Petroleum Reservoir Engineering

Dr. Pankaj Tiwari

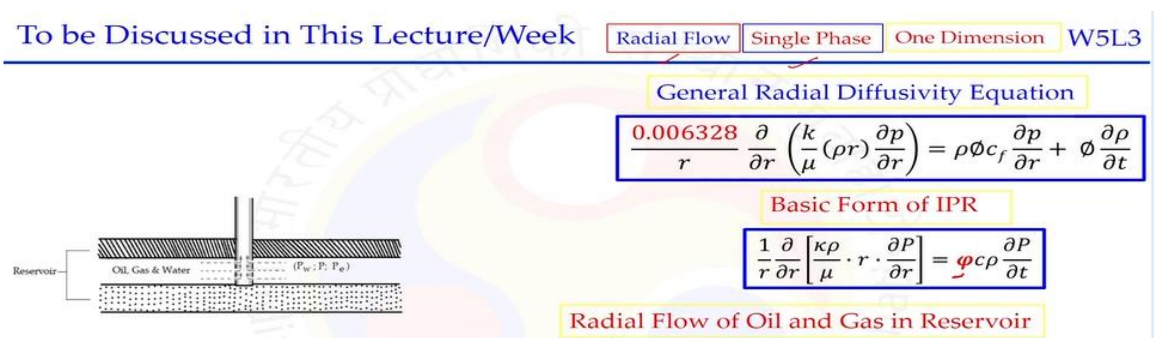
Department of Chemical Engineering

Indian Institute of Technology, Guwahati

Lecture 15: Inflow Performance Relationship for Reservoir Fluids

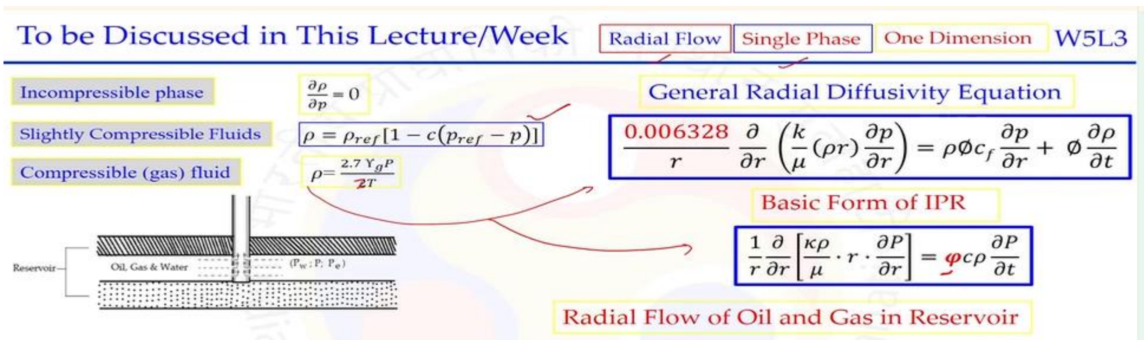
Hello, everyone. And I welcome you again to the class of petroleum reservoir engineering. This is lecture number 3 of week 5. In the previous lecture, we discussed some cases of developing IPR equation. In that we consider one dimensional model and in that one dimensional model, we consider only single phase flow and the fluid is flowing in the radial direction. So if in the summary sheet, you can see here we consider single phase radial flow and this is one dimensional model.

We developed the radial diffusivity equation in general that can be solved for different cases and similarly, we also developed the basic IPR equation. Both the equation general radial diffusivity equation and basic IPR equation developed in the previous class are applicable for one dimensional single phase radial flow system. Both the equations are identical in some senses. Only the way they are represented and the assumption are taken is slightly different.



For example, in general radial diffusivity equation, we did not consider even the porosity to be constant. While basic IPR equation, we consider this porosity to be constant. Another parameter considered constant in both equation is the page on thickness that is H. So in this lecture, we are going to continue utilizing either of this equation. Mostly we will start with the basic form of IPR to develop radial flow of oil and gas reservoir and we will consider single phase radial flow that is going to be common for all the cases we are going to discuss in today's lecture.

Then the types of the fluid that could be incompressible fluid, slightly compressible fluid or compressible fluid will be discussed. In case of the flow regime, we can solve this equation for steady state flow, unsteady state flow or pseudo steady state flow. As in the case of the types of the fluid, density is the parameter. In the case of flow regime, different flow regimes are characterized with different initial and the boundary condition those are going to be employed in solving the IPR equation. So in case of the fluid, we can say when the fluid is incompressible, fluid could be slightly compressible or compressible fluid.



So in case of incompressible phase, the density with respect to pressure is not changing, it is zero. Slightly compressible is having linear relationship with respect to pressure and can be expressed with this formula. This we already discussed in previous classes. Compressible fluid is little complex where the density is a function of pressure and temperature and then the compressibility factor Z is also appearing. So wherever we are getting the rho in the expression, either we are considering the radial diffusivity equation or basic IPR equation, we need to replace rho with this expression.

So in today's lecture that is named as inflow performance relationship for reservoir fluid, we are going to consider all these three types of the fluid and all these three flow regime to be used for developing the fundamental IPR equation dealing with different fluid and different flow regimes. So the basic IPR equation will be converted to actual inflow performance relationship depending on the types of the fluid and the flow regime. Two other parameter skin effect and non Darcy coefficient or non Darcy effect will also be discussed in today's lecture. So let us start with the first case that is the steady state flow condition. In steady state flow condition for the radial flow, we are going to consider one dimensional flow, single phase radial flow.

Fluid is travelling from all the direction towards the production well. This is happening only in the radial direction and steady state flow means the flow rate Q is constant and then the change in the pressure with respect to time at any location within the reservoir is 0. As I mentioned earlier, this is the situation when the pressure in the reservoir is maintained by external means or it is a hypothetical situation or the simplest situation that can be assumed when we are dealing IPR equation. So either pressure is maintained. In this case, no reservoir information is obtained with respect to flow condition because the flow is constant and the pressure is also not changing.

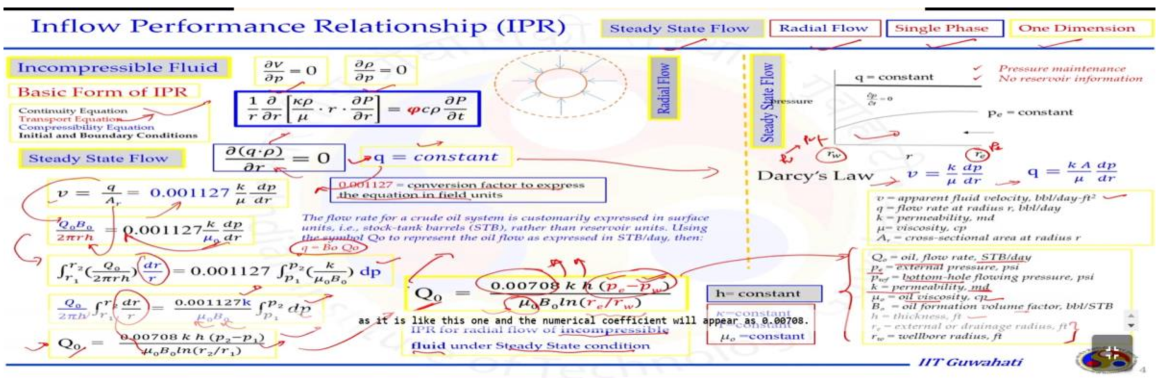
So what are the properties of the reservoir, how the flow is happening within the reservoir, such kind of the information cannot be obtained. So let us start with the one-dimensional single phase radial flow. Now steady state flow condition. So this is radial flow for the incompressible fluid and the basic IPR form of the equation that we derived in the previous lecture can be the starting point to discuss particular case of the incompressible fluid. So in steady state case, using this basic IPR equation that has been derived by including continuity equation, transport equation, fluid properties and initial and boundary conditions are required to solve this equation.

In this case of steady state flow, the situation would be pressure is not changing and this condition will be applied here that simply says the material balance equation or basic IPR equation will be reduced to this form and it means the Q is constant as rho is constant for the incompressible fluid to have this equations validity Q has to be constant and that is the condition for the steady state flow condition. Now this Q will come from the Darcy law as written here the transport equation required that is the Darcy law and in Darcy law we are having either the expression in form of velocity or in the form of volumetric flow rate. The usual meaning of this term appearing in the Darcy law are summarized here. So the equation Q can be represented in the form of velocity that is velocity is equal to Qy ar and then this is the Darcy law. The numerical value that is appearing here that is the conversion factor to express the equation in the field unit system.

Usually, the crude oil system is expressed in the form of STB that is stock tank barrel that is the amount of the volume or the volumetric flow rate measured at the STB condition. In that case the Q can be replaced with V0 Q0. V0 is the volume formation factor for the oil and Q0 is the volumetric flow rate of oil at STB. If we do so this Q is replaced by Q0 V0 area is 2 pi RH other side remains same and now this is mu is denoted as mu 0 it means we are solving the equation for the oil phase. If we integrate this equation by adjusting the variable del R can go to the other side and in that case when we are integrating this equation we are integrating on the left hand side with respect to R and right side with respect to pressure.

So on the left hand side 2 pi H and the Q also can be taken out while the R will remain inside the integral sign. On the right hand side we are now making the assumption of K is constant, mu 0 is constant and B0 is also constant inside the integral we will be having only the pressure term. Once we integrate this and adjust the equation in the form of Q0 that is the volumetric flow rate of oil at the STB condition it means it is measured at STB per day we are going to get this expression. Just simply del P became P2 – P1 ln form of the R will be appearing in the denominator. Adjusting this equation with respect to other parameter we can say the P2 and P1 can be expressed for the condition those are in our reservoir domain.

So RW is the radius of the valve bore and then the pressure at this condition we can say PW or sometimes denoted as PWF also. This is valve bore pressure or also called sand phase pressure. At the reservoir radius RE far away from the valve bore we are having the pressure PE. So the PE and PW are put up for P2 and P1 similarly in the radius also we put RE and RW. So this is the expression that we obtained with certain assumptions that assumptions we made H is constant permeability is constant flow is happening under isothermal condition means T is constant and Mu 0 that is the viscosity of oil that is also constant.

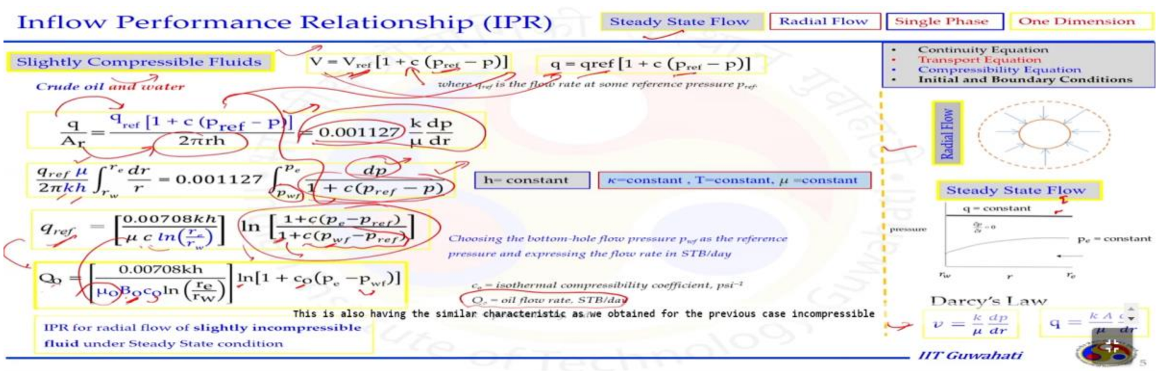


So by adjusting this equation we got the IPR equation that is for the radial flow of incompressible fluid under steady state condition. I am keep saying from last few lecture IPR means a equation that is going to characterize the production profile in terms of flow rate and the pressure difference. So this relationship that we obtained for a very simple case that is fluid is incompressible flowing under the radial flow conditions and steady state flow is happening we got the relationship Q0 versus pressure. In this expression we are getting properties of the reservoir domain like the permeability page on thickness and the properties of the fluid that is Mu 0 and B0 is also property of the fluid. The usual expression appearing here if we are changing the unit in other form accordingly this numerical factor that is appearing in the expression will be changed.

So if we are using the unit mentioned in this list you are going to get the expression as it is like this one and the numerical coefficient will appear as 0.00708. Let us move ahead next discussion for the same case where we are having the steady state flow but this time the fluid is slightly compressible. For the slightly compressible fluid we are having the expression how to account for the volume with respect to pressure. Where V reference is the volume at the reference condition that reference condition is P reference.

Similar can be converted to volumetric flow rate where we are having the Q as the volumetric flow rate at a time we are measuring the system or Qref is the flow rate with respect to reference pressure and that reference pressure again is P reference. C that is appearing in this expression is actually the compressibility coefficient for the fluid that we are going to take in the consideration. So slightly compressible fluid we need to know what is the compressibility coefficient for that fluid. We can consider crude oil and water as slightly compressible fluid. Actually, crude oil and water can fall in both the range both the categories in compressible fluid and the slightly compressible fluid.

The same methodology we are going to adopt start with the continuity equation, write the transport equation, compressibility equation, initial and boundary condition for the case of slightly compressible fluid flowing under steady state flow condition and then the flow is happening in the radial flow direction. So we are having the same set of the information that we are having for the previous case. Only difference between the previous case and this one we are having the fluid that is slightly compressible. In this case it means the density is not constant, density is changing linearly with respect to pressure and the expression with the Darcy law will be applicable. So same information is there Q is constant in that case Darcy law will be utilized to establish the relationship.



Darcy law simply say Q i A r is equal to this expression where A r is the area that is 2 by RH and then Q can be replaced using the expression mentioned for the slightly compressible fluid. In that case right hand side is same as the previous case we are having this numerical coefficient here because of the units adopted for the parameter appearing in this expression. Simply we will do the same methodology, integrate it considering Mu, K and H constant. Right hand side we will get integration with respect to r and numerator we are having the T r and denominator we are having the r. In the previous case in compressible fluid we were not having anything other than pressure derivative term here while in the case of slightly compressible we will get this term that is appearing from here.

So now when we integrate this under the assumption we had taken we will get almost similar expression except some terms are appearing here this is C appearing here and this ln term. This ln term is appearing because we are integrating this pressure term with respect to pressure from P1 to P2 or from reference pressure to reservoir pressure. Now that reference pressure can be chosen as a Pwf that we had taken here. In that case what will happen the expression will take this form. So what we did this Q is replaced with Q0 and P0.

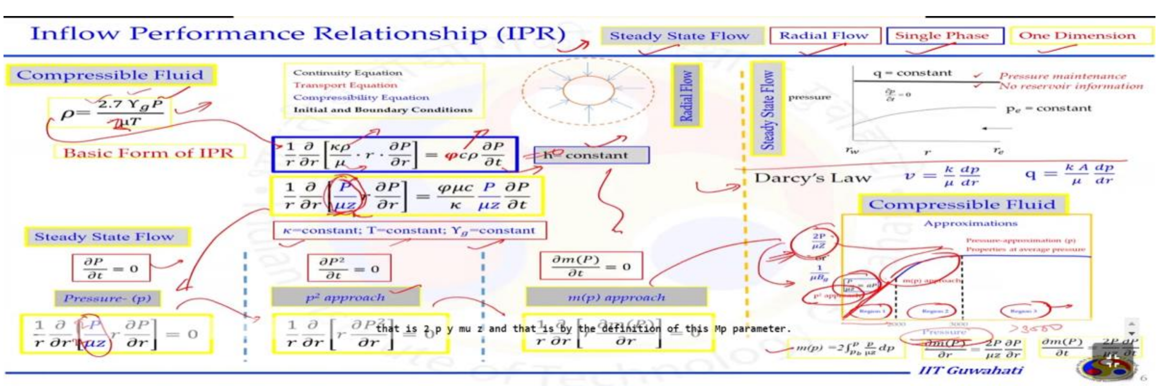
Q is replaced with U0 that is the oil on the other side when we are taking Pwf as the reference pressure this term will disappear and, in the denominator, we will just left with 1. That is why this expression will become ln 1 plus Co because this is for the oil Pe minus Pwf. So this is the expression we got in the form of Q0 that is the flow rate of oil measured at the STV per day unit. So this IPR for the radial flow of slightly incompressible fluid under a steady state condition is obtained. This is also having the similar characteristic as we obtained for the previous case incompressible fluid only difference is coming in the term of that we are going to get here.

When we talk about the compressible fluid the difference between the previous two cases and this case is our density is a function of temperature and pressure and that is nonlinear behaviour. We cannot assume it constant we cannot assume it is changing linearly while the definition says rho is equal to 2.7 gamma g py zt. So how pressure is changing accordingly the z in the denominator is changing and then the relationship between the density and the other parameter will be accounted as per that expression. For developing the IPR equation for the compressible fluid we will start with the same methodology one dimensional flow single phase radial flow.

So now these three terms become fixed for all these three cases we were discussing and the four terms also we are discussing only steady state flow condition. So in all previous two cases as well as in this case all these four conditions are fixed this is the way the radial flow happens we had seen in the previous slide also. For the case of steady state flow the similar thing is there we start with the continuity equation, transport equation, compressibility equation is included into it and then the resultant equation will be solved with respect to initial and boundary condition. So for the transport equation similarly to previous two cases we will be using the Darcy law and the only difference as I mentioned is the density changing with respect to pressure will be here and this is jet t. So this is basic form of the IPR equation when we substitute the density term here we are going to get the basic IPR equation form converted to the form that is applicable to compressible fluid.

When we look on the right hand side this term for the steady state condition is zero. So what will happen this expression of basic IPR we consider h is constant when we replace rho we are going to get this expression because 2.7 gamma g and temperature are constant they can be taken out of the derivative from the left hand side and the right hand side when we are replacing both density the term 2.7 gamma g by t will be cancelled out and we will just left inside p is the pressure that is from here and mu z that is mu was already here and z is coming from this density expression. And now if we take further assumption k is constant temperature is constant gamma g is constant we can just simplify this equation for the compressible fluid.

Now the compressible fluid are different than the incompressible and slightly compressible fluid and the resultant equation that we are having here we can solve it for the case of the steady state flow but in general this equation is not having any analytical solution because primary reason mu and z are the parameters those are dependent on the pressure hence it is not having any simplest solution for that purpose we need to consider certain approximation that is also we discussed in the previous lecture. So that Mp is defined like this py mu z is integrated from base pressure to the pressure of interest and we can use the integral method to integrate the system from base pressure to the pressure of interest by dividing it in a small small segment and that can be done with the help of the trapezoidal rule that we also discussed in first lecture. So for the steady state flow the condition will remain same change in pressure is 0 with respect to time and in the approach of p square when we are converting this into the form of p square approach when the pressure parameter py mu z is changing linearly with respect to pressure we will get the p square approach and in the third case when we are in reason to del Mp by del t is also 0. So the steady state condition simply say it does not matter what form of the pressure you are having p form p square form or the Mp form the pressure change with respect to time is going to be 0 and in that case the basic IPR equation will get simplest form it means like right hand side is going to be 0 and with respect to pressure py mu z will be here in case of p square what we did we divided and multiplied this factor with 2 and that we could convert this p form into p square form we could linearize the system with respect to p square and simply we can also convert this into Mp form. This we discussed in detail in the previous lecture so you can refer that lecture how the basic IPR equation can be converted to p form p square form and Mp form why it is important to discuss it here again because now we are solving the equation for different types of the fluid under different flow regime condition.

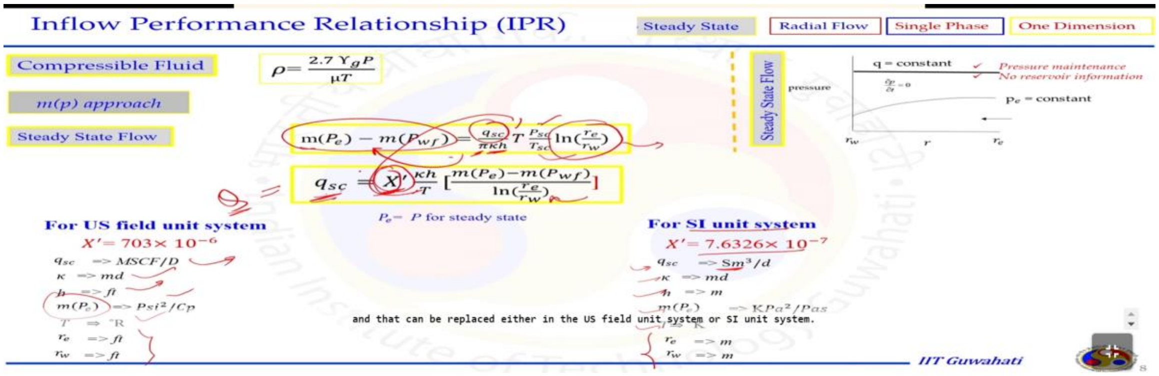


So for the case here the method that should be chosen is the Mp approach does not matter what pressure range is there it will consider entire pressure range it is more accurate compared to p approach and p square approach and Mp approach should be chosen but the Mp approach requires the integration with some computational method to be implemented to calculate the del Mp value with respect to pressure at different segment. So simplest form could be p p square but the complicated form del Mp form or just Mp form I can say is more accurate and it is going to account the changes those are happening in the properties of the fluid with respect to pressure in the form of lumped parameter that is 2 p y mu z and that is by the definition of this Mp parameter. So let us continue this our compressible fluid we know how density is changing the basic IPR equation in the form of Mp that I said Mp is the most accurate we can solve with respect to Mp similar methodology we are going to apply for the Mp form can also be implemented for the p approach and p square approach. So in this case we are having the similar condition for the steady state flow condition we are having this term is equal to 0 it means the right hand side we are having 0 it means Q becomes constant under the steady state condition for the case of Mp approach the similar form we are going to get for the Mp also. So now this equation will reduce to this form while keeping right hand side equal to 0 because we are considering the steady state flow condition when we integrate this with respect to R for one time we are going to get this term inside and the right hand side is going to be some constant.

In that case we need to calculate the value of the constant that is one of the boundary condition so at R is equal to R w it means at the bellwor condition what is happening there the flow rate Q can be given by the Darcy law. So this is area this is permeability viscosity and this is pressure gradient at R is equal to R w similar this equation can be represented in the form of p square or in the form of Mp if we convert in the form of Mp we will get this expression and then the constant value can be obtained like this. Now this constant value having Q that is at the reservoir condition we can use the real gas law at two points to translate this reservoir condition to standard condition. It means at any point of pressure temperature condition we are having Qp by Zt at one condition equal to Qp by Zt at condition 2 and similarly for the condition where the standard condition is prevailing. What will be the advantage of this because we know what is the value of the Psc that is 14.

7 psi the value of Tsc is 420 degree ranking and Zsc is considered 1 it means the compressibility factor Z for the real gas also at the standard condition can be assumed as 1. So if we put this part here all together we are going to get this expression. Now this expression is having Psc, Tsc we can put the numerical value it is having K and H those are the reservoir properties. We can integrate this equation from Pe to Pwf and finally we are going to get the expression in the form of Mp and that is having the relationship with respect to the radius and the fluid properties are already included in the form of Mp. So let us say the steady state flow condition for the compressible fluid using the Mp approach we got this expression putting the numerical value for the standard conditions

we are going to get this X prime a factor that will account for the unit chosen for different cases like for the Psc, Tsc and this is pi it also have the numerical value 3.



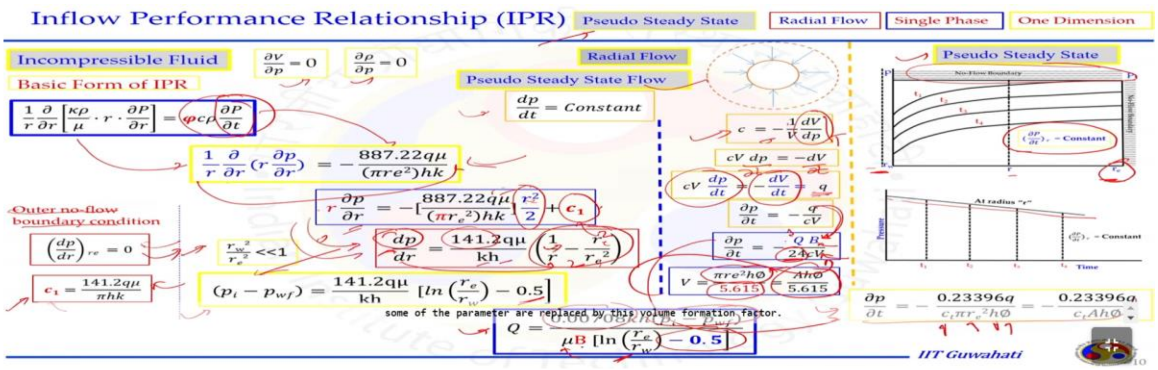
14. So if we put everything in this form we are going to get this Q that is here and then take everything on the other side. So this Qsc that is also mentioned capital Q so this is the IPR equation for compressible fluid under steady state conditions. In this expression what you see X prime that will depends on the unit system is chosen for the parameter which is representing this equation. So if we are choosing the US field unit system the K prime will be 703 10 to the power minus 6. If we are choosing SI unit system the numerical value of the X prime will be 7. 6326 *10 to the power minus 7. So what is the meaning of the SI unit system Qsc is defined as standard meter cube per day K is in milli Darcy height is in meter pressure is kilo Pascal square per Pascal second and the temperature in Kelvin and the radius are in meter. While in US field unit system for the compressible fluid the flow rate is Mscf per day permeability is similar milli Darcy H is in now feet pressure that is MP is psi square per centipoise and T is in degree ranking and the radius are in feet. So that is the way I mentioned here X prime because X prime will depend on the unit system and that can be replaced either in the US field unit system or SI unit system. We can move to the next condition that is here in our case we discussed already the steady state flow condition for all these three types of the fluid. We can discuss one case of pseudo steady state flow condition that one.

So what happens in pseudo steady state case we are having the radial flow similar to the previous cases. Early difference between the different flow regime is the initial condition or the boundary condition. So for example in the steady state case we were having the change in the pressure with respect to time is 0 while in the pseudo steady state case we are having the condition change in pressure with respect to time is constant. What does it mean the pressure at a particular location within the reservoir is not constant it is changing but the change is happening linearly. So if at a particular point radius R if I measure the pressure at different time t1, t2, t3 plot this then I am going to get the linear relationship.

It means the change in pressure with respect to time at a particular location is constant that can be obtained as one of the boundary condition to solve pseudo steady state flow condition. So let us start the pseudo steady state flow condition let us consider the incompressible fluid where the density and the volume are not changing the radial flow is same and now in the pseudo steady state flow condition this is going to be constant the change in pressure with respect to time. The similar diagrams shown here where the boundaries are having the no flow condition and within this reservoir domain the flowing condition between R w and R e we are having this condition where the change in pressure with respect to time at a particular location between R w and R e is going to be constant and that can be expressed by measuring pressure at different condition and plotting it. How to get this expression we can start with our compressibility coefficient definition that is C is called to change in volume with respect to pressure at constant temperature divided by the original volume. This equation can be adjusted simply like this and both the sides can be divided by dt and then this change in volume with respect to time is actually flow rate Q and now this flow rate Q can be related to this pressure change.

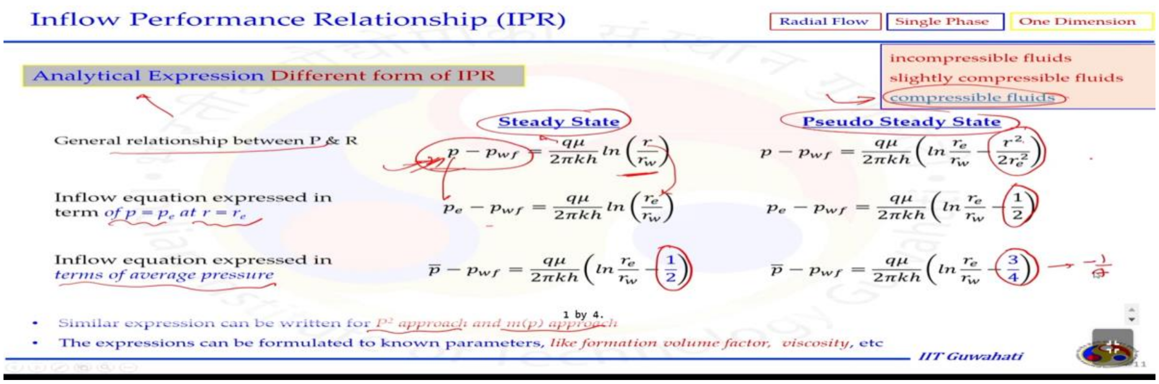
Now this flow rate Q can further be adjusted to Q and P I did not mention Qo or Bo if it is oil then Qo Bo if it is gas then it will be Q0 Bo. So let us say we are discussing in general any types of the fluid we are having the conversion of Q that is the reservoir condition to standard condition using the volume formation factor. In the denominator we multiply denominator by 24 to adjust the time unit and when we are saying the volume that is appearing here that is the pore volume pi re square h the bulk volume multiplied by the porosity and this 5.615 is appearing for the unit conversion that is for the radius we are converting from acre to feet or in the form of this also we can write the volume and when we are substituting this volume here we are going to get this expression for del p by del t and that will be like this. Now what is the use of this expression because our basic IPR equation is having this del p by del t on the right-hand side and this need to be replaced with some parameter those can be measured.

So now here you can see the Re, h, phi, Ct those are the parameter can be used to express the change in pressure with respect to time and that is going to be changed linearly. So let us say the another boundary condition that is required to solve for the pseudo steady state flow condition that is the outer no flow boundary condition means at any radius or at the r is equal to Re there is no change in the pressure with respect to r and that the coefficient C1 that will appear further in the expression can also be obtained. So let us start with the basic IPR equation we can replace this term here adjust the parameters and now this form by putting the row value everything we can adjust this expression this expression can be integrated and we are getting this C1 term here. This r will go here integrating this will become r square by 2 and the C that is appearing here can be calculated with this boundary condition where we are saying at r is equal to Re there is no flow of the pressure. So we will keep this is equal to 0 and r is equal to Re in this expression we are going to get this term.



Putting this constant value C1 here adjusting the equation we are going to get this on the left hand side del p by del r is equal to some numerical coefficient Q mu K and H and inside we are having 1 by r minus r by Re and this will happen by adjusting this r into the equation. Now we can integrate this equation one more time and in that case we are going to get this expression on the right hand side we will be having only the pressure difference this dou r will go to the other side and integrate with respect to r we will get three terms one of the term will be r by Re square that can be so this term will go to the ln form other form we are going to get where we will make the assumption of rw and Re and their power is going to be very very small compared to 1 so we can neglect that term only thing we are going to get this will become r square by 2 integral sign and when we are putting Re as the limit Re Re will cancel out we will get 1 by 2 that is 0.5. So the equation adjusting in the form of Q versus pressure draw down that is the IPR equation we are going to get the similar expression as we obtained for the case of steady state flow only difference is coming we are having this 0.5 factor in the denominator and that is getting subtracted with this ln Re by rw.

So if we compare pseudo steady state case and the steady state case for compressible fluid we are going to get same expressions similar expression we will obtain for the incompressible fluid compressible fluid slightly compressible fluid only difference will come how the parameters are getting adjusted what is the numerical coefficient is getting changed as per the unit chosen and the parameter like here in this case mu is appearing here and some of the parameter are replaced by this volume formation factor. So if I compare both steady state flow condition and pseudo steady state flow condition for the complex fluid that is the compressible fluid in terms of the analytical expression that we obtained we can see the expression written here are in the form of P as I mentioned earlier for the compressible fluid we can write in the form of P square approach MP approach and P approach. So the expression written here are in the form of P similar expression can be written for P square and MP approach. Let us compare the steady state and pseudo steady state case in case of general relationship between P and the r that is pressure and radius we are going to get this is pressure draw down this is flow rate and then the relationship with respect to r is like this. On the pseudo steady state case we are going to get this additional term that I was mentioning in the previous slide.

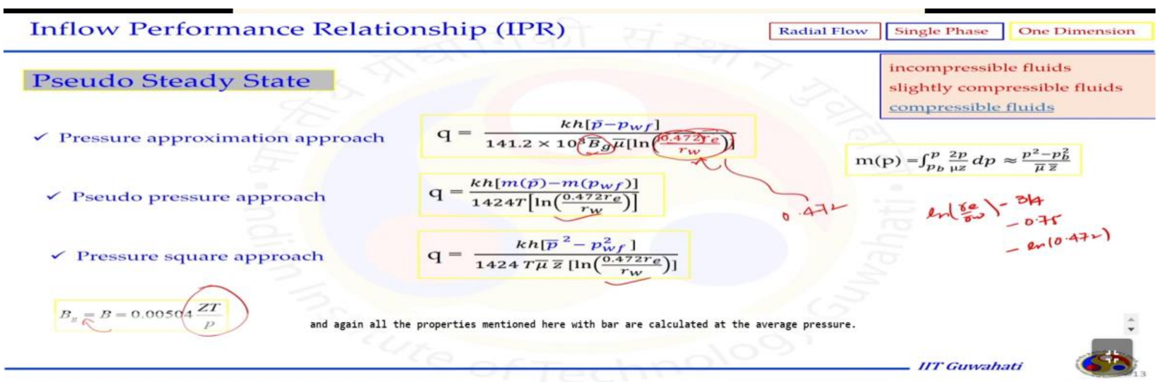


If the IPR is represented in the form of r is equal to Re where the reservoir pressure Pe will be equal to pressure P in this expression. So we will get this Pe minus Pwf here this r will become Re and this Re Re will cancel out you will get term 1 by 2. When the inflow performance equation is expressed in terms of the average pressure it could be volumetric average pressure then additional term will appear in both the equation for the steady state case and the pseudo steady state case. In a steady state case it is appearing minus 1 by 2 for this it will appear another minus 1 by 4.

So minus 1 by 2 minus 1 by 4 will become minus 3 by 4. So the expression for the steady state case and pseudo steady state case for the compressible fluid can be compared and more or less they are same additional term is appearing in the expression. The expression can be formulated to non parameter like the formation volume factor or the viscosity. So here we are seeing the expression r formulated in terms of mu we can replace this mu with the help of the volume formation factor for the compressible fluid that is b is equal to 0.005 we can express in terms of volume formation factor b. As for the compressible fluid so let us say for the compressible fluid as I was mentioning we can write the expression for P approximation, pseudo pressure approximation and then the P square approximation.

So for the steady state flow we can have this expression in the form of P minus Pwf for the MP approach we have MP minus MPwf and the P square. Now you can see in these three expression the other parameters got adjusted as per the need or the assumption taken in these three approximation. So for example P minus Pwf is a simplest form we are having we are having mu bar and bg bar in the expression while in the case of MP approach the mu and z are already accounted in the form of MP we do not need to have those expression in the this expression for the P square approach mu and z are appearing. Now here you will see the bar is mentioned over the properties or the parameters so we calculate these values at the average condition. So in the pressure approximation we calculate these values at the average pressure in P square approach we calculate these values at the average pressure that is calculated by this manner while in the case of the MP approach we do not need to calculate the average value because mu, z whatever the value those are changing with respect to pressure are already accounted in the form of MP and that is given like this.

The expression are usual here the only difference you will see are the Q that is actually should be capital Q but some books it is written as a small q also. So whenever you are seeing such kind of the expression you need to look what is the unit for that particular term is mentioned. So here Q although it is small case the unit is Mscf per day means it is a value of the gas production rate measured at the standard condition. Others are having the usual meaning only term that is here is MPw and MPwf so they are the real gas pseudo pressure and other P is the normal pressure in the reservoir so that is having the unit of psi. Similar expression can be summarized for the pseudo steady state case also.



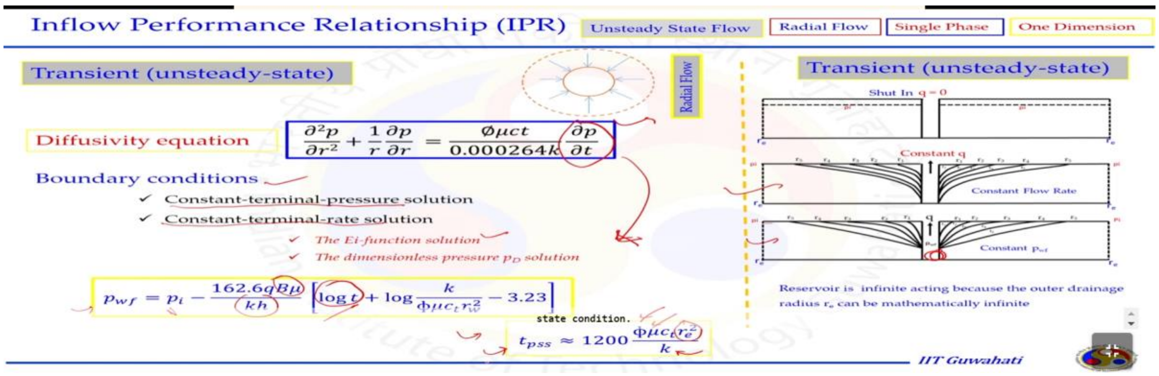
In this case the only difference between the previous case that is the steady state case and the pseudo steady state case we are seeing this term. Otherwise the expressions are similarly expressed in P approach P square approach and the MP approach. The next term that is making difference is in the denominator and this 0.472 actually appearing in this expression because for the pseudo steady state case we are having this Ln Re by Rw – 3 by 4.

Minus 3 by 4 is 0.75 and that is equivalent to Ln 0.472. So that is the way the term is adjusted and you are going to get this 0.472 into multiplication of Re by Rw and that is appearing in all three expressions. So the similar expression for the steady state and pseudo steady state case are expressed. Another part that is for the gas volume formation factor B is equal to 0. 00504 ZT by P. So either we can express the expression in the form of BG or BG can be replaced by ZT by P. In general whenever we are getting ZT by P in the expression we replace this with the BG and again all the properties mentioned here with bar are calculated at the average pressure. Again the expressions are having some term and the terminology is similar to the steady state case. The units are same, the nomenclature is same.

Now let us move to the third case that is unsteady state flow condition. In unsteady state flow condition for the radial flow we also call this the transient condition. We are having the well that is Q is equal to 0 under the certain condition no production is happening. When we start producing pressure surge happens near the wellbore region and slowly it reaches towards the boundary condition. When it reach the boundary condition or the condition when the change in pressure with respect to time at a location R is constant we reach the pseudo steady state condition. But before reaching pseudo steady state condition we are having the transient condition.

In that transient condition what is happening your pressure is changing at a particular location is a function of both position R and time T and that makes the problem more complex to solve it even for the simplest case of one dimensional single phase and radial flow. So under the transient condition the system is more complex and in this condition the reservoir is infinite acting because the radius mathematically can be considered spread to the infinity. So let us set up the IPR equation for the transient unsteady state flow condition under the radial flow and this is how the pressure is changing with respect to position and time and that is making the diffusivity equation. Either we can start with the diffusivity equation or the basic IPR equation. Now in this case this term neither zero nor constant it means we need the expression for this and then the boundary conditions are not that simple as for the pseudo steady state case where the del p by del R at R is equal to R e was constant.

Hence a very higher mathematical approach is required to solve this diffusivity equation for the transient flow condition and then the boundary conditions need to be considered to solve it. There are two approaches reported in the literature to have the constant terminal pressure solution and then the constant terminal rate solution. So either with respect to rate or with respect to pressure the boundary condition can be set up to solve this equation. Within terminal rate solution we are having two approaches EI function or dimensionless pressure solution. We are not going to discuss the solution under either the terminal pressure or the terminal rate solution.

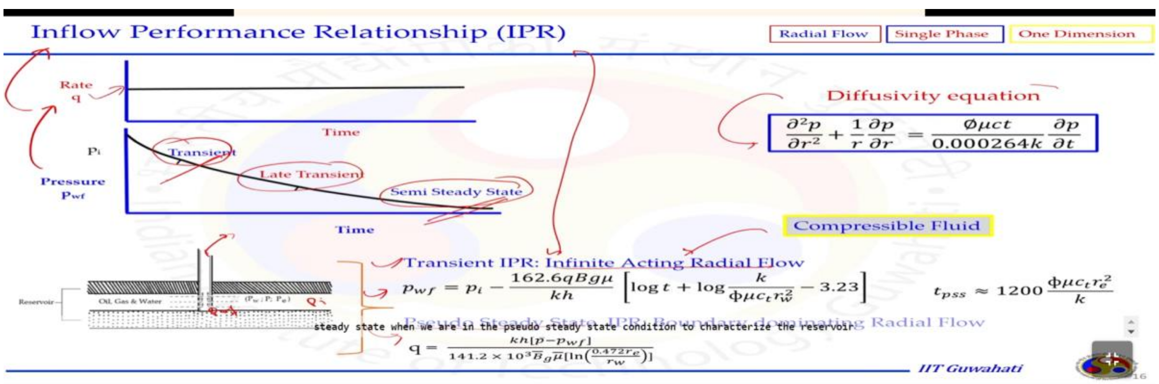


With the help of higher mathematics we can solve this diffusivity equation and that diffusivity equation will give us the solution that is PWF the pressure at the valve bore in the form of PI that is the average reservoir pressure and then we are having the fluid properties, reservoir properties in the expression. Now additional terms are appearing that is log T with respect to time at what time you are going to apply this transient flow condition. So the time will appear here because in the transient flow condition the pressure is changing with respect to time as well as with respect to location. So the diffusivity expression will give us this expression that is relating the Q with respect to pressure drawdown and that is the IPR equation. So this is the IPR equation for the unsteady state flow condition and now if we can write this for the oil and for the gas also.

If we are writing for the gas again P approach, P square approach and MP approach can be considered. The time to reach the pseudo steady state condition by this reservoir can also be calculated with this expression. This expression is having the properties of the reservoir like phi and K and the properties of the fluid that is mu and of course it will also depend on the radius of the reservoir Re because the time required to reach that radius is the time to reach the pseudo steady state condition. Here in this expression the time is given in hour, permeability is in milli darcy and CT is the total compressibility that is having the unit of pressure inverse. So understanding the IPR equation for the different fluid and different flow regime using the diffusivity equation can give us the better understanding about the IPR relationship.

So here in this diagram I am showing you if the production is happening at a constant rate Q we started producing from the well. So this is the well we started producing from this well what exactly happened with respect to pressure. So we are having the reservoir pressure Pi here we are having Pwf when we start producing the pressure changes suddenly and that sudden pressure change is happened and the flow regime is called the transient regime. So near the wellbore reason we are seeing the pressure is changing with respect to time as well as the position. When we spend more time in the constant production rate we reach to the late transient region still the change is happening but change is not happening with that drastically rate and after certain time the system reach to the semi steady state condition also called the pseudo steady state condition or quasi steady state condition.

In that case the reservoir is producing under the pseudo steady state condition and most of the reservoir produce under pseudo steady state condition for their long period of life. So the transient and late transient condition mostly happens in two condition first when the well just starting producing or second condition when the well just sudden in that time also either well just open or well just close for the production near the wellbore reason the pressure changes abruptly or means the pressure changes with respect to time and position and in that condition neither it is a steady state condition nor pseudo steady state condition. Semi state condition is actually kind of a hypothetical condition as I mentioned earlier also so the pressure relationship with respect to flow rate that is actually the IPR. Now we already set up the equation for both the cases transient flow condition and semi steady state condition using the diffusivity or the basic IPR equation. What those IPR equation for this reason in the reservoir fluid this is transient IPR where the infinite acting radial flow condition is happening, we are having the expression similar we are having the expression for the pseudo steady state condition.



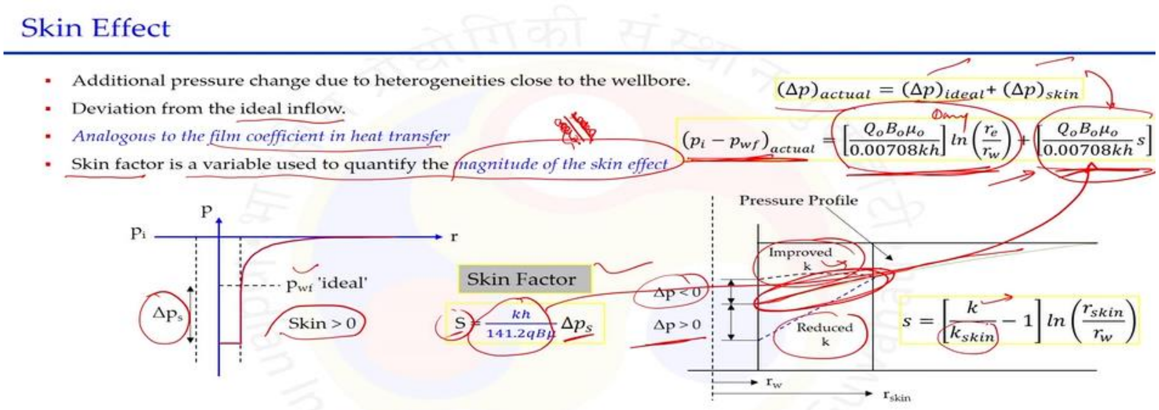
So when reservoir is under transient condition we can use this equation and when it is in the pseudo steady state condition we can use this expression to relate the pressure with the flow rate and the other fluid and the reservoir properties. These equations are written for the compressible fluid similar equation can be written for different fluid like the incompressible fluid and the slightly incompressible fluid. For the compressible fluid also the expression here are written in the form of P they can also be written in the form of MP and P square approach. Why we had written this expression because the IPR is actually used to develop the reservoir deliverability which means at what rate reservoir is delivering the fluid to the surface we can use either the transient if we are in the transient condition we can use pseudo steady state when we are in the pseudo steady state condition to characterize the reservoir performance. We will see the application of these equations when we are testing the wells in the next week.

Now we discuss one dimensional single phase radial flow we also discuss the types of the fluid and the flow regime. Another important aspects of the IPR equation is what is happening near this wellbore region. So two phenomena happens near this wellbore region one called the skin effect another called the non-Darcy effect. So far when we develop the IPR equation we consider the permeability is constant throughout the region. We also consider the flow is happening under the laminar condition it means we consider the Darcy law only or the flow conditions are considered the ideal condition to apply the Darcy law.

In all previous cases we discussed only Darcy law waste equation. But what exactly happened near the wellbore some pressure drop is happened because of the skin effect and because of the non-Darcy effect. The skin effect happens because of the permeability is different near the wellbore. Darcy effect happens because the violation of the Darcy conditions happen near the wellbore region we are not having the laminar flow condition specifically for the gas flow reservoir the non-Darcy effect becomes significant. So let us discuss one by one what these parameters are. So the skin effect is actually the additional pressure change due to heterogeneity close to the wellbore.

That heterogeneity close to the wellbore so for example this is my wellbore near this wellbore there is some additional deposition of some material that could happen because during the drilling operation during the completion of the wells or over the times of the production when we are producing the fluid some sand particles are getting deposited or some layer of the oil is getting deposited near this perforated region by any means the deviation from the ideal inflow is happening. Ideal inflow is the condition for the Darcy flow condition. The deviation happens and we need to account that deviation in the form of the additional pressure drop that is happening here. So the skin effect that is the additional pressure drop near on the surface or near the wellbore is happening is analogous to the film coefficient in the heat transfer and heat transfer also on the surface we are having the layer of any fluid deposition additional heat transfer coefficient is included in the expression. Similar we can include the pressure drop that is the driving force for the fluid to be produced in the reservoir.

We are having the additional pressure drop near the wellbore region and that can be accounted by the factor called the skin factor that is a variable used to quantify the magnitude of that skin effect. So for example in the ideal condition the pressure should be PWF but additional pressure drop is happening because there is some deposition of the skin or some damaged conditions are there near the wellbore. This damaged conditions are offering different permeability for the fluid to flow. So what actually happens compared to the ideal condition ideal means the Darcy law condition the actual pressure drop is the ideal pressure drop plus the skin condition and if we write the form of the flow condition the actual pressure draw down this is similar to the condition for the steady state flow condition under the Darcy law another term will appear that is accounting for this skin effect. This skin factor is additional pressure drop is denoted by S and then the parameter similar to that what we are appearing in our IPR equation is also appearing for the skin factor expression.

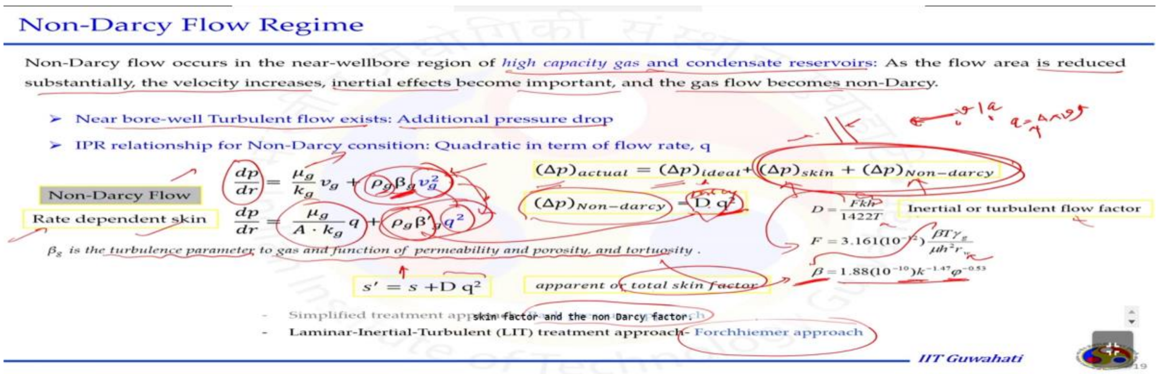


It means we can include this skin factor in our basic IPR equation and then the pressure drop total will be because of the Darcy law and because of the skin factor. Now this skin factor not necessary always on the negative side means it is having the damage condition also of course the damage conditions happens during the drilling operation during the production condition the permeability near the wellbore region is reduced and that is where the pressure drop is happening but over the time we also perform certain operation to stimulate the condition near the wellbore region and it might happens this is the ideal condition and from the ideal condition the improvement in the permeability is happening because of the stimulating scheme we are implementing either acidizing the nearby wellbore region or by creating more fracture or by any mean if we can improve the permeability near the wellbore region the improvement in the value of K will happen in that case the pressure drop will be negative and that should be adjusted in our equation. So any deviation from the ideal condition either it is improved or it is reduced should be because of the skin factor that is appearing in the expression of more accurate IPR for the steady state compressible fluid condition. Now this S can be related to both actual permeability that K is far away from the wellbore and this K is the permeability within this skin zone. So this skin zone could be few inches to few meter or feet from the wellbore surface so accordingly the radius of our skin and the RW that is the radius of the wellbore can be used to calculate this factor S that is appearing here or that is appearing here so we can calculate using the permeability value in the skin zone or away from the skin zone or other way if we know the value of S we can calculate the permeability within the skin zone and we can see the permeability is lesser than or greater than the permeability far away from the wellbore or the average value of the permeability in the formation.

So if del P skin is greater than 0 indicate an additional pressure drop is happening due to wellbore damage and then the K skin will be lesser than the K and that is happening here. Second condition when the pressure drop is lesser than 0 it means less pressure drop due to wellbore improvement by any mean we stimulated the condition near the wellbore and in that case the permeability is more than the permeability away from the well and then the fluid is having very easy passes near the wellbore region compared to the away from the wellbore region and the situation when the permeability away from the wellbore or near the wellbore are same it means K skin is equal to K means no additional pressure drop is happening and this term can be dropped we are having only the flow through the Darcy law conditions. So the skin effect is important to account in the expression similar another parameter that is the non Darcy condition that actually happens in the high capacity gas reservoir or the condensate reservoir as the flow rate is reduced substantially the velocity increases and the inertial effect becomes important and then the gas flow becomes non Darcy. So what does it mean so be away from the well this is our well away from the well gas is moving towards the production well and it is having like velocity V or the flow rate Q. So the change in the radius is not that much so the V and Q are changing not very high value as the radius is changing but when we reached near to the wellbore region suddenly the radius is changing fast or drastically it means Q is equal to area into velocity.

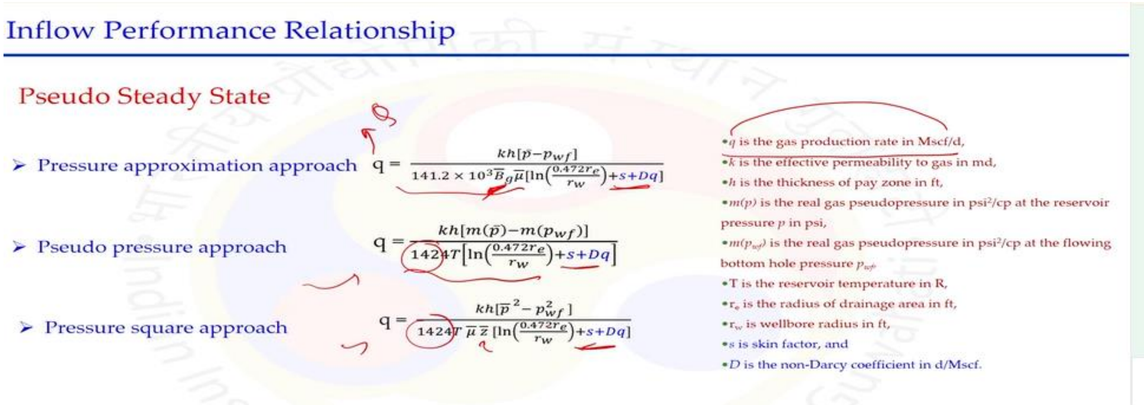
So if radius is changing the velocity will increase because change in the radius will also change the area. So the area will get reduced by the reduction of the radius and then the velocity will increase or the flow rate will increase. In that case what will happen the condition that we are assuming for the Darcy law the viscous flow or the velocity is not that high we are having only the laminar flow condition that will get violated near the wellbore region. So in the near wellbore region the velocity will be very high so pressure drop happening near the wellbore region will be because of two factor one is normally the Darcy law is happening and another the deviation from the Darcy law in terms of the non Darcy flow and this non Darcy flow is the rate dependent skin also called rate dependent skin another type of the pressure drop that is happening near the wellbore region.

So near the wellbore region turbulent flow may exist and additional pressure drop will be there. This is the situation that may happen or may not happen but in the case of the gas reservoir it is advisable to consider the non Darcy effect into the IPR equation to have the more accuracy in the expression. So the rate dependent skin that represent the non Darcy flow can be expressed with the velocity squared term or the flow rate squared term here. It means the pressure drop that is happening by the Darcy law just with respect to Q while in the non Darcy case it is having the quadratic relationship with respect to flow rate. The term that is appearing here Bg that is called the turbulence parameter to gas and it is a function of permeability and porosity and then the tortuosity of the reservoir domain.



So the Bg is actually defining the reservoir conditions and this rho g is for the property of the gas. Now the prime is mentioned here in the second expression because we adjusted velocity to flow rate. So in the case of non Darcy flow is happening in the reservoir the actual del p will be summation of del p under the ideal condition that is the Darcy law condition then the skin effect is there and then the non Darcy factor is also appearing there and then the non Darcy factor the change in the pressure because of this non Darcy phenomena that is happening here can be expressed in the form of D dot Q square. So this all is D for example along with other parameter and then the Q square is there. So the pressure drop by the Darcy law is in the linear relationship with Q while for the non Darcy it is in the quadratic relationship. This D is the parameter that is also called the inertial or the turbulent flow factor that is account for the permeability and the page on thickness and temperature.

The F factor that is appearing in the D is actually accounting some more properties of the fluid and the reservoir including the value radius and the beta that is appearing here that is actually K and phi this is the reservoir properties permeability and porosity. If we combine both skin factor and then the non Darcy factor this called the apparent or total skin factor. It means the deviation from the ideal behavior can be expressed as a total skin factor. This expression of S and T Q square for the skin factor and the non Darcy effect will be utilized when we are expressing our IPR equation in more generalized form in the empirical form in fact the back pressure approach and the force mirror approach to account for the skin factor and the non Darcy factor, So the IPR equation will get modified for the steady state case for example we got S plus D Q in the denominator for all P approach MP approach and P square approach.

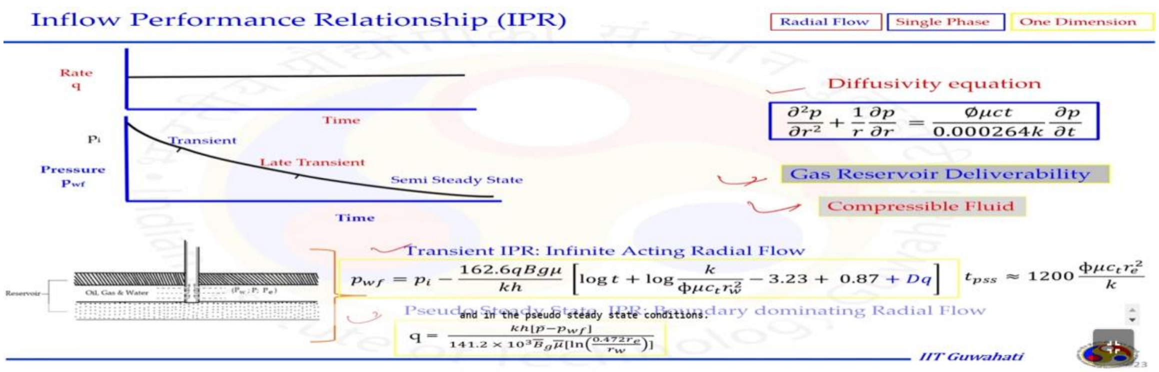


The S is defined like this that we discussed and then D is defined in the form of F and other parameter F defined in form of beta and beta defined in form of permeability and porosity. The quadratic form of this equation if we take everything on this side multiply this we will get Q square D and then other terms here. So that is where N multiplied by Q and of course divided by X. X is accounting for all these part and this is equal to pressure draw down that is P bar minus P W F. So that is the IPR again pressure difference relating to flow rate but in case of the non Darcy effect this equation will become quadratic form.

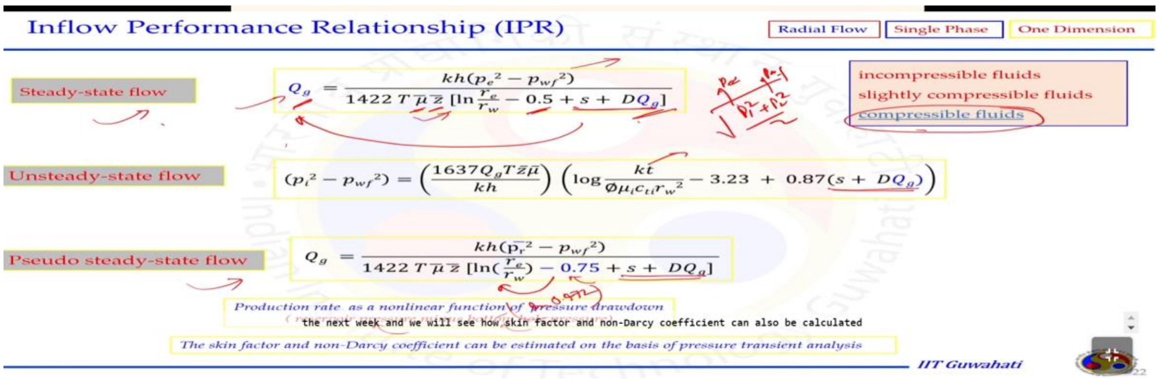
So compared to the previous understanding of the IPR equation we got two new terms S that is the skin factor and D that is the non Darcy coefficient and non Darcy coefficient is having the unit of inverse of the flow rate. Similar expression for the pseudo steady state case we are just going to get this term additional here for P approach P square approach and MP approach. So this I am showing for compressible fluids similar expression can be written for the incompressible and slightly compressible fluid. The same expression Q is expressed in MSCF per day the equations are having the same as previously they can be expressed either in the form of BG or in form of Z or Z and BG can be replaced numerical coefficient will get adjusted accordingly. It is having the relationship with temperature in this approach while here BG is accounting for the temperature and then compared to pseudo compared to steady state case pseudo steady state case is having this additional term in the denominator.

So for one dimensional single phase radial flow for the compressible fluid we can summarize what is happening under the steady state flow condition. Compressible fluid means for the gas and this is mentioned in P square approach all three expressions are mentioned in P square approach. So for a steady state flow we are having this additional term if the volumetric average reservoir pressure is taken otherwise this should not be appearing here. The value mu bar and Z bar are calculated at the average reservoir pressure on a steady state condition where we are having the time dependent phenomena and then time is expressed in hours. We are having this term in all three equations that is accounting for combined total skin factor that is skin effect as well as non-Darcy effect.

This term 0.75 comes in the pseudo steady state case. So ultimately we got the expression of pressure drawdown versus flow rate for all the possible cases we can get for different fluid regime, different types of the fluid and for the case of the compressible fluid accounting for the Darcy, non-Darcy and skin effect we get the production rate as the non-linear function of the pressure drawdown. Pressure drawdown means pressure of the reservoir minus pressure at the bottom hole and that makes the equation more complex to solve it. Although quadratic equation can be solved if we are solving in the current form. So these basic IPR equation can be deduced for different form. Now the IPR equation for a particular types of the fluid in particular flow regime is very beneficial not only to establish the relationship between the pressure drawdown and Q, but the unknown those are appearing in this term if we can test the well using this equation we can calculate the unknown parameter those are appearing in this equation by testing the well at different conditions.

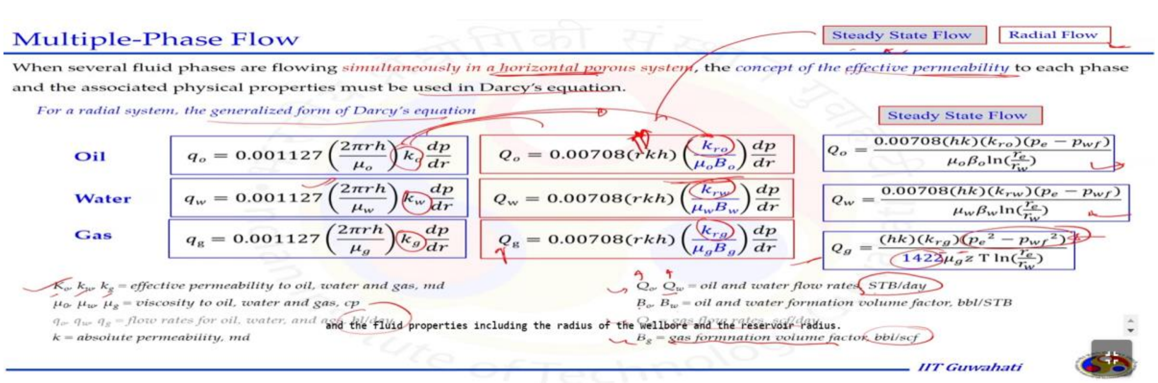


So the skin factor and non-Darcy coefficient can also be estimated on the basis of pressure transient analysis. Pressure transient analysis is a type of the well test analysis that we will discuss in the next week and we will see how skin factor and non-Darcy coefficient can also be calculated by performing pressure transient analysis on the well system. So what we do in that pressure transient analysis we are having this diffusivity equation for the gas reservoir dilute gas is a compressible fluid we got within this region our transient IPR equation and then the pseudo steady state condition equation. So the equation are for the compressible fluid it means they are applicable for the gas reservoir and using this equation we can have the gas reservoir dilute in the transient condition and in the pseudo steady state conditions. Another thing that we did not consider so far is the multi-phase flow system we consider always single phase. So now the single phase can be replaced by the multi-phase still we are in the one dimensional assumption and then the radial flow condition and then the multi-phase can also be considered for different types of fluid under different flow regime and then skin effect and non-Darcy effect can also be counted.



We are not going in that discussion we just saying when the multi-phase flow system is there how to deal multi-phase system. In that case of the multi-phase flow system let us consider the radial flow and the steady state flow condition. So when several fluid phases are flowing simultaneously in a horizontal porous media so we consider only the horizontal porous media the concept of effective permeability to each phase and the associate physical properties must be used in the Darcy equation. So instead of writing complex equation for the different phases are present we can write the equation for the phase that we are considering.

For example, we are considering all three phases oil, gas and water. We can write the Darcy law for oil, gas and water and then the permeability that was appearing can be replaced by the relative permeability in the Darcy equation. So for the radial system the generalized form of Darcy equation for the oil this is the generalized form of the Darcy equation. This can be solved for the steady state case and in this expression before solving for the steady state case what we did this term k o is replaced with relative permeability and then this k absolute permeability is also appearing in this term. Similar we can write for the water similar we can write for the gas and these are the nomenclature for different parameter those are appearing here. So the effective permeability of the oil is appearing in the oil expression for the water it is Kw for the gas it is Kg they can be transferred to K that is absolute permeability K is absolute permeability and k ro is the relative permeability of oil similarly for the water and similarly for the gas.



Now this equation can be solved for steady state condition before going that let us see what is q 0 and q w they are the condition at the fluid is measured at the standard condition so this is the standard tank barrel per day for oil and water and similarly for the gas m SCF per day. Now this expression from the Darcy law those converted to relative permeability part can be solved under the steady state flow condition. So this is the same expression we obtained for the oil similarly we will get for the water and for the gas we can get the expression in the form of p square that is written here we can get for the in the form of p and mp and then the numerical coefficient that is appearing here 1422 again depend on the unit system you are having and then the values are calculated at the average condition of the pressure. So to deal with the multi-phase flow system this is one of the way the expression or the flow rate of different fluid those are present in the reservoir can be utilized to get the IPR equation that is relating the flow rate to the pressure drawdown along with the reservoir and the fluid properties including the radius of the wellbore and the reservoir radius. So in summary what we did in today's class we developed the IPR equation for all three types of the fluid under the steady state flow condition.

We discussed one case for the pseudo steady state flow conditions solve the equation and we seen the expression for the unsteady state flow condition how the time is included into IPR equation to account for the unsteady state flow behavior or the transient flow behavior that happens near the wellbore condition or the start of the production condition. The factor skin effect and then the non-RSE effect are also included into the IPR equation and then the final expression that we obtained is having the capacity to include skin effect non-RSE effect as well as Darcy effect. So in the next class what we are going to discuss we will again take this inflow performance relationship IPR we will try to utilize this IPR equation that is developed for different fluid and different flow regime. Mostly we will focus on the complex fluid that is the compressible fluid and we will see pseudo steady state and unsteady state condition because those are the condition the reservoir gets operated and we will understand the concept of productivity index and absolute of the flow condition in the reservoir. Further we will go for the testing of the reservoir well using the empirical method those are based on some field test they can be the single point or the multiple point test methods and further the gas well testing, well performance and then the decline curve analysis.

So actually this is related to understanding the oil and gas reservoir their performance predicting their future profiles will be discussed in the next lecture. So with this I would like to thank you for watching the video we will meet in the next lecture thank you very much. .