integrated over the entire volume of the system will give you the total momentum that is present in the system. And if we since, we want d dt and this must be simplified this must be used Reynolds transport theorem to simplify this we will do this shortly.

The sum of all the forces is sum of body forces and surface forces. So, surface forces on the C V. Suppose, you have C V the system coincides with the C V at time t. What are the various forces that are being acted upon by the fluid or other solids that are present outside on this C V on the surface of the C V? And then there are body forces like gravity which act through the entire volume of the C V which coincides with the system at that time. So, these are the various forces that are acting on this system.

So, let us apply the Reynolds transport theorem, the Reynolds transport theorem. If you recall d dt of integral over volume of the system eta rho d V is nothing but, d dt of integral over the C V eta rho d V plus integral over C S eta rho V dot n d A.

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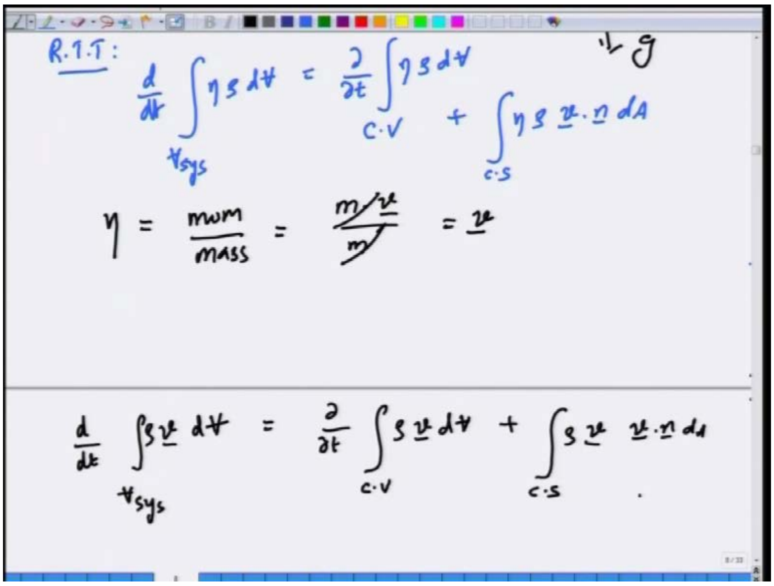

Now, we want conservation of momentum. So, eta is momentum per unit mass so eta will be mass times velocity divided by mass. So, mass mass cancels so eta will be simply the velocity vector itself.

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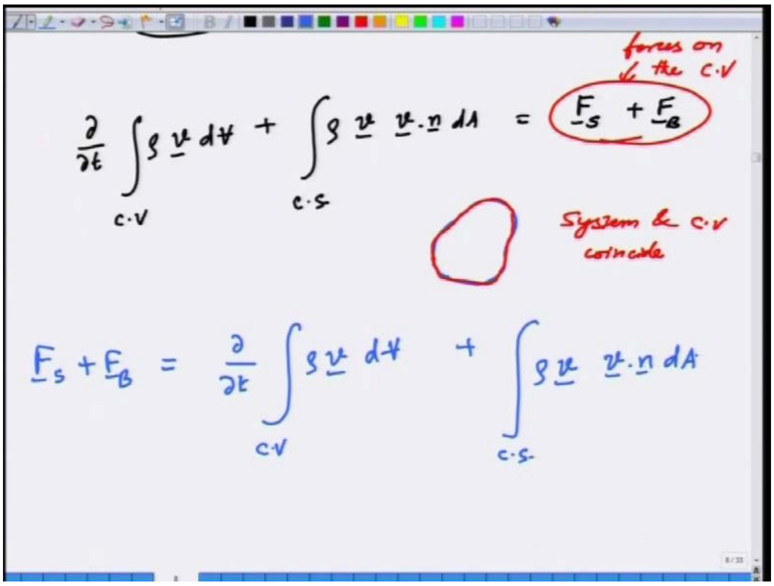

So, the Reynolds transport theorem, as applied to the momentum conservation principle is V d V is d dt over the C V rho V d V plus integral over C S rho V V dot n d A. V V dot n d A is sum of all surface forces plus sum of all body forces. Now, if you recall the system and the C V they coincide at a given time in the application of Reynolds transport theorem. So, the system and C V coincide. So, F S and F B are merely the forces acting on the on the C V because they are originally the forces acting on the system at time t but, since the system and C V coincide at time t. So, these are merely forces on the forces acting on the C V themselves. So, we are now able to obtain a version of the Newton second law to variables that pertain to a control Volume.

So, essentially what we have is this F S plus F B the sum of all surface and body forces is equal to integral rho V d V plus integral control surface rho V V dot n d A. So, this is the general form of integral momentum balance. Now, we can simplify it to some special cases.

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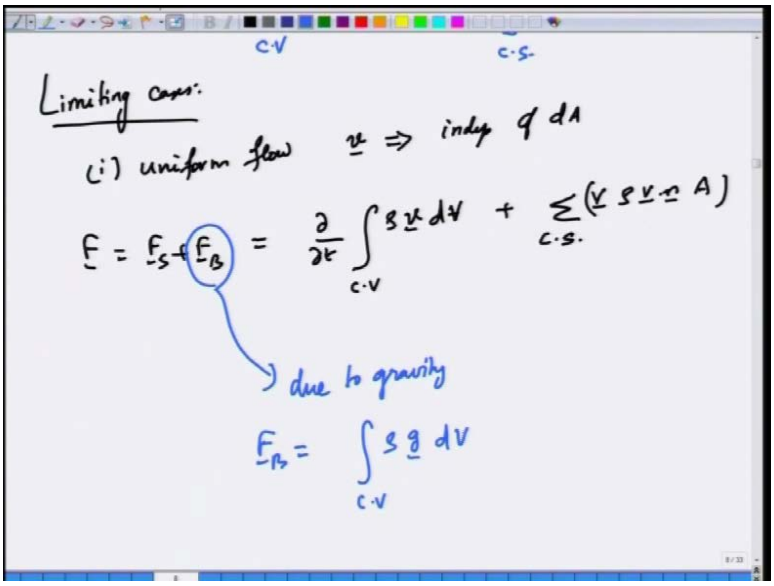

Suppose, you have limiting cases, you have uniform flow when the flow is uniform then V vector is independent of d A. So, you can pull it out of the in the of the area integral the surface term. So, you get F is F S plus F B sum of all forces on this C V is d dt rho V d V plus summation over C S V rho V dot A. Still there is a sign associated with the area let just put it as V dot n A.

Because if the flow entry is flow is entering the C V then n is unit outward so V dot n is negative and the flow is leaving then V dot n is positive. So, that is the usual convention that we follow. Usually what happens is the body force is due to gravity. So, the body force is merely integral over C V rho g d V.

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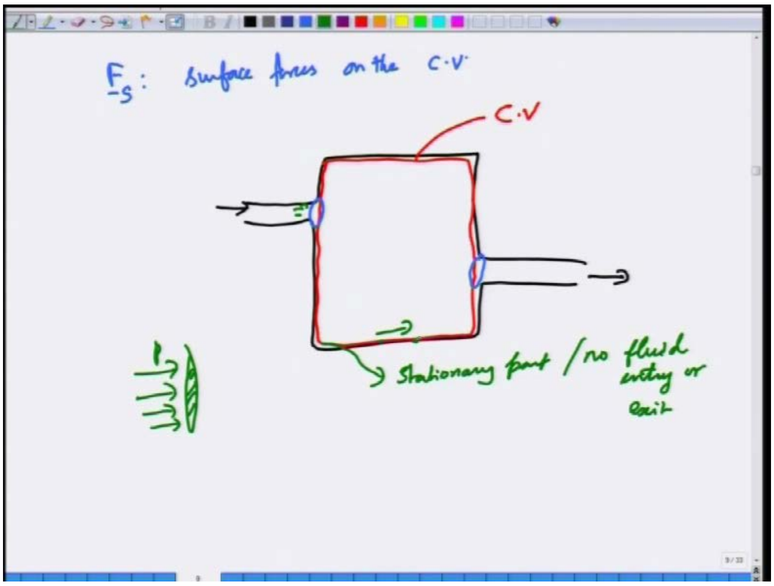

Now, what are the various causes of surface force F S? These are the surface forces acting on the C V because you have a system like this and this is our C V, red line is a control surface of the C V.

So, this is our C V, the red line is our C V and fluid is entering. Let us say like this and going like this now on the surface is the control surface can be divided into two parts, a part where flow is entering and the part where flow is leaving. This is one part where there is entry and exit. And there are the stationary parts here these are stationary parts where of the C S where there is no fluid entry or exit. So, various forces can act on the surfaces. For example, here whenever there is flow suppose I just exaggerate this entry whenever fluid is entering control surface into the C V then there are pressure forces that act on the control surface.

So, that forms a force and whenever there is fluid flow in inside this C V also. At the surface itself there can be resisting forces due to shear stresses exerted by the wall on the fluid. So, both these forces, surface forces are in fact possible.

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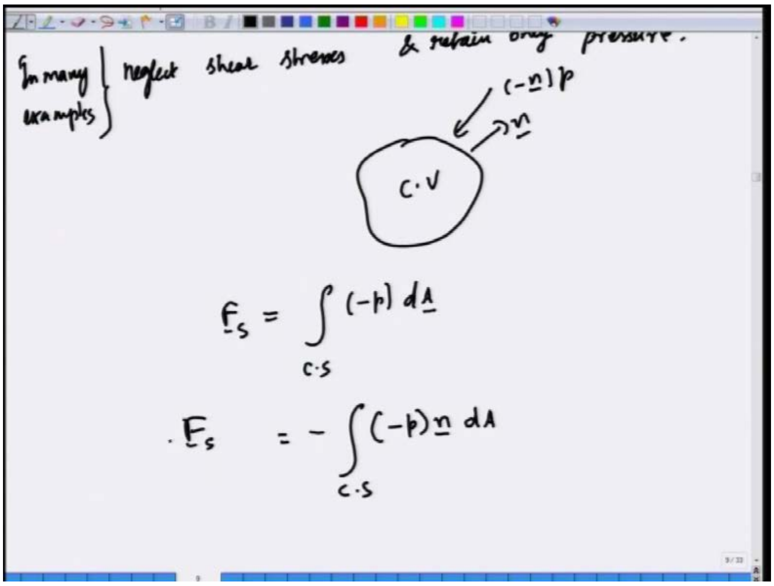

But, manier times in simple problems we neglect many cases, in many examples. It turns out that we neglect shear stresses and retain only the pressure forces. One has to look into the context and then make more judicious decision on whether shear stresses or in fact negligible or not but, in most cases it turns out that it can, one can get a rough answer by neglecting shear stresses just because of the fact that we do not have much idea about the drag forces that are exerted by the solid surfaces on the fluid.

So, we merely neglect them but, while doing examples I will try to indicate where one can actually get an estimate and so on. But right now, we retain only pressure. So, the pressure, suppose you have a C V and this is a unit outward normal. The pressure acts in the direction of minus n pressure is a surface force but, it acts it is compressive in nature So, F S is integral over C S minus p d A. This is minus so that is the way you compute the surface force due to pressure on the control surface.

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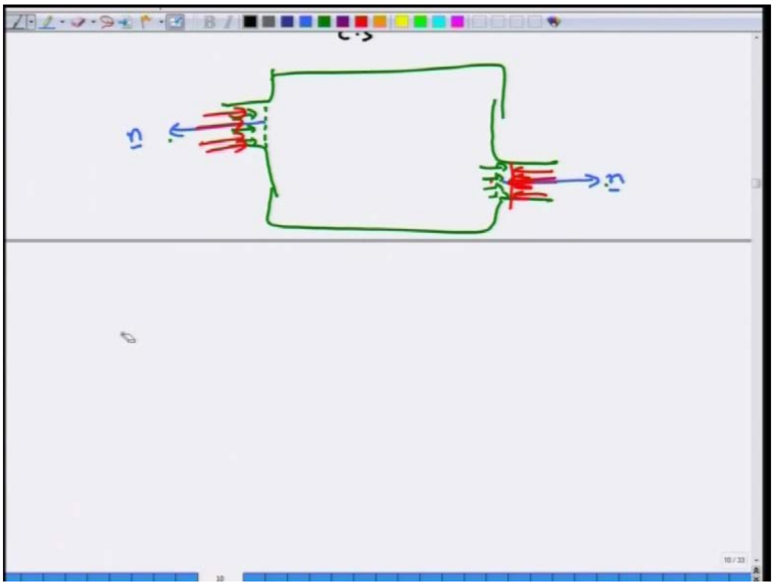

So, what is important here one when calculating the surface force due to pressure. Suppose, you have an entry and suppose you have an exit, this is your C V let us say. In the entry the pressure the outward unit normal to the flow is n while in the exit it is in the direction of flow. The flow is exiting like this while it is entering like this. So, what is important is the pressure acts in the direction opposite to the normal both here as well as here. So, the pressure here is acting like this here in the direction of minus n the pressure forces are acting like this here.

So, one should not think that just because fluid is leaving the C V through this C S pressure forces are in this direction. Whenever a fluid leaves the just fluid just outside the control surface will tend to resist it is motion by exerting a pressure. So, regardless of the direction of motion the pressure is always in the direction opposite to the unit normal. So, that is the key thing, one should not apply wrong intuition that since the fluid is flowing out the pressure is in this direction. What we are looking at is a force exerted or experienced by the fluid that is leaving out of the C V. Due to the fluid that is just outside which therefore, will be in this direction. So, this is something that we have to understand so the momentum equation is a vector equation.

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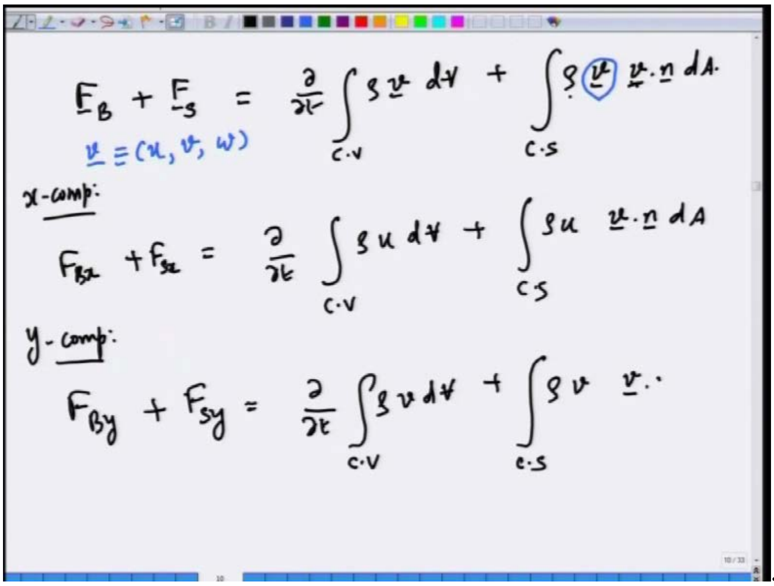

So, you can write it in terms of 3 scalar components the x component will be x component of the body force plus x component of the surface force is d dt. Let me first write the vector form of the momentum equation and then let me so F body plus F vector surface is d dt C V rho V d V plus C S rho V V dot n d A.

So, the x component of this equation is F B x plus F S x is d dt. So, in this term the only vector is a vectors velocity vector V. So, to take the x component velocity vector V has x component U, y component V, z component W. So, rho U d V. In this term rho is a scalar; V dot n is a scalar. Even though V is a vector once you dotted with n it becomes a scalar, so, V dot n is a scalar. So, this is the only vector that is available to U is V and the x component of width is U and V dot n remains as such.

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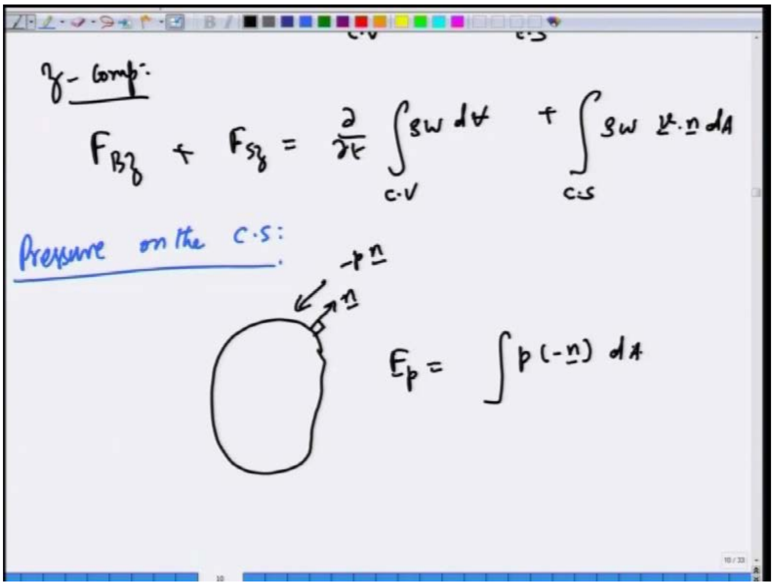

So similarly, the y component of F B y plus F S y is d dt over C V rho V d V plus C S rho V vector V dot n d A and the z component is F B z plus F S z is d dt rho W d V plus integral over C S rho W V dot n d A. So, these are three components of the momentum balance equation. When you apply the momentum balance equation to any problem, so, you will have to refer it to a particular coordinate system. Therefore, you will have to solve various components of the momentum balance. So, it is always useful to write it in a component form here I have the Cartesian coordinate system to indicate the various components.

Now, another thing is to notice the role of pressure on the control surface. Suppose, you have a C V like this and the unit outward normal is like this the pressure is in the direction of minus unit outward normal. Now, so F pressure is integral p times minus n d A.

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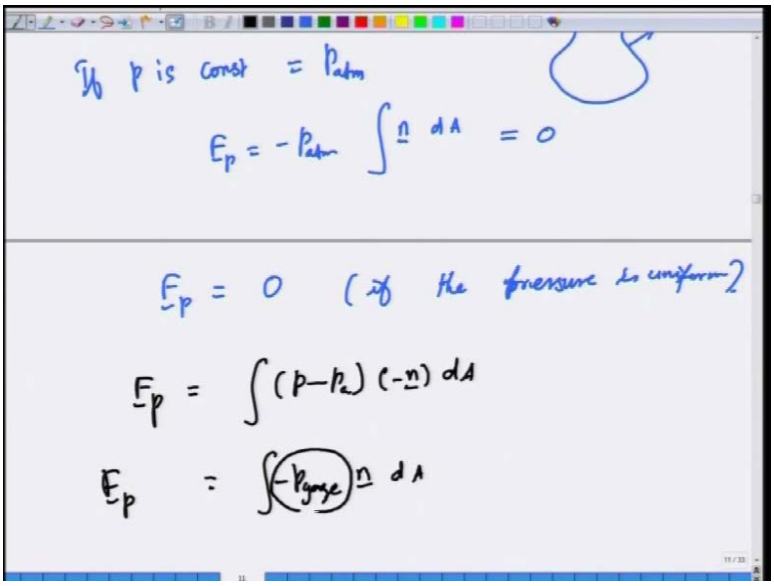

Suppose, if there is a uniform atmospheric pressure acting everywhere. If p is constant, let us say it is due to atmospheric pressure. In many applications you will find that you are C V is surrounded by atmospheric pressure and if that is an only pressure that is acting on a C V. If there is no other pressure that is acting then p is constant.

So, F p is since p is constant you can pull it out minus also let me pull it out and d A. Now for any close surface if you take n like this and integrated over the surface it will be identically 0 just by symmetry it has to be 0. So, the net pressure force acting on a C V due to if the pressure is uniform is 0, like an atmospheric pressure but, in many cases the pressure you will have atmospheric pressure over and above some other pressure.

So, the actual pressure force will be due to the difference between the pressure and atmospheric pressure in the direction of minus n d A. It is nothing but, the gage pressure because p minus p A is gage. So, the net pressure force is really because of the fact that there is a difference between the pressure force and the atmospheric pressure. If the only pressure is due to atmospheric pressure, it is completely constant it is uniform so it will give rise to 0.

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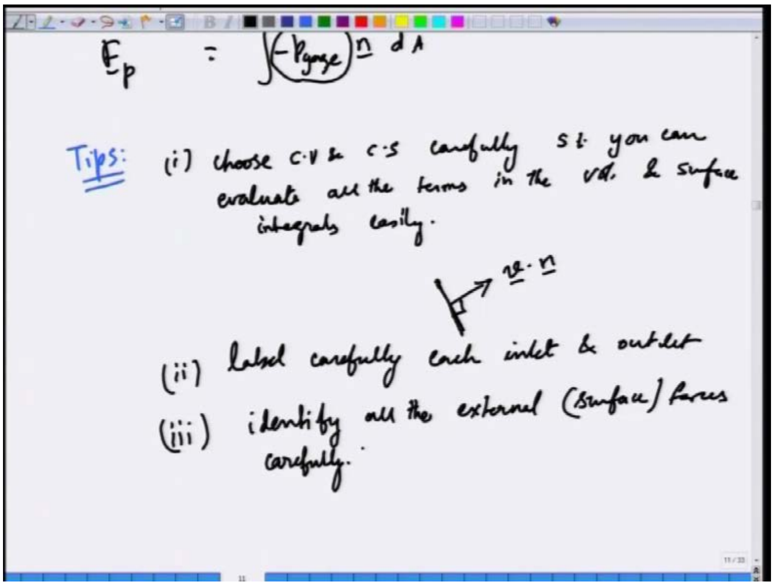

So, the gage pressure comes naturally when you want to compute the net pressure force contribution on any control surface in a given problem in momentum balance. So, let me just recap, first of all some tips and before I recap momentum balance, some tips and applying the momentum balance. So, firstly you choose the C V and C S carefully such that you can evaluate all the terms in the volume and surface integral easily. (No Audio Time: 48:37 to 48:44). So, one tip is that suppose whenever you have flow in direction you choose the C S such that it is normal to the flow so that V dot n becomes a simple quantity to calculate that is one thing.

So, the second point is, label carefully each inlet and outlet. And you must identify all the forces, all the external forces that are the surface forces importantly, carefully. Now, the surface forces as I told you are mainly due to pressure but, in some applications you will find that your C V will cut across a solid surface.

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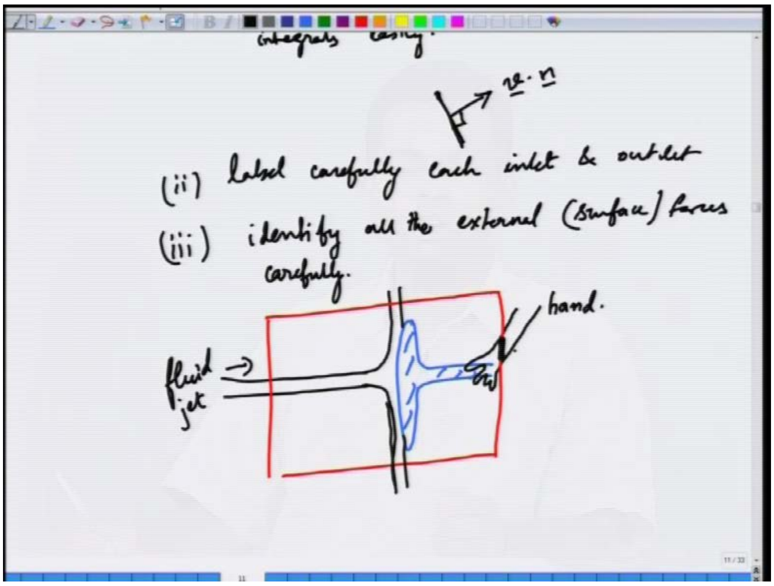

For example, let us say a fluid jet is impinging on a solid surface. Let us draw the solid surface with blue. This is the solid surface and a fluid jet is let us say coming and impinging on the solid surface and let us say it is going like this. Now let us say you are holding this solid surface in your hand and we choose the C V to be like this. So, this is the C V that you are choosing. Let us say fluid layer is leaving let us like this, like this and the C V is control surface is cutting across your hand. Now, this is the fluid jet that is coming impinging.

Suppose, you want to on the solid surface and it is leaving. Suppose, want to calculate the force exerted by the fluid on this solid surface. The C V is really cutting through your hand. You have to include the reaction force exerted force by your hand on the solid as a part of external force, external surface force on the C V because otherwise this C V will not this solid surface will not remain stationary. Since the solid surface is remaining stationary this fluid which is impinging on the solid surface will induce the momentum transfer to the surface, which will have to be resisted by an external force, which is a reaction force by your hand.

So, this must be included as a surface force because there is no other way we can account for this force this not a body force it happens only at the control surface where the C S the control surface cuts across your hand. So, that must be incorporated as a surface force. So, this is something that we will illustrate with the help of an example in the next lecture. So, we will stop here and we will continue from here from in the next lecture by illustrating how the integral momentum balance is applied to compute such forces. Thank you for your attention.