Petroleum Reservoir Engineering

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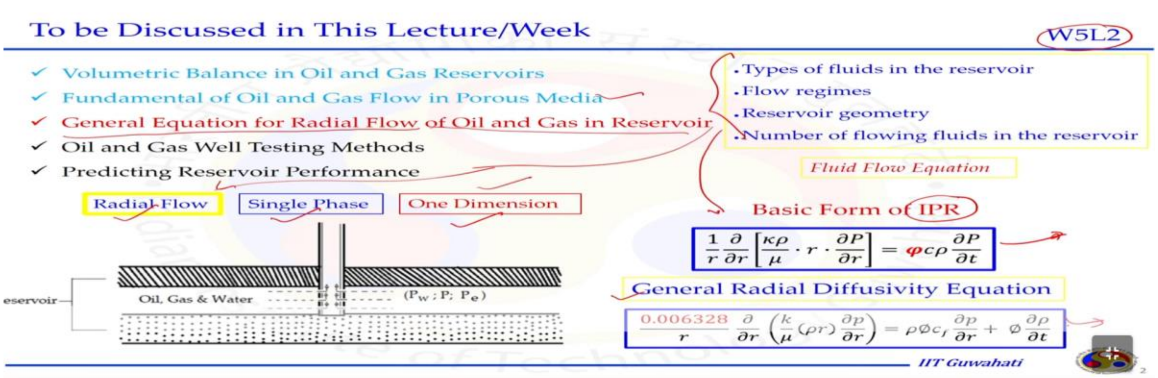
Lecture 14: General Equations for radial Flow in Reservoir

Hello everyone I welcome you again to the class of petroleum reservoir engineering this is lecture number 2 of week 5. So in the previous lecture, lecture 1 of week 5 we discussed the fundamental of oil and gas flow in porous media. In that lecture we discussed the types of the fluid flow regime reservoir geometry and number of the fluid those are flowing in the reservoir at the same time. So the combination of these 4 parameters they may vary because of the types of the fluid there could be compressible fluid, incompressible or slightly compressible fluid. Similarly for the flow regime it could be steady state flow, pseudo steady state flow or unsteady state flow. Reservoir geometry could be radial, axial, spherical, hemispherical and number of flowing fluid could be single phase, two phase or the three phase system.

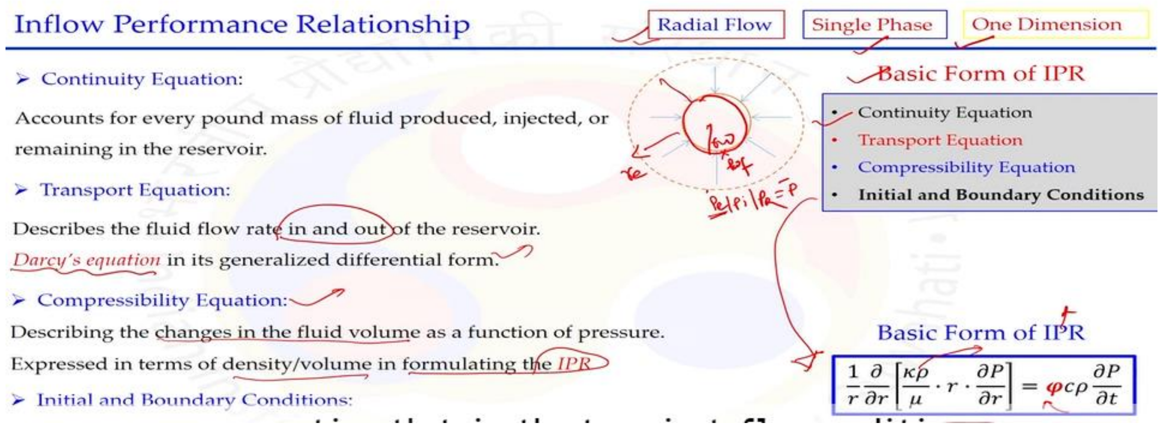
So we discussed the basic part of these 4 parameters along with the fluid flow equation that is the Darcy law the flow equation is used for the flow through porous media. So we developed our understanding to establish the one dimensional model in the previous case for a single phase system. In this single phase system we consider only one type of the fluid is there now it can be oil, gas or water or in terms of the fluid property it could be compressible fluid, it could be incompressible fluid or slightly incompressible fluid. We discussed one example of the axial flow so in today's discussion we are going to continue the same understanding where we will be setting up the equation in general the general equation for radial flow for oil and gas reservoir.



So the general radial diffusivity equation or the basic IPR equation will be established in today's lecture. And in today's lecture we are going to consider only the radial flow. So now we restrict our understanding to just one dimensional model single phase flow system and now the flow geometry here that is also fixed to radial condition. So the radial equation of the flow through porous media will be established in the form of inflow performance equation. And the basic form of the IPR will be deduced based on the parameter we discussed in the last class and then general radial diffusivity equation will also be discussed.

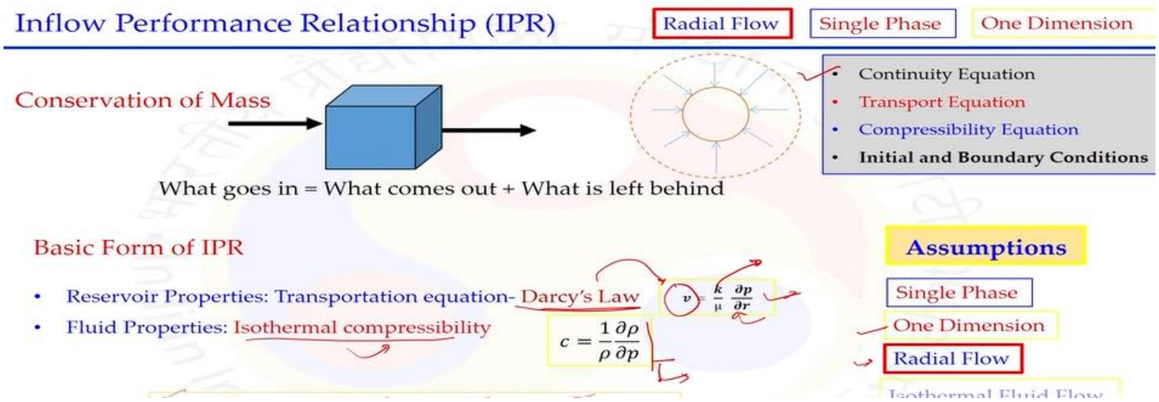
So both are same actually this is having certain assumption compared to this one we will discuss under what assumptions the form of the general radial diffusivity equation will take the shape. So let us start for a single phase flow system one dimensional flow under radial condition. So in radial flow the fluid from all direction coming towards this production well and then the radius of this production well is rw the radius of the reservoir is re the pressure at this condition is pwf and then the pressure at the reservoir we can consider this is too far from the well bore the reservoir pressure could be pe or we can name it pi and something it is refer as pr and sometime this is the average reservoir pressure pr also. So to set up the basic form of the IPR we have to understand from the fundamental that is the continuity equation. So the continuity equation account for every pound of mass fluid that is produced injected or remaining in the reservoir.

So this is mass conservation equation we account how much mass is going in how much mass is going out and how much mass remains within the control volume. So the continuity equation should be included to get the basic form of the IPR second is transport equation that describe the fluid flow rate that is in and out of the reservoir and that is done with the help of Darcy equation in its generalized form as now we are talking about the radial form of the Darcy equation will be included in the continuity equation to account for the flow behavior of the fluid. Then the properties of the fluid the fluid could be oil gas and water the properties of the fluid are included in the general IPR balance equation in the form of compressibility equation that equation describes the changes in the fluid volume as a function of pressure expressed in terms of either density volume in formulating the IPR in flow performance relationship. Once we get the basic IPR equation let us assume for a radial flow system we got this basic IPR equation this is the fundamental equation it is not for a particular type of the fluid not particular type of the flow regime it can be solved for any case. The certain assumptions are taken here that is like the porosity is taken constant the height or the Poisson thickness is taken constant but this equation is still including rho that is the property of the fluid so it is applicable for oil gas and water.



To solve this equation to get the relationship of IPR actually IPR is the relationship between flow rate and the pressure draw down or the pressure difference we have to solve this equation. To solve this equation we need some initial and boundary conditions so there are two boundary condition and one initial condition is required when we are solving the most complex part of this equation that is the transient flow condition. We can solve it for the steady state pseudo steady state and the unsteady state case in all the cases the number of the boundary initial conditions are required should be available to get the solution in the form of flow rate versus pressure difference and the assumptions so anytime when we are solving the equation we make certain assumptions to simplify the problem we need to explicitly write down or understand the assumptions are taken. So for example when we take the Darcy law into account for fluid flow behavior we are taking the assumptions under which the Darcy law does work. So we will see the assumptions also when we are establishing the relationship for basic IPR and radial diffusivity equation.

So let us start again with the same thing we are having the radial flow single phase fluid in one dimensional the mass conservation equation can be written for the continuity part first equation that simply what goes in is equal to what comes out plus what is left behind. In that case we can start with the transport equation from the Darcy law and the fluid property from the isothermal compressibility into conservation of mass. So this is Darcy law in terms of the velocity we can convert it into in the form of Q also and the isothermal compressibility that is C in terms of density could be in terms of volume also. Later on we see the application to deduce some other form of isothermal compressibility. So when we put all of them into conservation of mass equation we get the generalized form of the IPR and that generalized form of the IPR can be subjected what type of the fluid compressible incompressible slightly compressible and then the flow regime steady state pseudo steady state and the transient flow that is happening.

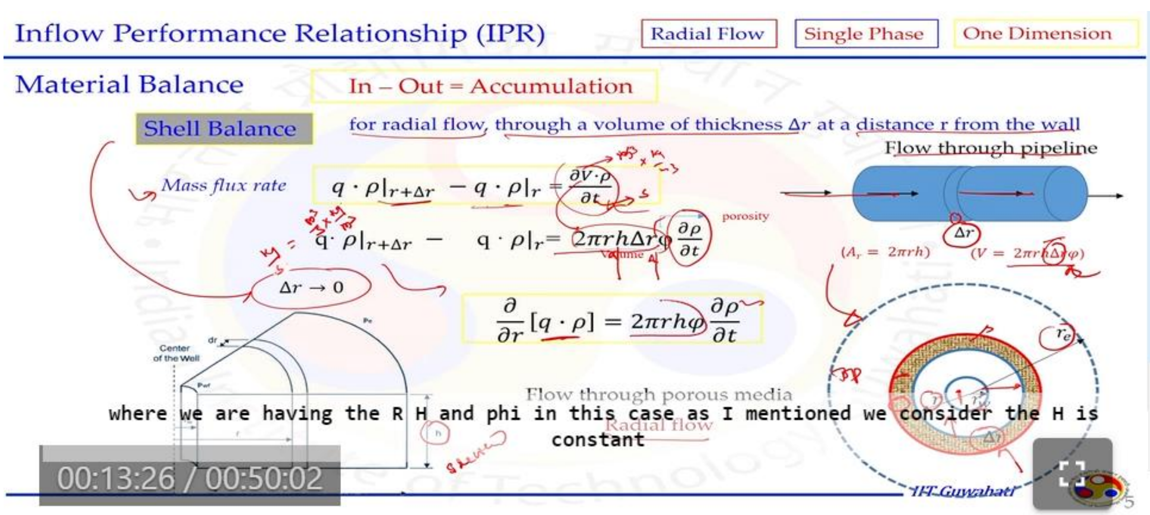


The assumptions that are taken should be listed out as of now just setting of the conservation of the mass equation so the assumption we had taken is the single phase so only single phase fluid is present because of that we had taken this K value here permeability for a single phase fluid that is flowing through the porous media. We had taken only one dimensional model that is where the Darcy law is written only for R direction. So the fluid is flowing only in the R direction however there is a possibility of fluid is flowing in the other direction also. But we consider the fluid is flowing only in one dimensional that is the R third assumption we had taken that direction is the radial flow we consider the isothermal fluid flow condition so in this set of so far we did not consider any energy equation what we are assuming the conditions within the reservoir is isothermal kind of uniform temperature and that is where the definition of the isothermal compressibility is also applicable under that condition only. And when the temperature is constant the definition of C is the isothermal compressibility another assumption that I mentioned Darcy fluid flow so whatever the assumptions of the Darcy fluid flow like the flow should be in the laminar condition and others are implicitly included into the assumption list.

So let us start the material balance that is in minus out is called to accumulation and we generally do the material balance equation as in chemical engineering or in some other stream also we start with the cell balance. We consider a small section of the control volume of very small thickness and we do the balance around it that is called the shell balance. So for the radial flow through a volume of thickness del R at a distance R from the wall for example you can see here when the flow through pipeline is happening we are having this thickness of del R of fluid is flowing from this pipe. So we consider this del R thickness within this small control volume we see how much mass is getting in how much mass is getting out or when it is a continuous flow system we see mass flux the mass flow rate in minus mass flow rate out is called to the accumulation term. So we can do the similar balance for our reservoir system where the radius R W is one boundary of the production well we can start from this boundary to distance R or we can measure it from the center also.

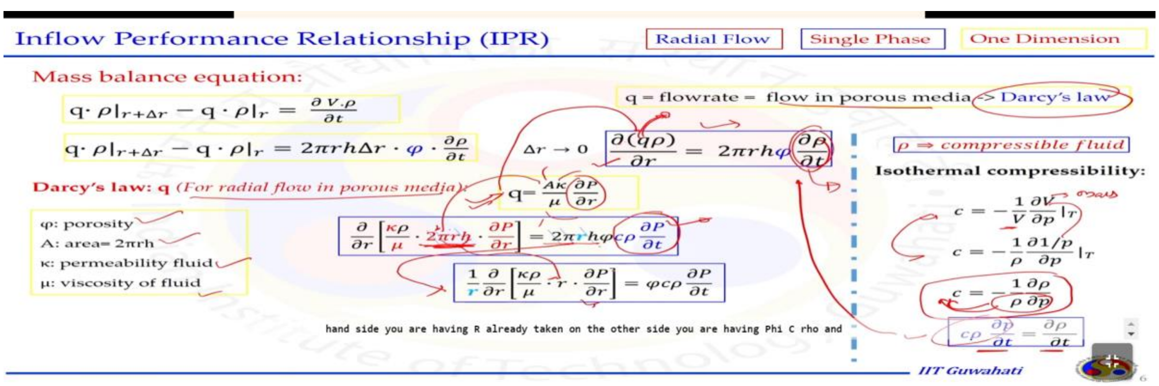
So the R distance from the center we can consider a slab of del R thickness and then it is like page on thickness H in the entire domain it is spreaded like this. So we are looking this from the top view when we are looking from the side view it will be like this. So we taken this slab of del R from center at R distance and then the reservoir radius is R e. Now within this shell of del R thickness we are going to consider the material balance by doing the shell balance. So the flow through porous media in the radial flow is considered and in that case the mass flux rate through this slab of del R thickness would be Q dot rho.

So the fluid is getting in fluid is getting out through this thickness del R so the fluid is getting in R plus del R and then the fluid getting out is R that is equivalent to the accumulation. So this Q is having the unit of volume let us say meter cube per second rho is having the unit of mass let us say kg per meter cube. So the unit is coming out as kg per second so that is why it is a mass flux rate and here also this is the unit of volume that is meter cube and then you are having the density that is kg per meter cube and then you are having the time that is in second for example. So the unit is balance so this is mass flux in mass flux out is equal to accumulation. Now this fundamental equation can be expanded to get the general IPR equation.



Now in this case the area is 2 pi RH and then the control volume that is we are having under consideration is 2 pi RH multiplied by the del R that is the thickness of the slab and then the phi is included to account the pore volume because the fluid flow is happening through this porous region only. So the more generalized form when we are replacing this volume here that is 2 pi RH del R phi for the volume and then phi is the porosity actually there and then inside of this is remain density because we are considering any types of the fluid could be there we are not specifying density is constant and something. So this is the general IPR equation or the basic IPR equation we are going to establish only things what we did we had taken out the H and the phi out of the derivative it means we had considered the H and phi are constant. Further when we do the shell balance we say the condition when this slab is going to be very, very small in that case the equation take the form of partial differential equation and in that case you will see this is the mass flux this is the density with respect to time it is changing and this is the part where we are having the R H and phi in this case as I mentioned we consider the H is constant and the phi is also constant. So the mass balance equation we could establish that is the way we can do just replacing the volume in its original form considering the thickness is going to be very small we are going to get this simple equation.

This simple equation now needs to be counted for the flow term and the flow through porous media is counted by the Darcy law. So this Q can be replaced in this equation by the Darcy law we can do that thing Darcy law for the radial flow in the porous media is given by this expression where A is the area K is the permeability Mu is the viscosity of the fluid and this is the pressure gradient. So all I mention here porosity area permeability and the viscosity so when we are going to place this Q here we have to take care on the right hand side also because it is not possible to measure the change in the density with respect to time at a particular location within the reservoir we can convert this into some term that could be measured or could be calculated. So let us see how this derivative of density with respect to time can be replaced by understanding the property of the fluid. So the isothermal compressibility definition that is change in volume with respect to pressure at constant temperature divided by original volume the form of this can be converted in the form of the density.

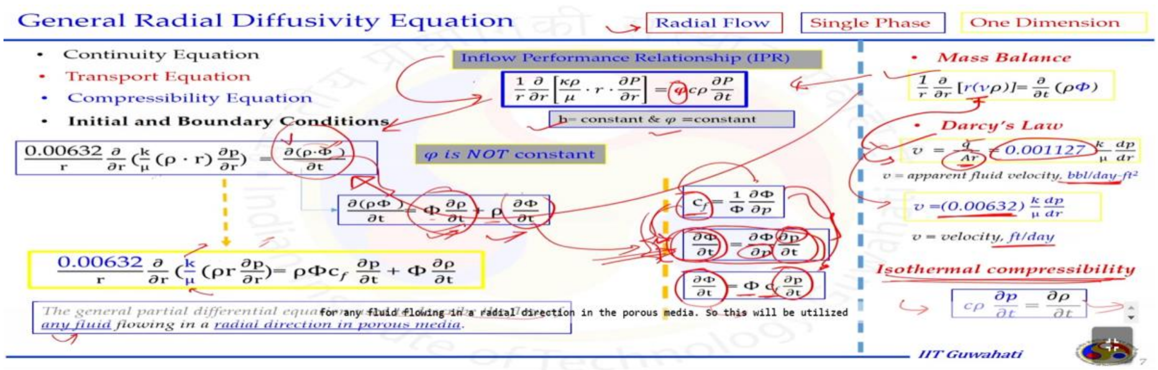


We can replace this volume by the density we can do this by multiplying the mass in the numerator and the denominator and we are going to get this form of C in terms of the density. If we adjust this equation taking the rho and del P on the C side we will get this term and then we can take the derivative with respect to time on both the sides we are going to get this expression. So now what we can do we can replace this term by this one so our equation will be in the form of like this when we replace Q in this equation using the Darcy law we are going to get this K rho Mu 2 pi RH is the area so the area is replaced by this one and then on the right hand side we are having 2 pi RH Phi and now this dou rho by dou t is replaced by this term this is C rho dou P by dou t. So now we convert that equation in the form of pressure we can solve this equation with respect to pressure to get the IPR so before we are going to do that let us consider already we made the assumption H is constant Phi is constant so 2 pi H from the left hand side will be cancelled out with 2 pi H on the right hand side the equation will be simplified this R can be taken on the other side so 1 by R and then the same term you are having here K rho Mu here only R remains here that is here del P by del R as it is on the right hand side you are having R already taken on the other side you are having Phi C rho and then the derivative of P with respect to time. Now this is the general IPR equation and this equation is not assuming any types of the fluid and any types of the flow regime so we can consider this as the basic IPR equation only assumption are taken H is constant and Phi is constant.

So to develop the radial diffusivity equation as the name suggest we are talking about the radial flow only the fluid is happening in R direction the difference between the inflow performance equation that we developed in previous slide this is this one is that where we consider Phi is constant the porosity is constant and if we do not consider that make more general form of the equation we are going to get the general radial diffusivity equation. In IPR equation we consider H and Phi constant so let us say when we consider this equation we can start again with the same mass balance equation using the Darcy law now Darcy law can be substituted if we are writing the equation in terms of velocity we can write this Q_i A_r, Ar is the area now some numerical value is coming into picture that is because of the unit consider for the velocity that is barrel per day per feet square if we are considering the velocity in feet per day the numerical value will be 0.00632. So the only difference between this and this the way the unit of the velocity or apparent fluid velocity is considered into the expression. We already know the isothermal compressibility so we can start with the mass balance include Darcy law in different form we will get different form of the IPR equation and in that form we also consider isothermal compressibility definition to account for the fluid properties.

So let us say when we are counting the Phi is not constant in that case the equation the same equation or the equation from the mass balance by including the Darcy law or isothermal compressibility we will ended up getting this kind of the expression. Here we did not consider the isothermal compressibility so far we consider the volume was here we had taken the other part outside but not taken Phi is still inside the derivative terms. So we can start solving the right hand side we can take two terms as A and B is density B is porosity and we can do the derivatives. So derivative of A and B is equal to A derivative of B plus B derivative of A so here we can replace this one with the help of taking the derivative of two variables. Now here we got the term how to replace this density we already know that term with the isothermal compressibility we can replace and similarly the compressibility of the rock can be used to account the change in the pore volume of the rock.

And that pore volume can be converted into porosity and the definition of the compressibility for the rock can be expressed in terms of the porosity. This we discussed when we are discussing the rock properties also. So similarly we can use the definition as we did for the fluid we can do for the rock sample also and we can replace this term in some terms that can be measured like the pressure. So with the help of this equation we can adjust using these two equation we can get change in the porosity is equal to porosity compressibility coefficient for the rock and then change in the pressure with respect to time. So these two equations are going to give us this thing.



The second equation just a chain rule where we are having derivative of one term we can multiply the both side by the pressure numerator and denominator to classify in two terms and later on using the definition of the first one we can replace this part. So now we know the expression for this term we know the expression for this term we can substitute both terms here in this equation and we can get this form of the IPR. In this form of the IPR we are still considering K and Mu inside the derivative terms means permeability and the viscosity of the fluid are still under the derivative terms we did not make any assumptions of making them constant or taking out. If we are doing that then the form of the general diffusivity equation will get changed. So let us see this general equation that we have is a partial differential equation that can be used to describe the flow of any fluid as I mentioned we did not consider the row as constant or other forms it can be applicable for any fluid flowing in a radial direction in the porous media.

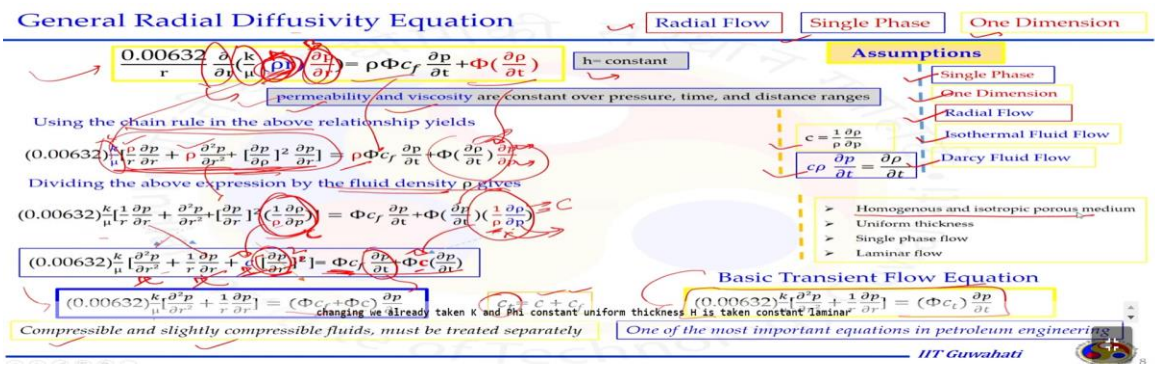
So this will be utilized for that purpose. So this is the radial diffusivity equation the same equation from the previous slide we develop for the radial flow single phase and the one dimensional only assumption we had taken in this is H constant while basic IPR equation we consider your Phi is also constant. Now this equation if you see inside of this first derivative we are having rho and R and both are changing correct and then we are having this del P by del R that may also change depending on the flow regime we are having at a particular location pressure is changing or not changing similarly on the other side we can replace the other terms. So here there is a type of mistake this should be P instead of rho as we already mentioned in the previous slide how to replace rho with the P so the only assumption we had taken is H is equal to constant and on the other side let me mention you again we consider single phase one dimensional radial flow isothermal fluid flow and the Darcy law. If we further consider the permeability and viscosity are constant over the pressure time and the distance range we are considering for the flow to flow then we can take these two terms out otherwise if they are also varying then the equation becomes so complex it cannot be solved analytically so easily.

So let us take these two assumption permeability and viscosity are constant so we can take both of these terms outside the derivative. Now inside we left with rho and R and then del P by del R so these terms can be differentiated with respect to R by considering the chain rules and when we are doing so we will get this expression. So first time we consider rho R as one variable differentiate this is A this is B we can differentiate A is remain constant derivative of B plus B constant derivative of A and then further this A is also part of rho and R we can differentiate one time with respect to rho another time with respect to R. And when we are doing that things we are going to get this expression. On the other side we are keeping this part as it is and the other part we can replace with the help of the compressibility of the fluid to do that we need to multiply both numerator and denominator with respect to P.

So dividing the above expression by the fluid density rho it will eliminate the rho from here it will put rho value here so that is appearing rho here on the right hand side this will be replaced and then this term will yield it like this kind of the things. So where the rho is replaced by the P and the other term that is appearing here dou rho by dou P divided by rho and this is equivalent to compressibility of the fluid. So in that way we can replace this complex term that is where we were supposed to know the change in the density with respect to time we can replace in a more simple form. So the left hand side is going to be the same only things what we did we replace here also the arrangement is done by dividing this term with the fluid density we are going to get similar term here that is appearing here also and this can also be replaced by C. So this is compressibility of the fluid here then this is compressibility of the fluid is appearing here.

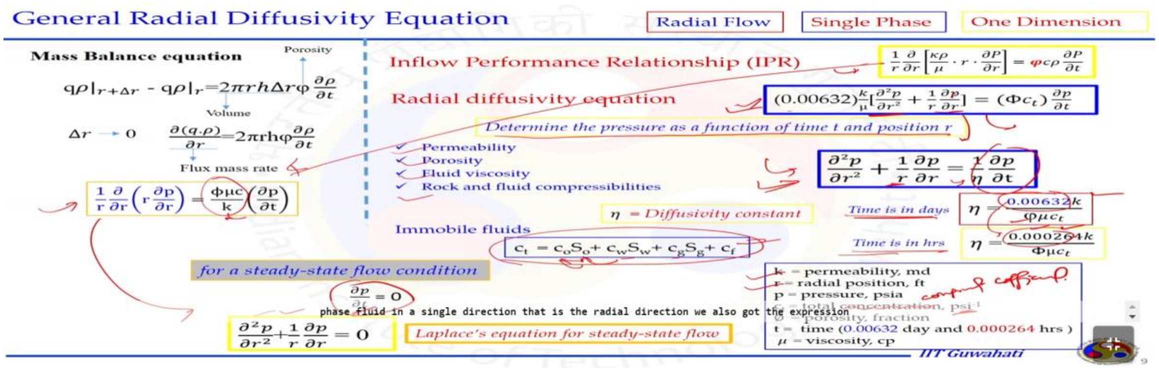
So this term is taken here just for the adjustment and this term is taken here so we are going to adjust our equation this is second derivative with respect to pressure first derivative with respect to pressure and here is first derivative is having square 2 this is power 2 of this derivative. Now the multiplication of C and then the square of the derivative is going to be a very small quantity that can be ignored in this expression and then the final expression we are going to get numerical value same ky Mu same this term is there this term is also there and right hand side we can take this term outside and then inside the bracket we are having Phi multiplied by compressibility of the rock and then

Phi multiplied by the compressibility of the fluid. So now this equation can be solved for different types of the fluid so the compressible and slightly compressible fluid must be treated separately. If we combined both the compressibility of the fluid and the rock we can denote this by the CT and then the expression would ended up like this one. So this is one form of the generalized radial diffusivity equation where we consider further assumption that the permeability and the viscosity are constant and this equation is also called the basic transient flow equation we can solve this equation for any flow regimes under the condition of assumption we had taken.



And this is one of the most important equation in petroleum engineering to understand the one dimensional single phase flow through the reservoir in the radial direction. And then the assumption we had taken to derive this basic transient equation homogeneous and isotropic reservoir properties are not changing we already taken K and Phi constant uniform thickness H is taken constant laminar flow and the single phase flow system is considered. So the same equation of IPR that we are having generalized form now we got the generalized form for the radial diffusivity equation both are having almost similar kind of the expression. And then these equation can be used to determine the pressure as a function of time and position when we are solving for the transient flow condition and this equation requires permeability porosity fluid viscosity and the rock and fluid compressibility to have the relationship or to solve this radial diffusivity or IPR equation for a special case of different flow regime and different fluid. So we started with the material balance equation included Darcy law so this is the same form of the generalized IPR equation we can obtain in a much simpler form.

So what we did we adjusted all the parameter of reservoir properties and the fluid properties on the right hand side we can do here also by taking certain assumptions or modifying this equation further. And in this case let us say this equation we are having we taken everything on the other side so this is second derivative here this is the first derivative divided by the radius and this is pressure derivative with respect to time here and then we are going to get 1 by Eta. And now this Eta is actually called the diffusivity equation in this radial diffusivity equation and if we are considering time is in days then the numerical value will remain same here for the Eta and then the permeability porosity fluid properties and the total compressibility are appearing in the expression and when time is considered in hour the numerical value will change accordingly and you will get different value. So accordingly the Eta expression will get change in terms of the numerical value other terms it is including is reservoir properties and the fluid properties. If the more phase are present oil gas and water along with the rock then the CT can be calculated in this manner which is accounting for the compressibility of individual component multiplied by the saturation of that particular fluid in the reservoir domain.

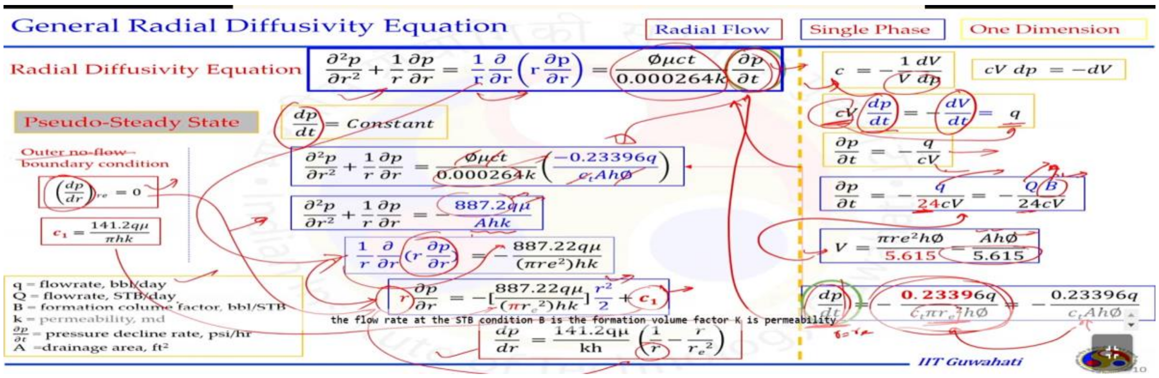


Here we already mentioned K is the permeability R is the radius P is the pressure CT is the total compressibility coefficient in the unit of PSI. So this is the total compressibility coefficient, Phi is the porosity which is having the unit as a fraction it has no unit time value depending on the unit chosen and Mu is the viscosity. So let us simplify this equation for a case for a steady state flow condition what is going to be the equation that we are having either we are considering this equation or taking the derivative of this equation in the second order form we are going to get the expression on the right hand side equal to 0. Because the change in pressure with respect to time will be 0 for the steady state flow condition and then this simple form of the radial diffusivity equation is called the Laplace equation for the steady state flow. So now in this form we get the Laplace equation for a steady state flow for a single phase fluid in a single direction that is the radial direction we also got the expression for the diffusivity coefficient that is appearing in this general radial diffusivity equation.

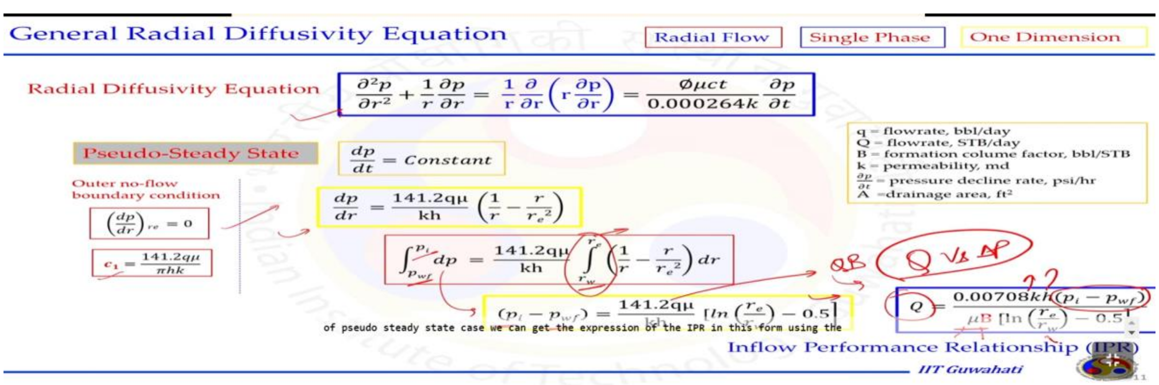
So the same equation either we can start from this expression or we can start with this material balance equation we are going to get this is equal to 1 by Eta and we are having radial diffusivity equation. Now again this change in pressure with respect to time we can replace for case wise if we are having different cases. So let us consider one of the case that is the pseudo steady state condition in pseudo steady state case change in pressure with respect to time at a particular location is going to be constant. So we are having this term as constant now we can get the expression of this term in some measurable quantity how to do that thing let us do it by side using the compressibility definition same definition we can adjust this in this form by taking this to other side. Now taking the derivative of both the side on the right hand side we are having this change in volume with respect to time actually that is equivalent to flow rate.

So now we got the expression for the flow rate in terms of dou P by dou t and then we can replace this here now this expression is still having the volume and the compressibility coefficient C here. So we can write in more general form that is if we are counting for 24 hours instead of day 24 will also appear here and this Q can also be written in terms of B where B is the volume formation factor and Q is the flow rate at standard condition. So with the help of B we can correlate the flow rate of reservoir flow to the standard condition and then this volume V is actually area multiplied by the height and then the porous volume that is available for fluid to flow or stored and the numerical part here is actually converting the acre means the unit converting the unit from one form to the other like the volume is converted from feed Q to barrel per day. So the expression by putting this value of V here we are going to get the expression for this term that is for our interest del P by dot t is equal to this form or we can write this is the area we can write just as a area or we can write in terms of the radius that is pi r e square and that is going to be at r is equal to r e that is the boundary condition the pressure change with respect to time at boundary condition we can get the expression in this form. Now we can replace this here and when we do so we are going to get the expression like this similar one previously we replaced here this term is having different form.

Now this equation can be solved further so this is the numerical part and the other part that is going to be cancelled like the phi can be cancelled out and you are having the CT numerator and the denominator that will cancel out ultimately will be having the Q and mu on the numerator A hk in the denominator. If we replace h with pi r e square that will be separate here. So and then this 887 came by the numerical adjustment of these two factor. Now we can adjust this equation or as I mentioned we can start either with the help of this equation this is more simple form to start the derivative otherwise we can assume here also this is second order derivative equation we can solve directly. So now we will do the derivative form of the equation once we do the first integration with respect to r we are going to integrate this is going to be the same r will be on the other side so that will become r square by 2 in terms of the integration and we will get one constant C1.



Now we have to find out the value of C1 that can be find out by the boundary condition means the outer boundary is having no flow condition in that case dp by d rho with respect to position at r is equal to re is also going to be go. So the C1 value when we are placing this condition here at r is equal to re we are going to get the value of C1 and when we are adjusting the C1 value with the value obtained here we are going to get the expression like this. In this expression we replace this r also that is came here and we also put the numerical value for the pi and that this expression finally is arrived. The terminology used in this slide Q is the flow rate capital Q is the flow rate at the STB condition B is the formation volume factor K is permeability then we are having the pressure decline rate with respect to time and then the drainage area is represented by A. So this radial diffusivity equation that we obtained for the pseudo steady state case we could establish the relationship by knowing the value of the constant C1 when we put we are going to get this expression.



So in this expression we have to integrate again to get the relationship with respect to pressure so we are integrating this from PWF well bore pressure to pi the reservoir pressure the well bore radius is rw the reservoir radius is re. So we are integrating this and then the expression we are going to get right hand side it is pi minus pwf and on the right hand side we will get the ln form and some other part by integrating this you can get the 0.5 by putting the integration and putting the limit rw to re you are going to get this expression. We will solve for particular types of the fluid later on that time I will be able to explain you how different form of the IPR equation happens. Now we can replace this Q with the help of Q square to Q and volume formation factor B if we are doing that and adjusting the equation in terms of keeping Q on the left hand side and keeping everything on the right hand side we are going to get the expression which is having KH mu B and then the radius of the reservoir and then the radius of the well bore and pressure draw down.

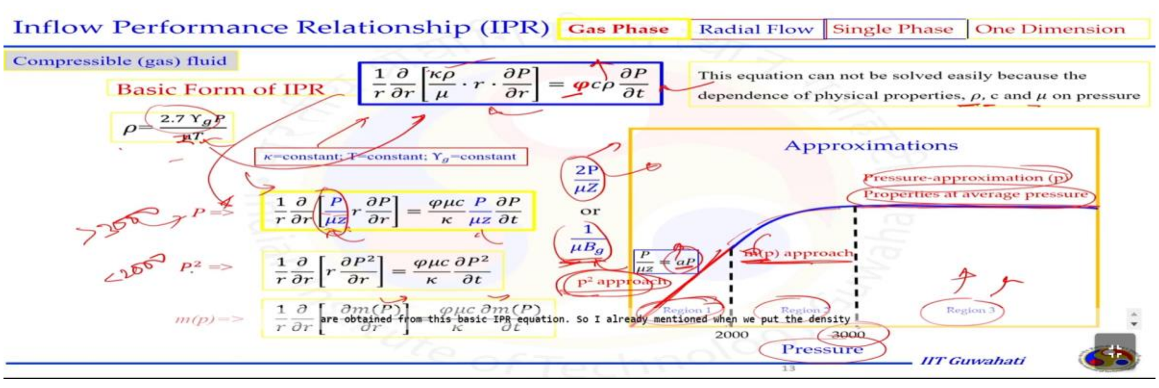
This is what the IPR definition we were keep talking about IPR is the relationship between the flow rate versus pressure draw down. So for the case of pseudo steady state case we can get the expression of the IPR in this form using the radial diffusivity equation. The basic form of the IPR can also be used for that purpose similarly as we did for that and this equation can be solved for the fluid properties for different flow geometry as mentioned here. So we consider the single phase we consider the radial flow to get this basic IPR we can solve it for the different fluid and different flow regimes. So what difference will come in the form of this basic IPR when you are having the incompressible phase the density will be constant when you are having the slightly compressible fluid the density will take this kind of the shape and when you are having the compressible fluid like the gas the density will be replaced by this expression 2.

7 gamma g P by ZT that we discussed in the properties of the natural gas. So this form of this basic IPR will get changed and accordingly the solution will be established for different flow regimes and then the final expression of the IPR equation for different fluid and different flow regime can be established under the radial flow condition. So let us see one case of compressible gas fluid where the basic form of the IPR can be used we can start with the generalized form of radial diffusivity equation. But let consider the case of basic IPR where we already taken phi constant and in that case we are supposed to replace the density as this is the compressible fluid or the natural gas we are considering rho is equal to 2.

7 gamma g P by ZT. So this equation that is here when we are replacing with this rho is going to be the equation that cannot be solved so easily it is actually not having the analytical solution for the compressible fluid and this happens because of the properties rho C Mu these properties depend on the pressure. So when pressure is changing the values of those properties of fluid cannot be taken constant. So far in this equation we consider K constant T constant and gamma g constant. So here T is taken constant if we are replacing rho here and rho here the T can be cancelled out further and when we are replacing this 2.7 gamma g is going to be constant that will also come out of the derivative and it will be cancelled out from left side to right hand side.

And then the basic form of the IPR for the compressible gas will take this form. Now Mu and Z they are there and they are the function of the pressure as we discussed in the properties of the natural gas. Now to solve this equation we have to make certain approximation and then the approximation can be taken to solve this equation we can see in this expression P Y Mu Z is appearing in the expression. So the P Y Mu Z or more precisely 2 P Y Mu Z that is equivalent to or 1 by Mu Bg another term can be seen how this lumped parameter 2 P Y Mu Z or 1 by Mu Z for the gas it is Bg is the volume formation factor for the gas is changing with respect to pressure. So when these kind of the analysis are done it was observed there are three reasons appear those can be distinguish clearly with respect to this lumped parameter is changing with respect to pressure and then the relationship in the reason 1 is the linear and then reason 2 it is non-linear while in reason 3 it is almost constant.

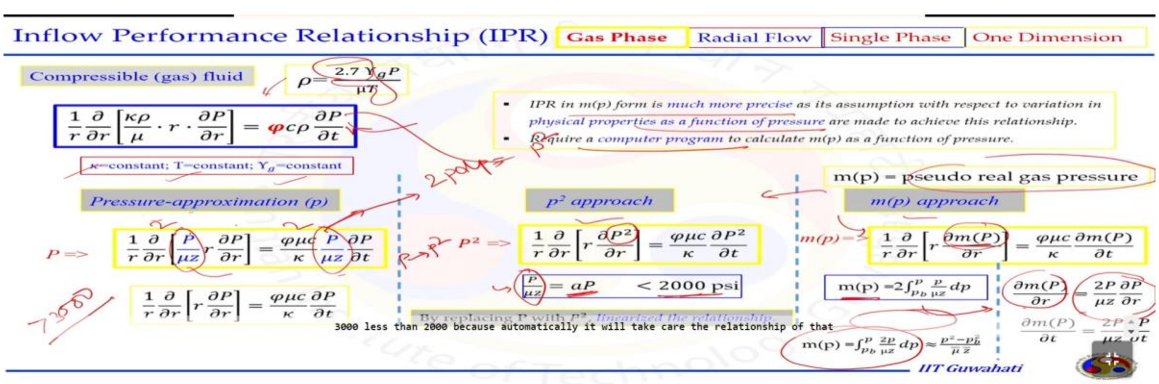
So the individual P individual Z are changing with respect to pressure in reason 3 but the ratio of 2 P Y Mu Z is going to be the constant and that is beauty of this approximation can be taken to derive the three forms of basic IPR equation for the gas system. Now basic IPR we are considering the types of the fluid so the compressible gas is considered and in that case we can get the first reason where P Y Mu Z can be a linear function of the pressure as it is appearing here and we can say the slope is A wherever P Y Mu Z is appearing we can replace this by AP and this reason is called the approach of P score approximation and then the shape of the IPR will be different. Similarly in this reason as pressure is changing we know one properties that is called the pseudo real gas properties we can replace the pressure form with the help of MP parameter in the expression and in the third one P Y Mu Z is constant. So for example here P Y Mu Z is constant we can take that out and that can be cancelled out. So let us see what form it will get in the pressure approximation the properties can be calculated average values and they will come out.



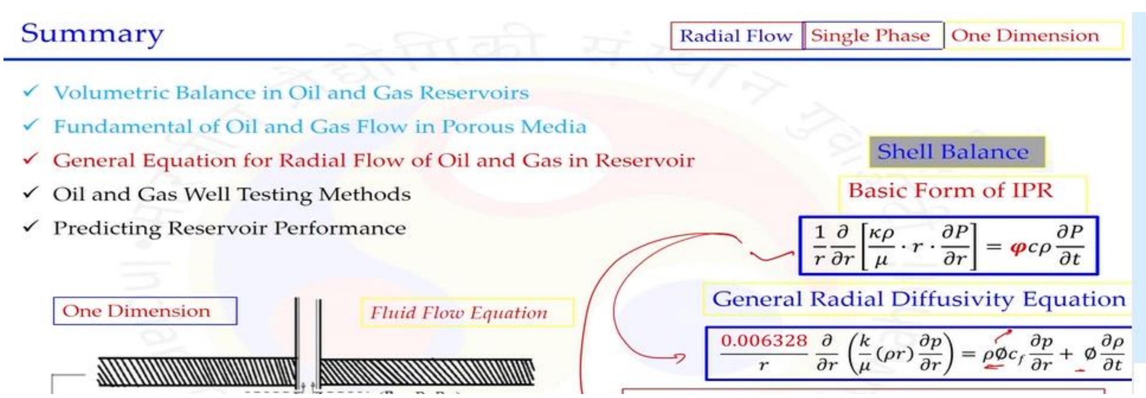
So this is the expression for the P approximation that is applicable when the pressure is beyond 3000 PSI P score is the reason where the approximation will be P score form means linear relationship of this P Y Mu Z parameter with respect to pressure in this reason one it means the pressure should be lesser than 2000 PSI and for this it should be pressure should be less greater than 3000 PSI and between range of 2000 to 3000 PSI as this lumped parameter is changing non-linearly we will get the expression in terms of MP that MP is pseudo real gas pressure. So let us see how this P score and MP forms are obtained from this basic IPR equation. So I already mentioned when we put the density in the basic IPR equation under the assumption of constant temperature constant permeability and constant value of gamma Z we are going to get the expression in pressure approximation P score approach or MP approach MP pseudo real gas pressure. So what we are going to get in the P form by putting this we will get P Y Mu Z other terms will cancel out what will cancel out 2.

7 gamma Z by T will be cancel out from both the sides. Now as we consider in the pressure approximation P Y Mu Z is also constant so this will also cancel out and this is applicable when the pressure is greater than 3000 PSI. What happens in the P score approach we assume P Y Mu Z is the linear function of the pressure and actually that holds good when the pressure is less than 2000 PSI. In that case here we can consider multiply both the side by the 2 and then this 2 P del P we can write this is equal to P square. So what we did the non-linear equation with respect to pressure that was here in the general IPR form we can convert that into linear form in the form of P square. So instead of P we are having the equation in P square form but this equation becomes the linear.

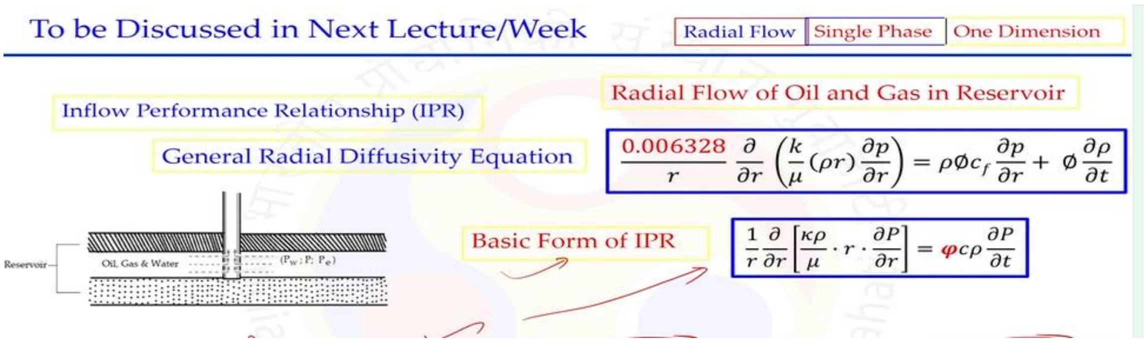
So we replace P with P square and linearize the relationship now we can solve this equation. We will solve this equation for different cases in the next lecture. What about MP we know the definition of MP if we are using the similar chain rules to replace the pressure in the basic IPR equation in the form of MP we are going to obtain the expression like this. So this P that was appearing here can be replaced in the form of MP by having the same relationship because del MP by del R will be equal to 2 P by Mu Z and then the derivative of pressure with respect to R by using this definition okay. So this is the way we can get 3 form of basic IPR equation for the system where we consider the flow to be compressible gas.



This IPR equation now we got 3 equation which one should be used of course pressure is greater than 3000 use P approximation pressure less than 2000 use the P square approach if in between 2000 and 3000 use MP approach but in general MP approach is considered more precise because it is not having the assumption. Here we assume P by Mu Z is constant here we assume P by Mu Z is a linear function of the pressure while in this case of MP we did not consider any assumption with respect to physical properties as a function of pressure hence it is more precise but it requires the computer program to calculate the value of MP because MP is the integral form of parameter 2 P by Mu Z and to calculate the MP value at a particular pressure you need to have the integration method available and then you need to calculate a different segment you divide the pressure range from pressure of reservoir to pressure of interest or pressure of interest to PWF and then the integration need to be performed. Hence it is complex but it is more accurate form the MP approach. MP approach can be applied for other pressure range also greater than 3000 less than 2000 because automatically it will take care the relationship of that parameter P by Mu Z with respect to pressure or in fact it is going to account the value of Mu and Z at different pressure by segmenting the pressure range in a small part and integrating over that pressure range. So in summary in today's lecture we discussed the shell balance to start with the material balance in the system including the fluid flow equation and the properties of the fluid we are dealing with we can get the basic form of IPR that will be in this form and then the radial diffusivity equation was also established and that the bigger form of the radial diffusivity equation which is not considering Phi as a constant although it is appearing here in this expression where Phi is not considered as constant and this form of basic IPR equation for the case of compressible fluid was deduced in the form of MP P square or the P approach.



We will see the expression when we are dealing with the compressible gas for different cases what those different cases will be there in the next lecture. So we are now having the influence performance relationship for the radial flow of oil and gas in the reservoir we are having general radial diffusivity equation either we can start with the IPR or general radial diffusivity equation both are kind of same but in a different form. In fact we will start with the basic form of the IPR in the next lecture and in that next lecture as we already converts to single phase and the radial flow situation we are going to get the expression for different fluid in compressible slightly compressible and the compressible fluid for different cases with respect to flow regime. So we will solve this basic IPR equation for a steady state flow and a steady state flow and pseudo steady state flow for all these three types of the fluids. And ultimately we are going to get the inflow performance relationship that is Q versus pressure difference that is responsible for the fluid to flow in the porous media.



Further that IPR equation will be modified to account for the skin effect that happens near the wellbore region and any deviation from the Darcy law is happening so the non Darcy effect or non Darcy coefficient will be included in the IPR equation in the next class. With this I would like to end my today's lecture thank you very much for watching the video we will meet in the next lecture thank you.