Petroleum Reservoir Engineering

Dr. Pankaj Tiwari

Department of Chemical Engineering

Indian Institute of Technology, Guwahati

Lecture 17: Well Testing and Performance-II

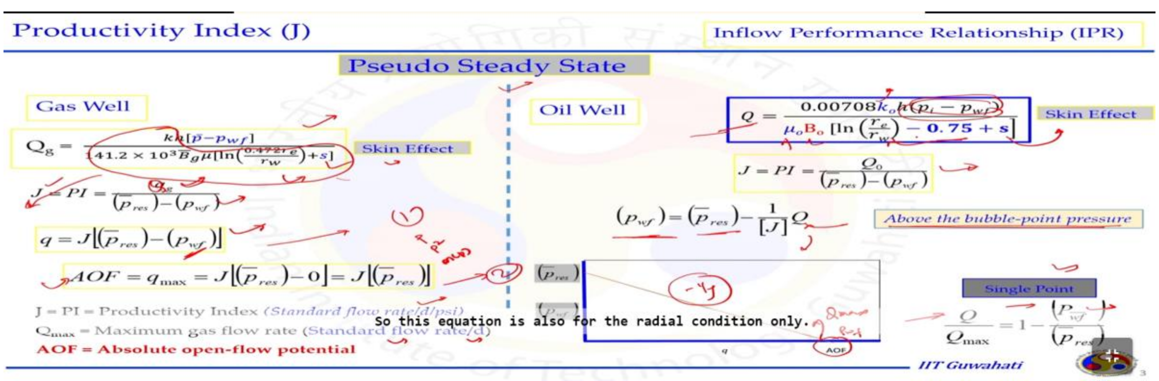
Hello, everyone and welcome to the class of Petroli. In this lecture, we will discuss the petroleum reservoir engineering. This is second lecture of week 6. In fact, this is the last lecture of section 2, where we will be covering about oil and gas well testing methods that we continue from the previous lecture and predicting the reservoir performance. In the previous lecture, we discussed in detail about the gas well testing method. We started with the productivity index, understood the concept of maximum flow production rate achieved from a reservoir well that is also called the absolute open flow condition.

Empirical methods based on the field data were also discussed in detail. In total, we discussed about the pressure transient analysis and then the dilute analysis to understand the gas well performance. Similar discussion we will continue in today's lecture about the oil well performance followed by the decline curve analysis also called the production decline analysis in today's lecture. Let us quickly compare the gas well with the oil well.

So, in terms of the productivity index, the IPR equation for the pseudo steady state condition can be represented by this equation for the gas well. We derive this equation from the fundamental understanding, this equation also including the skin effect. So, productivity index is a parameter that characterizes the performance of a well that could be the gas well or the oil well, but it is applicable only when the flow is happening under the pseudo steady state condition and non-Darcy factor is not included in the analysis. So, the gas well equation or IPR equation for the gas well can be adjusted in the form of productivity index that is defined as the flow rate divided by the pressure drawdown. So, the J is the symbol for that this is the flow rate for the gas we can write this Q capital because it is measured at STP condition or somewhere it is also mentioned at Q small we can check the unit that is the productivity index.

We can have this equation in the simplest form where everything that is appearing in the IPR equation other than the pressure drawdown is denoted by J and we discuss about the condition when the maximum flow rate can be achieved that is AOF that is the condition when the pressure at the bottom hole or the sand phase is 0. The unit of productivity index will depend on the form of the IPR equation we had chosen for the gas well we are having the P approach P square approach and MP approach accordingly the J unit will be adjusted. Similarly for the maximum flow rate this is a standard flow rate per day again the time unit could vary hour to day to month. AOF is the absolute open flow potential of a reservoir if we compare the similar thing for the oil well for the oil well that is incompressible fluid or slightly incompressible fluid we are having the equation for the incompressible fluid assumption in this form that also we derived in the IPR section of our discussion. This also including the skin factor here 0.

75 can be adjusted as a 0.472 that we also discuss. Now this equation is having the similar analogy here only the numerical coefficient got changed and then the symbol for permeability it would be KO viscosity will be Mu O and the volume formation factor will be Bo. So all the property related to the oil are mentioned in this IPR equation while in the gas well all the properties were related to the gas. Now this equation can also be adjusted in the similar manner we can keep this pressure here and remaining can go in the form of J that is the productivity index and then the ratio of flow rate divided by this pressure draw down will be our productivity index of a well.

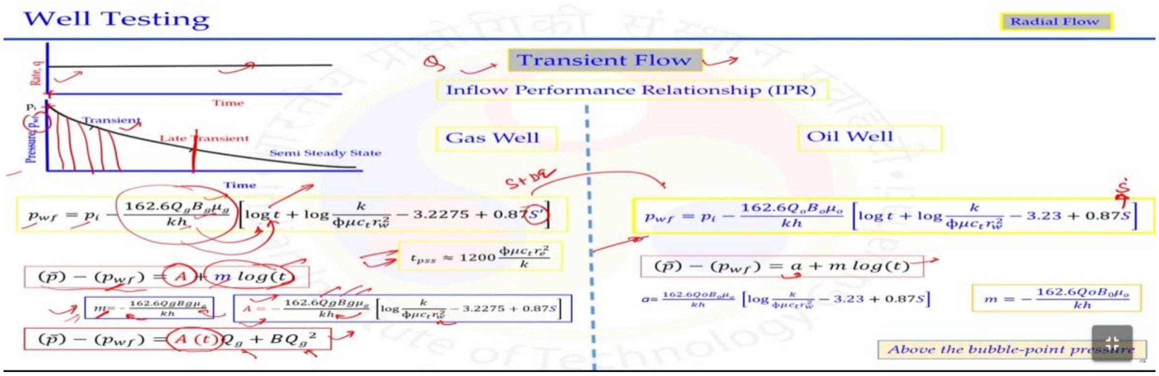


Now if we plot this pressure reservoir pressure and the bottom hole pressure data with respect to flow rate Q we are going to get slope that is minus 1 by J here. So both the equations are explained in terms of P approach. Now this equation can give us the productivity index J that is the characteristic of that particular well that could be the oil well or could be the gas well accordingly we will be using the IPR equation knowing the value of this AOF that is the Q max we can adjust the equation like this equation 1 and this equation 2 in the form of single point equation we are just knowing the one condition of P WF we can calculate the flow rate or knowing the flow rate we can calculate what is the value of P WF at the bottom hole condition for the oil reservoir as well as gas reservoir. So this is the way we can characterize oil and gas reservoir in terms of productivity index. This condition for the oil well is good when the pressure at the reservoir condition as well as the bottom hole condition is above the bubble point pressure.

What do we mean by the above bubble point pressure in that condition the oil will remain in the oil phase no gas is liberating out from the oil phase to the gaseous phase means we are having only the single phase that is the oil phase that is flowing in the reservoir under the radial condition. So this equation is also for the radial condition only. Next concept that we discussed in the previous lecture about the well testing so we can see here the graph is showing when we are producing the well at a constant rate Q this is the situation going to be for the pressure at the bottom hole so that is P WF. So when we are not flowing the well is in the certain condition the reservoir pressure and the P WF both are equal when we start producing first we get the transient condition in transient condition what happens the pressure changes with respect to time and that changes time as well as with the position. After the transient condition we are having the late transient condition and followed by the late transient condition we get the semi steady state condition that is the condition the well will be producing for a longer time.

This pressure versus time data are very important for the transient analysis. In transient analysis we use the IPR equation that has been derived considering the reservoir is flowing under the transient flow condition we use the boundary condition as well as time dependent condition to characterize the reservoir performance and then the final equation that we got for the gas well is like this where the pressure P WF and P I are related to the gas properties that is Mu G, B G and the reservoir properties like K and H this is little complex equation where the time is also appearing because this is for the transient condition in transient condition due to the pressure build up or pressure draw down kind of the test it means near the wellbore region we adjust the flow rate in such a manner that the pressure is varying with respect to time that can be done when the well is flowing we can shut in the well the pressure build up will happen at the P WF condition or when the well is in certain condition we start producing at a constant flow rate we are going to get the pressure change near the or at the P WF condition. So the transient flow test are done at the constant flow rate that is why it is mentioned here the constant flow rate now in this equation we can also estimate the time required to reach the pseudo steady state condition it means this equation should be applied only when we are in the transient flow condition or before this time is reached because once we reach the pseudo steady state time then the pseudo steady state equation should be used that we will discuss in the next slide. So in this case either the pressure draw down or pressure build up test we collect the data with respect to time how the pressure is changing and that data can be utilized to have the relationship between P WF and the time. So this equation is having the A that A is actually including everything other than this term where we are having the M log T.

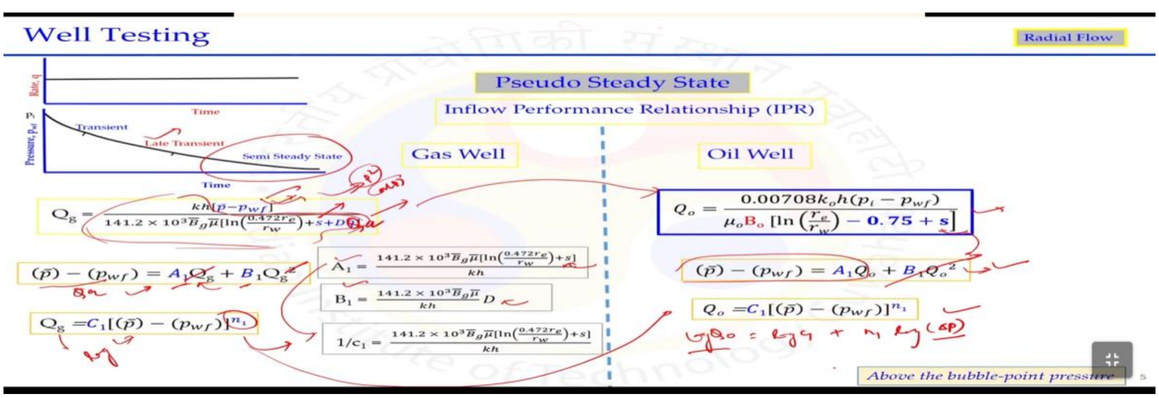
So M is actually represented by this part that is M this M is going to multiply by log T so this term we are going to get and then the remaining everything is in the form of A that also include the flow rate reservoir properties and the fluid properties. Once we are having the near the well bore reason when we are having the non-Darcy effect also prominent in that case we are going to get the quadratic equation and then the A will become the function of time because the equation is adjusted with respect to the flow rates Q and Q. Generally, this equation is used when we are having like different flow rate conditions and when we are utilizing non Darcy effect also into the equation. Now if we compare the similar analysis or we set up the similar set of the equation for the transient flow condition for the oil well the IPR equation for the oil well is going to be similar analogous to this one. The only difference you can see here is the S prime for the gas well that include both skin plus non Darcy coefficient while here in the oil well this is just S that is including the skin factor.



The primary reason for having this difference is because the non Darcy effect is more prominent in the gas well compared to the oil well but in case of where we are having the non Darcy effect prominent in the oil well also we can also modify this S to S prime with the similar definition. In this case also of the oil well under the transient flow condition we are going to get the similar equation where the A is including everything other than the time dependent part and then the similar case of the transient analysis means collecting the pressure P WF data at different time can be utilized to establish the equation and once we are having the equation for example we are able to get the slope M when we are plotting pressure versus log T data then the M is having the properties of the gas as well as reservoir and any property that is unknown for example permeability was not known we can calculate by knowing the value of M from here. Similarly from the A equation if any properties of the reservoir or the near the well more reason for example R W the radius that is known to us but if we want to calculate the value of page on thickness H we can get with the help of this equation. It means either the oil well or the gas well after performing the pressure transient analysis that is done by disturbing the well and measuring the P WF with respect to time plotting the data pressure versus log T data we will get two parameter A and M both are relating the reservoir and the fluid properties so any properties can be calculated by using these two parameters. Again the pressure transient analysis the equation written here is only for the gas well is only for the oil well it means the pressure should be above the bubble point pressure.

Next phase when the transient is passed out we are in the semi steady state case we call this is the dilute test or in that case we are having the different form of the IBR equation so for the gas well in the pressure form I have written here we can write in the form of P square as well as MP form. For the similarity purpose comparing the pressure of the gas well to pressure of oil well I had written this equation in the form of P accordingly the approach is chosen the parameter appearing here other than pressure will be adjusted. So, this QG that is the standard gas flow rate with respect to pressure is having all this term appearing here. Now the difference this would also be QG so the difference is here compared to the previous case we are considering the non Darcy factor here we are considering the skin factor and this equation is only for the pseudo steady state condition. It means the pressure at that condition is changing with respect to time at a constant rate it means the stabilized flow rate is achieved it means the QG value is at the stable condition now we can perform how the pressure is changing and at a different flow rate.

So, we can perform the dilute test we can collect the data at different flow rate at that different flow rate we can get the data for P W F minimum two data set are required because two unknowns are appearing here or we can get the more data set fit the linear equation and then get the parameter A1 and B1 in this LIT method and C1 and N1 in the method of back pressure approach. Of course, we have to adjust this equation this should be divided by QG so this will become linear equation similarly we have to take the log form of this equation that we discuss in detail in the previous lecture. Similar to transient test here also the parameter those we are able to estimate by performing the linear relationship are including the reservoir properties and then the non Darcy coefficient as well as the skin effect so the parameter can be calculated if they are not known to us in the form of back pressure equation this C is actually equal to 1 by B and the N1 is including for the non Darcy coefficient. If non-Darcy factor is not there the value of N1 will be 1 if it is prominently then the value will be 0.5 in between the value will vary from 0.



5 to 1. Similar thing we can see for the gas well where the IPR equation for the gas well under the pseudo steady state condition will be appearing like this. Now this equation is having the skin effect non Darcy effect is ignored that I said if it is there we can consider that part also if we are considering non Darcy factor then this quadratic equation will become otherwise this term will not be there we will get this simply equation. The back

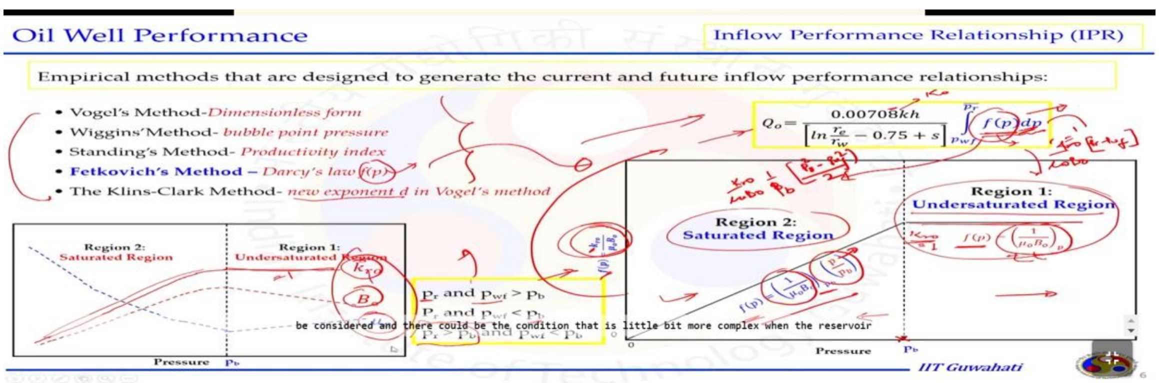
pressure equation this can also be put up like this and in that case the logarithmic of this equation log QO is equal to log C1 plus N1 log pressure difference will give us the linearity on the log log scale with respect to flow rate and pressure difference and then the slope and intercept will give us the value of the parameter those are appearing in that equation either A1 B1 or C1 and N1 those characterize the reservoir. So, this is the way the pseudo steady state and pressure transient analysis can be performed for the oil well also as we did for the gas well. The emphasis again the equation written here is only for the single phase and then the oil reservoir will be having the single phase only when we are having the pressure above the bubble point pressure.

Otherwise the two phase system may appear or the gas may evolved out or even the gas is not getting evolved out some of the properties those depend on the pressure they will be different than the properties those are above the bubble point pressure. What does it mean when we see the gas well performance we will see the difference appears in handling this IPR equation. So, there are several empirical methods are designed to generate the current and the future performance of the well like Vogel's method that is based on the dimensionless form of the IPR equation. Begin's method that is considered the bubble point pressure in the discussion. Standing methods they use the approach of the productivity index.

Fedkovich method they consider the Darcy flow and then the parameter those are pressure dependent or characterized by Fp. The cleanse Clark method they modify the Vogel's method just using another exponent D to characterize the IPR equation. In the equation of the oil well performance as I mentioned earlier if we are having the entire pressure range from above the bubble point pressure to below the bubble point pressure the factor Fp that is appearing in the IPR equation when we derived this Fp is actually having certain parameter as a combination of it. So, what parameter it is having K ro that is the relative permeability of the oil. So, this K o that should be the effective permeability for oil can be related to this K ro and the K.

So, this K ro mu 0 is mu o and P o are the part of this Fp function. Now this Fp function will vary depending on the pressure condition. So, if the pressure condition is above the bubble point condition we are in region 1 and then the reservoir is called the under saturated oil well reservoir. In this case the Fp simply will say 1 upon mu 0 b 0 at P any particular pressure beyond the bubble point pressure will be constant and we will get the straight line for this Fp because in this region K ro is going to be just 1 the effective permeability and absolute permeability will be the same and then the K ro value will be 1. Now what about the mu o and P o we discussed in detail when we were discussing about the oil properties how the volume formation factor changes with respect to pressure and how the viscosity of the oil changes when we are making the change in the pressure.

So, the collective factor of all these three K ro that is going to be the 1 in this region constant and then decline when we are having the pressure below the bubble point because now the gas evolved out there is a competition between the oil and gas to flow. So, considering all these three factor as a lumped parameter in Fp we are going to get the constant value in the region 1 but when we come to the region 2 that is below the bubble point pressure in that case the Fp is having evaluate the value of mu o and P o at the bubble point pressure means at this pressure and then the functionality of pressure is also appearing. So, what will happen when we are integrating this equation integral of this term will give us mu 0 V 0 and K ro also let us say that is equal to 1 and then the pressure part that is del P appearing here that will give me P r minus P wF while in region 2 this term will become like 1 by mu 0 V 0 K r 0 and then P r having this 1 by P v and integration of this P will give me P square so P r square minus P wF square divided by 2. So, this will be the form of the IPR equation so the measured difference in the oil well is as the pressure is changing the system is going from a single phase to two phase system below the bubble point and then the IPR equation need to be adjusted accordingly. So, the Shatkovich method that is explained here based on the Darcy law simply for the pseudo steady state approach we are going to get this IPR equation that IPR equation can be solved for the region 1 and region 2 considering the lumped parameter that is Fp that is accounting how the properties of the pseudo steady state condition changes with respect to pressure that could be the relative permeability viscosity of the oil and then the volume formation factor of the oil.



So that is the way the oil well performance equations are modified by different researcher considering different approach to account this part there could be three conditions in the oil well performance equation when the P r and P wF both are above the bubble point pressure means the well is just operating in the region 1 under saturated condition in that case whatever the IPR equation B wF for the oil well that is exactly applicable but there could be the second condition where the P r and P wF both are below the bubble point case in that case this kind of the modified equation as for the Fatkovich method should be considered and there could be the condition that is little bit more complex when the reservoir pressure is above the bubble point pressure and the P wF is below the bubble point pressure. So the reservoir pressure the phase is in the oil phase but when the fluid is travelling towards the production well and when it is reaching P wF the pressure is below bubble point and then the two phase system is appearing that is where the complexity in the oil well performance evaluation comes into picture. So we are not going to discuss in detail but these three condition will be there in the oil well reservoir. There are certain other things that we can discuss about the performance of the reservoir so far we consider only the single well that is also the vertical well we did not discuss about the horizontal well and horizontal wells are kept away from the slavers of this course so the vertical wells that we consider is in the reservoir field. In reservoir field there could be many certain situations for example there are many wells and then the performance of one well is going to affect the performance of the other well because internally they are connected underneath the surface.

So the principle of superimposition is considered to handle such kind of the situation and then the other situation also those may arise I will discuss couple of them. The real reservoir system usually have several wells that are operating at varying flow rate a more generalized approach is needed to study the fluid flow behavior during the unsteady state flow period. So when the pressure is changing with respect to time at a location as well as time then that is the condition of unsteady state flow condition. So the principle of superimposition are applicable for the unsteady state or transient flow condition. So what the superimposition principle theorem states that any sum of individual solution to diffusivity equation is also a solution to that equation.

So we can combine the effect of individual wells and then we can or individual wells or individual factor and we can superimpose one effect on the other effect. That is for example when we are having the multiple wells in the field so the performance of well 1 will also be affected by well 2 and well 3 they are at certain distance let us say R1 and R2. So the pressure drop that is happening at well 1 that is equal to pressure drop happening due to well 1 that is the production is happening here. Similarly, the production is happening here pressure will decline here also that will also affect this one and similarly at the well 3. So that is the principle of superimposition we can set up the equation for the transient flow condition combine their pressure drop into the big form of the equation and we can superimpose the one effect on the other.

For example here we had superimposed the pressure difference. Effect of rate change there might be the situation where the well is performing let us say a single well a well that is performing at varying flow rate over the time. What does it mean we are having this flow rate Q versus time let us say the well is performing for certain duration then it change the flow rate then it change the another flow rate then it change the another flow rate that could happen in the hours or days for example. Now so far the method we discussed we are having the constant flow rate for the pressure transient analysis but now in this pressure transient analysis when we are performing we are having the multiple flow rate. In that case also the principle of superimposition can be imposed and that simply says every flow rate change in a well that will result in a pressure response which is independent of the pressure response caused by the previous rate changes.

That means the pressure change in total for that particular well can be combined pressure drop happens in this first case pressure drop happens in the second case pressure drop happens in the third case and so on. There could be multiple flow rate change and then the pressure change would be different. So we can apply the principle of superimpose on the effect of rate change also. Similarly the effect of the boundaries there might be some fault the reservoir is very heterogeneous it is not having very clear cut boundaries there could be very varying boundaries similar principle of superimposition can be implemented to account for the effect of the boundary. The concept of amazing the boundaries can be applied for that purpose.

The effect of pressure change if the pressure change is happening in a particular well the similar as we did for the flow rate we can do for the pressure change also. So the principle of superimposition simply says superimpose one effect on the other effect but this method is only applicable when the reservoir is flowing under the transient flow condition. Next topic of today's discussion is production decline analysis. So this is the symmetric diagram for the conventional production system we are producing the fluid that could be the oil and gas. So what we measured from this well production profile we predict cumulative production versus decline reservoir pressure.

So this is reservoir pressure PE this is Pwf over the time when we are producing the reservoir pressure will decline and similarly with respect to time the production is also decline as the pressure energy responsible for the flow will decline. So we can collect the two types of the data where we are having the cumulative production versus the pressure and then the time dependent production data. These data are very useful to characterize the reservoir or evaluating the future performance of the reservoir well. The material balance equation that we did for the volumetric balance and IPR equation that also did the similar thing and then the instantaneous GOR can also be used to calculate the performance of the oil and gas well reservoir. Now more data set are generated that data set you are simply collecting the flow rate with respect to time and you are collecting the cumulative flow rate with respect to time and now you are having the data that is rate versus time data and you are having the rate versus cumulative production. Now this could be the gas well or could be the oil well and we are denoting for the oil and G for the gas P is saying the cumulative data. So this data set will be utilized to understand the decline behavior of the reservoir. So the production decline analysis is the analysis of past trend of declining production performance that is rate versus time and rate versus cumulative production plot for wells as well as reservoir. It means it can be performed for a just single well or it can be performed for a reservoir where multiple wells are present in the field area. There are two types of the approach are adopted for that purpose the classical curve fit approach of historical production data.

Now the data collected for production versus time can be utilized to understand the performance of the reservoir or predicting the future in two approaches. First one is classical curve fit of historical production data that method is given by Earps in 1945 and then the second is type curve matching technique that is proposed by Agrawal in 1970. Lot of modification has been done in 1970 model proposed by Agrawal at all. Actually, this type curve matching technique is you are having the trend of the data that you are collecting from a particular well or the reservoir. The data we are collecting is production rate versus time then we are accumulating the production rate also.

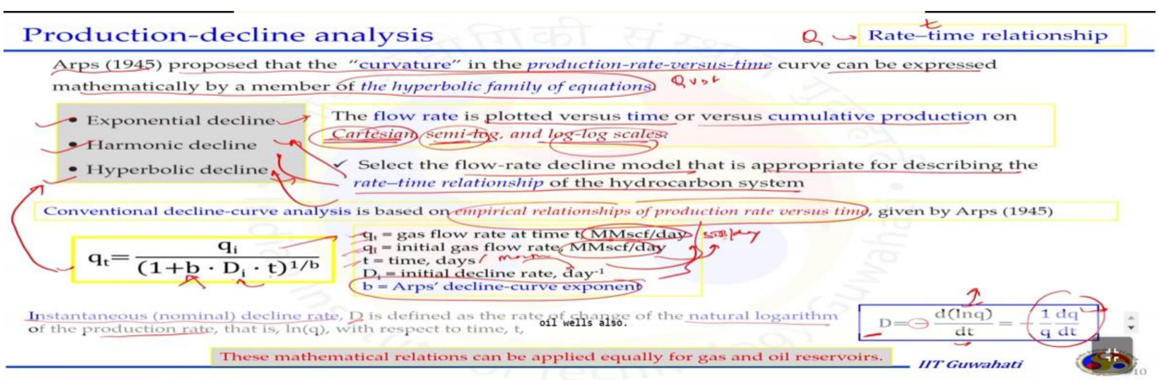
Now we are having the several mathematical expressions with respect to different functionality and now we fit the data for those already established mathematical expression that is called the type curve matching and we see which curve is matching with respect to the data collected from a particular well or the reservoir. So the decline curve analysis technique is based on the assumption. What assumption this decline curve analysis is considered? Past production trends and their contributing factor will continue in the future and therefore can be extrapolated and described by a mathematical expression. It means the assumption has been made when we collecting the data in the past extrapolating those data for the future we are considering no external forces or the changes are happening within the reservoir domain. Reservoir is going to be the performed as the identical condition it is performed in the past similarly it is going to perform in the future.

In other sense we can explain the skin factor and then the damage near the well bore are not considered in this approach. Similarly the identical system that was in the past is extrapolated for the future performance by the mathematical expression. The factors those influence the decline curve analysis is the initial production rate or the rate at some particular time. The curvature of the decline at what way the decline is happening in the production profile and then the rate of decline. The decline in the production rate at what rate the decline is happening in the production rate.

So these are the factor to be considered when we are having the production decline analysis. Other important factor of the production decline analysis is the well is performing under the stabilized condition. It means it is not fluctuating we are in the pseudo steady state kind of the situation where the stabilized flow rate is achieved. This decline curve analysis is applicable for both oil and gas wells. So the similar concept which we are going to discuss in the next slides will be applicable for both oil and gas wells we may consider one of the case.

So the production decline analysis that is also the rate time relationship rate is flow rate and time is the T. The approach of production decline analysis is proposed by Earps in 1945 and he proposed that the curvature in the production rate versus time that is Q versus T data can be expressed mathematically by a member of hyperbolic family of equation. So he considered the data of flow rate versus time and fit them for the hyperbolic family of equation those are exponential harmonic and hyperbolic curvature and he examined one of the condition the reservoir should be performing either in the exponential way the flow rate is declining or in the harmonic manner it is declining or hyperbolic manner it is declining. So the flow rate is plotted versus time or versus cumulative production. So we are having cumulative production also we can integrate the flow rate over the time that will give us the cumulative production and we can plot that on the Cartesian coordinate system semi log plot and then the log scale and can understand the nature of the curve and the nature of the curve is going to suggest us under what condition either it is oil well or the gas well is going to follow the curvature that could be the exponential harmonic and the hyperbolic.

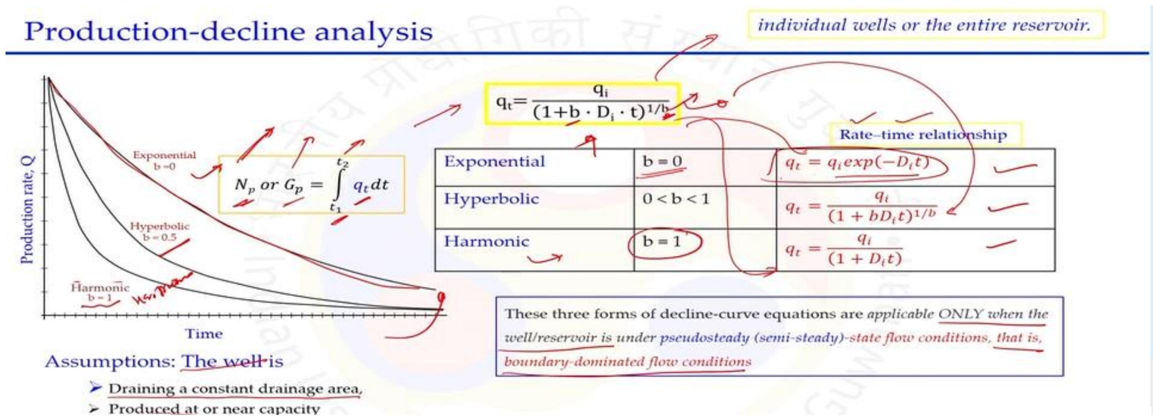
So the select the flow rate decline model that is appropriate for describing the rate time relationship of the hydrocarbon system. So we can choose the model that is matching with the production versus time data and then we can appropriately designated that particular well to be followed this kind of the curvature in the decline. Conventional decline analysis is based on the empirical relationship of production rate versus time given by this or 1945. So what is that empirical correlation that says Qt is equal to Q divided by 1 plus b multiplied by Di multiplied by T to the power 1 by b. So this is the empirical formula that relate actually the flow rate at any particular time T initial flow rate of the fluid why I said fluid because this expression mentioned here is having the nomenclature that is given for the gas but it is equally applicable for the oil also.



So in the case of the oil the unit will be STV per day. So Qt is at any particular time Qi is the initial flow rate T is in time that is in days it could be in months also depending on accordingly the unit of other part will be adjusted and Di is the initial decline rate that unit will also depend on the time unit is chosen. So the Di is having the unit of time inverse and the factor that we are seeing here b that is actually the arc decline curve exponent that is characterized the equation for different curvature or different form of the equation for the family of hyperbolic equations. Another factor that is also defined when we are having the data flow rate versus the time we can characterize this in different form as mentioned here we can do in the Cartesian semi log log scale. Another factor that is also utilized to understand the nature of the well performances instantaneous or called the nominal decline rate denoted by d that is defined as the rate of change of the natural logarithmic of the production rate that is lnQ with respect to time. So when we are having this lnQ with respect to time to keep the value of d positive we put the negative sign here and then when we do the differentiation of this we are going to get minus 1 by Q dQ by dt.

So this is instantaneous decline rate with respect to the flow rate and time at a particular time that is also one of the parameter that is characterized the reservoir performance. We will discuss this in detail at the later slides. So let us see this mathematical relationship can be applied equally for the gas and oil reservoir although the units are mentioned for the gas but it can be applied for the oil wells also. So the data that we collect with respect to time and then the flow rate can be utilized in different manners. So the IPR equation that we consider can be utilized in a different manner similar way this decline curve analysis can also be utilized in different manner.

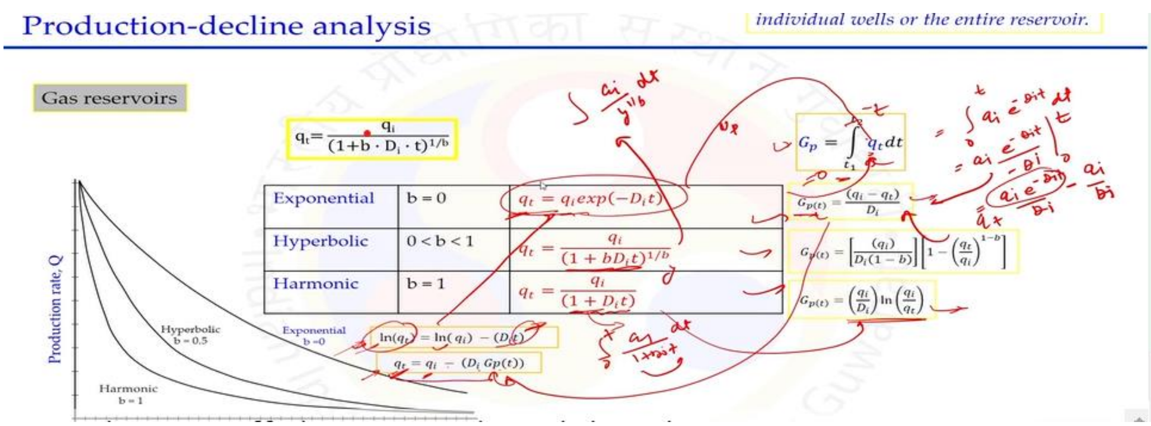
So for example this is our empirical correlation. This empirical correlation can be applied for the individual well or for the entire reservoir domain. Now in this equation when we see the exponent b can have different value. So for example when this b is having the value of 0 we can take the limit of this term either with the initial value theorem or by the L hospital rule we can convert this in the form of simplified form where b tends to 0 and that is equal to Q2 is equal to Qi exponential minus T it means the exponential decay in the flow rate is happening with respect to time. So this is that situation when the exponential decay is happening and that is b is equal to 0. Second condition could be the harmonic flow condition in that case b is going to be 1 and when b is equal to 1 the simplified form of this equation will be like this just we can keep b is equal to 1 b is equal to 1.



When the b value is between 0 to 1 the same formula will be applicable to understand the rate time relationship because now all these three equations are having the rate and time in the expression. Harmonic equation will give us this kind of the plot and when the harmonic equation this would be the harmonic that will give me b is equal to 1. So the flow rate versus time can be plotted and can see the trend of exponential hyperbolic and the harmonic form of that equation. Now the second concept we had keep talking about is the cumulative gas production that cumulative gas production can be obtained for gas well similarly the cumulative oil production can be obtained for the oil well that is integrating this Q t that is the flow rate at a particular time we can record at different different times and then we can integrate between the time of interest from t 1 to t 2 that will give me the N p and G p. Now important aspect here these equations that we mention here for different cases are applicable only when the well or the reservoir is performing under pseudo steady state condition we also call this semi steady state or quasi steady state condition that is the boundary dominated flow conditions and these equations are not applicable for the transient flow conditions.

What are the other assumptions are taken to understand this form of the equation the well is draining a constant drainage area means the boundaries are defined the well is producing at or near the capacity at what capacity this should be producing and then the production is happening at a constant bottom hole pressure P W F is also considered as constant when we are performing this production decline analysis. Now take the case of the gas reservoir I said equally applicable for the oil reservoir also what we can do we can integrate between the time of interest t 1 to t 2 of this flow rate that is measured at different time and we can get the G p form of the equation for the gas well N p will be appearing for the oil well. How we can get that we can place this expression under this integral sign and then integrate it with respect to time t 1 to t 2 if we take t 1 is equal to 0 and t 2 is equal to t the expression we are going to get for the exponential term that is G p t that is the cumulative production of the gas till time t from the initial condition and then the expression will simply take this form. You can do that integration just keep this here Q t so you are going to get integration 0 to t and then you are having Q t is equal to Q i exponential to the power minus d i t d t Q i is constant you can take out Q i e to the power minus d i t by minus d i and you can put the limit 0 to t when you are doing this you will get this expression. In this expression let us say Q i when you are putting t e to the power minus d i t divided by d i and then you are going to get minus 0 when you put 0 then you will get Q i by d i now this is equivalent to Q t again so by adjusting the equation you will get this form.

Similarly you can integrate for the harmonic case where the Q t equation will be replaced by this and then you can do the integration, integration will give you the expression in this form and similarly for the hyperbolic little bit complex but you can do for that also where you can assume this term is equal to by so you will get the integral form let us say if I show it here integral form as Q i by to the power 1 by B d t and then you can integrate it. Similarly, here when we are integrating this you will get you can consider this as 0 to t Q i upon 1 plus d i t d t you know how to integrate this equation you are going to get the ln form in this form. So now we got the flow rate versus time data we could integrate them with respect to different form of the hyperbolic family or we are able to get the expression in the cumulative production also. Now why it is important to calculate the cumulative production because now we got additional component to analyze the data. So let us say for example here when we are having this equation for the exponential term this Q t and Q i are having the exponential relationship if we take the logarithmic of both the sides ln Q t is equal to ln Q i minus d i t.



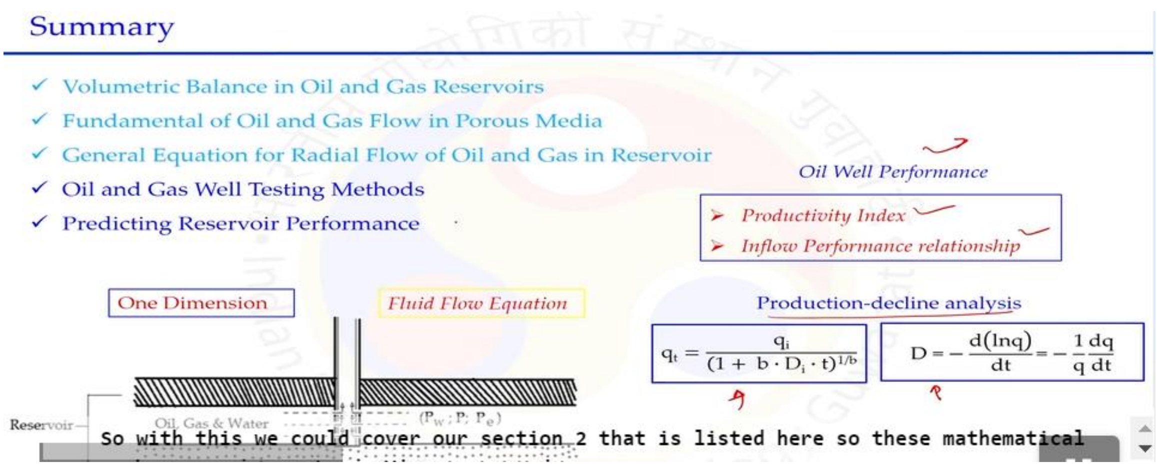
So now if I plot this data flow rate versus time on the semi log plot Q t on the log and t is on the normal scale I am going to get the straight line. Similarly for the case of the cumulative gas production that is here Q t is equal to Q i minus d i G p t. So if I am plotting the Q t versus G p t on the Cartesian coordinate system I am going to get the straight line. So that is the way the curve can be characterized with respect to Cartesian semi log scale log log scale as well as in the parameter form of flow rate versus time or flow rate versus the cumulative production. So this production decline curve analysis method that we discussed for different cases having limitations and that limitations is the method completely ignore the flowing pressure data.

So in all these equation that we had seen here we are not seeing any pressure data and that may produce lot of the error in the analysis it may overestimate the situation or underestimate the situation. Hence these methods could be good to characterize the things but they are ignoring the flowing pressure data and that should be considered taking the data with this decline curve analysis compare with the IPR or the productivity index because those are considering the production profile also into the characterization of the reservoir. So let us say the decline curve analysis as mentioned in the previous slide we are having the rate versus time data we can plot that rate versus time we are having the rate versus cumulative data those can also be plotted here and those can be plotted in the coordinate system where both the axis are having the normal scale. So when we do rate versus time I give the color coding also blue color is showing for the exponential decline red is for the hyperbolic and the black for the harmonic decline. So we are going to get rate versus time data and all are declining in this manner but when we plot rate versus cumulative data the other two equation will still not be the straight line but the first the exponential decline will give me the straight line.

Now straight line can be extrapolated to get the initial condition once I am having the initial flow rate condition Qi that will be at the condition when the production is equal to 0 cumulative production is 0 and in that case so this is that situation here in that case we are able to get the Qi and then I can get the Di also running the one condition and then utilizing the empirical correlation proposed by air. Now similar analysis can be done on the log log scale where the same data are plotted on log log scale and we see none of the decline curves are giving us the straight line when we are plotting the rate versus time on the log scale similarly rate versus cumulative flow rate is also not giving any straight line when we are plotting the log log data. On the semi log we can see the curve one is giving us the straight line for the data when we are plotting the rate versus time and on this rate versus cumulative production, we are having the straight line for the case 3 that is the harmonic decline curves. So that is the way we are having the data we can plot those in the coordinate log log semi log conditions with respect to rate and time or we can plot rate versus the cumulative rate and can see which kind of the curve is going to get fitted and we can see the decline nature of the production for a particular well or for a reservoir domain. So for the exponential decline as mentioned here semi log scale that is here will give us the straight line and then the Cartesian will give the straight line when we are having the cumulative production.

So these two conditions are giving the straight line when we are having the hyperbolic decline none of the curves. So for the case of the hyperbolic decline for the red color we are not having any relationship on any scale either Cartesian semi log or the log it is not resulting the straight line. For the harmonic decline on the semi log scale we are having the straight line that is here. So other curve will not give us the straight line only the semi log log versus cumulative production should give us the straight line. So that is the way we can characterize the data and can understand the nature of the production decline that is happening and then we can extrapolate this for the future time.

So another form of the decline curve analysis as discussed the instantaneous or the nominal decline curve that is we are having this expression where the flow rate is changing with respect to time we can take the derivative of this and this empirical correlation can be fitted with two parameter b and d those are appearing in this expression. And we can see the exponential decline harmonic and hyperbolic decline with respect to these data this is the flow rate Q. So in the case of exponential decline model this d will be equal to 0 and we will get simple this relationship when this instantaneous decline rate is not changing with respect to flow rate and when d is equal to 1 we are going to get the harmonic decline model and that is d is equal to 1 here and when we are having the value of d between 0 to 1 this will be the hyperbolic decline model. In that case in the exponential decline this will be the straight line harmonic decline it will be the straight line with certain slope and hyperbolic will be having the deviation from the straight line and it will not be having the constant slope. So this is the way also we can characterize the reservoir performance in terms of b and d those are the empirical constant those are appearing in this equation.



So in summary of today's lecture, we discussed the similar analysis for the oil well performance in terms of productivity index IPR equation production decline curve analysis was discussed for this air empirical correlation and then the instantaneous decline curve analysis approach. So with this we could cover our section 2 that is listed here so these mathematical relationship mentioned here those are empirical but can be applied equally for the oil and gas reservoir or individual well and also for the entire reservoir where the multiple wells are there. So with this the section 2 of our syllabus is completed we discussed the volumetric wellings IPR equations understood the performance of the oil and gas well reservoir under different conditions and at the end in today's lecture we also discussed the production decline curve analysis. So in the next lecture we are going to enter in section 3 that is mostly on after the primary recovery we will be discussing about the secondary and tertiary recovery methods followed by a brief introduction about the reservoir simulation and then the unconventional natural gas production specifically gas hydrate. Thank you very much for watching the video we will meet in the next lecture thank you very much.