

## **Petroleum Reservoir Engineering**

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### **Lecture 11: General Material (Volumetric) Balance**

Hello everyone. Welcome again in the class of petroleum reservoir engineering. So in the last lecture we discussed about several natural dry mechanism. Those are responsible to let the fluid flow in the reservoir because of the additional energy that can support the reservoir production for a longer time. The dry mechanism we discussed about are liquid and rock compressibility, solids and gas dry. Gas dry means a cap of the gas on the top of the liquid formation where the gas is pushing the oil.

Aquifer water dry means encroachment of the water from the nearby aquifer. Gravity segregation and the combination of these dry. We discussed these in terms of how these dries are supporting the reservoir and the indicator to understand the effect of these dries or the presence of these dries towards the production of the fluid. We discussed in terms of reservoir pressure, gas oil ratio, water production and the ultimate recovery factor that can be obtained through these individual dry. And there could be the combination of the dries also or within the combination dry one dry may be effective than the other dries. So the relative effect of the dry could also be understood in the last lecture. In the last lecture we discussed mostly the qualitative nature of these dries. How they perform there and the indicator behaviour like how pressure of the reservoir decline, how GOR ratio changes over the time when that particular type of the dry mechanism is supporting the reservoir production. In this lecture that is now we entered in section 2 of our defined slavers.

So in section 2 mostly we will be discussing about the mathematical analysis of the reservoir. We will start with the volumetric balance of the reservoir in today's lecture. In that volumetric balance or the material balance we will be considering reservoir as a tank and what happening in the reservoir in terms of the influx and outflux we will set up the mathematical equation for that that is called the volumetric performance equation. Of course under certain assumption these equations are valid but these equations for different cases allow us to estimate hydrocarbon volume in place underneath the surface, predict future reservoir performance in a mathematical sense and the prediction of the primary dry mechanism that is supporting the reservoir for the production or providing the natural energy in terms of the pressure can also be predicted with this mathematical analysis. So let us start with the volumetric balance that is the basic principle of the

material balance equation along with the other governing relationship that can be used to predict volumetric performance of oil reservoir as well as gas reservoir.

So in today's class we will discuss the general volumetric balance or material balance. In the next class we will discuss about specific cases of the oil reservoir and the gas reservoir. So the material balance based on the volumetric basis is given by Cille Tui in 1941 and that volumetric balance is a very simple way of understanding what is happening in the reservoir in terms of the volumetric changes. So the various recovery mechanism way affecting the reservoir performance the overall performance of the reservoir can be understood with the help of this volumetric balance. So in simple language the initial volume of the reservoir is equal to volume remaining in the reservoir plus volume recovered.

So what we recovered from the reservoir oil gas and water. So this is the symmetric diagram of the reservoir domain where the oil gas and water are present along with the rock. If we compare this with the volumetric balance where the volumetric region or volume of the reservoir can be considered as a tank that is why this model is also called the tank model. In the tank model we consider the boundaries of the reservoir and consider within this boundary whatever happening in the reservoir in terms of the changes in the volume. So this is the tank model at any condition.

So let us start with the initial condition. In initial condition this is the boundary of the reservoir that are shown here and as time progress production happens some influx may also happen and over the time when the reservoir performance is evaluated at any time we are still having the same dimension of the reservoir. So the reservoir volume remains constant in this volumetric balance. Within these two whatever is happening we understand in terms of the volumetric changes and as there is front mechanism may be responsible for the production happening or the encroachment of the water or the injection of the external fluid we will discuss in detail about all the possibilities those could happen in this initial condition and how that initial condition is reaching to any condition with respect to time. So when we are evaluating with respect to time at any time what are the time mechanism there in terms of the index we can predict them so like depletion drive, segregation drive, water drive or the expansion of rock and fluid a drive.

So all these indexes can be established. How we do that considering the volume as constant with other certain assumptions are taken. So let us start this model that we also called zero-dimension model. So we are not considering the flow of the fluid in a particular direction or in multiple direction. This is just a tank model where the zero-dimension model concept is applied.

In that case we consider the average pressure of the reservoir. So initially when the pressure is there  $p_i$  where the reservoir pressure is uniform within this boundary. So the

average pressure is taken and that average pressure initially can be  $p_i$  or at any time that pressure will be  $p$ . The  $p_i$  and  $p$  actually represent the average pressure. Similarly for the saturation of the fluid, oil, gas and water the average value within this volume or tank is considered as  $S_O$ ,  $S_G$  and  $S_W$ .

So we can take out the bar just  $S_O$ ,  $S_G$  and  $S_W$  are the average value within this control volume. When we are talking about initial condition this will become  $S_{Wi}$  initial water saturation  $S_{Oi}$  initial oil saturation and  $S_{Gi}$  initial gas saturation. So with this zero dimension model we can relate how the volume is changing within this tank model or in the control volume. So this tank model that is actually having oil, gas, water and the formation  $F$  represent for the rock, rock formation. So these are the component those are making this tank model or the volume model.

This is considered as a one grid cell in terms of the computational language. So this is just one grid where everything is happening within this grid or one grid of large size and the performance of the reservoir is evaluated with respect to that grid. Cross fluid flow into or out of the reservoir. So when the production is happening we are not separating how much oil, gas and water is producing. Cross fluid flow into or out of the reservoir domain.

We considering the gross flow not with respect to the direction of the flow that is why it is a zero dimension model. If we represent this reservoir into a tank model we can see this is the reservoir domain. In this reservoir domain we are having oil, gas, water and the rock that is composing this reservoir. We can drill the well to that location start getting the production after well completion process. In this production well what we get at the surface oil, gas and water as a production fluid while the rock remains in the reservoir domain.

If we consider the water influx from some nearby aquifer or from water bearing rock that is encroaching in this domain that should be considered separately that is getting in into the reservoir depending on the pressure depletion or the production profile. Once the production start pressure decline and that pressure declination allows the nearby water body to encroach in the reservoir domain. If we are having the gas produced at the surface and that gas is reinjected into the reservoir domain for that purpose we should have the injection well and that injection well allowing gas or the water to reinject in the reservoir domain to maintain the pressure. This is the situation mostly comes in the secondary recovery process. Let us consider most of the possibilities in the reservoir domain to develop the general material balance equation.

So the injection fluid could be gas or the water. The aerosols this is getting in into the domain upward aerosols the production of the fluid is happening to the surface. We understand how to relate the reservoir fluid condition to the surface condition or the STB

condition standard tank viral condition using the volume formation factor  $B_o$  for the oil,  $B_g$  for the gas and  $B_w$  for the water. So the volumetric balance since the volume of the reservoir is constant change in one type of the volume is happening that should be accommodated by the second fluid volume or the rock volume. So let us see what happens in terms of the volume.

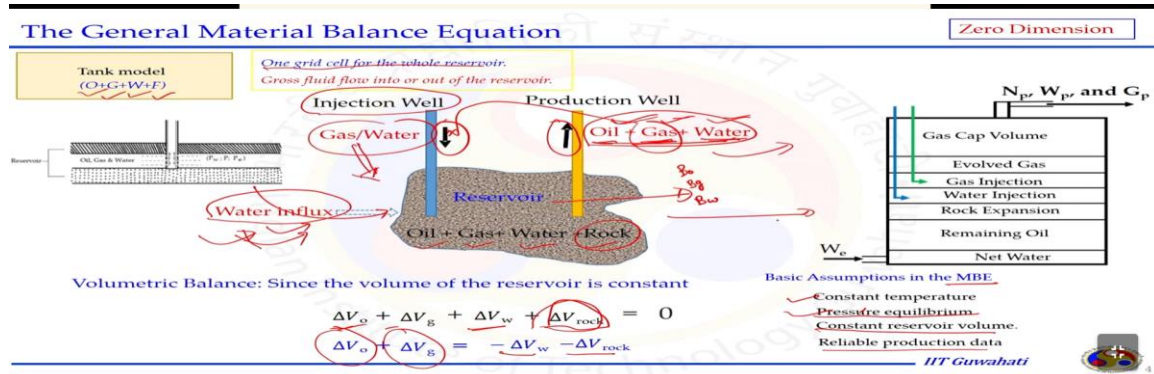
We are having the change in the volume of the oil that is  $\Delta V_o$  change in the volume of the gas change in the volume of the water change in the volume of the rock that is equal to 0. All the changes in the different segment of the reservoir should be equal to 0. How the volume of the oil changes because of the production because of the gas that is soluble in the oil that is liberating out volume of the gas if there is a gas cap that is allowing the fluid to produce the gas is also getting produced to the surface. So because of this production oil and gas volume will change. Similarly for the water, water may get influx from here and water is also getting produced to the surface.

So the volume of the initial water will also change with respect to time because of the influx and the production. The compressibility of the rock that will allow the volume of the rock also change because initially when the reservoir is very high pressure the rock is compressed and when the production happens pressure declines and rock also start expand to occupy the pore volume that is made available because of the production. So the rock is also occupying some of the volume change that is happening in the reservoir. So the summation of the changes of all these volumes should be equal to 0. If we adjust this equation so change in the volume of the oil change in the volume of the gas because of the production of the oil and gas to the surface should be equal to minus change in the volume of the water and change in the volume of the rock.

Certain assumptions are taken to develop this mathematical equation. We call this MBE material balance equation or the volumetric performance equation. We consider constant temperature. So in this tank model we are not seeing any effect of the temperature. Although temperature may vary slightly little bit in the reservoir domain as reservoir domain is large but we are considering isothermal conditions and the volume pressure change is happening relatively no effect of the temperature is considered in that case.

Pressure equilibrium means pressure is considered uniform across this tank or within the reservoir domain or the control volume we are considering. There might be slightly variation but we consider at a particular time when we are talking about the pressure of the tank it is a constant value or the average value that is represented by the pressure equilibrium concept. Constant reservoir volume so the tank volume is constant. Reliable production data as we depend on the production data in this material balance equation how much oil, gas and water is getting produced to the surface. Similarly, how much water is getting influx into the reservoir domain and if there is any injection well that is pumping gas and water to the reservoir domain all these injection and production data

must be very reliable to establish accurate model and predicting the dry mechanism naturally present in the reservoir domain.



If we convert this into the tank model we will see what is present in the reservoir and how the production is happening. So let us say this is the tank where we are focusing to develop the volumetric balance equation. Here we are having gas cap on the top because of the gravity segregation. We are having the rock expansion that may happens remaining oil after the production. So NP is representing the cumulative oil production. WP is cumulative water production. GP is cumulative gas production. Oil, gas and water those are getting produced. P subscript here denote the cumulative.

So NP is cumulative oil. WP is cumulative water and GP is cumulative gas production. W is water injection that may happens in the reservoir and here the downward arrow shows we might be injecting the gas from the external sources into the reservoir domain or we are injecting the water. Whatever the possibilities there we will consider those other can be eliminated. For example if there is no external injection of the gas and water into the reservoir domain they will not be considered they will become 0. But in general mathematical equation we are going to consider the possibilities.

What are the other possibility within the reservoir domain will be? So the gas that is initially present in the reservoir but there might be possibility when the reservoir pressure is declining and it is coming below the bubble point pressure the gas will also evolve out from the oil and then the new gas cap volume may be formed. So that primary gas cap plus secondary gas cap. So the evolved gases will also be there and the net water so some water is getting produced some water is getting in. So at a particular time when we are estimating the performance the tank model is having some net water within the pore volume of the reservoir or the tank model. So the production of oil and gas water, average reservoir pressure, water influx and the accurate measurement of initial oil and gas are some of the pre-requisites of this.

We need these data to develop the mathematical equation quantitatively and perform the polymetric balance to understand the different time mechanism those are supporting the

reservoir and which time mechanism is having more contribution which is having the less contribution towards the production. All this information help us to predict the performance of the reservoir as well as setting up the conditions which are kind of a screening criteria to implement the secondary and tertiary recovery processes for the oil reservoir. So the material balance in general let us see this is the tank model where oil, gas, water and the rock is there. This is the situation when we are producing the water, oil and gas from the reservoir some water influx is happening in terms of  $W_e$  within the reservoir domain or in the tank domain we are injecting some water or gas they are also considered and the nomenclature here used like  $n$  for the oil,  $W$  for the water,  $G$  for the gas subscript  $P$  with the letter denoting the cumulative performance,  $W_e$  here is the encroachment of the water. So all these nomenclature or the abbreviation used are with respect to society of petroleum engineering.

So society of petroleum engineering approved that nomenclature and it is worldwide acceptable nomenclature to consider for the material balance equation. So let us see what we are having in this tank model oil that is denoted by  $n$  we are having gas that is  $G$  we are having water that is  $W$  and the rock let us say it is  $F$ ,  $F$  is for the formation so we can say this is representing whatever happening with respect to rock in our control volume. So let us see what is in the rock, rock is having the pore volume so  $P \cdot V \cdot$  is the pore volume in terms of  $VBL$  this pore volume is actually accommodating oil, gas and water. So when it comes to the oil  $n$  is representing initial or original oil in place, gas is initial gas cap that is actually present in the reservoir at the reservoir pressure condition that is on the top of the formation because of the gravity difference it moves up and form the gas cap. Here you will see it is showing in terms of STB so this is standard tank barrel it means it is measured at the surface at the STB condition.

To relate this with the reservoir condition we can use volume formation factor for the oil that is actually relate the reservoir condition and the surface condition together. So  $n$  that is actually initial oil in the place but measured with respect to surface condition. Similarly for the gas we measured this in SCF standard cubic feet so at the surface but we can relate this with the help of the volume formation factor for the gas to the reservoir condition.  $NP$  is the cumulative oil production that is again measured at the STB condition,  $GP$  is the cumulative gas production,  $BOI$  is the initial oil formation factor for the oil and that initial oil formation volume factor will be useful to relate this  $n$  and  $BOI$  to measure the amount of the oil that is present at the reservoir condition. Similarly for the gas volume formation factor can be used,  $BO$  is the oil formation volume factor at a particular time or at a particular pressure when we are evaluating the tank performance.

Standard value will be for the gas also there. Cumulative water production is given by  $WP$ ,  $WE$  is the cumulative water influx that is getting into the system. The injection of the gas from the out source is represented by  $GING$ ,  $GING$  that is cumulative gas

injected standard cubic feet into the reservoir. So at the reservoir we need to know the volume formation factor for the gas at the injection conditions to calculate the amount of the gas that is going to be in the reservoir. On the rock side we are having the formation compressibility or the rock compressibility  $C_F$  that is accounting the change in the volume with respect to pressure.

Here we are having the data for the water  $C_W$  that is the water compressibility both compressibility of water and the rock is having the unit of inverse of PSI. This is the water injection into the reservoir if we are injecting any water. So this nomenclature shown here is very standard and we are going to use the same nomenclature to develop our mathematical equation. Some more interrelated properties also need to be known. For example, oil and gas they are having one common property that is the gas solubility in the oil.

So at the initial condition when the reservoir pressure is  $P_i$  the solubility of the gas is represented by  $R_{Si}$  with respect to standard conditions SCA per STB.  $R_S$  is the gas solubility at any pressure,  $R_p$  is cumulative gas oil ratio. So when we are producing the cumulative oil, cumulative gas the ratio of these two oil and gas cumulative profiles can be represented by a common factor  $R_p$  that is cumulative ratio of these oil and gas.  $GR$  is the instant  $GR$  means at a particular time when we are measuring the gas oil ratio that is represented by  $GOR$ . To relate the initial oil and initial gas at the reservoir condition a parameter  $M$  is used to eliminate one of them.

So if we are knowing only one parameter that is  $M$  that is actually showing the ratio of initial gas cap gas reservoir volume to initial reservoir oil volume. Actually, mathematically this is the ratio of initial volume of the gas and the volume of the oil initially present at the reservoir condition. So  $M$  relate actually the oil and the gas together at the initial condition and  $W$  is actually the initial water that we missed out. So initial water could also be present in the reservoir but in general we consider the change in the water volume. So the initial volume is reflected in terms of how much water is getting in by the influx and how much water cumulatively is getting produced.

So the  $W$  is actually not required but initial water should also be known when we are talking about the pore volume occupied by the initial water to know the initial water saturation  $SW_i$ . So the same model we are having where the oil and gas initial condition are related by a parameter called  $M$  and if we look the parameter those are required to establish this thing in terms of the pressure we need to know the initial pressure, average volumetric pressure at a condition we are evaluating the performance, change in the reservoir pressure during this time and  $P_b$  is the bubble point pressure that is actually define the types of the reservoir we are dealing with in terms of the oil reservoir. So if the pressure of the reservoir is above bubble point reservoir it is under saturated oil

reservoir if it is lesser than bubble point pressure the reservoir is saturated reservoir. We will discuss different cases of oil reservoir and the gas reservoir in our next classes.

These are the pressure things those need to be known. These are the initial conditions of the oil, cumulative oil production, cumulative gas production and cumulative water production. Similarly to relate the reservoir condition to standard condition we are having the formation volume factor. So we need to know the formation volume factor of oil, gas at any time or the moment we are going to investigate and the initial oil and gas volume formation factor. The relation between oil and gas, cumulative gas oil ratio  $rp$ ,  $gor$  for the instant,  $rs_i$  initial gas solubility,  $rs$  is the gas solubility at any time and on the external injection side we are having the water injection from the surface, gas injection from the surface and  $w$  is water influx that is happening by the nearby aquifer. In the rock part we are having the pore volume and the compressibility of the formation and then the compressibility of the water also need to be known.

Compressibility of the water formation and oil are of the same order of magnitude actually they are very low compressible substance compared to the gas. So in general like the effect may be neglected but as I mentioned we are going to develop the general material balance equation. So we are considering all these possibility into our balance equation. So the balance equation that we are going to develop initial reservoir pressure should be known. Average reservoir pressure at successive interval after the start of the production is required.

The STB of oil produced measured at STP at any time during the production interval. So at the surface we are measuring the oil and gas. The total standard cubic feet of the gas produced at the surface when gas is injected into the reservoir externally there will be a difference between the total gas produced and that gas returned to the reservoir that is actually the total volume of the gas that is going into the domain. The ratio of initial gas to the volume of the initial oil volume that is  $m$  that is need to be known. The gas and oil volume factor so the volume formation factor like  $B_o$ ,  $B_g$ ,  $B_w$  and their initial condition also and we discussed this in our class of fluid and rock properties how to estimate the value of formation factor using the differential or flash liberation methods.

So let us say laboratory measurement are giving us the volume formation factor for oil, gas and water at any condition that could be initial condition or any condition when we are evaluating the performance. The quantity of water that has been produced so  $W_p$  cumulative production. The quantity of water that has been encroached into the reservoir that is  $W_e$ . So all these information are required or the knowledge of all these information are essential to establish the relationship that is going to help us to understand what are the natural dry mechanism those are supporting the reservoir production. Two parameter that is I am keep saying  $m$  that is the initial volume of the gas to initial volume of the oil in the reservoir condition that can be established.



So this is  $g$  that is the amount of the gas produced at the surface. If we multiply this with the initial volume formation factor of the gas we will get the value of the gas that is originally in the reservoir condition. Similarly  $n$  is the oil that is initially in place measured at the STP condition. If we multiply this with volume formation factor we are going to get the amount of the oil that is initially present in the reservoir at reservoir condition.

### The General Material Balance Equation

- ✓ The initial reservoir pressure and the average reservoir pressure at successive intervals after the start of production.
- ✓ The stock tank barrels of oil produced, measured at STP at any time or during any production interval.
- ✓ The total standard cubic feet of gas produced. When gas is injected into the reservoir, this will be the difference between the total gas produced and that returned to the reservoir.
- ✓ The ratio of the initial gas cap volume to the initial oil volume, symbol  $m$ .
- ✓ The gas and oil volume factors and the solution gas-oil ratios. These are obtained as functions of pressure by laboratory measurements on bottom-hole samples by the differential and flash liberation methods.
- ✓ The quantity of water that has been produced.  $W_p$
- ✓ The quantity of water that has been encroached into the reservoir from the aquifer  $W_e$

$$m = \frac{\text{Initial volume of gas cap}}{\text{Volume of oil initially in place}} = \frac{GB_{gi}}{NB_{oi}}$$

$$P \cdot V_r = \frac{NB_{oi}(1+m)}{1-S_{wi}}$$

So the expression for  $m$  will be in this ratio form. Similarly with respect to the formation we are having the pore volume and pore volume is given by this expression that includes oil initially present in the reservoir this all together  $1 + m$  actually  $m$  now replacing the parameter required with respect to gas. So if we know  $m$  we do not need to worry about the gas or the oil one of them is required by using this ratio we can calculate the other one. So  $1 + m$  is actually representing oil and gas together and  $1$  minus  $SWI$  in the denominator is the volume of the rock occupied by oil and gas. So how we get this thing? So the volume of oil and gas in the reservoir domain that is equal to pore volume into  $1$  minus  $SWI$ . So if we consider this volume of the oil, volume of the gas, volume of the water that is actually the pore volume.

It means like the pore volume in the reservoir domain available to accommodate the fluid either occupied by the oil, gas or the water. Now this water volume can be shown in terms of pore volume multiplied by the saturation of water at initial condition. So the initial condition of the water and the saturation at that condition actually allow us to find out how much pore volume is covered by the water within the reservoir domain. Similarly knowing the saturation for oil and gas we can also convert that into the amount of the volume occupied by the oil and gas in the pore volume. But let us consider this oil and gas in terms of these volumes which we are getting from here and the pore volume on the right-hand side if we take this water part here we will be getting pore volume is coming out of the bracket and we are getting  $SWI$  as the factor to be multiplied with the pore volume and that is equivalent to the volume of oil and gas in the pore volume.

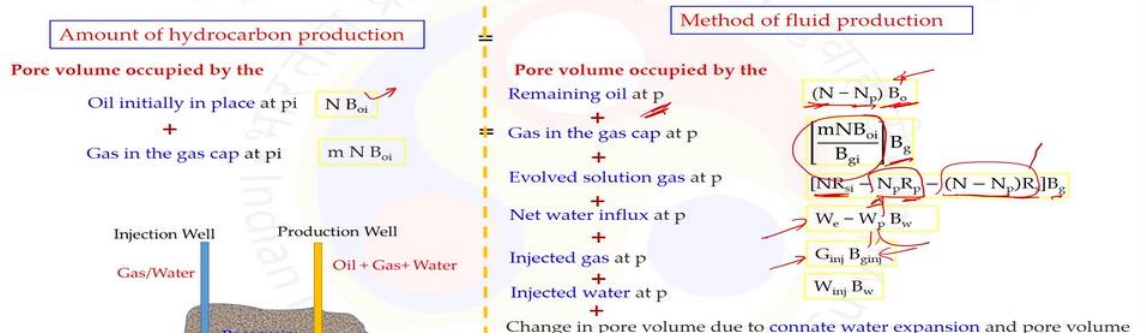
So the expression will be useful for pore volume at several places when we are setting up the equation. So in general material balance equation we are having 9 terms that actually relate the production PVT properties of the fluid and the rock properties within the tank model. So let us say the amount of the hydrocarbon produced on one side and that is equivalent to the methods of fluid production how the method how the fluid is getting produced in different ways. This is the reservoir domain where all the possibilities are depicted.

So the pore volume initially occupied by the oil and gas. Let us consider water will be balanced by the difference of water encroachment and water produced. What we are getting on the other sides the pore volume occupied by at any time when the pressure is  $p$  so the remaining oil that is initial oil minus the amount of the oil that got produced. So here we are considering the volume formation factor  $B_o$  is not changing that much within that pressure changes only difference is coming initial oil minus the oil already produced so that is the cumulative oil multiplied by the volume formation factor for the oil to measure the remaining oil at pressure  $p$ . So here it becomes  $B_o$  so this is initial volume formation factor here the volume formation factor at the condition that is adjusting at the reservoir when we are making this evaluation and that at that time the pressure in the reservoir is  $p$ . Similar we can do for the gas so this is the amount of the gas  $g$  that is getting remaining in the gas cap multiplied by the volume formation factor of the gas.

Similar we can have the evolved solution gas that evolved solution gas will be having 3 or 4 factor we will discuss in detail later on. So this is the initial amount of the gas that is present in the reservoir but soluble in the oil and this is  $n_{p,r}$   $p$  is the amount of the gas that has been produced to the surface so  $n_p$  is the cumulative production of the oil multiplied by the cumulative gas oil ratio. So if we multiply both of them we are going to get cumulative gas production  $n$  minus  $n_p$  is the remaining oil that is also having some solubility of the gas so we are measuring the gas remain in the soluble condition in the reservoir and multiplied by the volume formation factor. Net water influx so this is water getting encroached minus water getting produced the difference of these two is the amount of the water that is present in the reservoir domain or in the tank model. Injected gas if there is any multi so this is the injection gas rate multiplied by the volume formation factor of that gas similarly for the water and the change in pore volume due to connate water means remaining water the water that remains in the reservoir so the expansion of water may happens and then the pore volume reduction due to rock expansion.

## The General Material Balance Equation

*Nine terms composing the MBE can be separately determined from the hydrocarbon PVT and rock properties*



So both water expansion and rock expansion can be combined together and with the help of the pore volume that is going to be changed because of the water expansion and the rock expansion or I mean to say the amount of the pore volume that is going to be occupied by the water expansion and rock expansion can be represented by this expression. We will see how we reach to this expression in the coming slide. So the general material balance equation we are having the oil, gas, water and the rock we can divide this entire thing in four part where what is happening with respect to volume for oil, gas, water and the rock. So oil we are having the initial oil  $N B_{oi}$  oil remain in the reservoir  $N - N_p$  multiplied by the  $B_o$  volume formation factor and the change in the volume that is what we want change in the volume is the difference of these two. Similar thing will happen for the gas initial gas in terms of gas data or in terms of the oil data we are multiplying with  $m$ .

So free gas at any time  $T$  can be calculated initial gas free and dissolved gas already produced minus gas remain in the solution. So if we represent free gas at time  $T$  that is by  $G_f$  so this is the amount of the gas that is initially present so this is initial gas and this is the gas soluble in the oil at the initial condition some gas got produced so either we can say it is  $G_p$  or  $N_p \cdot R_p$  and this is the remaining oil multiplied by the solubility of the gas that is showing the gas is still present in the soluble condition in the reservoir at time  $T$ . So the change in the gas volume is  $G - V_{gi}$  initial gas -  $G_f$  multiplied by the volume formation factor so we are going to get the change in the gas volume. We want change in the oil volume, we want change in the gas volume, we want the change in the water volume also so this is change in the water volume that is actually initial reservoir water volume so we say  $W_i$  and water volume at any time  $T$  so that is initial water, water coming through the encroachment, water getting produced and then this additional term if the water expansion is happening because of the pressure changes. So the compressibility factor or isothermal compressibility coefficient of the water is coming into the expression.

## The General Material Balance Equation

Oil	Water
Initial reservoir oil volume = $NB_{oi}$ Oil volume at time $t$ and pressure $p = (N - N_p) \cdot B_o$ Change in Oil Volume = $N \cdot B_{oi} - (N - N_p) \cdot B_o$	Initial reservoir water volume = $W$ Cumulative water produced at $t = W_p$ Reservoir volume of cumulative produced water = $B_w \cdot W_p$ Volume of water encroached at $t = W_e$ Change in Water Volume = $W - (W + W_e - B_w W_p - W_{cw} \Delta p)$
Gas	Rock
Initial free gas volume = $GB_{gi} = NmB_{oi}$ Free Gas at $t = \left[ \begin{array}{l} \text{Initial Gas, free and dissolved} \\ \text{Gas Produced} \\ \text{Remaining in Solution} \end{array} \right]$ Change in Gas Volume = $G \cdot B_{gi} - G_t \cdot B_g$	Initial void space volume = $V_f$ Change in Rock Volume = $- \text{Change in the Void Space Volume}$ Change in Rock Volume = $-V_f c_r \Delta p$

so how we know all the changes that is required to do the material balance equation or the volumetric balance equation

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On the rock side we are having initial void volume let us say  $V_f$  change in the rock volume is initial voidage that is  $-V_f$  plus the change in the voidage happening because of the expansion so in that case we are going to get the change in the rock volume as the initial void multiplied by the compressibility factor multiplied by the pressure. So this is actually the compressibility definition where we say isothermal compressibility is change in volume with respect to pressure at constant temperature divided by the original volume using that expression we get the change in the rock volume also. So now we know all the changes that is required to do the material balance equation or the volumetric balance equation. One more time we can see how to estimate the changes in all these four oil, gas, water and rock.

## The General Material Balance Equation

Initial Volume	Change	Volume at any time $t$ and $p$
Oil: $NB_{oi}$	$NB_{oi} - (N - N_p)B_o$	$(N - N_p)B_o$
Gas: $GB_{gi} = NmB_{oi}$	$NmB_{oi} - \left[ \frac{NmB_{oi}}{B_{gi}} + NR_{si} - N_p R_p - (N - N_p)R_s \right] B_g$	$\left[ \frac{NmB_{oi}}{B_{gi}} + NR_{si} - N_p R_p - (N - N_p)R_s \right] B_g$
Water: $W$	$-W_e + B_w W_p - W_{cw} \Delta p$	$(W + W_e - B_w W_p + W_{cw} \Delta p)$
Rock: $V_f$	$-V_f c_r \Delta p$	$V_f + V_f c_r \Delta p$

$\Delta V_o + \Delta V_g + \Delta V_w + \Delta V_{rock} = 0$

$\Delta V_o + \Delta V_g = -\Delta V_w - \Delta V_{rock}$

Injection Well: Gas/Water  
Production Well: Oil + Gas + Water  
then,  $G_{inj}B_{inj} + W_{inj}B_w$   
Water influx:  $G_{inj} + Gas + Water + Rock$

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So we can club oil and gas and then they are related through  $m$ . Water is  $W$ , rock is  $V_f$  and at any time this will be the remaining oil, this will be the remaining gas in the reservoir, this will be the water balance or change in the water balance can be calculated with considering the water present, water encroached minus water produced plus conic water expansion is happening and then this is the change in the volume formation factor at any time. So if we want to calculate the difference between change at any time with respect to initial volume we are going to get these expressions just by making the differences of this part with respect to this part. So we are having this reservoir domain where we are considering the changes in the volume of all these four factors. The balance equation can be adjusted like this. Expansion of gas and water cumulatively we

can write like this and if they are present then only they will appear in the expression otherwise we can make them 0 or we can nullify them.

So the change in pore volume that I said earlier can be represented by this equation. This expression for the pore volume comes by knowing the pore volume occupied by the oil, gas and water. So the change in pore volume due to expansion of the connate water can be given by this expression. So this is the pore volume, SWI is the amount of that pore volume occupied by the water and this is CW is the compressibility coefficient that is explaining how the expansion of the water is happening when the pressure is changing and this is changing with respect to pressure.

So the CW came with this definition and the pore volume came by this definition. Similarly change in pore volume due to expansion of the rock we can have this pore volume multiplied by the compressibility factor multiplied by the pressure change will give us the changes in the pore volume due to the expansion of the reservoir rocks. So total change in the pore volume because of these two factors can be obtained by summing up both the equations and when we do that we are going to get this expression. Actually this expression appears in the material balance equation to account for the changes in the pore volume because of the connate water and the rock expansion. So the injection of the gas and water cumulatively can be shown like this. If required then they only be appearing there if not we can make them 0 and the cumulative gas production can be related to the oil with the help of RP that is cumulative gas oil ratio.

**The General Material Balance Equation** **Change in pore volume**

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$$P \cdot V_o = \frac{NB_{oi}(1+m)}{1-S_{wi}}$$

$$\text{Change in pore volume due to Expansion of connate water} = \frac{NB_{oi}(1+m)}{1-S_{wi}} S_{wi} c_w \Delta p$$

$$\text{Change in pore volume due to Expansion of reservoir rock} = \frac{NB_{oi}(1+m)}{1-S_{wi}} c_r \Delta p$$

$$\text{Total changes in the pore volume} = \frac{NB_{oi}(1+m)}{1-S_{wi}} (S_{wi} c_w + c_r) \Delta p$$

$$c = -\frac{1}{V} \frac{\partial V}{\partial p} \bigg|_T$$

$$\Delta V = V c \Delta p$$

**Material Balance Equation**  $\Delta V_o + \Delta V_g = -\Delta V_w - \Delta V_{rock}$

$$N = \frac{N_p B_o + \left( \text{terms of all the parameter those are participating in the material balance equation.} \right)}{(B_o - B_{oi}) + (R_{si} - R_s) B_g + m B_{oi} \left[ \frac{B_g}{B_{oi}} - 1 \right] + B_{oi} (1+m) \left[ \frac{S_{wi} c_w + c_r}{1-S_{wi}} \right] \Delta p}$$

**Injection of gas and water**  

$$G_{inj} B_{ginj} + W_{inj} B_w$$

$$G_p = R_p N_p$$

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So if we place all the information that we gathered for change in volume of oil, gas, water and rock we are going to get this big expression and this big expression with respect to the initial oil present in the reservoir measured at STP condition can be shown in terms of all the parameter those are participating in the material balance equation. This big expression is obtained just by doing this simple volumetric balance equation where the changes in the volume of oil, changes in the volume of the gas, changes in the volume of the water and changes in the volume of the rock are considered. So the change in the volume of the rock is accounting for both change in the water expansion and change in the rock expansion. So if we place all of them considering the expression



be knowing to relate the gas and oil cumulative production we can obtain this big expression that is the general material balance equation. So if you place this equation here, this equation here, this equation here and this equation here you will see like the expression that is this can be eliminated as it is already shown in terms of here.

### The General Material Balance Equation

$$\begin{aligned}
 & \Delta V_o + \Delta V_g = -\Delta V_w - \Delta V_{\text{rock}} \\
 & \text{NB}_{oi} - (N - N_p)B_o = -W_e + B_w W_e - W_{cw} \Delta p \\
 & \text{NB}_{oi} - \left[ \frac{N m B_{oi}}{B_{gi}} + N R_{oi} - N_p R_p - (N - N_p) R_s \right] B_g = \frac{N B_{oi} (1+m)}{1-S_{wi}} (S_{wi} c_w + c_f) \Delta p \\
 & N = \frac{N_p B_o + (G_p - N_p R_s) B_g - (W_e - W_p B_w) - G_{inj} B_{ginj} - W_{inj} B_w}{(B_o - B_{oi}) + (R_{si} - R_s) B_g + m B_{oi} \left[ \frac{B_g}{B_{gi}} - 1 \right] + B_{oi} (1+m) \left[ \frac{S_{wi} c_w + c_f}{1-S_{wi}} \right] \Delta p}
 \end{aligned}$$

$G_p = R_p N_p$

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The rock expansion is accounting for the expansion due to the conduit water expansion. So we can eliminate this here but when we are putting all this information making sure  $n$  is kept on the one side and remaining on the other side we are going to get this expression because of the oil, this because of the gas, this is because of the water. So this is oil production, gas production, water and this is external injection of the oil and gas and in the denominator we are having the several parameter those actually relating how the changes is happening in the oil, gas, water and rock as the pressure is changing. So the general material balance equation can be represented in multiple forms. So one of the form is here where the GP is just replaced by RP NP. So if we do so the color coding shows where the changes are made we are going to get NP out of this bracket and everything is here that is Bo plus RP because of here and the RS from here and the remaining expression remains same.

So this expression can be represented in terms of mostly the oil production parameter. So we do not need to worry about how much gas is getting produced from here. We replace the gas with the oil with the help of this RP parameter here. Initially we already replaced GP with the help of this M that is showing like how much initial gas by initial oil is present in the reservoir. So this expression can further be modified considering the two phase formation volume factors we are having oil and gas we can combine the volume formation factor for oil and gas. So this is the volume formation factor for oil this is the volume formation factor for the gas and this needs to be adjusted at the amount of the gas will change from initial solubility to the solubility at the point of investigation.

So whenever we are getting this expression Bo plus difference in the solubility multiplied by the BG we can replace this with BT. So this expression can further be adjusted for the BT here RSI was not there so to bring this RSI in the expression we have to add and subtract RSI. So the changes from this expression to this expression

happening in terms of total volume formation factor or the two phase volume formation factor and the initial solubility of the gas in the reservoir RSI. Similarly on the denominator we have to adjust this in terms of two-phase formation factor at any time  $t$  at initial condition so  $B_{Ti}$  is appearing whenever we were getting  $B_{oi}$  and  $B_{gi}$ . So we can eliminate the need of  $B_o$  and  $B_{oi}$  with the help of adjusting the equation by this BT two phase formation volume factor and that is BT at any condition  $B_{Ti}$  at the initial condition and in this case if we further modify this equation considering there is no gas injection there is no water injection.

So these two terms will drop out 0 and this is 0 the same expression can be written in a little bit simplified form considering only these two conditions where there is no external injection from surface to the reservoir domain is happening. So the general material balance equation without considering any injection of the water and gas we like this and this is a zero dimension model. The drive mechanism and their contribution to the production can be established using this expression so we can further do the adjustment in this equation multiply all of that with this  $n$  and what we are going to get a equation where we are having the effect of different drive mechanism. So let us simplify this thing let us this is entire term is A this is B and then the all that is appearing in the denominator is C.

So I can write this equation  $n$  is equal to  $A$  minus  $B$  by  $C$ . So what is  $A$  is representing the cumulative production of oil and gas what is  $B$  is representing the net water flux so encroachment minus the production and what is  $C$  the different mechanism those are actually contributing towards the production that is change in the volume of the two phase system change in the volume of the rock due to water expansion and the rock expansion that is here. So now if I adjust this equation that is  $nC$  is equal to  $A$  plus  $B$  so now this  $A$  I can keep this  $A$  here and then I can do the adjustment like this is minus  $nC$  plus  $B$  is equal to  $A$  and that  $A$  is actually the production of the oil and gas to the surface. So this  $A$  can be considered as a factor in both side and on the right hand side we are going to get 1 and left hand side we are going to get  $n$  multiply in all the terms  $C$  plus  $B$  divided by  $A$  is equal to 1. So this is the same way the expression has been adjusted on the right hand side we are just getting 1 and on the left hand side we are having several terms so this is with respect to oil we are having  $B_t$  minus  $B_{ti}$  so this is two phase consideration when oil and gas both are present and we are having  $n$  multiplied by the change in volume formation factor. Second term is having the gas the change in the volume of the gas due to gas cap expansion here we are having the water related things and here we are having the expansion of the remaining water or connate water and the rock formation.

If we consider these four parameters with respect to this  $A$  that is actually the cumulative production of oil and gas to the surface we will get different drive index first one is showing  $D_{di}$  that is depletion drive index. So this is the part that is contributing as a

production mechanism when the oil is present and the gas is liberating out from that oil due to the pressure below the bubble point pressure and this is contributing towards the production. So this Ddi index will be represented by this one and this is the expansion of the original oil volume with all its original dissolved gas and this is actually less contribution is there in general balance equation the Ddi contribute very less in the overall production drive it may be neglected but when the reservoir is having the condition where this cannot be neglected significant amount of the gas is getting evolved out that is maintaining the pressure supporting the production Ddi should also be considered. So there could be different cases when one particular type of the drive index should be considered or should be ignored. Second one is SDI that is segregation or the gas cap drive index that is the gas cap that is formed on the top and that is supporting the reservoir and that is happen expansion of the original free gas cap that is contributing towards it.

**The General Material Balance Equation** Zero Dimension

$$N = \frac{[N_p(B_t + (R_p - R_{si})B_g) - (W_e - W_p B_w)]}{(B_t - B_{ti}) + mB_{ti} \left[ \frac{B_g}{B_{gi}} - 1 \right] + B_{ti}(1+m) \left[ \frac{S_{wi}c_w + c_f}{1 - S_{wi}} \right] \Delta p}$$

*Driving mechanisms and their contribution to the production*

$\frac{N(B_t - B_{ti})}{A}$ 

DDI

DDI = depletion-drive index

$\frac{N m B_{ti} (B_g - B_{gi}) / B_{gi}}{A}$ 

SDI

SDI = segregation (gas-cap)-drive index

$\frac{(W_e - W_p B_w)}{A}$ 

WDI

WDI = water-drive index

$\frac{N B_{oi} (1+m) \left[ \frac{S_{wi}c_w + c_f}{1 - S_{wi}} \right] (p_i - p)}{A}$ 

EDI

EDI = expansion (rock and liquid)-drive index

$A = N_p [B_t + (R_p - R_{si})B_g]$   
 expansion of the original oil volume with all its original dissolved gas - minimum  
 expansion of the original free gas cap -- minimum  
 net encroachment of water, maximum  
 the contribution of the rock and fluid expansion to the oil recovery is too small and essentially negligible and can be ignored

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It might be minimum in some cases but may be very prominent in other cases. So we will discuss several cases the oil and gas reservoir to understand which drive mechanism can be prominent and under what conditions those drive mechanisms should be considered and we cannot neglect them. Third one is Wdi that is water drive index and that is net encroachment of the water actually that is very good drive mechanism we also discussed this in the previous lecture because it support the pressure for a longer duration and the recovery factor is also very high so this could give us the good amount of the oil produced to the surface and the reservoir should be maintained to have the water drive index but again that will depend on the water encroachment is happening or not happening in the reservoir domain. If it is happening this is a very good candidate reservoir to produce the good amount of the oil. Last one is the EDI that is expansion of rock and fluid drive index so we considering liquid means oil and the water but the oil compressibility and the water compressibility factor are almost same order similar for the rock also in that case we can combine oil and water together and just representing in terms of the water in this expression but the EDI include both rock and liquid and the drive index through this one is due to the contribution of the rock and fluid expansion to the oil recovery it is in general is very small and most of the time it is neglected or



ignored in the consideration but in general balance equation we need to consider all these drive mechanism. The combination of drive mechanism could be present there in this general material balance if we compare with respect to what we discussed in the primary recovery mechanism we did not consider the gravity-based drive mechanism because we considered the tank volume is not having much effect of the gravity but if it is present there that effect should also be included but in this balance equation we did not consider that part.

So the general material balance equation can be represent as a equation in the form of a straight line that a straight line equation can be obtained by adjusting this equation in the form of f is equal to n multiplied by some factor plus another factor so this is equal to y is equal to mx plus c and this expression in the straight line form give us lot of flexibility to develop the understanding of the different drive mechanism. This is given by Havelina and Ode in 1963 to understand the better understanding of the different drive mechanism in the form of straight line where the term Eo Eg and Fwo similarly for f are representing

**The General Material Balance Equation** Havlena and Odeh (1963) Zero Dimension

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**MBE : Equation of a Straight Line**

$$N = \frac{N_p[B_o + (R_p - R_s)]B_g - (W_e - W_p B_w) - G_{inj} B_{ginj} - W_{inj} B_w}{(B_o - B_{oi}) + (R_{oi} - R_s)B_g + mB_{oi} \left[ \frac{B_{gi}}{B_{gl}} - 1 \right] + B_{oi}(1+m) \left[ \frac{S_{wi}C_w + c_f}{1 - S_{wi}} \right] \Delta p}$$

$$N = \frac{\{N_p[B_t + (R_p - R_{si})B_g]\} - (W_e - W_p B_w) - G_{inj} B_{ginj} - W_{inj} B_w}{(B_t - B_{ti}) + mB_{ti} \left[ \frac{B_{gi}}{B_{gl}} - 1 \right] + B_{ti}(1+m) \left[ \frac{S_{wi}C_w + c_f}{1 - S_{wi}} \right] \Delta p}$$

Three unknowns in MBE

- The original oil in place **N**
- The cumulative water influx **We**
- The ratio of original gas cap to oil **m**

$$F = N[E_o + mE_g + E_{f,w}] + (W_e + W_{inj} B_w + G_{inj} B_{ginj})$$

$$F = N_p[B_o + (R_p - R_s)B_g] + W_p B_w$$

**Production**

$$E_o = (B_o - B_{oi}) + (R_{oi} - R_s)B_g$$

**Oil Expansion**

$$E_g = B_{oi} \left[ \left( \frac{B_g}{B_{gl}} \right) - 1 \right]$$

**Expansion of Gas Cap Gas**

of connate water and rock.

$$E_{f,w} = \frac{B_{oi}(1+m)}{1 - S_{wi}} (S_{wi}C_w + c_f) \Delta p$$

**F** = Total fluid (oil, gas and water) withdrawal

**E<sub>o</sub>** = Expansion of oil and its originally dissolved gas

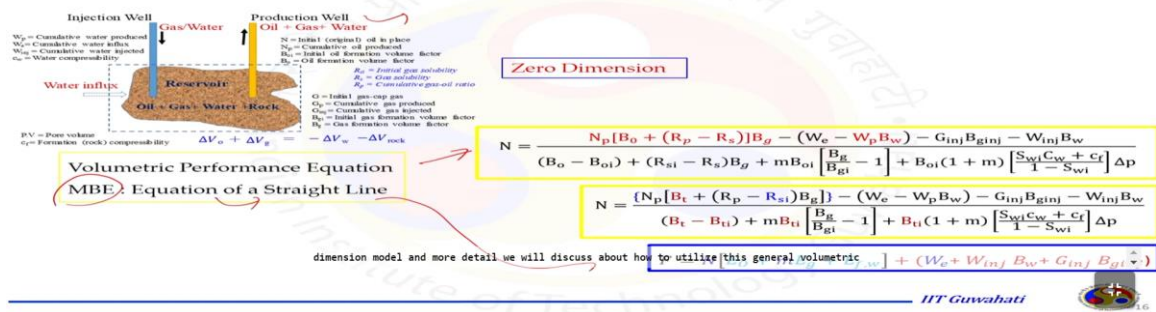
**E<sub>g</sub>** = Expansion of the gas cap

**E<sub>f,w</sub>** = Expansion of connate water and rock

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with respect to what is happening in the reservoir domain. So for example f is total fluid that is oil and gas water so this is the production that is happening Eo is actually representing the expansion of oil and its original dissolved gas so this is oil expansion Eg is the expansion of the gas cap and Efw is actually the expansion of connate water and rock. So the all four factor production oil expansion gas expansion and the rock expansion are considered we can represent this in the form of Bo and Bg or can convert this in the form of two phase volume formation factor and this straight line which is having three unknown if we see n, m and We essentially these are the three unknowns those can be obtained if we can fit the equation in the form of y is equal to mx plus c and the slope and intercept can give us the value of n We and m. We will discuss all these things in the next class so in summary today we discussed the general volumetric balance we developed the overall volumetric performance equation for a tank in model

## Summary



kind of the system where the production and injection wells are present the material balance equation can be represented as a straight line so this is the volumetric performance equation that can be represented in the form of straight line this is a zero dimension model and more detail we will discuss about how to utilize this general volumetric balance equation and equation of a straight line to evaluate different reservoir those are either classified as oil reservoir or gas reservoir or whatever the conditions are there in terms of the injection and production we will discuss in the next class and that next class we will focus on oil and gas reservoir to understand the volumetric performance equation in terms of straight line.

So with this I would like to end today's lecture so in the coming lecture we will also doing the mathematical analysis of our flow through porous media develop the radial flow equation in general for oil and gas reservoir and perform the evaluation criteria to understand the performance of oil and gas reservoir based on several parameters. So with this I would like to end my today's lecture on general material balance equation thank you very much for watching the video thank you.