Petroleum Reservoir Engineering

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Lecture 13: Fundamentals of Reservoir Fluid Flow

Hello everyone. And I welcome you again to the class of petroleum reservoir engineering. So, in the last lecture, we discussed about material balance in oil and gas reservoir. In fact, in week 4, we discussed the volumetric balance. And that volumetric balance is a tank model that is actually zero-dimension model. With the help of that model development in the volumetric balance, we could establish the relationship to understand the dry mechanism, those are responsible for the fluid production to the surface.

And straight line concept was discussed in detail to estimate the unknown terms those are appearing in the zero dimensional model or the volumetric balance model. For example, here it is shown there are different processes like the gas cap, aquifer, solution gas drive and other drive mechanism those are responsible for maintaining the pressure in the reservoir. So, with the help of the volumetric balance, we could establish that relationship. But the limitation of the volumetric balance was it was a zero dimension model.

We considered the principle of pressure equilibrium. And we did not consider any flow equation. While in the actual condition in the reservoir, the fluid is flowing from the reservoir domain towards the production well, or very precisely we can say towards the well bore. The pressure at the well bore is Pwf, while in the reservoir the average pressure we can say either Pi or Pe or P bar we mentioned like this. So, the because of this pressure difference the fluid is flowing, while in the case of volumetric balance, we did not consider the fluid flow equation.

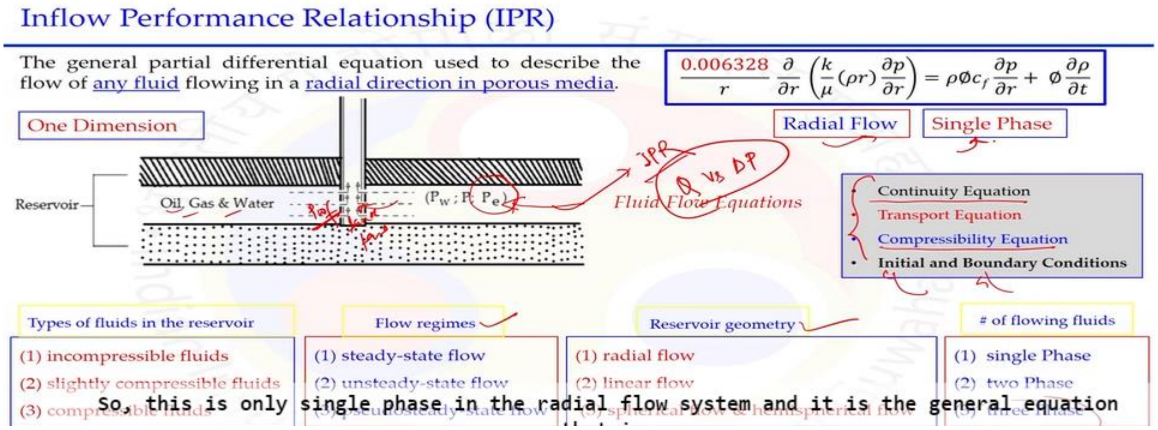
Hence, let us develop the one-dimensional model where we are counting the fluid flow equation in our balance model equation. So, before going into discussion of the one-dimensional model, let us understand the fundamental of oil and gas flow in porous media. So, our reservoir is a porous media, where oil, gas along with water also are flowing through this porous and permeable region. So, the fundamentals of this flow will be discussed in today's class and that will be discussed in terms of what types of the fluid is present in the reservoir and the other associate terminology that are required to establish the fluid flow equation in terms of fundamental understanding of the flow of reservoir fluid in the reservoir domain, means in the porous media. So, the fluid flow equation will be included in several cases to establish that one dimensional model.

Why we are calling it one dimensional model? Because in this case, we are considering the fluid is flowing from the reservoir towards the production well and it is just flowing in one direction either it is linear. So, for example, this is my production well, this is reservoir fluid it is traveling only in this direction. While in the actual situation the fluid may be traveling to y and z direction also, but we are assuming for the simplicity purpose, this is one dimensional model flow equation, where the fluid is just traveling in one direction. Similar for the radial flow condition, when the fluid is flowing from all the direction towards the production stream, but that all direction is actually the r direction. Again, it is a one dimensional model, where we are setting up the equation only in the r direction, we are not counting for the theta and the z direction or we are not considering in the spherical coordinate system, we are not considering the phi and theta component.

So, let us see in the one dimensional model, the reservoir pressure PE, where the PWF is the valve pressure or we called it sand phase, this also called the sand phase condition or the pressure at the valve where the production is just going through this valve over towards the surface. The fluid present is oil, gas and water, they all are present or one of them are present or two of them are present. So, in this one dimensional model, the fluid flow equation can be included now because we understand the pressure energy that is the driving force for the fluid to flow from the reservoir position to the valve over position. So, the types of the fluid in the reservoir could be incompressible fluid, slightly compressible fluid or the compressible fluid. This is the classification of the reservoir fluid based on the compressibility.

What could be the second condition is the flow regime, under what condition this production is happening, that could be steady state condition, unsteady state condition or the pseudo steady state condition. The third component to establish the relationship is the reservoir geometry, means how the fluid flow is happening within the reservoir domain that could be happening in the radial flow direction or in the linear flow condition, spherical flow or semi spherical flow. Number of flowing fluid, there could be single phase fluid, two phases are present, oil and gas for example or multi phase system where more than one phase is present. It means all three phases can also be present in the reservoir domain. Establishing the relationship of particular types of the fluid in a particular type of the flow regime for a particular geometry, the relationship that comes out in terms of flow rate versus pressure draw down means pressure difference.

What is that pressure difference is PE reservoir pressure, PWS is sand phase pressure or the valve over pressure. So, the relationship is called the inflow performance relationship, that is the condition here IPR in short form for the inflow performance relationship, that actually establish the relationship between Q versus del P. With some other parameter those are related to the fluid that is flowing through the reservoir domain as well as the properties of the reservoir domain for example porosity, permeability, page on thickness and others. So, in this one dimensional model setup what we need to do, we have to start with the continuity equation that is the mass balance equation and the transport equation need to be included in this flowing system where the fluid is flowing and that is done with the help of Darcy law. Darcy law is applicable for the fluid flow through the porous media.



And then the fluid type that is flowing in the reservoir, the characteristic of that fluid can be included with the help of the compressibility equation. And after setting up this equation together we have to have the initial and the boundary condition to solve that PDE equation that is resultant by combining all these three equations together. This is just an example of one of the IPR equation that we also call the diffusivity equation in the radial flow condition for a single phase fluid system. So, this is only single phase in the radial flow system and it is the general equation that is applicable for any types of the fluid. Fluid could be compressible, incompressible or slightly compressible and any types of the flow regime.

This equation can be solved for the steady state case, unsteady state case or pseudo steady state case. What this equation is saying, how the fluid properties and the reservoir properties are related to each other, those are going to give us the expression that will result in the form of Q versus del P, how the pressure draw down is contributing towards the production of the fluid from this one dimensional model. Again, this general radial diffusivity equation that is shown here is for the one dimensional model only. The parameters of interest are varying only in the direction of R. So, let us discuss these types of the fluid in the reservoir.

There could be incompressible, slightly compressible and the compressible fluid. The characteristic of the fluid is determined by a factor or a coefficient we call the isothermal compressibility coefficient. This is defined as the change in the volume with respect to pressure at a constant temperature divided by the original volume of that fluid. This can also be represented in terms of the density as mass is going to be the constant. So, the volume can be converted into density or in some other form also it can be converted.

We will see that later on. So, the compressibility C that can be the characteristic to define the types of the fluid in the reservoir. If we see the behavior of the density with respect to pressure, if it is not changing at all while changing the pressure, we can say the fluid is incompressible. It means the value of C is 0 because the change in density with respect to pressure is 0. When the non-linear behavior is obtained for the change in the density with respect to pressure, then we call the fluid is compressible because its behavior is changing as we are changing the pressure.

In fact, when we are increasing the pressure, the density of the fluid is increasing. While we are in between where the linear relationship can be obtained with respect to the density and pressure, then we call the fluid is neither incompressible nor the compressible, but somewhere in between and we call the fluid as a slightly compressible fluid. Similar relationship can be established for the volume versus the pressure data where the incompressible fluid is having no change in the volume with respect to pressure. Slightly compressible is having the linear relationship while the compressible fluid is having the non-linear relationship. We can discuss in more detail about this isothermal compressibility coefficient that is defined at constant temperature.

So, every time whenever we say the compressibility coefficient for a fluid, it is isothermal compressibility coefficient, it means the parameter C is calculated at a constant temperature. So, for a fluid of interest, if it is incompressible fluid, it means the fluid whose volume or the density does not change with the pressure. In that case, what we are going to get change in volume with respect to pressure is 0, change in density with respect to pressure again 0, it means the fluid is incompressible. But the incompressible fluid do not exist in reality, every fluid is having certain type of the compressibility. But within the practical range or within the pressure range we are dealing with the fluid or for simplicity purpose of the calculation, we can assume the fluid nature is incompressible, its density or the volume is not changing with respect to pressure.

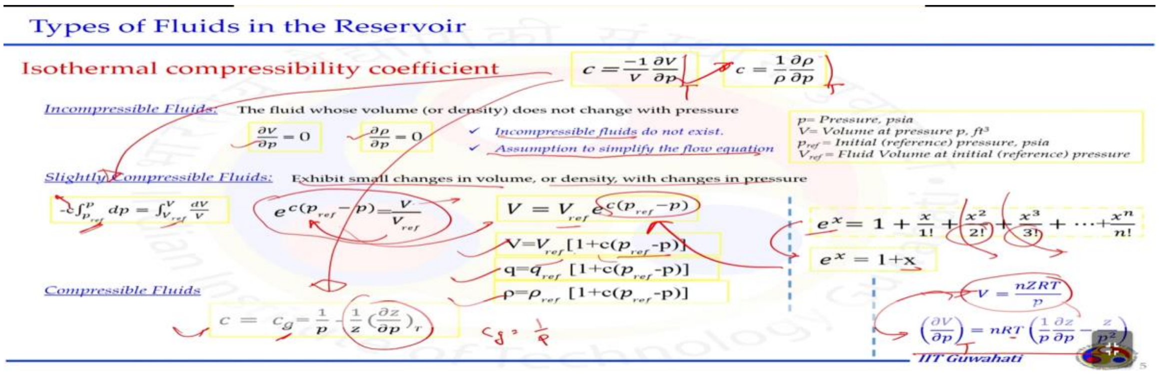
So, this is an assumption to simplify the flow equation and we can understand the nature of the fluid flow assuming the fluid is incompressible, its density is not changing much with respect to pressure and the C value is 0. While in the case of the slightly compressible fluid, the expression for C can be derived. This type of the fluid exhibit a small change in volume or density with change in the pressure. It means we are having this expression, that expression can be integrated with respect to reference pressure to pressure of interest. On the other side, we can integrate from reference volume at the reference pressure to the volume of the system at the interest pressure.

C is taken out and, in that case, when we integrate this, we are going to get this exponential relationship with respect to pressure and volume. We can adjust this equation in the form of V remains here, V is equal to V reference that is taken on the other side, e to the power C multiplied by the pressure difference. Now, this is showing the exponential relationship, but this exponential relationship can further be simplified with the help of the Taylor series expansion. As we know e to the power x can be expressed as 1 plus x factorial 1 plus x to the power 2 divided by factorial 2 plus x to the power 3 divided by factorial 3 so on till x to the power n divided by factorial n. As the power is increasing, the denominator number is also increasing.

And if the parameter of interest is not having a strong functionality, we can consider by simplifying this Taylor series expansion only for two terms by not considering the higher order terms in the expression. So, in that case, e to the power x will be equal to 1 plus x. If we apply the similar concept here, we are going to get V is equal to V reference 1 plus C pressure difference that is x is equal to e to the power this term. Now, this volume can also be converted into volumetric flow rate with the same concept and that can also be converted to the density. So, all three are expressed in terms of reference pressure, reference flow rate and reference density.

And we can use this expression as and when required in the model equation that we are going to develop for one dimensional form. In case of the compressible fluid, for example, natural gas is the compressible fluid, where the C will be denoted as CG, G denote for the gas. And we are going to get this expression. This expression can simply be obtained with the help of the definition of the compressibility coefficient. What does it mean? We know for the natural gas, the real gas law will be applicable and then the volume will be expressed like V is equal to n z RT by P.

If we take the derivative of this volume with respect to pressure at constant temperature, assuming T is constant, we are going to get this expression considering both pressure and compressibility factor are two variables. So, we have to use derivative of A and B. By that principle, we can get this expression. And when we are putting both V and derivative in this expression here, we are going to get this C is equal to CG is equal to 1 by P minus 1 by z derivative of z with respect to pressure at constant temperature. For example, if the gas is or the compressible fluid that is following the ideal gas law, the CG will be equal to 1 by P only because this factor will not be there in that case, in the case of the ideal gas.



So, now we understand how to correlate the nature or the types of the fluid with respect to its density, how density or the volume is changing when the pressure is changing. Second important part is the flow regime. So, we are having the steady state flow, unsteady state flow or the pseudo steady state flow. Pseudo steady state flow actually also called the semi steady state flow or the quasi-steady state flow condition. So, this flow regime actually tells us the fluid flow behavior and reservoir pressure distribution as a function of time.

So, in case of a steady state, we are not having that time variation, while in the unsteady state and pseudo steady state case, we can understand how fluid flow behavior is changing when the time is changing. So, we can discuss all these three cases one by one. So, let us take the example, when time is changing, pressure is also changing within the reservoir domain. And let us consider a particular location within the reservoir domain.

So, this is my reservoir domain. Here, I am considering one point or the observation point. At that point, we are measuring the pressure with respect to time. So, we can get the linear behavior or the nonlinear behavior or just as a constant line. In the case of when pressure is not changing with respect to time, this is pressure P at particular location I, then we will call this is a steady state flow condition. It means the pressure is not changing with respect to time. This situation can be arises in the reservoir domain when we are having the constant support that is maintaining the reservoir pressure at particular location with respect to time. However, this is the hypothetical situation. In most of the cases, a reservoir does not operate under the steady state condition. What could be the second case when we are having the nonlinear behavior or unsteady state flow condition, the pressure change with respect to time at that particular location is changing with respect to time and this is also a function of a particular location. So, if you are changing the location, the change in the pressure with respect to time will also be changed.

While in case of the semi steady state case, the change in the pressure with respect to time at a particular location is going to be constant. So, this is the way the fluid behavior or the flow of the fluid behavior can be characterized within the reservoir domain. Now, let us discuss in more detail what exactly happened in the reservoir. So, we are having the flow rate Q that is constant. So, we are producing the fluid from the reservoir at constant value of Q with respect to time.

Now, to maintain this Q, the pressure drop down should happen in the reservoir and let us see on the other plot where we are having the Pwf, it means the reservoir pressure Pi on the y-axis and we are seeing the behavior of Pwf on the y-axis with respect to time. So, when at time t is equal to 0, we are not producing anything, means the reservoir pressure is Pi that is the uniform reservoir pressure and that is the average reservoir pressure we can say is Pi. What as we start producing over the time the pressures would decline and at the early stage we get the transient behavior means suddenly the change in the pressure happen or non-linear behavior of the change in pressure with respect to time happens. And then we obtain the semi steady state case or the pseudo steady state condition where the change in pressure or the drawdown in the pressure we can say Pi minus Pwf is changing but that is not changing non-linearly but in a linear form, it is changing with a constant rate. Between transient condition and the pseudo steady state condition, there is also one condition called the late transient condition.

So, these three kinds of the things happen in the reservoir, mostly reservoir produced under the semi steady state or the pseudo steady state condition for most of the time. Transient or late transient conditions appear in the reservoir production domain when we are starting the well or when we are shutting down the well. During both the conditions, the pressure near the well bore changes suddenly or non-linearly. So, let us discuss the case of pseudo steady state and steady state condition. So, for example, in the steady state condition, the pressure is here, this is rw is actually the location at the well bore, rw is the radius of the well bore, Re is the radius of the reservoir, so it might be very far from the well bore.

And in this case of the steady state case, your pressure Pe is constant. It means the derivative of Pe with respect to time is going to be 0. It means the pressure either maintained by some aquifer or the situation is not existing. In the case of the steady state flow condition, we are not going to get any reservoir information because in this case, you are producing at a constant flow rate or constant velocity. It means the reservoir conditions are not going to affect the production profile under the steady state condition and we do not get any information about the reservoir formation.

In case of the pseudo steady state case, here the reservoir pressure Pe and Pwf, they are having the linear relationship. Means they are changing, but they are changing in a linear form. For constant production of Q, there should be a pressure drop, but that pressure drop is also changing with respect to time at a constant rate. This kind of the conditions actually also emphasize that the flow is happening under the boundary dominating condition. While the steady state, it is having no reservoir information, while pseudo steady state, the boundary condition is actually dominating.

So, in this case, what is going to be at r is equal to Re, that is the boundary of the reservoir, the pressure change with respect to r is going to be 0. While at other cases, the change in the pressure with respect to time is constant, it is having a constant slope. And at r is equal to rw, the constant flow rate of the fluid is happening through this reservoir. So, this Q is equal to constant throughout the case, it will also be constant at r is equal to w, the condition when the production is just happening or the fluid is reaching to the valuable conditions. While in the case of the transient or unsteady state case, what exactly happens in this case, for example, this well is in setting condition, we are not producing anything.

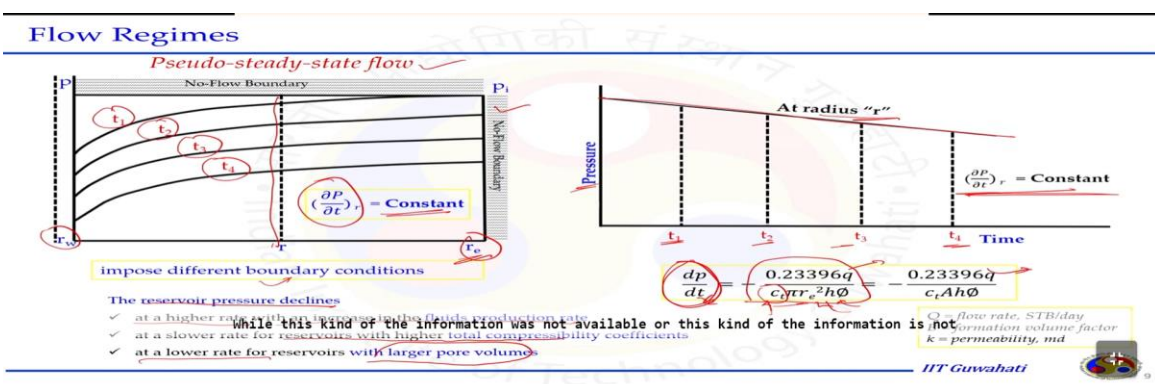
Everywhere the reservoir pressure is same, that is Pi, when we start the production either at the constant flow rate or keeping the constant Pwf value. In both the cases, the pressure near the well bore or at the sand phase just started getting disturbed. And with respect to time, if you see the pressure reaches to different radius, and when the pressure reaches near to the boundary condition, we actually obtain the pseudo steady state condition. In that case, there is no transient flow is happening, the flow is happening under the pseudo steady state condition. What are the other information about the transient flow condition or the unsteady state condition? This is the condition when the initial or the certain or shutdown time which is happening.

In that situation, the reservoir behave like a reservoir that is in infinite in size. And the boundaries are infinitely away from the well bore until the disturbance reach to the boundary condition. Once it reach to the boundary condition, pseudo steady state condition is obtained and the production is happening under pseudo steady state case. But before that reservoir will behave like a infinite acting reservoir or we can see infinite acting radial flow if it is happening under the radial flow conditions.

And it means the Re is also infinite mathematically. The nature of the pressure distribution or the rate at which the pressure distribution is propagating from the sand phase to the boundary of the reservoir will depend on the fluid and the reservoir domain properties. For example, porosity, permeability, fluid viscosity and then the total compressibility that is the compressibility of the fluid as well as the rock. So, these properties will determine how slow or fast the pressure distribution is happening in the reservoir domain and how early and how late we are going to attend the situation of pseudo steady state case. In this case a very large reservoir and it is producing at the constant flow rate that is Q and it requires longer time to reach the pseudo steady state case. In this case also when we are solving either we are solving the steady state, pseudo steady state case or unsteady state case we have to start with setting up the continuity equation including the transport equation means the flow equation into it, accounting the properties or the types of the fluid we are dealing with, with the help of the compressibility equation and then setting up the problem with respect to initial and the boundary condition that will allow us to make the solution of that equation.

The system is little complex so there could be several possibilities. We will simplify our problem and discuss them one by one in the coming lectures. So, let us relate that unsteady state condition to the pseudo steady state condition. This is the situation in the reservoir which is going to be exist in the reservoir for most of the production time. So, this rw is the value radius, r is the reservoir radius, no flow is happening from this radius away and no flow is happening in the vertical direction also.

Whatever is happening within this domain and we can correlate that at different time what is happening at a particular location r either we can say it r or i and when we are having this relationship for the pseudo steady state case what we see the change in pressure with respect to time at that particular location is going to be the constant. What does it mean? At a particular radius r the pressure is changing with respect to time t1, t2, t3, t4 it is changing but the change is having in the linear form. So, that is where the transient condition reach to the pseudo steady state condition. When we are setting up the mathematical equation for the transient condition we can solve that equation to obtain the time when the pseudo steady state condition will be achieved in a particular reservoir and that will depend on reservoir properties as well as fruit properties. So, this is the pseudo steady state flow condition is the condition where boundary are dominating and the situation when we are saying this equal to constant we can get the expression for that that at r is equal to Re how the pressure is changing with respect to time at r is equal to Re we will get that expression that expression is having CT h these are the rock properties phi this is also reservoir domain properties and then the area and this Q is the flow rate at which the flow is happening in the reservoir domain.



From this expression we can get some understanding about how the reservoir pressure declines under the pseudo steady state flow condition. So, if the reservoir pressure declines at a higher rate it means with an increase in the fluid production rate it means the production rate is increased so the pressure will decline at a higher rate and that is happening the change in the pressure is happening faster when the Q is increased it will happen at a slower rate for a reservoir with having higher total compressibility coefficient. So, CT is in the denominator the rate of the change in the pressure will be lower if the CT value is high and for a third case when the pore volume is large the pressure of the reservoir will decline at a slower rate within the reservoir domain. So, these are some of the information we can obtain about the reservoir pressure with respect to the properties of the reservoir domain. While this kind of the information was not available or this kind of the information is not available when the reservoir is considered to be flowing under the steady state condition.

Because under the steady state condition the Q is constant throughout the time throughout the reservoir domain. So, there is no variation in the Q no variation in the pressure we are not having any information about the reservoir. The reservoir geometry the third important aspects of setting up the mathematical equation or the inflow performance relationship is consideration for the geometry. Most reservoir are having very irregular geometry but for the simplicity purpose we can consider this is a radial flow, linear flow, spherical or hemispherical flow situation within the reservoir. So, this is the way the radial flow is happening from all direction the fluid is traveling to the reservoir, balloure where the fluid will get start producing through the production string to the surface.

We discuss all these in the introduction lecture in more detail. So, let us recap them quickly. So, if we are having the linear flow it means the flow is happening only in one direction and the fluid is flowing only in one direction. The situation could be when the perforated zone or the production range is smaller than the page on thickness. So, we will be having the spherical flow and when the fluid is flowing through the bottom of the production string we will be having this hemispherical flow situation. Mostly in the further discussion we will be considering the radial flow in the detail.

So, let us see what exactly happen in the radial flow and the linear flow condition. So, this is the radial flow if we are looking from the side this is our production string which is having the perforation means hole through which the fluid will be entering to the production string and when we are seeing the reservoir radius is far away from the wellbore and at a particular location at Re we can see how the pressure is changing. Now the change in pressure could be established with steady state, pseudo steady state and unsteady state condition. But at a particular time when we are seeing the fluid is moving from all direction towards this production string and the side view will appear like this.

This is your page on thickness. If we see from the side view with respect to pressure so this is reservoir pressure at radius Re, Pwf is the pressure at the sand phase it means at R w position and in the case of radial pressure change in pressure with respect to R is always greater than 0 because the pressure is declining where the radius is also declining and this H is actually the page on thickness. So, this kind of the pressure relationship would be there when we are considering the radial flow. When we are talking about the linear flow system it means the flow is happening only in one direction not from all direction like in the radial flow but radial flow also in the cylindrical coordinate system is just one dimensional flow situation. While in the case of the linear flow the pressure is declining this is your pressure inlet pressure this is the outlet pressure. Consider this is a pie flow situation fluid will flow from higher pressure to lower pressure.

In that case the change in the pressure with respect to distance will be negative because the pressure is changing in the direction or decreasing in the direction where the length is increasing with respect to position of P1 to P2. So, both the situation we will discuss but mostly discussion will be around the radial flow in the coming lecture. So, last component that is the number of flowing fluid in the reservoir. So, the single phase means you are having only the oil you are having only water or only the gas or there could be the two phase flow situation where you are having the oil and water, oil and gas and gas and water and then the three phase situation all three phases are present in the reservoir domain and they are producing with the competitiveness with respect to the saturation we can establish the relationship how they are spreaded within the reservoir domain and with the help of the effective permeability as we discussed previously we can establish the same flow rate equation for each phase of the fluid that is present in the multi phase flow system. So, the description of fluid flow and subsequent analysis of the pressure data becomes more difficult as the number of mobile fluids increase.

If multi phase system is there we have to find out how the total pore volume is shared by these phases what is the critical saturation beyond that the fluid will start moving and then the effective permeability that can be related to relative permeability how one fluid will be having the preference over the other in terms of passing through the permeable zone. So, the situation could be more complex. So, with this common understanding about these four important parameters those actually characterize this reservoir we can establish the relationship between the parameters involved in the flow of the fluid because of the pressure energy or because of the pressure drawdown that is existing in the reservoir. The relationship is called the inflow performance relationship IPR. This also called the reservoir deliverability it means at what rate the reservoir is delivering the fluid to the surface and that could be established. So, the reservoir deliverability is the same as the IPR this determines the flow capacity of a well by establishing the relationship between the inflow rate and the sand phase pressure or the bottom pressure. It means the relationship between Q and the pressure difference between the reservoir pressure minus Pwf. The production rate is a non-linear function of the pressure drawdown. So, this is not having the linear relationship in most of the cases it is the non-linear relationship and the production profile that we are going to obtain through this reservoir for a particular type of the fluid will be the function of the fluid properties as well as the reservoir properties. The IPR that is going to be the mathematical relationship that is relate flow rate to the pressure drawdown will depends on the flow regime, fluid properties, formation properties and the geometry.

Some of them we already discuss about how to decide the types of the flow geometry that is existing in the reservoir. The flow regime steady state and steady state and pseudo steady state we can consider fluid properties can be determined with the help of the compressibility coefficient. So, establishing this relationship will give us the IPR. The IPR will depends on the nature of the fluid is compressible, incompressible or slightly incompressible.

It is single phase only present or the multi-phase is present. What types of the flow behavior is existing in the reservoir domain. All sort of the things could be there like the well is just vertical or the horizontal well. It is with or without the hydraulic fractures. Two important aspects here are non-Darcy or the Darcy flow and then the skin factor.

These are the two phenomena those happens near the bellwor region. Why these two happens over the time the sand phase or the bellwor section at the bottom is having additional pressure drop because of some skin effect means because of some deposition of unwanted things. Because of that there will be additional pressure drop that is need to be counted in the IPR that is the relationship between Q and the pressure drop. Second is Darcy and non-Darcy flow. So, in the mathematical equation that we set up to get the IPR we consider the flow of the fluid in the porous media is going to follow Darcy flow equation. While near the bellwor region the Darcy law is deviate because of the assumptions those are used to establish the Darcy law gets violated.

In that case the non-Darcy flow exists near the bellwor and we need to account the non-Darcy flow behavior in the near bellwor region. That is also related in terms of additional pressure drop. So, the total pressure drop from reservoir pressure to PWF will also be having the pressure drop because of the skin effect, pressure drop because of the non-Darcy flow behavior. We will set up those equation at the later stage. Let us have the simple case where we are establishing the IPR relationship that is the flow rate versus the pressure draw down relationship for the simplest cases or the cases where we are not considering Darcy case or the skin effect.

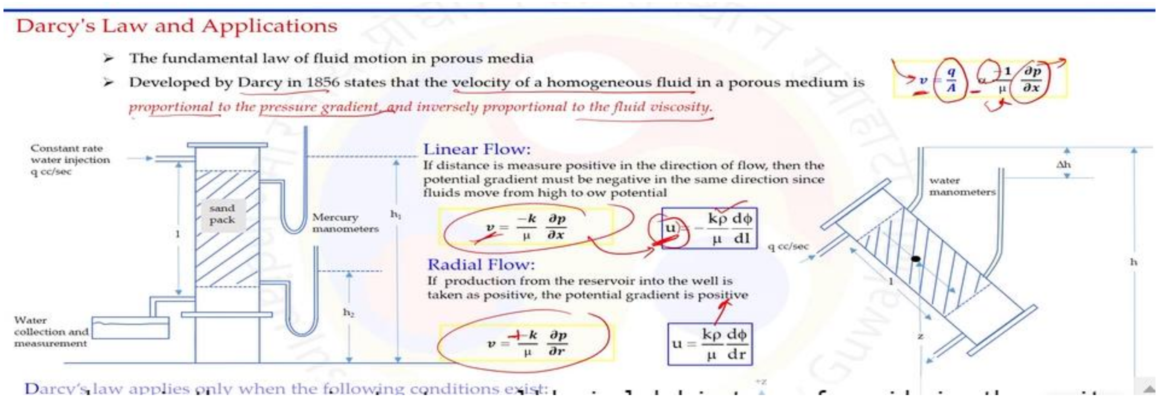
We can discuss the cases in different conditions considering skin effect and non-Darcy coefficient to be included in the equation will be discussed at the later stage. So, let us see how to get this IPR equation. We can start with the material balance equation within this reservoir domain. So, the material balance equation will give us the relationship between the pressure drop and the flow rate Q. The relationship could be solved by analytical or the empirical correlation can also be established.

So, we can start with the fundamental understanding to get the analytical expression for the IPR or we can have the well producing at different conditions and establishing the relationship that is the empirical or field data relationship to get the expression for IPR that is the relationship between the pressure drop and the flow rate. How we get that mathematical expression considering certain cases. So, for example, if we are considering single phase flow that is going to define the number of fluids present that we are seeing only single phase and single fluid is present in the reservoir. The geometry of the flow is radial.

The types of the fluid could be incompressible or the compressible. We can set up the equation accordingly and then the flow behavior could be steady state, pseudo steady state or unsteady state case. Accordingly, we can solve that equation. The skin effect and non-Darcy effect near the wellbore region will be included later on in the established IPR relationship without considering the skin effect and the non-Darcy effect. Symbol same terminology we have to use we have to start with the continuity equation that is the material balance equation include the Darcy law that is the transport equation, compressibility equation that is going to define the types of the fluid and then the initial and boundary condition are required to solve that resultant partial differential equation. The developed inflow performance relationship or IPR can further be simplified to understand the bulk parameter effect on the performance of the reservoir in terms of the productivity index that is also deal to a situation when we are having the absolute open flow means the maximum flow rate that can be obtained from a particular well.

We will discuss this later on once we establish the inflow performance relationship. So, the equation we are keep talking about the transport equation that is the Darcy law for the fluid flow through the porous media it was established by Darcy in 1856 that is simply says the proportional behavior of a velocity of a homogeneous fluid in a porous media with respect to pressure gradient and inversely proportional to the fluid viscosity. So, the velocity of the fluid in a homogeneous porous media that is equal to Q ya flow rate divided by the area is equal to velocity is proportional to pressure gradient that is del P by del x for the linear flow system and inversely proportional to the viscosity of the fluid minus sign is appearing here because del P by del x is pressure is changing in the opposite direction as the length. So, the linear flow could be there and could be the radial flow the law of Darcy flow is having certain assumption like the flow should be the viscous flow or the laminar flow that is the condition near the valor where the non-Darcy coefficient need to be included because the laminar flow condition gets violated. The incompressible fluid but the same equation can be included for the compressible fluid also we will take care of the compressibility of the fluid in terms of C. The steady state flow this expression was given for the steady state condition but can be applicable for the unsteady state condition also or we can include it or homogeneous formation.

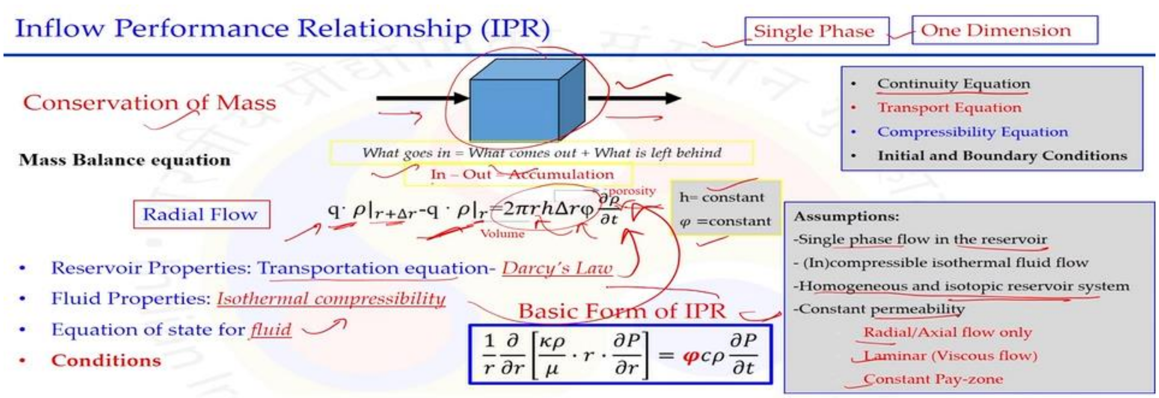
So, if practically we see none of the conditions are existing within the porous reservoir medium where the fluids or multi-phase fluids or more than one type of the fluid are present together reservoir is not homogeneous it is not flowing under steady state condition laminar flow far away from the valor we can consider the laminar flow is there but the near valor it is also not true situation and then the fluid could be incompressible and compressible. So, as and when the situation is coming we will modify the Darcy law equation but we will start including the transportation equation or the fluid flow equation in the porous media considering the Darcy law. The expression Darcy law could be for the linear flow it could be like this one and for the radial flow it could be like this one this should be positive here in that case because the pressure and radius changes in the same direction this could be expressed in some other unit also here you see the density is appearing in the expression. So, this is actually the velocity that is mass flux rate velocity while here this is the velocity meter per second.



So, the expression will get changed when you are arranging the equation in different form. This Darcy law can be applied to the inclined or the tipped reservoir also where the datum change in the reservoir structure could be included in terms of considering the gravity effect. So, in our case that we are going to solve the problem for establishing the inflow performance relationship IPR for one dimensional model where the reservoir is having pressure Te, Pwf is the pressure at the sand phase what situation we are going to have to include the fluid flow equation in this porous media. The types of the fluid that we discuss could be like this flow regime could be like this reservoir geometry could be radial linear spherical and hemispherical number of flowing fluid could be single two and the three phase. So, the relationship that we are going to establish with the help of all these four step by step procedure we will be having different expression when we consider the different case. So, for example when we are considering the single phase flow the equation will be just for the single phase but when we are going to the multi phase flow system each phase will be having its equation within that model development.

So, let us start considering just a single phase system for a one dimensional flow of that single phase single fluid in the reservoir domain. As mentioned we can start with the continuity equation that is the conservation of mass. Let us consider a control volume the volume of the investigation through which the fluid is getting in getting out. So, the mass balance equation simply says what goes in is equal to what comes out plus what is left behind in the control volume.

If we say it in terms of in minus out is equal to accumulation. For the radial flow if we are considering the radial flow of the single phase fluid the resultant material balance equation will be like this is Q dot rho that is the mass flux and Q dot rho that is actually the mass flux at location r plus del r that is the out condition here and the mass that is getting in or the mass flux that is getting in into the system is Q dot rho that is at the r condition and within the control volume if we are having this control volume the accumulation term can be expressed like this. We will discuss this concept of material balance with the shell balance in the next lecture. Here let us say to establish this relationship this is to have this relationship we have to make certain assumption. What assumption we made it is a single phase flow in the reservoir the fluid still could be compressible or incompressible because we did not take any decision with respect to density. Homogeneous and isotropic reservoir we had taken the porosity out of the derivative means we are considering the constant porosity of reservoir is having the homogeneous nature and the properties are not changing with respect to the position only we are considering the one dimensional flow so the reservoir is isotropic.

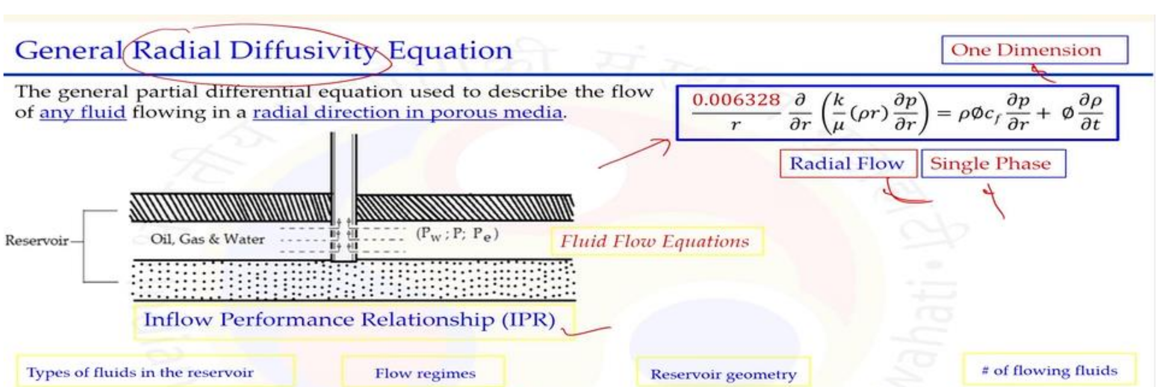


like the constant H and constant phi. If we solve this equation we will get the basic form

Constant permeability it is radial flow, laminar flow and the constant page on thickness H is the constant page on thickness that is also taken out. So, the radial flow equation can be simplified by including the second set of the equation that is the transport equation Darcy law into it. Now the fluid behavior we can include the isothermal compressibility coefficient here and if required to understand the nature of the fluid when it is subjected to different pressure condition we can use the equation state for the fluid especially for the gas phase. And then the condition what condition could be there those will help to get the relationship get the final shape. In the current equation that we establish here we make these assumption like the constant H and constant phi.

If we solve this equation we will get the basic form of IPR that is we are going to get and that equation is not assuming what type of the fluid it is under certain assumption only we consider that still the compressibility or the fluid nature is not included into it. So, this is the general IPR equation or the basic IPR equation that can be used for any types of the fluid and can be solved for any flow regime for the steady state case, pseudo steady state case or unsteady state case. But let us say for the radial flow we got this basic form of IPR this IPR can be solved for all types of the fluid compressible, incompressible and slightly compressible and all types of the flow regime steady state, pseudo steady state or the transient flow regime. And that is going to give us a resultant equation that can be used to establish the relationship between the flow rate and the pressure drop relationship is established we are having multiple application of that equation that we will discuss later on. So, what we are having in the IPR different cases with respect to types of fluid flow regime reservoir geometry, flowing of the fluid including the fluid flow equation we are considering the one dimensional flow only and in that case if we are considering the radial flow we are going to get the equation that we call the radial diffusivity equation.

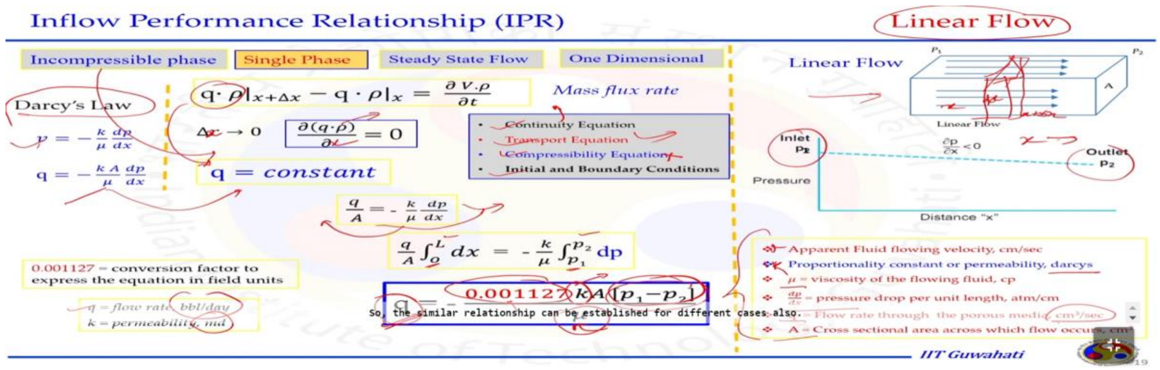
If it is linear flow only we can call it linear diffusivity equation. So, the diffusivity equation and IPR are interchangeable. In the next lecture we will discuss about setting up the general diffusivity equation for the radial flow of a single phase in the one dimensional equation and we will see how this expression that is appearing here on this slide can be obtained with just fundamental understanding of the oil and gas flow or the reservoir fluid flow in the porous media. The cases we will discuss about we will consider only the single phase as I mentioned for the multi phase system we can use the concept of effective permeability that can be related to relative permeability that will take care of setting of the equation for each phase in the reservoir domain. We will mostly consider the radial flow for larger discussion and we will set up the equation of the radial flow for all three types of the fluid and all three types of the flow regime.



But for the simplicity purpose let us discuss the case of linear flow as an example. In linear flow what is happening fluid is flowing only in one direction we can take a control volume here in this control volume of the del x thickness the fluid is flowing from x to x plus del x and the pressure change is happening from p1 to p2. If we are considering this linear flow we have to start with continuity equation, transport equation, compressibility equation, initial and boundary condition. Now depends on the situation we recalling this equation to set up the IPR. So, let us take this example of linear flow if we are considering the fluid is incompressible it means the density is not changing it means we do not need this equation to be included because the density is constant with respect to pressure. We are considering the single phase if we are considering the steady state flow equation in that case we do not need to worry about how pressure is changing with respect to time or the other parameter are changing with respect to time.

It is just one dimensional only happening in the x direction only we are not counting behavior of the fluid or the direction of the fluid in y and z direction. So, how to start it writing the mass balance equation that is the continuity equation we can write this in mass flux rate mass in minus mass out is equal to accumulation and if we take this slab of the control volume is very small we are going to get resultant equation like this. So, the change in this mass flux with respect to position is going to be 0 as we are saying the fluid is incompressible means rho is going to be the constant it means Q is also going to be the constant. Now, this Q can be related with the help of the Darcy law so we are going to include the transportation into it that is here this terminology that is to define the variable that is happening in the Darcy law are here. So, the V is the apparent fluid flowing velocity this K is the proportionality constant here and mu is the viscosity this is the pressure drop Q is the flow rate is the cross-sectional area.

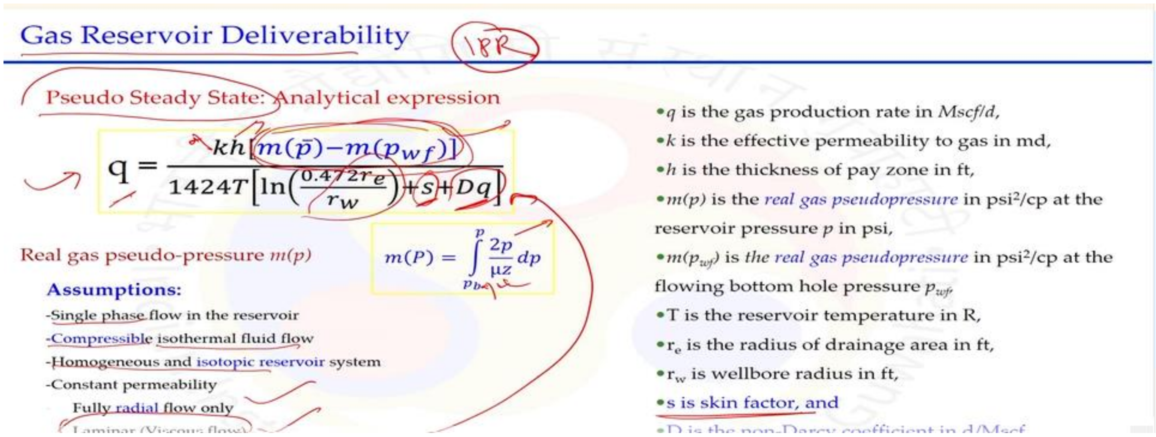
So, simple terminology that we use for the Darcy law are mentioned here and now when we are saying for the incompressible fluid the resultant continuity equation is going to be Q is equal to constant. Now, when we include the Darcy law we can just take Darcy law for Q for the flow rate here the IPR equation is going to be like this and now this equation can be solved we can take the variable of interest means with respect to x on the one side and pressure on the other side we can integrate from length 0 to L where the pressure is P1 to P2 the resultant equation is going to be like this. Here you will see some numerical coefficient is appearing that is the conversion x factor to express some of the parameter in the field unit. Specifically flow rate here it is in BBL per day and the permeability is in milli Darcy while in the Darcy law the permeability is in Darcy and the flow rate was in centimeter Q per second so the numerical coefficient is appearing for that. Otherwise, if we see we got the IPR that is the flow rate relationship with respect to pressure draw down that is the pressure P1 to P2 in the reservoir and that is having the properties of the fluid properties of the reservoir domain.



So, this is the IPR for incompressible fluid single phase flow under steady state condition for the one dimensional only in the direction of x the fluid is flowing and now we can correlate certain parameter here like to increase the Q you need to increase the K and create the more permeability of the reservoir is having more permeability the flow rate will be high. If the pressure difference is high flow rate will be high. If the more viscous fluid is present in the reservoir the flow rate will be less. So, the similar relationship can be established for different cases also. Let us take the example of gas reservoir dilute it means the IPR for the gas phase that is under the pseudo steady state case.

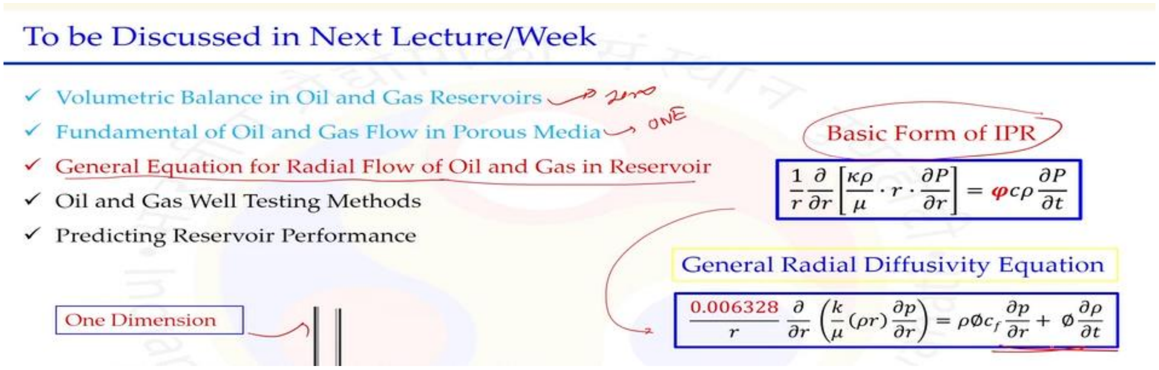
So, if you are setting up the material balance equation for the gas flow under the pseudo steady state case solving it the resultant equation is going to be this much complex. Now this equation is having the expression for the pressure draw down in the form of MP. In the case of reservoir properties we discuss about the MP that is the real gas pseudo pressure a lumped parameter that is defining how the pressure viscosity and the z value are changing together with respect to pressure. So, this is actually the pressure difference. So, the Q similar to the previous case is the direct relationship with the pressure draw down if pressure draw down is more Q will be more, page 1 thickness is more, Q will be more, K permeability more, Q will be more and then it is having the relationship with respect to the radius of the reservoir and then the radius of the bell bore.

Other two terms are appearing here those two terms are including near the bell bore phenomena that as for the skin effect that is happening in the reservoir if it is happening, we have to include it. If it is not happening, we can take it out or can make it 0 there could be the situation when we are stimulating the bell and then the S is not only the negative the pressure drop is not only negative it is going to be positive also we will see later on and then dQ combinedly called the factor that is including for the non-Darcy flow behavior. So, D is actually the non-Darcy coefficient that is accounting the deviation of the fluid flow in the porous media from the Darcy law. So, this complex expression can be obtained from the analytically balancing the equation that we will do later on or maybe next or the next-to-next class. To establish even this relationship that appears to be big one certain assumptions were made only single phase fluid is present fluid is compressible so the gas is compressible in nature but isothermal flow is happening homogeneous and isotropic reservoir system is considered fully radial flow only laminar flow and the constant page on thickness are considered the laminar flow condition if it is getting deviated the non-Darcy coefficient will account for the deviation.



So, we can get similar expression for different cases and we can also get the expression by the empirical form also from the field data. So, in summary what we discussed in today's class there could be several cases within the reservoir domain under the flowing condition we need to establish to get the inflow performance relationship that is the relationship between the flow rate and the pressure draw down and that relationship should include the fluid properties as well as the reservoir properties. The situation could be with respect to the types of the fluid flow regime flow geometry and the single phase. The starting point will be the continuity equation transport equation including the nature of the fluid and then resultant equation need to be solved under the initial and the boundary condition. So, single phase will be considered in most of the cases for the multi-phase we will discuss with respect to relative probability only.

We discussed the case of linear flow in the previous slide in the remaining discussion with respect to setting up the IPR or the radial diffusivity equation we will consider the radial flow similar can be applied for the inflow performance relation for linear flow condition also but we will be discussing the radial flow system for all these three type of the fluid and two situation steady state and pseudo steady state in more detail while the unsteady state situation would require little bit higher mathematics. So, we will just see the solution of that with certain assumption and see how the equation of IPR appears to be when we are having the unsteady state flow situation in the reservoir. So, to be discussed in the next lecture what we discussed so far in this section 2 we discussed the volumetric balance that is the zero dimensional model. Today we discuss the fundamental of oil and gas flow in porous media that is going to help us to set up the one dimensional model and in the next lecture we will discuss the general equation of radial flow of oil and gas in the reservoir and then further we will discuss about the testing method and the performance of the reservoir. So, we are going to set up one dimensional model only and first we will obtain the basic IPR for the radial flow and then the general diffusivity equation for radial flow one dimensional model single phase is present.



Now this is general equation that is applicable to any fluid in the radial direction and it can be solved for any flow received steady state pseudo steady state or unsteady state case that we will set up this general radial diffusivity equation in the next lecture. So, with this I would like to end today's lecture.

If you are having any query please write to this email id- reservoir.iitg@gmail.com. With this I would like to thank you. Thank you very much.