

Petroleum Reservoir Engineering

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Lecture 18: Secondary Oil Recovery Methods

Hello everyone. Welcome to the class of petroleum reservoir engineering. In today's lecture, we are going to continue our discussion about the petroleum reservoir system. Now, we enter in section 3. We already completed the slavers of section 1 and section 2. And in this section 3, we are going to discuss the improvement in the oil recovery from the reservoir in the form of secondary recovery and in the process of tertiary recovery that is also we called enhanced oil recovery.

In today's class, mostly we will focus on secondary recovery process and in that secondary recovery process also our focus will be on water flooding. In the next lecture, we will discuss briefly about different ER schemes followed by in the next week introduction to reservoir simulation and then a lecture on some unconventional source of the hydrocarbon fluid. So in this lecture 1 of week 7, we will discuss about secondary recovery process. When we look around the crude oil production system, crude oil production from a reservoir well happens in three stages.

Primary oil recovery, that oil recovery mostly happen because of the naturally pressure is there in the reservoir domain that is pushing the fluid towards the production well and then followed by from production well to the surface. This happens by the naturally occurring pressure into the reservoir domain. Around 5 to 10 percent of the oil is recovered means the original oil in place out of that OOIP up to 5 to 10 percent is recovered by this natural process only. And then there is a need to maintain the pressure in the reservoir domain to recover the remaining oil. For that purpose, the secondary oil recovery processes are implemented in the reservoir domain to recover the additional oil.

In the secondary oil recovery process, the fluid is injected into the reservoir domain by either converting one of the production well into the injection well or drilling a new injection well that is the decision made by the administration in terms of the profitability of the reservoir production process. By any mean one is the injection well, another is the production well. In the injection well, the water or the gas is injected into the reservoir domain and that injected gas or the water is pushing the oil towards the production well. This is done just by maintaining the pressure in the reservoir domain. So the interaction of the injected fluid that we also call the displacing fluid is not happening with the displaced fluid that is the crude oil in the reservoir domain.

From the secondary oil recovery process also we get from 15 to 30 percent additional oil recovery only and after this maintenance of the pressure still be left significant amount of the crude oil in the reservoir domain. Together primary and secondary account for 30 to 40 percent of original oil in place. In the tertiary oil recovery process also called the enhanced oil recovery process, What we do? Different methods are implemented in the system not just only to maintain the pressure or pushing the oil towards the production well from its original position but also altering the properties of either the crude oil or the interaction between the crude oil and the rock that we will discuss in detail in the next lecture when we will be covering different process of enhanced oil recovery. Those could be the miscible flooding. In miscible flooding we inject the gas but the gas is injected above the minimum miscibility pressure means the interaction of the crude oil and the gas is considered both are miscible and then they are forming a miscible front churn.

The difference between the secondary oil recovery process and then the miscible flooding process in the tertiary recovery is that in the secondary recovery process we are injecting below the minimum miscibility pressure means the gas is injected into the reservoir domain but that still remains as the gas phase. The primary purpose of injection of the gas is not the interaction but the force or the pressure that is maintained in the reservoir that is pushing the oil towards the production well. Your processes are also there chemical, thermal, microbial we will discuss those in detail later on. The recovery with this tertiary or enhanced oil recovery process ranges from 5 to 30 percent depending on lot of the factor. Together secondary and tertiary oil process are called the IOR or improved oil recovery processes.

If we look the crude oil production in terms of the flow rate and then the time we can say the primary recovery reaches to certain production range and then it is start declining and that is the time we need to decide when to start the secondary process. Secondary means injecting the water or the gas into the system. The time to decide the secondary process is very crucial and lot of the factor needs to be considered in that. At the end of the secondary process also we need to decide the time when the tertiary process should be implemented. So, both the time when to start the secondary process when to start the tertiary process are very important.

There is a concept also instead of doing the secondary process we can directly go to the tertiary process. But the cost of the operation involved is the deciding factor. In general after the primary recovery process secondary recovery process are implemented and instead of waiting the end of the primary to start the secondary similarly end of waiting for secondary to start the tertiary process are not considered. Before the process primary is reached to its end point we start the secondary. Similarly we do not wait secondary to end we start the tertiary process and those depend on lot of the factor primary factor is the

saturation of the oil in the reservoir domain how much oil is still remain and at what rate that oil can be produced to the surface.

If we see the oil recovery data with respect to time primary will give us the cumulative oil recovery in this manner. And instead of waiting that I said we can start the secondary process and we can see the difference in the primary and secondary process recovery. Of course, the primary is the natural process secondary will cause something so the difference in the additional oil recovery will be judged based on the cost involved in the process. Similarly, from the secondary to tertiary we will get additional oil recovery again the method that can be implemented into particular reservoir well and the cost will determine the time and the implementation of the tertiary process. Otherwise after the primary well if we are not including the secondary and tertiary process the well is abundant but that well is having significant amount of the crude oil remains in the reservoir domain.

So let us briefly recall about the primary recovery process this is the first stage of the production in which the natural reservoir energy such as the gas drive, water drive, gravity drainage displace hydrocarbon from the reservoir into the well bore and then further to the surface. We discussed in detail about various types of the drive mechanism those are naturally present in the reservoir domain for the crude oil system as well as for the gas well system. For the crude oil there are six types of the drive mechanism those are responsible by maintaining the pressure in the reservoir domain to produce the oil to the surface. Each reservoir is composed of different types of the geological formation, porosity, permeability, pore on thickness are different these are the physical characteristic of the reservoir. Hence the implementation the primary drive mechanism that is responsible for the production may vary from reservoir to reservoir.

Each drive mechanism has certain key marker those are utilized to understand the drive mechanism ultimate recovery, pressure decline rate, gas oil ratio, water production. So we have discussed all these factor in detail in our previous lectures. The primary recovery mechanism is important to understand because it help us to understand or estimate the hydrocarbon reserve that is present in the reservoir domain. It also help us to understand the future production profiles of the reservoir domain at what rate the fluid can be produced in the future as well as it help us to understand what type of the EOR methods or the IOR method can be implemented in the reservoir domain to get the better recovery of the oil. When it comes to the secondary recovery process as I mentioned over the time of the well the pressure declines and some point there will be insufficient underground pressure to force the oil to the surface.

So the oil is there but the driving force the pressure is not there and that is where the secondary processes are implemented. The secondary processes are also called the pressure maintenance recovery process. In these processes we inject the water or we

inject the gas. That gas could be the natural gas, could be CH₄, could be nitrogen, could be CO₂ depending on the cost and availability and of course the reservoir characteristic. So the diagram showing here for the water so we are including the water into reservoir domain by injecting the water through the injector well.

So now we can classify our wells, production well those are producing the reservoir fluid and then the injector well through which we are injecting the substance or the fluid into the reservoir domain. So here this is our injector well through which we are injecting the water into the domain and then the production well is giving us the oil. It will give us the oil as long as the water is not reaching towards the production well and when it is reaching then we will also get the water produced at this production well. So what happens if we see in this diagram the water is injected at injection point and then the front of the water start moving towards the production well. Now between the injection and production there is oil so this water will provide the pressure to push that oil towards the production well.

Now a phenomena that is called the viscous fingering happens in the water injection process that happens because of the viscosity of the crude oil that remains in the reservoir is higher than the viscosity of the water and the mobility of the water is faster than the mobility of the oil. We will discuss that what is mobility and mobility ratio. In general we can say because of the difference in the viscosity of the water and the crude oil remain in the system the water will start finding out the easy path or form the channel bypass the oil and reach the production well significantly earlier that is called the water breakthrough and before that the viscous fingering phenomena happens in the reservoir domain of course lot of the factors are there the characteristic of the reservoir the characteristic of the fluid both displacing fluid that is the water and the displaced fluid that is the oil. So the kind of the fingers or the easy channels find by the water to move from injection well to production well called the phenomena that is viscous fingering. So the factor that is characterized the viscous fingering is the mobility ratio that is the ratio of mobility of the displacing fluid that is water to the mobility of the displaced fluid that is the oil.

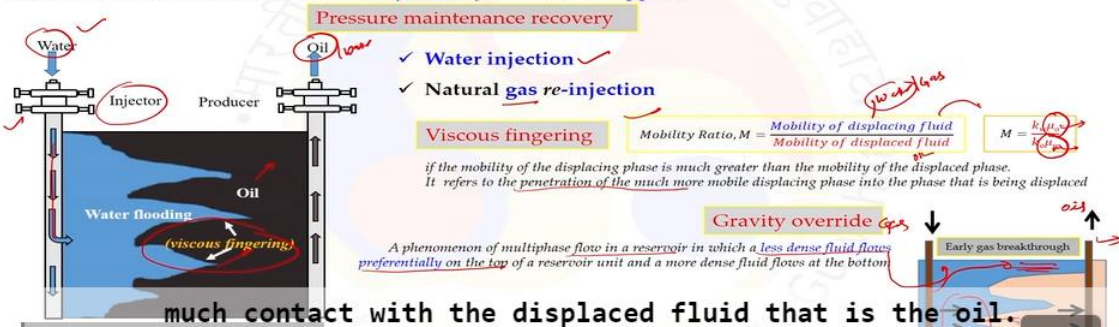
Now our fluid of the displacing fluid could also be the gas as we are discussing about the water in this diagram let us focus on the water. So the expression for the mobility ratio will turn out like this by placing the definition of the mobility for both the fluids. In this expression we can say viscosity of the oil is on the numerator and the viscosity of the water is in the denominator. Of course the permeability for the oil and water within this reservoir domain are also the part of the mobility ratio. But if we see with respect to the viscosity if the viscosity of the water is not higher than the viscosity of the oil the mobility ratio will be greater than 1 and that is the unfavorable condition.

So the water will be injected it will bypass the oil reach the production well and it is not contributing significantly towards the oil production. Hence to have a better recovery by the water injection the mobility ratio should be lesser than 1 but as we know the viscosity of the oil remain in the reservoir after the primary process is higher than the viscosity of the water that is why the viscous fingering happens. But still some of the water that is injected push the oil towards the production well and we get the recovery by the secondary process. The mobility ratio can be improved. Improved means we can create the situation when the mobility ratio is lesser than 1 means we are having the favorable condition that could be done by changing the viscosity of the water that is we are injecting or by changing the viscosity of the oil.

Crude Oil Production

Secondary Recovery Process

Over the lifetime of the well the pressure will fall, and at some point there will be insufficient underground pressure to force the oil to the surface. Then secondary recovery methods are applied.



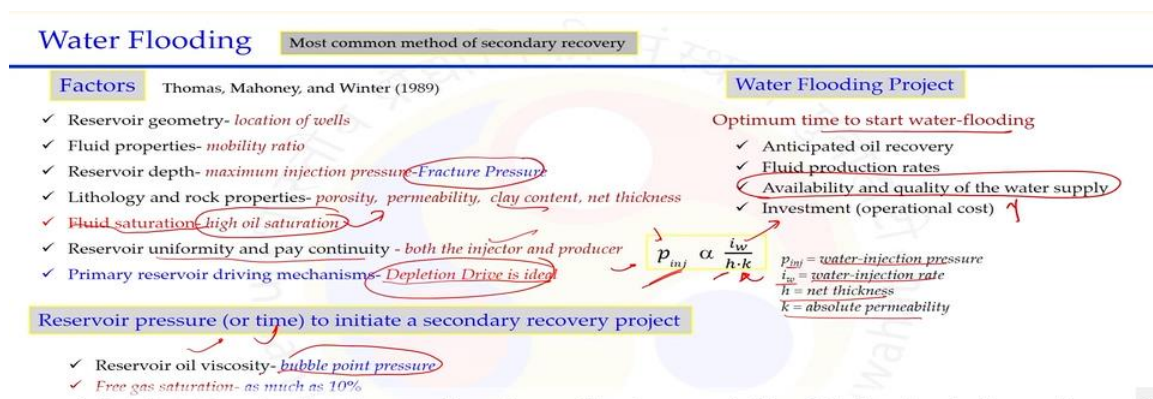
Those phenomena we will discuss in the ER process. So the viscous fingering happens in the water injection. Similarly for the gas injection gas is lighter than the oil by the density. So when we are injecting the gas in this reservoir the gas will settle on the top what the oil will be at the bottom because of the gravity and then the gas will find easy path to travel from the injection well to the production well on the top of the page on thickness. And that is where the early break breakthrough of the gas will happens when we are injecting the gas although some gas will try to push the oil towards the production well but this is the advantage of the gas injection that is where the gas shows the early breakthrough.

In the gas injection this phenomena of the early breakthrough or the setting of the gas at the top of the oil just passing from injection well to the production well without contacting the oil or without pushing the oil is called the gravity override. So this is a phenomena of multi-phase flow system in which a less dense fluid flow preferencedly on the top of the reservoir. So our gas is less dense that flow on the top of the reservoir and it is not having much contact with the displaced fluid that is the oil. Other important features or the parameter that characterize the performance of the secondary oil recovery process or assess the success of the water flooding process or the gas injection process is

the location of the injector and producer wells. Of course there could be many wells in the reservoir some are converted to the injection wells some are still under the production well.

If needed more injection well are also drilled in the reservoir domain to have the better connection between the injection and the producer wells. Other than the location it also depends on the properties of the oil and characteristic of the reservoir rock the success of the secondary recovery process. So let us say the water flooding process we can discuss in more detail that is the most common method for the secondary process. The factor those characterize the water flooding process or determine its success are the reservoir geometry means location of the well, how many injection wells are there, how many production wells are there, what is the arrangement of the injection well and production well has been selected for the water flooding. Fluid property as discussed the mobility ratio is important parameter.

Reservoir depth as we are having the reservoir at a high depth means we have to inject the water at a very high rate and in that case the pressure of the reservoir we have to inject the water at a high pressure and then the pressure may exceed the fracture pressure of the rock that is present in the reservoir domain. So hence the care should be taken it is not fracturing otherwise the fracture may be in any direction in the reservoir domain and the crude oil may migrate through this fracture to the other nearby zones. Lithology and rock properties like the porosity permeability, age on thickness, the types of the geological formation all will matter to decide the water flooding. Another is fluid saturation that is very important for the water flooding as we discussed in the primary recovery process there are several recovery process and at the end they leave the residual oil remain in the reservoir. So if the oil saturation in the reservoir domain is high that is the favorable conditions for the water flooding because if the large quantity of the oil is there then the percent of the water flooding recovery may be improved.



Similarly the oil saturation is high then the residual oil saturation is a parameter that characterize the water flooding success. So the favorable condition is having the high oil

saturation in the reservoir domain and that happens in the depletion drive case where the significant amount of the oil remains in the reservoir and that is the ideal case for implementing the water flooding process in the reservoir domain. Of course, the uniformity and the pay continuity are also important both for the injector and the producer what are the types of the porosity and permeability near the injection well as well as the production well. In general the pressure of the injection of the water is defined by this proportionality relationship so if you want to inject the more water or the injection rate should be high the injection pressure will also be high. If the reservoir domain is having less permeable region near the well bore then the injection rate or the injection pressure should be high to inject the same amount of the water and the similar proportionality with the permeability on thickness.

So P_{inj} is the water injection pressure i_w is the water injection rate h is the net thickness and k is the absolute permeability of the reservoir domain near the well bore. So when we decided to have the water flooding project the optimum time to start the water flooding again depends on the reservoir conditions like what is the anticipated oil recovery from that particular well or the reservoir domain what is the fluid production rate that can be achieved from that water injection flooding and the availability and the quantity of the water supply when we are injecting the water in terms of the pore volume injection very significant amount of the water is required and the subleakage must be ensured continuously to implement the water flooding scheme and of course the investment the total cost of the project will be the deciding factor. So the reservoir pressure or the time to initiate the secondary recovery project will be determined by the reservoir oil viscosity the reservoir is at bubble point pressure it is above or below bubble point pressure because if the reservoir is above bubble point pressure it means it is having only the oil phase while if it is below the bubble point pressure it might have the gas phase also along with the oil phase. Free gas saturation is required as much as 10 percent is there this will be the favorable condition for the water injection now these two are contradictory and many other parameter are contradictory to each other when we are talking about the favorable condition for the water injection and then the non favorable condition for the water injection. So for example the depletion drive that will be taken out the gas out of it the only oil remains or the high content of the oil saturation is required for the water flooding but it is also observed the free gas saturation is also important to pressurize the system.

Cost of the injection equipment productivity of the producing well at what rate those can produce and of course the overall life of the reservoir how long that reservoir is going to perform under the secondary recovery process and at how much quantity of the oil can ultimately be produced from that particular reservoir. So some parameter are contradictory as I said about it hence the optimization of the various factor is required in order to develop the most favorable conditions for the overall economy of the water

flooding scheme. If we list out the properties of the flute, rock and then the reservoir properties are required to implement the water flooding so the understanding of all these properties are required to optimize the process. So let us discuss in detail about the location of wells in the next slide. So the location of injector and the producer wells are very important to have the contact of the displaced fluid and the displacing fluid together.

So the arrangement of injection and the production wells depend primarily on the geology of the formation and then the size of the reservoir. Maximum possible contact with the crude oil is the aim of choosing the pattern. The geometry of the common pattern I can listed out here. So the maximum possible contact with the crude oil of the displacing fluid is the objective of the water flooding pattern and that is where the specific pattern should be chosen to decide which are going to be the injection well which are going to be the producer wells. There are four types of the well arrangement peripheral, regular, irregular and crustal and vessel type of the arrangement those could be arranged in the reservoir field and the arrangement is also characterized by the ratio of producer to injector wells in the arrangement.

So let us say the triangle is showing the injection well and then the circle is showing the production well. This is the peripheral flooding arrangement of the wells. So let us consider this is our reservoir field where we are having the several production well. The production well at the peripheral of the reservoir field can be converted to the injection well and this is very common and then the effective process because from all the peripheral injection well the displacing fluid is in the contact of all the remaining wells those are still under the production condition and the efficient contacting between both the fluids displacing fluid and the displaced fluid can be achieved by the peripheral flooding. When it comes to the regular flooding there could be the arrangement of different ways this arrangement is called the regular four spot.

In this case there are three injection wells and then the one is production well at the center. So every arrangement is having the three injection well and the one production well. The injection well is contributing towards all the directions if we see this is our injection well this is also contributing towards this production well, this production well, this production well, this production well, this production well and this production well. So around six production wells are covered by this one injection well. But when we look in the regular four spot pattern the production well is one and this is supported by three injection well in the triangle shape.

The second is skewed four spot in this also similar arrangement three injection wells are supporting one production well and then the shape is little bit skewed. Five spot is the arrangement where the four injection wells are supporting one production well and total we are having four injection and one production well that is why it is a five spot and then the arrangement end of like the square shape. Seven spot we are having the arrangement

one production well supported by six hexagonal shaped injection well and remember this injection well is also supporting the other one. So the production well is supported by six injection well while each injection well may be supporting some other production well in the field. In inverted seven spot we are having one injection well in the center this is supporting six production wells those are in the hexagonal shape.

So the regular or normal means we are having these production well in the center while in the inverted we are having the injection well in the center. Similarly, we can see for the normal nine spot and inverted nine spot system. There could be the arrangement in the direct drive line arrangement where all the production well are in single line at certain distance in the same line the injection wells are aligned so they are supporting these production well in both the directions. And then arrangement is production well injection well production well injection well. Staggered line drive in this case the injection wells are not aligned with the production well while they are at a certain distance and the distance vertical distance is actually the half of the distance of these two production wells.

So if distance is d the vertical position far away from this point but this point will be d by 2 and this is the same manner we are having the production well injection well production well injection well only difference is the injection wells are not aligned they are somewhere at the center point with certain distance. There could be some other cases called the monoclinical arrangement where the oil is on the upper side and water is injected in the lower domain and then the one side we are having only the injection well another side we are having only the production wells. Anticlinic could be where we are having the surrounding by the water injection well at a depth and then the center arrangement is or the production well. So this is like we are having the injection well and then this are the production wells. So the arrangement could be in many irregular shape also most of them I saw on here are the regular shape.

Irregular shape also could be planned those again depend on the geological structure of the reservoir and the fluid and reservoir characteristic. The value of the production and injection ratio for different cases. So for example normal 4 spot we are having the 2 ratio 1 inverted 1 ratio 2 regular spot 1 for the 5 cases and inverted 5 is having the 1 regular 7 is having 1 ratio 2. So the ratio is also important to identify how many total wells in the domains are converted into the injection well and how many wells are still under the production condition. The ratio could be greater than 1 or could be lesser than 1 depending on the reservoir conditions.

So if we see physically what happens in the water flooding this is the use of the

injection of water to the production well. Let restrict our discussion to only one injection well and one production well. So what happens at the injection well we injected the water into the domain that movement of the water from the injection to the production will be determined by the characteristic of the reservoir as well as the fluid characteristic. Fluid means what is the viscosity of the water that is injected and what is the viscosity of the crude oil and the reservoir characteristic means the permeability primary factor how much relative permeability for the oil and relative permeability for the water. After certain time if we see from this condition to this condition the front moves little bit further as the time passes.

So the water is moving towards the production well now the more viscous fingering phenomena is happening. The front that is having the easy channel that will move faster than the front that is not having the easy passes and the movement is characterized by the mobility ratio and this mobility ratio should be around 1 for a good condition. Favorable condition is of course lesser than 1 but that the viscosity of the oil is higher than the viscosity of the water. A good condition for the water flooding could be when it is around 1. After certain time you will see the front that was leading it will reach preferably faster than the other viscous fingering point in the injection to the producer wells.

So which is having the front leading faster rate will reach to the production well because it is having the easy path. And in that case the sum of the area is left un-swept that is not still in the contact of the water that is injected through the injection well. So, the recovery efficiency of the water flood is largely a function of the sweep efficiency. How much this water injected is in the contact of the oil and how much it is able to push it or sweep it towards the production well. And then the ratio of the viscosity of the oil and water that is I mentioned in terms of the mobility ratio.

So if the reservoir oil is extremely viscous then the mobility ratio will likely be much greater than 1 and that is the unfavorable conditions. When this is the case the water will finger so it will find out the channels and this kind of the finger phenomena will be seen through the reservoir and bypass much of the oil. So the recovery by the water flooding can be characterized by the overall recovery efficiency parameter. This definition of overall recovery efficiency is valid for secondary recovery as well as tertiary recovery process.

For tertiary we will discuss separately. So the overall recovery factor also denoted by RF of any secondary or tertiary oil recovery method is the product of a combination of three individual efficiency factor. What those three individual efficiency factors are ED, EA and EV. EA and EV collectively called the volumetric sweep efficiency. So in fact

RF is a product of ED and the volumetric sweep efficiency. RF is actually the cumulative oil produced divided by the NS that is initial oil in place.

So we can adjust this equation mathematically in the form of cumulative oil produced. Again, this part will remain same here. So RF is the overall recovery factor. NS initial oil, NP cumulative oil produced all are measured at STV condition.

ED displacement efficiency. Now the heterogeneity is there in the reservoir. So, the aerial sweep efficiency and vertical sweep efficiency will be different. It means how the water is contacting the crude oil in the aerial condition and in the vertical conditions that is why the efficiency of the aerial sweep and then the vertical sweep will be different hence both are independently accounted in the calculation of the recovery factor. As I mentioned the product will be the volumetric sweep efficiency. So instead of having the individual we can also characterize this with the volumetric sweep efficiency.

Water Flooding

Secondary or Tertiary Oil Recovery

Overall Recovery Efficiency

The overall recovery factor (efficiency) RF of any secondary or tertiary oil recovery method is the product of a combination of three individual efficiency factors

$$RF = E_D \cdot E_A \cdot E_V$$

$$N_p = N_s \cdot E_D \cdot E_A \cdot E_V$$

$$\text{Volumetric Sweep efficiency} = E_A \cdot E_V$$

RF = overall recovery factor ✓
 N_s = initial oil in place at the start of the flood, STB ✓
 N_p = cumulative oil produced, STB ✓
 E_D = displacement efficiency ✓
 E_A = areal sweep efficiency ✓
 E_V = vertical sweep efficiency ✓

The displacement efficiency E_D is the fraction of movable oil that has been displaced from the swept zone at any given time or pore volume injected. Because an immiscible gas injection or water flood will always leave behind some residual oil, E_D will always be less than 1.0.

The areal sweep efficiency E_A is the fractional area of the pattern that is swept by the displacing fluid. The major factors determining areal sweep are:

• Fluid mobilities • Pattern types • Areal heterogeneity • Total volume of fluid injected

So in the vertical sweep efficiency similar vertical

So the displacements efficiency ED is a fraction of mobile oil that has been displaced from the swept zone at any given time for the pore volume injector. So we are injecting the water into the porous region and then the time we are evaluating the displacement efficiency that is actually how much oil is displaced from its original position. Because an immiscible gas injection or the water flood will always leave behind some of the residue that is why the ED will always be lesser than 1. So the recovery factor will always be lesser than 1.

In the case of water flood it is viscous fingering. In the case of immiscible gas it is the gravity override. Those are not have the process very efficient. The aerial sweep efficiency EA is the fractional area of the pattern that is served by the displacing fluid. It means how much area is contacted by this water or gas injection. The factor determine the sweep efficiency in the aerial direction, fluid mobility, types of the flow pattern distribution, aerial heterogeneity and then the total volume of the fluid injected into the

system.

Similar for the vertical sweep efficiency EV it is the fraction of the vertical section of the page on thickness. So this is our page on thickness. This is the vertical position how much it is contacted by the water and this is aerial position how much it is contacted by the displacing fluid in the aerial direction. So in the vertical sweep efficiency similar vertical heterogeneity degree of the gravity segregation now because the vertical position will be having gravity into the picture fluid mobility and then the total volume injected similar to the aerial sweep efficiency will also be the factor in the vertical sweep efficiency to determine the recovery by these two efficiency factors.

So let us discuss displacement efficiency in little bit detail. Buckley and Labouard developed a well-established theory called the frontal displacement theory to understand the displacement mechanism. Mathematically the displacement efficiency or the ED is defined as volume of the oil at the start condition. Oil remains means how much total oil swept out from this region divided by the volume of oil at the start of the flood. If we write this in the mathematical term the initially SOI is the oil saturation in the reservoir and at that condition the volume for mission factor for the oil is BOI. So, this is the ratio that is saying the volume of the oil at the start condition.

Water Flooding
Secondary or Tertiary Oil Recovery

Displacement Efficiency

$$E_D = \frac{\text{volume of oil at start of flood} - \text{Remaining oil volume}}{\text{Volume of oil at start of flood}}$$

$$E_D = \frac{S_{oi} - \bar{S}_a}{S_{oi}}$$

Assuming a constant oil formation volume factor during the flood life

$$E_D = \frac{S_{oi} - \bar{S}_a}{S_{oi}}$$

initial oil saturation

$$E_D = \frac{S_w - S_{wt} - S_{gt}}{1 - S_{wt} - S_{gt}}$$

in the swept area, the gas saturation is considered zero


$$E_D = \frac{S_w - S_{wt}}{1 - S_{wt}}$$

Buckley and Leverett (1942) developed a well established theory, called the frontal displacement theory

S_{oi} = initial oil saturation at the start of flood
 B_{oi} = oil FVF at start of flood, bbl/STB
 \bar{S}_a = average oil saturation in the flood pattern at a particular point during the flood

\bar{S}_w = average water saturation in the swept area
 S_{gi} = initial gas saturation at the start of the flood
 S_{wi} = initial water saturation at the start of flood

So now we got the mathematical definition of ED if we know the saturation of the water



Similarly the volume remain at the conditions can be given by SO by BO. SO is put as the bar means average value and divided by the same factor here the original volume of the oil at the start of the flooding. Now we can have certain assumptions to simplify this equation if we assume a constant oil formation factor during the flooding life. So the formation factor is not changing means BO and BOI remains same that is where the BOI and BO will cancel out from this equation and we will get simple equation of the ED in terms of the oil saturation. Now oil saturation is difficult to measure we can convert this initial oil saturation to some other form that is the saturation of the water and then the saturation of the gas. We know the summation of the saturation is equal to 1 and in that case we can place this SOI and SO using this definition we will get the ED in terms of water saturation as well as the gas saturation.

If we further assume in the swept area from where the oil is getting displaced the gas saturation may be considered 0 in that case only the oil and water are present in the reservoir domain and then the definition of ED will be simplified that is average saturation of the water saturation of the water at the initial condition and this is $1 - SW_i$ in the denominator. So this would also be subscript WI and all these are subscript actually. So now we got the mathematical definition of ED if we know the saturation of the water every saturation of the water in the reservoir domain we can calculate the displacement efficiency of the flooding process. The water flooding arrangement could be in different form as discussed previously also we did not discuss the two well patterns we can see here the crystal and the vessel pattern. So, let us say these are our production well we may inject the gas or the water the injection can be done above let us say this is the inclined surface and we are having the place where we can convert this upside well as the injection well.

This condition called the crystal well pattern where the injection well are at a higher height compared to the production well and the vessel flow pattern calls when the injection wells are below in terms of the height of the producer wells. So in general the gas injection is always done in the crystal well pattern and then the water injection is preferable when we are having the vessel well pattern because of the gravity segregation will also assist in the flooding. Now the position of the well consider this is a linear displacement we are having the production well here and then the injection well here we are considering the water injection it could be the gas injection also. Now the inclination of this linear displacement system is having the angle α from the horizontal position. If the α is in this direction this is called the up dip flow means the injected fluid and then the oil will move from injection well towards production well in this direction.

So let us say the length is L height is H this is a linear system where the cross sectional area is A the oil is in this domain water is injected both oil and water will be moving upward that is why it is called the up dip flow. If we reverse the position these if we make this as a injection well and this is a production well in that case α will be negative and then this condition called the down dip flow condition. So to understand the mechanism of the movement of the oil in the water flooding or the injection flooding system of two immiscible displacement fluid that could be the water and oil or gas and oil both are under the immiscible condition. Buckley and Leavitt proposed the displacement mechanism theory and that theory is widely accepted and implemented to understand the immiscible displacement process. This is a one dimensional mathematical system for two phase system designed for the linear system.

So not for the radial system we discussed like several cases the Buckley and Leavitt given this theorem for the linear system for the two phases both the phases are immiscible in the nature so that is only for the one dimension. So let us say if there is no

dip angle this is the way the horizontal linear system is there which is having the cross-sectional area A the movement is happening in this direction this is our X direction this is injection well this is production well and both oil and water will move towards the production well like this. This X is equal to 0 this is X is equal to L in this case only fluid is considered to flow in the one dimension and that is the X direction this also called the unidirectional flow of the system. So the Buckley and Leverett developed a well established theory called the frontal displacement theory for characterizing the immiscible displacement of two phases. This classic theory consists of two equations fractional flow equation and then the frontal advance equation.

Immiscible Displacement Processes

One dimension mathematical linear system of two-phases

Buckley and Leverett (1942) developed a well established theory, called the frontal displacement theory.

This classic theory consists of two equations:

- ✓ Fractional flow equation
- ✓ Frontal advance equation

Fractional flow equation (Leverett 1941)

For two immiscible fluids, oil and water, the fractional flow of water, f_w (or any immiscible displacing fluid), is defined as the water flow rate divided by the total flow rate, or:

$$f_w = \frac{q_w}{q_t} = \frac{q_w}{q_w + q_o}$$

f_w = fraction of water in the flowing stream, water cut

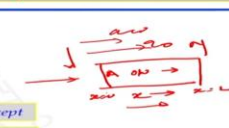
q_t = Total flow rate, $\frac{bbl}{day}$

q_w = water flow rate, $\frac{bbl}{day}$

the water cut in the well.

water cut equation

The Buckley and Leverett Displacement Mechanism



Displacement based on the relative permeability concept

Gravitational and capillary forces may be neglected

The displacement based on the relative permeability concept and then the gravitational capillary forces may be ignored we will discuss first considering the gravity and capillary forces and later on when we neglect this term how the equation will take the shape. So the gravity primarily comes with the dip angle if there is no dip angle gravity can be ignored and capillary pressure is when the two phases are immiscible both the phases will experience different pressure and the difference between wetting and non-wetting phase pressure is the capillary pressure. If it is not significant we can ignore it but we will see in general and see how capillary pressure can be more accurate in certain cases and when we can ignore it. So let us see the frontal flow equation developed by Leavitt in 1941 it is a very simple equation for the two immiscible fluid let us consider oil and water we consider oil and gas also the fractional flow of water or any immiscible displacing fluid could be the water or could be the gas is defined as water flow rate divided by the total flow rate or mathematically so simple F_w that is the fraction flow of the water is the flow rate of water and then the flow rate of total oil and water into the system. So the Q_t can further be written as Q_w plus Q_o so this is both oil and water are flowing so this is Q_w this is Q_o now we can implement Darcy law to understand the flow of the water and the oil in the linear system.

Here the terminology is very simple F_w is the fractional of water V_{bl} per V_{bl} Q_t is the total flow rate in per day Q_w and Q_o for the water and oil flow rate. This equation that is very simple equation also called the water cut equation that determines the water cut in the well. So let us consider this immiscible displacement process as I already mentioned

this is the cross sectional area A this is X equal to 0 we are having X equal to L length dip angle is α so the cross sectional area could be measured in feet square X is the distance in feet this is dip angle and if $\sin \alpha$ is positive means we are having the up the flow conditions if negative then we are having the down dip flow conditions. Both the up dip and down dip are having certain advantage and disadvantage we will discuss later on. This is the way we are having the arrangement for the linear system of two immiscible fluid oil and water in the reservoir domain.

Now we can make certain assumption assuming a homogeneous system we can implement the Darcy law. So the Darcy law properties are included actually for the two phase system we already discussed the concept of the relative permeability if we include the relative permeability in the Darcy law this equation called the modified Darcy law. So we can write the modified Darcy law for oil and water system considering the effective permeability converted to relative permeability that we will do later on. So this is the simple equation for the oil this is the effective permeability for the oil this is the effective permeability for the water. Cross sectional area we are keeping constant we are saying the cross sectional area is same for oil and water they are having the competition to flow or the percent of the area shared by the oil and water will be characterized by the permeability or effective permeability.

So let us say for the oil we are having this modified equation where we consider this is the pressure gradient experienced by the oil this is the pressure experienced by the water viscosity of each fluid are included the additional term is here compared to previously what we discussed about the Darcy law is because of the gravity term that is because of the tilting of the linear system or having some slope. Now this equation for the oil we can readjust it taking the μ and a k_o on the other side so this is one part and then the other part we are having the pressure gradient as well as the gravity part. Similar we can write for the water equation also and when we substrate both the equation we are going to get the simple part. So this subtracted by this left hand side is so simple right hand side is also we club these two terms in the bracket and this term is so simple. Now this is the density difference between the displacing fluid and the displace fluid and this is the difference in the capillary pressure.

So the difference experienced by the oil layer and then the water at the interface we can characterize this by the capillary pressure and the we discussed in detail about the capillary pressure in one of the previous lecture. So the wetting and non-wetting phase could be characterized like this we can take the derivative with respect to the position when the fluid is moving from this direction to this direction we can take the position x and then the derivative will give us the change in the capillary pressure with the position. So this part can be replaced by the simple this term and we got this equation. The terminology is as usual so I am skipping this defining different terms again and again.

Now this density difference is simply density of the displacing fluid to the density of the displace fluid.

So the same equation I had written here for the oil and water same terminology mentioned here what we can do we can use this water cut equation to develop the equation that will be more in terms of the properties are known or in terms of the fractional flow or the water cut F_w . So we know this water cut equation by the definition of this fractional flow water we can have the similar equation for the oil this will become $1 - F$ because we know this Q_o plus Q_w will be equal to Q_t total flow and we know this fractional flow of oil and water will be equal to 1. So we can either express in terms of fractional flow of oil or the fractional flow of water. Let us choose fractional flow of the water because it characterizes the equation that is we called water cut equation. So if we substitute this Q_w here and Q_o here and adjust the equation we are going to get the expression in the form of F_w this part will remain here you can see this is the remain part only adjustment is happening in this form and what we are going to get the equation in terms of Q_t .

Immiscible Displacement Processes

water cut equation

$$q_w \mu_w - \frac{q_o \mu_o}{A k_o} \left[\frac{\partial p_c}{\partial x} - g \Delta \rho \sin(\alpha) \right]$$

$$q_w = f_w q_t \quad q_o = (1 - f_w) q_t$$

$$f_w = \frac{1 + \left(\frac{k_o A}{\mu_o q_t} \right) \left[\frac{\partial p_c}{\partial x} - g \Delta \rho \sin(\alpha) \right]}{1 + \frac{k_o \mu_w}{k_w \mu_o}}$$

In Field Unit system

$$f_w = \frac{1 + \left(\frac{0.001127 (k_o) A}{\mu_o q_t} \right) \left[\frac{\partial p_c}{\partial x} - 6.433 \Delta \rho \sin(\alpha) \right]}{1 + \frac{k_o \mu_w}{k_w \mu_o}}$$

If the gravity and capillary forces are not taken into account, the fractional flow of the injected fluid varies only with pressure and saturation

$$f_{inj} = \frac{1}{1 + \frac{k_o \mu_w}{k_w \mu_o}}$$

Oil

$$q_o = -\frac{k_o A}{\mu_o} \left[\frac{\partial p_o}{\partial x} + g \rho_o \sin(\alpha) \right]$$

Water

$$q_w = -\frac{k_w A}{\mu_w} \left[\frac{\partial p_w}{\partial x} + g \rho_w \sin(\alpha) \right]$$

f_w = fraction of water (water cut), bbl/bbl
 k_o = effective permeability of oil, md
 k_w = effective permeability of water, md
 $\Delta \rho$ = water-oil density differences, g/cm³

q_t = total flow rate, bbl/day
 μ_o = oil viscosity, cP
 μ_w = water viscosity, cP
 A = cross-sectional area, ft²

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So we are replacing Q_w and Q_o in this equation by Q_t and then the expression will be in the form of total flow of the fluid that is include for the oil and water. So this is very general equation that can say the fractional flow or how much water flow rate per total flow rate in the form of F_w this is including the capillary pressure this is including the gravity this is including the permeability for the oil and water effective we can convert this effective to relative also and then we are having the fluid properties in terms of viscosity cross sectional area is also into the picture. Now this equation can be written in the field unit system if we do so we will get this numerical factor at both the places we discuss this in detail how to get this numerical factor when we convert into US field unit system. If the gravity and capillary forces are not taken into the account so if we are saying this term is not taken into the account both the phases are experiencing the same pressure the wetting phase and non-wetting phase are not having the capillary effect at the interface and then there is no gravity means the plane is horizontal in the if we cancel out these two term what we are going to get a simple equation this F_w this is F_w it is written as F_{inj} because the injection fluid could be the water or could be the gas

and we will get here this term is 0 simply 1 in the numerator and in the denominator. So we are having the permeability and the viscosity of both displacing fluid and then the displaced fluid injection is actually the displacing fluid that is the gas or the water.

Now this simple equation is saying the relationship how the viscosity of the two phases can affect the fractional flow of the injected fluid within this linear system from injection welds to the production weld. Let us further we can simplify this equation if we are having the same equation without ignoring the capillary and the gravity term the equation will be here in this form for the USU field unit system we can convert this ratio of the effective permeability to the relative permeability we know the fractional flow can be characterized either in the terms of F_w or F_o the summation of both will be equal to 1. So for the two phase flow system the flow rate Q_t is essentially equal to the injection rate so we are having the injection rate and then the amount of the injection of the water is actually the pore volume that we are injecting into the system and that is moving so the flow rate Q_t that is the total flow rate we can assume is equal to the injection rate if we do so this Q_t that is appearing here can be replaced by the injection rate. So that is done here and only difference from this equation to this equation is this term I_w is included in terms of Q_t .

Immiscible Displacement Processes

Buckley and Leverett (1942)

$\frac{k_{ro}}{k_{rw}} = \frac{k_o}{k_w}$
 $f_o + f_w = 1$
 $f_o = 1 - f_w$

For two-phase flow, the total flow rate q_t is essentially equal to the injection rate i_w .

$i_w = q_t$

a more generalized form to describe the fractional flow of any displacement fluid

$$f_D = \frac{1 + \left(\frac{0.001127(kk_{rD})A}{\mu_o I_D} \right) \left[\frac{\partial p_c}{\partial x} - 0.433 \Delta \rho s n(\alpha) \right]}{1 + \frac{k_{rD} \mu_D}{k_{rD} \mu_o}}$$

* water injection rate, i_w * water viscosity, μ_w * direction of the flow, i.e., updip or downdip injection when the displacing fluid is immiscible gas

$$f_g = \frac{1 + \left(\frac{0.001127(kk_{rD})A}{\mu_o I_D} \right) \left[\frac{\partial p_c}{\partial x} - 0.433 (\rho_g - \rho_o) s n(\alpha) \right]}{1 + \frac{k_{rD} \mu_g}{k_{rD} \mu_o}}$$

is in the range of 0 to 1.

i_w = water injection rate, bbl/day
 f_w = water cut, bbl/bbl
 k_{ro} = relative permeability to oil
 k_{rw} = relative permeability to water
 k = absolute permeability, md

The effect of capillary pressure is usually neglected because the capillary pressure gradient is generally small

$$f_w = \frac{1 + \left(\frac{0.001127(kk_{rD})A}{\mu_o I_w} \right) [0.433(\rho_w - \rho_o) s n(\alpha)]}{1 + \frac{k_{rD} \mu_w}{k_{rD} \mu_o}}$$

i_g = gas injection rate, bbl/day
 μ_g = gas viscosity, cP
 ρ_g = gas density, g/cm³

$$f_g = \frac{1 + \left(\frac{0.001127(kk_{rD})A}{\mu_o I_g} \right) [0.433(\rho_g - \rho_o) s n(\alpha)]}{1 + \frac{k_{rD} \mu_g}{k_{rD} \mu_o}}$$

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So this is I_w water injection and other terms are having the similar meaning. So a more generalized form to describe the fractional flow of any displacement fluid can be written F_t is the displacing fluid where this is the density difference for the displacing fluid to the pile and then it will depends on water injection rate, water viscosity, direction of flow this is the characterized by the alpha, up dip or the down dip and that is the way the fractional parameter F_t for the displacing fluid can be calculated by knowing the relative permeability of the displacing fluid and then the injection rate of the displacing fluid. When the displacing fluid is the immiscible gas remember the gas is below the minimum miscibility pressure it is in the immiscible condition only for the secondary recovery process. Same equation can be converted instead of D we can write the G wherever the subscript was coming. So this is the way we can write the expression for F_g . The effect of capillary is usually neglected because of the capillary pressure gradient is generally very small.

The equations for F_w by ignoring the gravity term by ignoring the capillary term will be simplified to this part similarly for the gas. Similarly part is we ignore the capillary pressure and then the sign of this change because this was negative here so when it is taken out this will become negative. It also implies like the value of F_w and F_g could be maximum up to 1 because this 1 minus something and divided by 1 plus something that will make sure the F_w or the F_g value is in the range of 0 to 1.

We will discuss that later on. So let understand this relationship between the saturation and water cut. What do you know saturation and water cut? In this equation when we are calculating the fractional flow we are seeing the relative permeability is coming into the picture and that relative permeability is a function of the saturation of that particular fluid. So to calculate this F_g we can know certain viscosity of the both the phases tilt angle density but how to know the relative permeability that is the part of the saturation and that saturation is changing from the injection well towards the production well how this front is moving from injection well towards production well at different time the front will be at different location and that is where the saturation will be the different. So this F_w is actually a function of K_{rw} or K_{ro} both are the function of saturation and that saturation is a function of position as well as time. As I mentioned this is our injection well this is our production well when we injected water this water will be moving and when we are seeing the system at a particular time the water movement could reach up to certain distance x only. So that is where the T and x both determine the location of the waterfront of course we are having the viscous fingering also there that is where the water saturation is calculated as a average water saturation in the domain at particular time in particular location but we need to know this S_w to calculate the relative permeability.

So before going to understand that let us understand the relation between the saturation and then the water cut we know the water cut that is F_w could have the limit between 0 to 100. So either no water is getting produced or only the water is getting produced. So let us say this relative permeability curve that we discussed in detail in the two phase relativity diagram where this F_w can vary from this position to this position the value will be varying from 0 to 1 but the saturation of the water if we plot on the x axis from 0 to 1 the movement of the fluid will be there between this range only that is we call the conic water saturation or residual oil saturation. So these are the boundaries between this boundary only the fluid will be moving and accordingly the relative permeability of the oil and relative permeability of the water can be read out with this kind of the chart by knowing the value of the water saturation. We can convert water saturation to the oil saturation also because oil saturation plus water saturation will be equal to 1.

So similarly in this diagram also the oil movement will be here very high because mostly the oil is there the saturation of oil is high this is the conic water saturation and

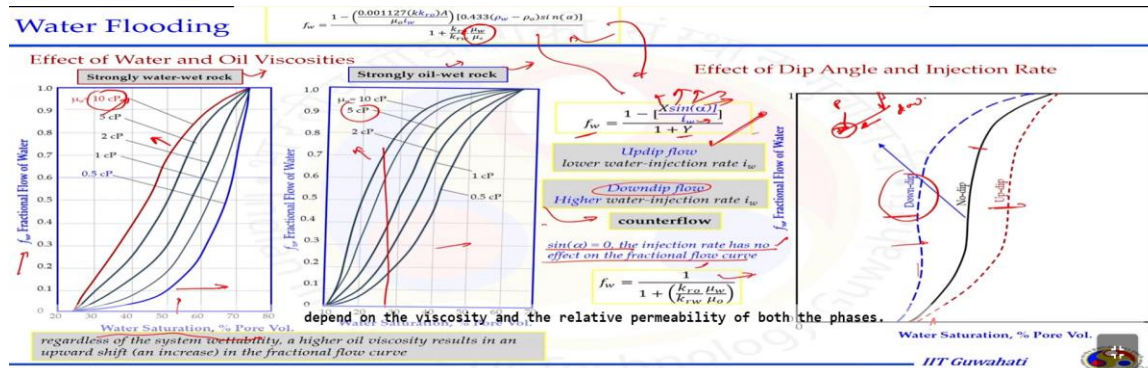
when the water saturation increases the permeability of the water will improve and it will keep improving as the saturation is increasing there will be a condition when it will become maximum that is the residual oil saturation condition. So let us say what we discussed previously also we can recall that detail at the irreducible or conic water saturation at this point the water flow rate Q_w is 0 because there is no permeability or relative permeability for the water therefore the water cut is 0 and that is we are seeing here at this condition this is 0 water condition. Second as the residual oil saturation S_{OR} reached the oil flow rate is 0 the water cut reaches its maximum limit. So at the residual oil saturation oil is not moving only moving fluid is the water and that is where the value of water cut will reach to 100 percent because only the water is moving and only the water will be produced that is this condition and that happens suddenly when the reservoir well is producing suddenly there is water cut reach to very high value let us say 80 percent 90 percent because the front has moved from injection well towards production well and then this is the condition when the front reach the production well suddenly now most of the water is getting produced. The shape of the water curve versus water saturation curve is characterized by the S shape that is here and then the limit of the F_w will be between 0 to 1 and defined by the relative permeability curve.

So these are the end point of relative permeability curve. The objective of the water flooding is reduce the water fractional flow we do not want the water is just reaching very fast towards the production well it should spend time it should push the oil and then the front of the water should not be moving with the faster rate. So the reduce the water fractional flow is the objective by adjusting certain parameter this can be done by altering the viscosity of the water injected formation dip angle water injection rate on the water cut or some of the parameter those control the movement of the water frontal phase from injection wells towards the production well. Similar things we can discuss for the gas also so the similar analysis the S_g value will also vary from 0 to 1 depending on the permeability nature of both the fluid it means the relative permeability of oil and gas in the reservoir domain. So when we use the same equation to understand the more features of the water flooding let us see the effect of water and oil viscosity. So the oil and water viscosity are coming into the picture of this expression to calculate the F_w we can see the effect of water and oil viscosity for two cases when the system is strongly water wet and then the system is strongly oil wet.

In this case the strongly water wet system we will see the trend is like this the water saturation or the percent of the pore volume versus the fractional flow rate of the water shifted towards the higher saturation side compared to the strong water wet system. And when we are varying the viscosity of the oil if the more viscous oil is there the water front will move towards the production well or achieve the value 1 at very low water saturation. So the water saturation is not high still the water is having the more preference or the fractional flow of the water is more compared to the oil. So that is not

the favorable condition favorable conditions are when the viscosity of the oil is less but that is not the case. So if the very high viscous fluid then the viscous fingering will be more and then the by passing of the water from injection well to the production well will be faster.

While in case of the strong oil wet system the movement of the water will be much faster compared to the oil and we will get even at a low saturation the fractional flow value is high even for a not very high viscous oil. So regardless of the system water wet or the oil wet the higher oil viscosity result in upward shift in the fractional flow curve and that is undesirable. So if the more viscous oil is there the water flooding will be less effective. Similarly for the effect of the dip angle we can say the fractional flow will be achieved at a low saturation if it is down dip it will take time if it is up dip and when there is no dip the situation would be in between. What does it mean if we modify this equation in the simple form taking everything as X other than the dip angle and similarly in the denominator we are taking 1 plus Y we can simplify this equation like this.



So the fractional flow F_w is depending on $\sin \alpha$ divided by the injection rate also there. So let us say if up dip flow means the positive value of the $\sin \alpha$ then the lower water injection rate is more preferable in the case of the up dip flow condition. While in the down dip flow condition higher water injection rate is more preferable because in this case up dip flow the fluid will be traveling against the gravity and then the even the lower water injection will be preferable and then the fluid will not moving fast towards the production well. While in the down dip even at a lower saturation the fractional flow value will be high so the higher water injection rate may be favorable for the down dip flow condition because the higher flow rate will help us in taking the gravity effect into the account and then the down dip flow condition is like this only because in this case the injection well is here and then the production well is on the other side so the angle is reverse. So this is our production well this is our injection well for the down dip condition. So if we are injecting at a higher rate this will push the fluid towards the production well because of the gravity effect also.

So the higher water injection rate is favorable. There could be the situation of the counter flow conditions and that should be avoided it means like we are injecting the water and then the water is moving towards the production well while the oil is moving towards the injection well and that should be avoided and that avoid could be done by maintaining the higher injection water rate. So when sine alpha is 0 the injection rate has no effect on the fractional flow curve it will remain same and then the simple form when putting the sine alpha is equal to 0 the equation will be simplified like this and that is the case we already discussed depend on the viscosity and the relative permeability of both the phases. Now the immiscible displacement process can be utilized to have the relationship between different parameters I will just listed out quickly some of them. So the reservoir flow F_w and the W_{or} that is the water oil ratio can be established we are having this equation for the F_w by the definition of F_w we can write for the water and for the oil if we divide this by the oil we will get the fractional of Q_w by Q_o and this is Q_w by Q_o is actually W_{or} water oil ratio we can replace that so we got the established relationship between the fractional flow and the W_{or} .

Buckley and Leverett (1942)

Reservoir f_w - Reservoir W_{or} Relationship

$$f_w = \frac{1 - \left(\frac{0.001127(kk_{ro})A}{\mu_o l_w} \right) [0.433(\rho_w - \rho_o) \sin(\alpha)]}{1 + \frac{k_{ro} \mu_w}{k_{rw} \mu_o}}$$

$$f_w = \frac{q_w}{q_w + q_o} = \frac{\left(\frac{q_w}{q_o} \right)}{\left(\frac{q_w}{q_o} \right) + 1}$$

$$f_w = \frac{W_{or}}{W_{or} + 1}$$

water cut f_w at any point

water saturation at the point is known

$W_{or} = \frac{f_w}{1 - f_w}$

Reservoir f_w - Surface W_{or} Relationship

$$W_{or_s} = \frac{B_o f_w}{B_w (1 - f_w)}$$

Reservoir W_{or} - Surface W_{or} Relationship

$$W_{or_s} = W_{or_r} \frac{B_o}{B_w}$$

Surface f_{ws} - Surface W_{or_s} Relationship

$$f_{ws} = \frac{W_{or_s}}{W_{or_s} + 1}$$

divided by W_{or} measured at the surface plus 1.

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So water cut at any given point can be calculated just adjusting this equation in the form of F_w . So the water saturation at the point is known when the water saturation is known we know the relative permeability when we know the relative permeability we can calculate the W_{or} . Similar relationship can be established for the other like the reservoir F_w to surface W_{or} here we consider the reservoir W_{or} here we can consider the surface W_{or} the difference will come only converting the flow rate at the reservoir condition to surface condition using the volume formation factor B_o for the oil and B_w for the water and then all these are very simple expressions so I am not going to derive them. Reservoir W_{or} to surface W_{or} can also be established with the similar concept so the definition will remain same only the term we are including could be modified F_{ws} is actually the fractional flow measured at the surface and that will be the ratio of W_{or_s} measured at the surface divided by W_{or} measured at the surface plus 1. So this

expression in a different form can be adjusted by just simply using the volume formation factor and then the flow rate at the reservoir condition and flow rate at the surface condition we can measure or we can establish the relationship for different parameters. Question terms how to determine the water saturation and if we know the water saturation we can calculate the relative permeability we can calculate the fractional flow F_w .

For that the frontal advance equation is given by the Buckley and Leavitt this is the second set of the equation of the classical theory given by Buckley and Leavitt. The frontal displacement equation is designed to determine the water saturation profile in the reservoir at any given time during the water injection. So the Buckley Leavitt equation is basic equation for describing the two phase immiscible displacement in linear system for gas and water the two phase could be the oil and water and could be the oil and gas based on developing a material balance for the displacing fluid at its flow through the porous media. So we already discussed briefly some part of it let us say this is our condition where this is our injection well this is our production well both the fluid water and the oil are moving towards the production well and we can take a small section Δx on this x unidirectional pathway of the fluid movement and that is spanning from 0 to L other dimensions are there we can take the cross sectional area A and ϕ of this domains are constant. So the frontal advance equation is based on the volumetric balance so the volume entering in this element of Δx minus volume leaving the element is equal to change in the fluid volume.

Immiscible Displacement Processes Buckley and Leverett (1942)

Frontal Advance Equation

The frontal advance equation is designed to determine the water saturation profile in the reservoir at any give time during water injection

Buckley and Leverett (1942) - basic equation for describing two-phase, immiscible displacement in a linear system. \rightarrow $q_1 \rightarrow$ 0.75
 - based on developing a material balance for the displacing fluid as it flows through porous media: 0.75

Volume entering the element - Volume leaving the element = change in fluid volume

$$q_t f_w \Delta t - q_t (f_w - df_w) \Delta t = A \phi (\Delta x) (dS_w) / 5.615$$

$$\left(\frac{dx}{dt} \right)_{S_w} = (v)_{S_w} = \left(\frac{5.615 q_t}{\phi A} \right) \left(\frac{df_w}{dS_w} \right)_{S_w}$$

at a particular saturation.

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Now the difference here the things are changing with respect to position as well as with respect to time. So we have to consider both the parameter we can have a more complex equation we are having the change with respect to time on the parameters like the saturation that will depend on the time least the front is moving at what location it is reaching at a particular time the saturation will vary. But let us go with a simplified form this is simply the amount of the volume entering into this domain that is total flow rate multiplied by the F_w this is Q_w actually and this is leaving the change in fractional flow is happening during this time and this is the volume change area multiplied by the length

this Δx that is giving the volume multiplied by the ϕ is the pore volume and this is the saturation change during that time from this position of this x to x plus Δx . So this material balance equation can be written in more complex form considering not many assumptions are made but here we are assuming the ϕ is constant area is also constant this is the numerical factor that is appearing because of conversion of area. Now this equation can be simply adjust in this form taking this Δx from here Δt from here we are going to get this Δx by Δt at a particular saturation and that is the value of the movement of the front with respect to time. So the front is moving at a particular time it might be at this position then this is moving it might reach to this point it might reach to this point and ultimately when it travels all the distance reach to the production point that becomes x is equal to L condition.

So x is equal to 0 condition we are having the water because water is injected mostly the water phase is there and when we reach x is equal to L before front is reaching to that point we are having only the oil but when the front reach there we are having the oil and water and as the front is moving with a high rate continuous injection is happening the water percent will keep increasing and that is where the fractional flow of the water at the production well increases the water cut increases when the front is moving. So this is the actually kind of the velocity of that frontal phase that is moving through this porous media and in this equation you will see another term that is also in the form of derivative that is how the fractional flow is changing with respect to saturation at a particular saturation. So we want to calculate the value of the front movement we need to know the value of this change in the F_w with respect to saturation. So this is the velocity of the fluid this is cross sectional area Q_t is total flow rate and this the slope of F_w versus S_w curve at particular S_w . So let us see further how we can further modify this equation we still saying this part is required we can assume this is constant and can take the integration of this equation under certain consideration we can take at a particular saturation this will be the constant otherwise the fractional flow is changing with respect to saturation and we already assume the injection rate is constant, ϕ is constant, area is constant if we integrate this from 0 to x length that is travelled by the front from 0 to t time we are going to get this simple equation this x is a function of S_w the saturation and then we are having the injection rate of water here time and then the other parameter.

To calculate the position the front will be moving with particular injection rate within this time frame we can use this equation. In simplified form we can write in terms of the injection also because this theory is applicable for both oil and gas. So under a constant water injection rate the cumulative water injected can be written e multiplied by I_w at that particular time. So this is cumulative water injection into the system if we place that the simple equation will get modified like this. So the position of any value of water saturation S_w at given cumulative water injection is proportional to this slope and we need to calculate this slope and this slope will be at a particular S_w .

For two-phase flow, the total flow rate q_t is essentially equal to the injection rate i_w

$$\left(\frac{dx}{dt}\right)_{sw} = (v)_{sw} = \left(\frac{5.615 i_w}{\phi A}\right) \left(\frac{df_w}{dS_w}\right)_{sw}$$

$$\int_0^x dx = \left(\frac{5.615 i_w}{\phi A}\right) \left(\frac{df_w}{dS_w}\right)_{sw} \int_0^t dt$$

$$(x)_{sw} = \left[\frac{5.615 i_w t}{\phi A}\right] \left[\frac{df_w}{dS_w}\right]_{sw}$$

i_w = water injection rate, bbl/day
 W_{inj} = cumulative water injected, bbl
 t = time, day
 $(x)_{sw}$ = distance from the injection for any given saturation S_w ft

$$\left(\frac{dx}{dt}\right)_{sw} = (v)_{sw} = \frac{5.615 q_t}{\phi A} \left(\frac{df_w}{dS_w}\right)_{sw}$$

under a constant water-injection rate, the cumulative water injected is

$$W_{inj} = t \cdot i_w$$

$$(x)_{sw} = \left[\frac{5.615 W_{inj}}{\phi A}\right] \left[\frac{df_w}{dS_w}\right]_{sw}$$

The position of any value of water saturation S_w at given cumulative water injected W_{inj} is proportional to the slope (df_w/dS_w) for this particular S_w

Given Time:
 Position of front
 Position of any saturation
 Average oil/water saturation behind front

and suddenly we will get lot of the production of the injected fluid is happening at the time of Breakthrough

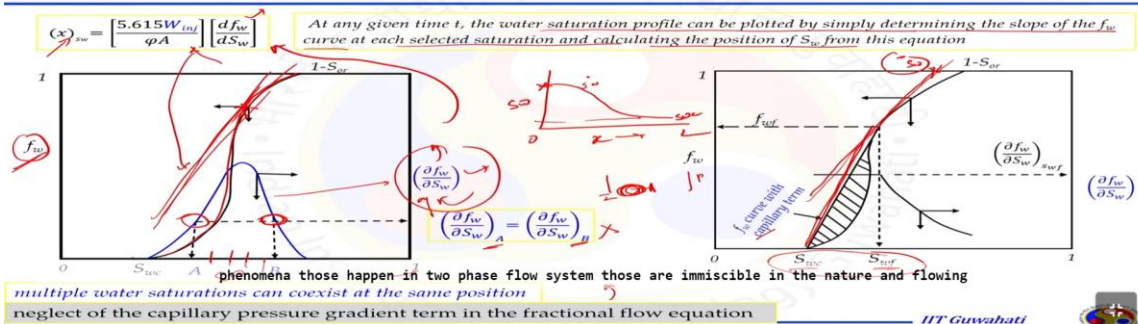


So as saturation is changing the slope will also change. By doing this frontal advance equation we can calculate now the position of the front where it is moving the position of any saturation at particular time particular location what is the value of saturation and average oil water saturation left behind this front. So the front is moving this is the front that is moving from injection well to the production well what is the average value of the saturation behind this front can be calculated and then the time to reach this front at the production well that is called time of breakthrough that can also be calculated by considering this equation that will be the case when X is becoming L that is the length or the distance between the injection well and the production well we can calculate the value of T the time required to travel injection point to production well for the injected fluid and that is the time when the breakthrough of that fluid will happen and suddenly we will get lot of the production of the injected fluid is happening at the production well. So let us understand that in terms of this curve where we are having these F_w with respect to the water saturation and this is the curve that we are expecting for F_w at different saturation of course front is moving different velocities at different time the position may be different. Now we can take the derivative of this or we can calculate the tangent on this at a particular saturation we want so this is actually the tangent at that particular saturation. Now this tangent will give us the value that is required in this formula to calculate how much distance a front has travelled in the reservoir porous dormant.

So at any time the water saturation profile can be plotted by simply determining the slope of F_w curve at each selected saturation and calculating the position of S_w from this equation. So we can calculate this and we can get this kind of the relationship where we are having this x from 0 to L here the saturation of the water will be high and as the front is travelling the saturation will reduce and it will reach to the conduit water saturation okay. This is the point and this is the saturation of the water so at the time travels the front is travelling and then the saturation is also changing at different location of S_w we can calculate the slope or we can have the derivative also to get the value of this slope. Now in this equation we can see the equation is giving us simple mathematical thing and

that mathematical thing is saying there could be the two saturation where the value of this is going to be the same and that is mathematically not appreciable things for the front is moving and then the saturation is also changing so there could not be the different saturation where the value of this x_w will remain same because now this x_w is just depending on this part only. So if the value is same at two different distance that is mathematically not correct or even physically also front is keep moving then the saturation should also be changing and this happens because of the capillary pressure are neglected in this analysis.

Immiscible Displacement Processes



So the multiple water saturation can coexist in the same position neglecting of the capillary pressure gradient terms in the fractional flow equation is the responsible for this situation that we arrived with respect to saturation and the position. Hence if we consider the capillary pressure this curve will be having the straight line in this range of where we are having the conic water to the fractional flow condition and when we are taking the slope of this condition it is not going to give us this kind of the situation. And how to know the average value of this s_w ? So we can increase this tangent at that fractional flow and when it is intersecting f_w is equal to 1 that is the average value of s_w just before the front means just left behind the front so if our front is moving from injection to production like this this is the front move up to this point then this s_w average is just behind this front what is the average water saturation that we can calculate using this slope. So the immiscible displacement process are very useful to calculate or understand many phenomena those happen in two phase flow system those are immiscible in the nature and flowing in the uni direction. So with this I would like to summarize today's lecture in today's lecture we discussed the secondary recovery process primarily the water flooding and in that water flooding we were having the viscous fingering phenomena.

Gas flooding is similar to the water flooding as a secondary recovery process where the immiscible gas injection is considered that is actually affected by the gravity over right. The immiscible displacement process was understood with the fractional flow equation and then the frontal advance equation proposed by the Buckley and Leavitts. The similar analysis as we did for the water flooding we can do for the gas injection also the gas injection rate and the time period should be sufficient enough to inject the gas and get the

oil recover continuous monitoring and the measurement are essential for the gas injection part and then the sufficient bed time before and after gas injection is required to monitor the progress of the secondary recovery by the gas injection. Gas injection pilot test to be conducted early in the life of the developed field to understand lot of the reservoir domain characteristic like the porosity permeability and the other parameters. Similar to the water injection the gas injection property can also be listed out and as in the case of the gas injection we could inject the hydrocarbon gases that is CH₄ could be the nitrogen or the flue gases or could be CO₂ or any other gases.

There is a criteria has been established for choosing appropriate gas injection depending on the fluid properties and then the reservoir parameter. So for example at what depth we are injecting what is the oil saturation present in the domain and what is the viscosity of the oil and then density of the oil accordingly the choice of the gas injection can be selected and implemented as a secondary recovery process it means below the minimum miscibility pressure means immiscible displacement of the fluid in the reservoir domain from injection well towards the production well can be done with the gas injection also. So with this I would like to end my today's lecture so in the next lecture we will discuss the tertiary process or the enhanced oil recovery process there are variety of the schemes have been implemented in the reservoir domain to recover the remaining oil after the secondary recovery we will first discuss the basic concept and then briefly the different processes those are listed out can be discussed briefly in the next lecture. Thank you very much for watching the video we will meet in the next lecture with the concept of EOR for additional oil recovery. Thank you.