

## **Petroleum Reservoir Engineering**

**Dr. Pankaj Tiwari**

**Department of Chemical Engineering**

**Indian Institute of Technology, Guwahati**

### **Lecture 20: Introduction to Reservoir Simulation**

Hello everyone welcome to the class of petroleum reservoir engineering. In today's lecture we will continue our discussion of section 3 in today's lecture we will discuss about briefly on the reservoir simulation. So this is the introduction lecture on the reservoir simulation reservoir is a place which is underneath the surface so this is subsurface phenomena where the hydrocarbons are stored in a very complex geological formation. Hence the reservoir simulation is a complex process the computer modeling approach is often used to simulate the reservoir to understand the more efficient process and simulating the reservoir process for scaling up to field operation. In general the experiments are performed at the lab scale and then the reservoir simulation are performed in the computer and then the field scaling up is implemented to understand the integrated reservoir modeling approach. So reservoir simulation is a process of creating the replica in the computer the simulation of petroleum reservoir is necessary in the development of more efficient technology or the techniques to recover the maximum oil possible from the subsurface phenomena.

It is a combination of several processes so the physics involved in the reservoir simulation include geological formation data geological data petro physics data we will discuss separately and these complex interrelated physical processes need to be understood in the mathematical terms reservoir engineering concept and then the knowledge of computer programming is often required to develop such kind of the simulation tool. The reservoir simulation help us to gain inside in the reservoir domain that is the subsurface phenomena it is kind of a black box underneath the surface but we can create a replica in the computer with as much as information can be provided to the simulation tool and that simulation tool requires data in different form. So the developed reservoir simulation can help us to assert the evaluation that means estimate of the asset means the reservoir domain how wide how depth and then what is the potential of that particular reservoir followed by the reserve estimation means how much oil gas and specifically oil in place is present in the reservoir domain and third one is deliberately at what rate this oil can be produced economically to the surface and then the oven down meant means the condition when the well will be left out without any further production because it is not economically good condition. So how long the reservoir is going to be in the economical production condition will also be assessed with the help of the reservoir simulation.

So it means the reservoir simulation give us the opportunity to forecast the data for the future production of the reservoir performance. So when it comes to setting up the reservoir simulation the physical models need to be developed first. So as much as physics with the accurate information can be provided the more realistic model can be developed. So this is the essential feature of the underlying physical phenomena those are happening in the process or the part of the physical model. This physical model need to convert it into mathematical language and that mathematical language is actually the coupled system of time dependent non-linear partial differential equation and the physical model converted into mathematical model by applying the conservation law.

So the mathematical model include the conservation of mass conservation of momentum constitutive equation for different physical properties equation of state and the other relationship those are required to define the physical system. So this constitutive equation need to be provided to define the physical properties or physical phenomena those alter with respect to the other parameter. If the heat equation is also need to be included that should also be included for the thermal reservoir process. This mathematical model need to be converted into numerical model and that is done by basic properties of both physical and mathematical models are considered to develop this numerical model and numerical methods for converting this partially differential equations can be of two types like spatially discretization. In spatially discretization we are having the finite volume methods finite element methods finite difference method in the time discretization we are having the explicit and implicit method.

One of the method that is very popular is IMPS method in which implicit pressure and explicit saturation is defined. Once the numerical model is developed we need a computer algorithm that can solve these models and that is more efficiently the system of linear and non-linear algebraic equation can be solved with the help of the computer model. So what numerical model does it converts the partial differential equation into the linear algebraic equation. So the numerical methods for linear equation could be used direct method that is the Gaussian elimination method iterative methods means Gauss Seidel and Jacobi iteration method. There are several other methods also in both like numerical method for the PDA and the numerical method for the linear equation there is a array of the mathematical schemes those can be implemented to solve this equation.

Once the computer model gives us the result we can assess those result understand the efficiency of the model developed and if we record this could be put in the iterative cycle where we are modifying the physics including more features those are not incorporated earlier and then sequentially we are going again to the mathematical model numerical model and then computer codes. So this is a sequential process it means the number of iteration among these stages are sometimes required to adjust physical mathematical and numerical model along with the computer algorithm to get the more accurate reservoir

performance and that can be forecasted for the future production. The steps to develop a reservoir simulator could be seen from this flow chart so first we choose the recovery process formulate the problem once we formulate the problem in terms of the conservative equation and constitutive equations we get the non-linear partial differential equation those can be discretized to get the non-linear algebraic equation further linearization of those equation we can get the linear algebraic equation. These linear algebraic equation can be put for the solution in terms of the pressure and saturation and some other parameter and finally what result we get we need to validate those result for different application and finally we get the reservoir simulated process it means the physical process those are happening has been put up in the mathematical sense those mathematical equations are solved numerically to get the reservoir simulator. Now we can use this developed reservoir simulator for forecasting the future performance of course before making the future performance we need to validate that model it means we need to have the matching with the history and the validation of the model with the available data those data could be from the lab scale or could be from the field data.

Same formulation can be seen here in more representative way so the differential equation for mass flow for example mass flow this is a set of the conservative loss equations those are put for the further step that could be in 2 step finite difference approximation or grading system where we are making the grades of the developed model and then putting for the numerical model where the sensitivity of the model that is implemented for the solution needs to be understood. So what is the stability means numerical stability of that model chosen and then discretization error how much error will be produced when we are discretizing at a different size. So discretization can be done in a different size and different shape and once we solve this reservoir equation we get the reservoir performance. So the workflow to set up a reservoir simulation that is the part of the reservoir modeling and the simulations we first need to set up the objective of the simulation study what we want to do it is just developing a model that is going to help us in forecasting the production profile or it is that what we want to know more about the reservoir domain we want to see how the reservoir will perform when we are having the different configuration how the reservoir will perform when we are implementing from primary to secondary recovery or secondary to tertiary recovery. So by any mean the objective of the simulation study must be defined and then second step comes the selection of the simulator.

So there are variety of the simulators based on different parameters those are included to develop those simulator so the selection of the simulator is also very essential component. Once we have this selection of the simulator we need to develop the physical model for the reservoir. The development of the physical model for the reservoir requires lot of the

data as I mentioned earlier as accurate as information can be provided to develop this physical model the chances of the accuracy of reservoir performs will be high. So the information required is the seismic data means the size of the reservoir domain the geology of that reservoir domain, backlog data, core data for characterizing the rock that is there and spatial core analysis in terms of the saturation, porosity, distribution, permeability. And we can include this data also to develop the physical model along with the conservative equations so this all data are required first one is the dimension of the reservoir domain and what shape this could be the regular shape or irregular shape what are the XYZ dimension of that shape and how porosity, permeability are distributed in that how the phase saturation is present that should be included in this physical model along with the conservative equations.

So the fluid data what is the porosity, permeability of the fluid that is present and then once we provide all these five set of the data to develop the physical model then finally we also need to specify the locations of the well injection well, production well this is just a combination of one injection and one production well or there is a configuration of injection and production well as we discussed in the water flooding class that could be the regular shape or irregular shape of the configuration of the injection and the production well that could be normal sequence or could be the inverted sequence also. So to develop the reservoir physical model we have to start with the geological model that means the seismic data to define the structure of the geological model how the geological formation is there are there any fault how the layer wise distribution of different mineralogy or geology present that is provided by the geologist. Petrophysicist provide us the petro physical data based on the well log data or the core plug data for example the porosity, permeability, water saturation the average value or the distributed value of this parameter can be included in the model. Geological modeling means gridding of the reservoir domain is large in size in several meters in all the dimension so the gridding is required to discretize the system and then the size of the grid cell is a point to be discussed for the accuracy of the model developed as well as the computational time required to simulate the process. So often it happens geologist want very fine grid so they are able to represent as accurate information as possible while engineer wants to limit the number of the grid so the computational time to solve the simulation problem can be reduced.

Third is PVT properties so number of the fluids present the types of the fluid present accordingly the PVT behavior in terms of equation state or in some other parameters need to be included for example the dissolved gas ratio volume formation factor viscosity density of the fluid present. So the viscosity and density of the fluid should be provided in terms of the PVT properties special core analysis also called the SCL data for the multi-phase system we also need to provide the relative permeability of the front fluids those are present in the domain capillary pressure and other information. So this is not

the comprehensive list but the example that showing the types of the information required for developing the physical model of the reservoir process. So the reservoir after providing the information could have been like this is shown as the regular geometry it could be very irregular geometry also with different up and downs with different layers of the geological formation with different values of the permeability porosity saturation distributed in this and then we can fix the location of the injection or the production well. Similarly so in different configuration we can do the combination of injection and production well.

So let us consider this is one injection well here this is we are injecting and this is one production well. So in this case when we are injecting some fluid in the secondary recovery process or tertiary recovery process it has to pass through from this injection well towards the production well because of the pressure difference. Now the pressure will be distributed between the injection well and the production well. Similarly the distribution of the porosity, permeability and water saturation will provide us more accurate information. So let us say we could develop the simulator in terms of the size in terms of the physical properties geological properties fluid properties well rock properties now we have to initialize the simulation that can be done with the providing initial value of the pressure or the saturation.

Well location and completion is also required as I mentioned the location of the well and between the injection and production well how they are connected will be the matter of the recovery process. And then schedule for well control either specified as a rate that is STV per day at what rate we are producing so this is the well is controlled at constant flow rate or well is controlled at constant PWF that is the bottom hole pressure conditions. The time required to run the simulation will also be specified so we are seeing the production profiles of the particular reservoir for a period of the time and then we can see at what rate the cumulative production profile is going to follow the curve and when it is going to produce significant quantity of the well when we have to abundant the well which means we can forecast the data. Before forecasting the data we need to have the history matching it means tuning of the parameter as I mentioned earlier we are having the physical model that is going to mathematical numerical model and then the computer algorithm. In this process we might have to tune all these parameters so if the physics is not defined properly we are not getting a good match with the already collected data either from the field or from the lab.

So let us say this is our history match time so where we are matching the simulation data with the already available data if the match is not happening it means we have to adjust our physics. Physics is not properly defined that could be in terms of the fluid properties rock properties the well locations or in terms of the lithology that is present. By adjusting that physics we might have to also adjust the numerical scheme that has been taken so sometimes the finite element method is good sometimes it is not good

depending on the physical problem that is setup by providing the physical conditions. So we will tune that condition also and then finally the numerical method that is used for the iteration purpose like the Jacobian and others that may also need to be tuned. So by tuning the models we can have the history match and when we are getting good history match we can forecast this production profile here it is on oil recovery versus the time for a future time.

And when we are doing this future time there will be some uncertainty in the physical model, mathematical model, numerical model and that is why the data are always forecasted with some uncertainty and the uncertainty may increase as we are going future time so the uncertainty would be like this in the data. But within the uncertainty data the reservoir model that is developed by performing certain iteration will be able to provide us the forecast of the production profile. Based on that techno economical analysis can also be performed when we talk about the techno economical analysis lot of the other factors are need to be included that is make the problem more complex along with the complexity of the problem the risk associate in the operation provide us the integrated reservoir simulation tool and that can be utilized for the techno economical analysis it means how the oil well is going to perform in terms of the revenue generation. So the money input to have the secondary or tertiary process for example are good enough to get the good recovery of the oil. So that kind of the techno economical analysis can also be performed with the help of the developed reservoir simulation tool.

So the physical model that is developed for the reservoir can have different features so the classification of the models in the reservoir simulation can be initially fixed as we fix the objective as we choose the simulator we can also define some of the physical conditions. For example the types of the reservoir fluid that could be the compressibility and the composition of the fluid. So the fluid is slightly compressible compressible or incompressible and what are the composition of that fluid that is present in the reservoir or that is injected into the reservoir also. The dimension of the reservoir we may solve the problem for the 1D problem, 2D problem or the 3D problem actual representation is 3 dimensional but for the simplicity purpose or for understanding the basic feature of the reservoir we can start with 1D problem. Number of phases present so the single phase only oil is present oil and water is present or oil water and gas is present could also be specified in the setting above the simulation and then the coordinate system chosen.

So the rectangular system where we are having the X, Y, Z or we may go with the cylindrical coordinate system where the R, Z and theta and the spherical where we are having the R, theta and Phi. So the choice of the coordinate system depend on the geometry we define. Along with this the classification of the reservoir simulation model are also done on the types of the reservoir fluid. So the reservoir simulator could be the gas simulator which is good for simulating the gas process. Black oil simulator this is

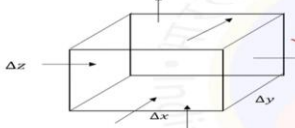
good for simulating the primary and the secondary recovery process because the recovery process are not sensitive to the compositional changes in the reservoir fluid.

It means it does not require the information of the composition of the fluid only the physical properties of the fluids are important. Third one is compositional simulator it is very sensitive for the compositional changes happening with respect to the fluid that is considered in the simulation so the compositional simulator is also there. Based on the recovery process the conventional recovery process simulator that is good for the primary and secondary recovery process such as water and gas injection. The simulator could be specifically setup or design for the ER process specifically for the chemical flood simulator ASP alkali surfactant polymer waste process system where the mobility ratio displacement or mobilization of the residual oil is considered and then the miscible displacement process that include the gas and oil system could also be simulated with this kind of the model. Thermal recovery simulator use the heat equation or the energy equation to simulate the process so the steam flooding or the institute combustion process can be simulated with this thermal recovery simulator these kind of the simulator use an equation of energy conservation in addition to the mass conservation and momentum conservation equation.

### Reservoir Simulation

**Three Dimension**

**3D Single Phase Flow Equation**



Mass conservation

$$-\frac{\partial}{\partial x}(\rho u_x A_x) \Delta x - \frac{\partial}{\partial y}(\rho u_y A_y) \Delta y - \frac{\partial}{\partial z}(\rho u_z A_z) + \frac{q}{\alpha_c} = \frac{v_B}{\alpha_c} \frac{\partial(\phi \rho)}{\partial t}$$


Momentum conservation

$$\frac{\partial}{\partial x}(\beta_x A_x k_x \frac{\partial P}{\partial x}) \Delta x + \frac{\partial}{\partial y}(\beta_y A_y k_y \frac{\partial P}{\partial y}) + \frac{\partial}{\partial z}(\beta_z A_z k_z \frac{\partial P}{\partial z}) + q_{tc} = \frac{v_B}{\alpha_c} \frac{\partial(\phi \rho)}{\partial t}$$

having the derivative with respect to X, Y and Z here is the flow rate at the standard

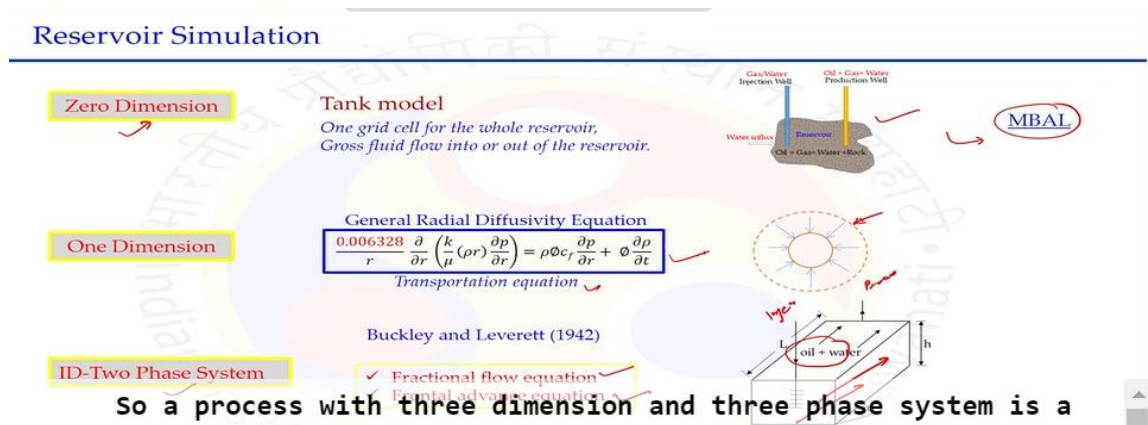
**Laws Governing the Flow of Fluid in a Reservoir**

- ✓ **Mass conservation**  
Continuity equation:  $\frac{\partial}{\partial t}(\phi \rho_\alpha) + \nabla \cdot (\rho_\alpha \vec{U}_\alpha) = 0$
- ✓ **Momentum conservation**  
Darcy's law:  $u = -\frac{k}{\mu} \frac{\partial P}{\partial x}$
- ✓ **Constitutive Equation**  
Equation of State:  $c = \frac{1}{\rho} \frac{\partial \rho}{\partial p}$



We already discussed some of the cases of reservoir simulation but not in these computational language but in a simple language those can also be simulated in the computer model. For example zero dimension model we also called this a tank model this is a one grid cell for the whole reservoir it is considered a fluid is present in a tank because of the pressure difference the fluid is getting produced we do not consider the flow equation we just consider the fluid at a time t within the tank volume after the production what remains in the tank after passing time t. So this is a representation of that tank model and this is zero dimension model can be simulated or dedicated software called the M-BEL there are several other simulator also there you can also write your own model equation because this is based on the linear equation that help us to understand the primary recovery mechanism process present in the reservoir domain. Then we discuss about the one dimensional radial diffusivity equation that include the transportation equation also for the radial flow condition could be for other flow

condition also be set up then we discuss about the one dimensional but two phase system given by the Buckley and Labot that is having the two set of the equation fractional flow equation and the frontal advance equation where the oil and water two phase are flowing in one direction that could be from the injection well to production well. Now when it comes to reservoir simulation actually in the computer model this is a 3D and 3 phase system so oil, gas and water are present there and then the fluid can move in all direction XYZ direction or R theta Z direction and that make the problem more complex.



So a process with three dimension and three phase system is a very complex process let us consider the case of three dimensional single phase flow equation. So this is the representation of the fluid flow in all three dimensions XY and Z the equations or the law governing the flow of fluid in the reservoir could be given by the mass conservation equation that is the continuity equation momentum conservation equation including the Darcy law for the flow and the constitutive equation that could be the equation of a state or the definition of isothermal compressibility coefficient. So these three equations are going to represent the physical system in the three dimensional for a single phase flow system. When we talk about just a single phase flow system the mass conservation equation for the three dimensional will take this shape where the fluid is flowing X direction Y direction and then the Z direction also. Accordingly the other parameters are adjusted the momentum conservation equation is also having the derivative with respect to X, Y and Z here is the flow rate at the standard condition and this is the properties of the fluid on the change with respect to time.

Now this is for the single phase flow system when we are talking about the three phase flow system we have to write the conservation equation for each phase. Here in this equation beta C is representing the transmissibility conversion factor that is like this and the alpha C is showing the volume conversion factor that could be 5.615. We already discussed these values when we adjust the unit of the parameter they come into the



picture. Now in this equation if you see we are having the properties of the fluid we are having the properties of the reservoir that is in terms of the permeability and then the porosity also here and the equation is saying fluid flow of a single phase is happening in three dimension.

## Reservoir Simulation

### Types of Fluid

### Three Dimension

#### Single-Phase Flow Equation for

##### Slightly Compressible Fluids

$$\frac{\partial}{\partial x} \left( \beta_c \frac{A_x k_x}{\mu B} \frac{\partial P}{\partial x} \right) \Delta x + \frac{\partial}{\partial y} \left( \beta_c \frac{A_y k_y}{\mu B} \frac{\partial P}{\partial y} \right) \Delta y + \frac{\partial}{\partial z} \left( \beta_c \frac{A_z k_z}{\mu B} \frac{\partial P}{\partial z} \right) \Delta z + q_{sc} = \frac{V_b c}{\alpha_c B^0} \frac{\partial P}{\partial t}$$

##### Compressible Fluids

$$\frac{\partial}{\partial x} \left( \beta_c \frac{A_x k_x}{\mu B} \frac{\partial P}{\partial x} \right) \Delta x + \frac{\partial}{\partial y} \left( \beta_c \frac{A_y k_y}{\mu B} \frac{\partial P}{\partial y} \right) \Delta y + \frac{\partial}{\partial z} \left( \beta_c \frac{A_z k_z}{\mu B} \frac{\partial P}{\partial z} \right) \Delta z + q_{sc} = \frac{V_b}{\alpha_c} \frac{\partial}{\partial t} \left( \frac{\phi}{B} \right)$$

##### Incompressible Fluids

$$\frac{\partial}{\partial x} \left( \beta_c \frac{A_x k_x}{\mu B} \frac{\partial P}{\partial x} \right) \Delta x + \frac{\partial}{\partial y} \left( \beta_c \frac{A_y k_y}{\mu B} \frac{\partial P}{\partial y} \right) \Delta y + \frac{\partial}{\partial z} \left( \beta_c \frac{A_z k_z}{\mu B} \frac{\partial P}{\partial z} \right) \Delta z + \mu q_{sc} = 0$$

So based on the fluid we can set up the equation when we talk about the multi phase flow equations.

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Now when we talk about the types of the fluid the equation shown in previous slides need to be modified so for example same case single phase flow equation for the slightly compressible fluid the right hand term would be like this. The compressible fluid this will still remain Phi and beta G will still remain inside the derivative of time while in case of the incompressible fluid the change in pressure with respect to time or time dependent parameter here will be 0. So based on the fluid we can set up the equation when we talk about the multi phase flow equations we will be writing the same thing continuity equation for oil water and gas we will be writing the momentum equation for each phase oil water and gas here the relative permeability of that phase will come into picture. So the 3D flow equations would be like this okay and then this is the equation for the oil component this is the equation for the water component and this is the equation for the gas component. So the equation for a single phase system shown in the previous slide will be further modified for each phase we have to write for the oil water and gas component.

## Reservoir Simulation

### Boundary and Initial Conditions

- Initial condition (pressure and saturation conditions)
- Boundary conditions

Dirichlet condition:  $p = p_1(x)$

Neumann condition:  $\frac{\partial p}{\partial x} = p_2(x)$

Mixed boundary condition:  $ap(x) + b \frac{\partial p}{\partial x} = p_3(x)$

Cauchy boundary condition:  $u(x) = \text{constant}$  and  $\frac{\partial u(x)}{\partial x} = \text{constant}$

- For 1D single phase flow Dirichlet boundary condition is given by:  $P(x=0, t > 0) = P_i$  OR  $P(x=L, t > 0) = P_r$
- A pressure condition: bottomhole pressure of a production or injection well.
- In Neumann boundary condition, the flow rates at the end faces of the system are specified.
- Using Darcy's equation the conditions become: interest that is called the Neuman boundary condition:  $\frac{\partial p}{\partial x} = 0$  at  $x=L$

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Now you can see in the gas component there are two terms for each system here okay and this would be dy this would be dz and here you can say this is the gas free gas

Multiphase Flow Equations

Continuity equation for each fluid phase flowing:  $-\frac{\partial}{\partial x}(A\rho_l u_l) = A \frac{\partial}{\partial t}(\phi\rho_l S_l) \quad l = o, w, g$

Momentum equation for each fluid phase flowing:  $u_l = -\frac{k k_{rl}}{\mu_l} \frac{\partial P_l}{\partial x} \quad l = o, w, g$

3D Flow Equations

> Oil component  $\frac{\partial}{\partial x}(\beta_c k_x A_x \frac{k_{ro}}{\mu_o \theta_o} \frac{\partial P_o}{\partial x}) \Delta x + \frac{\partial}{\partial y}(\beta_c k_y A_y \frac{k_{ro}}{\mu_o \theta_o} \frac{\partial P_o}{\partial y}) \Delta y + \frac{\partial}{\partial z}(\beta_c k_z A_z \frac{k_{ro}}{\mu_o \theta_o} \frac{\partial P_o}{\partial z}) \Delta z = \frac{V_D}{\alpha_c} \frac{\partial}{\partial x}(\frac{\partial S_o}{\theta_o}) - q_{osc}$

> Water component  $\frac{\partial}{\partial x}(\beta_c k_x A_x \frac{k_{rw}}{\mu_w \theta_w} \frac{\partial P_w}{\partial x}) \Delta x + \frac{\partial}{\partial y}(\beta_c k_y A_y \frac{k_{rw}}{\mu_w \theta_w} \frac{\partial P_w}{\partial y}) \Delta y + \frac{\partial}{\partial z}(\beta_c k_z A_z \frac{k_{rw}}{\mu_w \theta_w} \frac{\partial P_w}{\partial z}) \Delta z = \frac{V_D}{\alpha_c} \frac{\partial}{\partial x}(\frac{\partial S_w}{\theta_w}) - q_{wsc}$

> Gas component  $\frac{\partial}{\partial x}(\beta_c k_x A_x \frac{k_{rg}}{\mu_g \theta_g} \frac{\partial P_g}{\partial x}) \Delta x + \frac{\partial}{\partial y}(\beta_c k_y A_y \frac{k_{rg}}{\mu_g \theta_g} \frac{\partial P_g}{\partial y}) \Delta y + \frac{\partial}{\partial z}(\beta_c k_z A_z \frac{k_{rg}}{\mu_g \theta_g} \frac{\partial P_g}{\partial z}) \Delta z = \frac{V_D}{\alpha_c} \frac{\partial}{\partial x}(\frac{\partial S_g}{\theta_g}) - q_{gsc}$

component and this is the gas that is dissolved in the oil and in each direction x, y and z we will get these two terms similarly we will get these terms in this part also. Now once we set up the equation of the conservation including the constitutive equations we need to solve this equation and for that the boundary and initial condition need to be specified. The initial condition could be the pressure and the saturation condition given at time t is equal to 0 the boundary conditions are classified into four parts so this is in general for any numerical problem where the boundary condition could be of four types Dirichlet boundary condition for our case it is the pressure so P is specified at a particular location that is called the Dirichlet condition. So the value of the variable at a particular location is called the Dirichlet condition. Newman condition is the second one that is the derivative of that variable with respect to position is specified. So the derivative boundary condition is called the Newman boundary condition when we combine both the condition with some factor A and B that is called the mixed boundary condition and when both the boundary conditions are specified like the value of that parameter and then the derivative of that parameter at a particular location that is called the Cauchy boundary condition in our case that could be pressure here this could also be pressure.

So for 1D single phase flow the Dirichlet boundary condition will be given like this at position X is equal to 0 the pressure value is PL while at X is equal to L the pressure value is PR what does it mean for example we are having the one dimensional flow this is our left position this is our right position at X is equal to 0 here the length is L for example here the pressure is PL here is pressure PR that is the way we can define and this boundary condition is called the Dirichlet boundary condition because the value of the parameter of interest is defined at particular location. A pressure condition for example the bottom hole pressure of the production or injection well at R is equal to RW is defined that is P is equal to PWF that is an example of the Dirichlet boundary condition. In Newman boundary condition the flow rate at the end phases of the system are specified so we can define the flow rate here we can define the flow rate here and flow rate means we are having some term that is having the derivatives in our Darcy equation we get this pressure difference or pressure gradient we can define this is at X is equal to 0 we can define this at X is equal to L when we are defining the derivative of

the parameter of interest that is called the Newman boundary condition. The example is Darcy equation at  $X$  is equal to 0 and  $X$  is equal to  $L$  after defining the boundary condition we need to discretize the system so the discretization with finite difference method is shown here there could be finite volume method finite element method and some other a numerical method used to obtain an approximate solution for a given boundary value problem. Linear system of equation are solved using the Gaussian elimination methods once we discretize the system we make them the linear equation those linear equation can be solved with a appropriate mathematical tool for example Gaussian elimination method.

The finite element methods works on a large number of discretization element and also on different kinds of grid within the domain. So this is the advantage of the FT method finite difference method it can be used for different size different shape it can easily handle complex geometry and also provide good result for the coarse grid. Coarse means we are not having very fine the size of the grid is little large compared to the fine size there are various size of the grid from very fine to fine to coarse system it means the accuracy can be compromised but the simulation can be done in a lesser time frame. So that is actually the engineers choice where the simulation time can be reduced. So the discretization can be point discretization system for example we are having this point that can be discretized in a point distribution system each point is having certain node through which the fluid is flowing and we can define the location as  $I$  and  $J$  notation.

Similarly there could be the block center grid so our grid is here that is within this block that is  $I$  block and the other one is  $I + 1$  block the point between this is half of this so the location of  $X$  is  $I + 1$  by 2 or  $J - 1$  by 2 if it is  $I$  if it is  $J$ . So here the point distribution grid system is shown for the regular shape that is the corner point or could be the center point but it could be for a very different regular or very well defined shape for example triangle and other shape. The methods based on truncated Taylor series can be utilized to have the different schemes for discretizing by the finite difference method. So for example forward difference method where the derivative between  $I$  and  $J$  is used using this formula  $H$  is actually the distance between  $I$  and  $J$ . Backward difference where the backward step is used to calculate the value of the derivative.

## Reservoir Simulation

### Discretization with Finite Difference Method

- > A numerical method used to obtain an approximate solution for a given boundary value problem.
- > Linear system of equations are solved using Gauss elimination method.
- > The finite element method works on a large number of discretization elements and also on different kinds of grids within the domain.
- > It can easily handle complex geometries and also provides good results for a coarse grid.

#### Method based on truncated Taylor Series

##### Forward Difference

$$\left. \frac{\partial u}{\partial x} \right|_{i,j} = \frac{u_{i+1,j} - u_{i,j}}{h}$$

##### Backward Difference

$$\left. \frac{\partial u}{\partial x} \right|_{i,j} = \frac{u_{i,j} - u_{i-1,j}}{h}$$

##### Central Difference

$$\left. \frac{\partial u}{\partial x} \right|_{i,j} = \frac{u_{i+1,j} - u_{i-1,j}}{2h}$$

$$\left. \frac{\partial^2 u}{\partial x^2} \right|_{i,j} = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2}$$

partial differential equation to linearize the system. So the discretization using finite



Block-centred Grid



So for example here U I J we are calculating we are calculating this is U 1 plus so if we are calculating forward means we are taking this U I plus 1 minus this value divided by H in the backward we take the previous value so this U I J minus U I minus 1, J. So in forward we take this side in the backward we take this side while in the central forward we take both the sides to calculate the value of derivative at this point we take forward point as well as backward point. And similarly for the second derivative can also be done so all these are based on the truncated Taylor series these approximation can be substitute for differential in the partial differential equation to linearize the system. So the discretization using finite difference approximation of spatial derivative for a uniform grid here we are talking about uniform grid as I mentioned the grid size could be non uniform also in some section where the physics is more important we can go to have the very fine gridding where the physics is not very complex it can simulate easily within the numerical stability we can go to coarse grid system. So let us consider this uniform grid system if we are having no boundary conditions means we are having just 1, 2, 3, 4 this is not a boundary system where we are not considering this block and this block with no finite difference approximation for time derivative we are not talking about the time derivative of finite difference approximation also in that case the 1D single phase flow equation for the compressible fluid let us take example that is given by this can be discretized using the spatial derivative.

## Reservoir Simulation

### Discretization using finite difference approximation of spatial derivative for a uniform grid

- ✓ With no boundary condition
- ✓ With no finite difference approximation of time derivative



1D single phase flow equation with a compressible fluid:

$$\frac{\partial}{\partial x} \left( \frac{B_c A_c k}{\mu B_l} \frac{\partial p}{\partial x} \right) \Delta x + q_{isc} = \frac{V_b \phi c_l}{\alpha_c B_l} \frac{\partial p}{\partial t}$$

Discretization using spatial derivative:

$$\left( \frac{B_c A_c k}{\mu B_l \Delta x} \right)_{i+\frac{1}{2}} (P_{i+1} - P_i) - \left( \frac{B_c A_c k}{\mu B_l \Delta x} \right)_{i-\frac{1}{2}} (P_i - P_{i-1}) + q_{isc} = \left( \frac{V_b \phi c_l}{\alpha_c B_l} \right) \frac{\partial P_i}{\partial t}$$

$$T_{ix, i+\frac{1}{2}} (P_{i+1} - P_i) - T_{ix, i-\frac{1}{2}} (P_i - P_{i-1}) + q_{isc} = \left( \frac{V_b \phi c_l}{\alpha_c B_l} \right) \frac{\partial P_i}{\partial t}$$

$$\text{Where, } T_{ix, i+\frac{1}{2}} = \left( \frac{B_c A_c k}{\mu B_l \Delta x} \right)_{i+\frac{1}{2}} \text{ and } T_{ix, i-\frac{1}{2}} = \left( \frac{B_c A_c k}{\mu B_l \Delta x} \right)_{i-\frac{1}{2}}$$

equation. The transmissibility between two blocks is the measure of how easily fluids flow between them.

Spatial derivative means we are talking about the position not with respect to time so in this case what will happen this will remain same here for example and now this part that

was here can be put like this so this is  $I + 1$  minus  $I$  so forward position is taken  $I + 1$  and minus the current position and in this case what we are talking about the center grid system so the location of the point is for example  $X$  or  $I$  then we are talking about  $I + 1$  by  $2$  so this is  $I$  this is  $J$  it will be  $X$  is equal to  $I + 1$  by  $2$  or  $X$  will be equal to  $J$  minus  $1$  by  $2$ . So in this case we can get the discretization of the spatial derivative we are not talking about the time derivative so the pressure can be discretized with respect to this forward position with respect to this backward position and similarly this equation can further be written like this where this term can be represented by this  $T$  part here the pressure difference here the pressure difference so with respect to spatial derivative we could convert this equation into linear algebraic equation. And now the part that we did not consider is the time derivative that can also be discretized here this  $T$  that is appearing in this equation is called the transmissibility between the two blocks that is actually the measure of how easily fluid flow between them. So we are talking about this from point 1 to point 2 how easy it is flowing is considered by this factor  $T$  that can be substituted here and that is actually this part within the equation. So now we could discretize the system in linear one dimensional equation for the 1D single phase flow equation without boundary.

Now if boundary condition are also imposed there for example the newman type of the boundary condition where the pressure derivative is constant we can do this at boundary condition so that boundary condition here what is the value here is  $1$  minus half. So this is  $1$  minus half or it could be done here at point 5 that is actually  $4$  plus half so we are talking about here and in that case the pressure derivative can also be written like this so  $\frac{\partial P}{\partial X}$  will be  $P_1$  at this point minus  $P_0$  at this point divided by  $X_1$  minus  $X_0$  so this is  $X_1$  this is  $X_0$  condition. Similarly for the other boundary condition this is  $X_4$  this is  $X_5$  so at this condition this will be  $P_5$  minus  $P_4$  divided by  $X_5$  minus  $X_4$  or we adjust this equation this will give us the linear relationship for the pressure with respect to position. So we can discretize the system converting the partial differential equation into linear equation. So after defining the physics setting up the boundary condition choosing appropriate discretization system and then using the appropriate numerical scheme or the computer algorithm we can simulate the reservoir process.

Now there are various reservoir simulator software available in the market those are classified as the commercial software those are actually based on the integrated reservoir modelling system where they are providing lot of the information with respect to development of the field. Those commercial software are CMG computer modelling group suits which is having the three simulator IMAX for the black oil simulator, JAM for the compositional waste simulator and STARS that is the thermal simulator. Schlumberger is software suit is also including Eclipse, petrol, intersect they are having their own features how they are discretizing the system what types of the physics can be defined and what numerical scheme is chosen to make the iteration and the final result

can be simulated. Baker Hughes is having the software called the dual suits, Halliburton is having the landmark nexus. Not only the commercial software there are certain open source software also there those are kind of free but those are having the basic function not the integrated reservoir modelling features.

The open source software can be modified as per the need so the black oil applied simulation tool also called the BOST based on the IMPS schemes is there MRST that is also a free software and open porous media that is open source code that can be utilized to develop the reservoir models and simulate the process. This is not the comprehensive list for the commercial software also there are some more commercial software available similarly on the open source also there could be some more user friendly software could be available. So the classification of the simulator as we seen earlier the black oil simulator and the compositional simulator they are based on the type of the reservoir fluid. The thermal recovery process or chemical polymer flooding process they are based on the recovery process so the simulator can also be classified. So for example jam here is actually the compositional simulator that is based on the type of the reservoir fluid star is recovery waste process simulator that is used for the thermal process simulation.

This list is showing some of the study already performed in the literature considering for a case of nano fluid. So I said the simulator can be set up for different application your objective should be well defined. Now you can define the physics according to the objective chosen. Now this list is showing the study conducted by some of the researcher recently so if you see all these papers are within one decade they simulated the process using different software for more detail you can refer these articles. So what outcome we get from this reservoir simulation development we can get the information about the individual well as well as for the average value for the reservoir field.

A reservoir field could include many injection and the production well in different configuration so what value we get for the individual well and the average field pressure as a function of time at a particular location. Production profile with time that could be the cumulative production profile could be daily, monthly, weekly or annual production profile and for each phase we can get for oil, water and gas. Water and gas injection rate that we are injecting water cut and then the gas oil ratio with respect to time at what rate the water is getting produced what is the value of the water cut at appropriate time all these information can be obtained by developing the simulator with different information that is required. So water cut or gas oil ratio with time can also be obtained. Spatial distribution of oil, gas and water within the reservoir domain with respect to time spatial distribution of pressure as a function of time can also be obtained by simulating the reservoir.

This is not a comprehensive list many more features of the reservoir simulation tools are utilized for development of the field. Here the example I am showing you if we are

developing the reservoir with a distributed porosity and the permeability that can be done so instead of using the constant value as we do in our HENDS calculation we can have a distribution within the reservoir domain the value could be distributed for the porosity and permeability in each grid as we discretizing we can assign the value of the porosity and permeability. The outcome could be the distribution of the pressure within this porous permeable region similarly the velocity profile within this porous permeable region can be obtained. So these are the input parameters those we can provide very complex input parameter and this is the output parameter those we can obtain with respect to time as well as position within the reservoir domain. So the developed reservoir simulation tool can help us to understand various features for example model sensitivity to estimated data we are having the data estimated from the field we can check those by matching with the simulation data how model is sensitive to capture the physics that is defined it may require some additional data need to be provided for the model to modify the model with a better accuracy.

The developed model can be used for estimating project life recovery with respect to time comparing different recovery processes within the same reservoir domain plant development or the operational changes those are required for the field development and overall maximizing recovery of the oil in the economic manner can be achieved with well established reservoir simulation tool. So when it comes to validation of the reservoir simulation often the simulation tools validate the model against the data obtained from the well controlled experiments conducted at the lab scale. As the reservoir is a subsurface phenomena happens in a large scale we can do the lab scale experiment up to certain validation purpose only and then the lab scale experiments are broadly classified at the microscopic level or the lab scale where we are using the core. So in the microscopic study we consider mask or replica of the reservoir in a very small domain we visualize the flow how the flow is happening under different condition in the lab scale we also recovered the oil but the visualization is often not done. So let us say in the lab scale core flooding experiment we are having this core holder where the core is placed that could be of certain diameter and length usually 1 to 2 inches in diameter and 4 to 10 inches in length and that is representing our reservoirs as I said reservoir is wide but we are trying to mimic within this dimension of the core sample and that core is placed here inside this core holder now we are having the pressure measurement device at the inlet and outlet and we are pumping at a constant flow rate the fluid in this core.

So first we try to replicate the reservoir condition means we first flood with the water then with the crude oil let the water saturation crude oil reach up to the saturation condition then we start flooding with the different schemes that could be alkali surfactant and polymer followed by the chase water or could be the gas injection also be performed in this kind of the system. So this core flooding data can give us the permeability relative permeability value oil recovery value and those data can be utilized for



understanding the performance of the simulation model that is developed. If there is a mismatch then certain things need to be modified with respect to the scaling up process with respect to defining the physics more accurately in the reservoir simulation tool. So there are certain features those cannot be captured with this core flooding data setup we need to go to even smaller scale to understand how exactly the flow through porous media is happening when multi phase flow system is present that could be done by the microscopic study. In this microscopic study a porous media of regular shape is fabricated with certain beads within the single dimension domain and it is placed below the inverted microscopic camera.

Now we are having the same scheme we are injection pump injecting the fluid flowing through this porous media observing the recovery or the discharge is collected. Now what is happening within this region can be captured with the help of the camera that gives us more accurate physics or physical phenomena those actually happening at the pore scale. So this is kind of the system microscopic analysis beacon design with the kind of PDMS sheet where this regular or irregular this picture is showing the regular shape of the rocks is assembled in terms of the obstacle. So these are actually the rock and then the space between this is the porous and permeable path you can see. And then we can get the information by flooding the water and oil or the chemicals in a logical sequence we can see how the oil is getting displaced from this porous and permeable region.

So the microscopic or the macroscopic data along with the field data are often utilized to compare the performance of the reservoir simulation tool. So in summary I would like to say the reservoir simulation is an engineering problem which needs physical models to define the problem as accurate information or as more as information we can provide to this one that will help us to develop the more accurate model at the end. The engineering problem needs some mathematical models those can be supplied by conservation laws or constitutive equations along with the conservation laws. We need to do the numerical modeling by discretizing the system and then finally the computer algorithm is written to get the engineering solution of the problem. Why I am saying the engineering problem because we need to keep iterating keep adjusting the parameter to get the model that is able to predict the reservoir performance accurately model can be used for forecasting the prediction profile.

Accurate mathematical representation of a system for the physical phenomena is very important for example we did not consider in previous case the energy conservation equation if we are simulating the thermal process of course energy conservation equation need to be included. Other physical processes those we did not consider if those are happening there we need to modify the equation for example when we are injecting the chemical the IFT value the adhesion model also need to be included in the developed



mathematical equation either modifying the conservation law equation or supplying this information as a constitutive relationship. At the end history matching is required means validating the performance of the reservoir and then it is done the prediction means the forecasting of the reservoir production is achieved with the reservoir simulation tool and that help us to understand the production profile of a particular reservoir field. So in the literature it has been compare we also discuss this method for example we discuss the volumetric method that is the zero dimension model we discuss the material balance equation that is the 1D model and we also discuss another mathematical tool that is decline curve analysis with respect to time how the flow rate is declining and now we discuss in today's lecture about the reservoir simulation. Now what are the uses and the accuracy of this model is compare in this table so for example the exploration of the hydrocarbon which one could be useful only the volumetric method could be useful but for that also we need lot of geophysical geological and the regional trend data are required for utilizing this tool for the exploration purpose.

What about with respect to discovery again only the volumetric method could be useful but again with the certain information for the discovery purpose while the decline material balance and reservoir simulation are not applicable for the discovery and for the exploration. Delineation we are having the volumetric that can be used and reservoir simulation can also be utilized for that purpose. Development of the field volumetric could be used decline in material balance cannot be used while the reservoir simulation is good for the development of the field. The important feature of the reservoir simulation is in the production volumetric can also give us but with a good accuracy decline can give us with the fair accuracy material balance fair to good while the reservoir simulation can predict the production profile with a good to very good range. So the reservoir simulation tool are very good when we are talking about the production profile.

So with this I would like to end my today's lecture in next lecture we will discuss the last component of our slavers that is the unconventional natural gas production I would be considering unconventional hydrocarbon production and more emphasis would be on the natural gas production. With this I would like to end my today's lecture thank you very much for watching the video we will meet in the next lecture thank you. .