

# **Petroleum Reservoir Engineering**

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## **Lecture – 1**

### **Introduction to Petroleum Reservoir Engineering**

Hello, and welcome to this first lecture on petroleum reservoir engineering. I am Dr. Pankaj Tiwari, Associate Professor in the Department of Chemical Engineering, at IIT, Guwahati. Petroleum reservoir engineering is a course that is very much needed for petroleum reservoir engineering students or petroleum students and the people those are working in flow through porous media. One of the standard definitions of reservoir engineering is the art of developing and producing oil and gas fluids in such a manner to maximize hydrocarbon recovery and increase revenue generation. The reservoir or petroleum reservoir is different than the water reservoir bodies.

The petroleum reservoir is underneath the surface. It is a porous and permeable lithological unit or set of the units that holds the hydrocarbon reserves. Hydrocarbon reserves means oil that is crude oil and natural gas. There are many important properties that needs to be understood performing the reservoir operation or recovering the hydrocarbon fluid to the surface in a profitable manner.

Out of those two important properties are porosity and permeability that differentiate petroleum reservoir than the water body reservoir on the surface. So, the porosity and permeability are two important properties. Those actually characterize how much hydrocarbon reservoir fluid is present and how easy those are accessible. So, the porosity is the amount of the hydrocarbon reserve those are present in the permeable rock and permeability is the property of that rock that calculate how easily hydrocarbon fluid can flow out of this porous region. Petroleum reservoir are broadly classified in two terms conventional and unconventional reservoir.

The conventional reservoir is the reservoir those are producing at profitable rate or a very high rate naturally when the hydrocarbon fluids are produced naturally from the reservoir. While in the unconventional reservoir, the flow rate is too low, and additional techniques or unconventional techniques need to be implemented to get the better recovery or the profitable recovery. The conventional reservoir and unconventional reservoir are different in terms of the technology that can be implemented. Conventional

it is pretty much standard techniques. we can drill the well at the identified location and start producing the fluid. The unconventional reservoir could be different not only in terms of the technology but the types of the fluid present, the permeability present in the reservoir. By any means, the flow rate is not profitable and additional techniques need to be implemented that is unconventional reservoir.

In the conventional reservoir to produce the fluid or hydrocarbon fluid to the surface, there are various steps that need to be implemented. It starts with the exploration of the site, needs to be identified where the exploration should be performed or the site where the potential reserve of hydrocarbon fluid are present. A decision needs to be made at a precise location where the drilling should be done. Initially, the drilling is done in the vertical manner from the surface to the surface to reach the hydrocarbon reservoir pool. Once the drilling is started, the extraction of this hydrocarbon reservoir fluid from underground surface to the surface needs to be brought out. Surface to the surface needs to be brought out.

So the assembly or the drilling completely processed needs to be implemented. Once the production starts, the fluid comes to the surface, they go through the several processes, unit like the separator and refining, transportation from the well head and processing unit using the extensive network of pipelines and other modes. The crude oil or the natural gas needs to be transported further processing and storage facilities need to be installed and finally this is the distribution of the crude oil to the end user and of course all this model should be in the marketing manner which is generating the profitable revenue. The conventional petroleum production system is shown in this diagram, you can see there are three components, bottom part that is actually we call the reservoir where the hydrocarbon fluid are stored. In this picture, it is looking very systematically oil, gas and water, these are the three fluids that are present in the reservoir but they are not in this simple manner, there is a very complex porous and permeable region where these fluids are stored, we will discuss this in later slides or in other lectures. So, the reservoir is at very high pressure, very underneath the surface, around in the order of several thousand feet where the reservoir fluid are stored. To reach this reservoir, we need to drill the well and the section called the well bore.

At the surface, what we see is the Christmas tree and that Christmas tree is having the monitoring, controlling and safety feature for this production system. Our focus will be on the reservoir side but in general, what happens in the reservoir that is at the reservoir pressure  $p_r$  or the average reservoir pressure because the pressure will be varying when we drill the well, the pressure of the reservoir varies from the reservoir pressure to well bore pressure that we call PWF. Because of this pressure energy, the fluid travels towards the production well or production string and then they move upward. When they are

moving upward pressure energy further required to bring them from subsurface to the surface and they reach to the well head with a pressure of PHF and then the fluid those are produced pass through the several unit one of the unit is the separator. The separator could be horizontal, vertical, two-phase separator, three phase separator ultimately the job of the separator is to separate the fluid in water, oil and gas.

So at the reservoir we were having the oil, gas and water we produced them to the surface and separated them oil, gas and water. The characteristic performance relationship that relate flow rate to the pressure at the reservoir condition is called IPR inflow performance relationship. This is the relationship that relate flow rate  $Q$  of the hydrocarbon fluid with pressure drawdown or it is called the pressure difference or pressure drawdown. Similar set of the equation is developed for the fluid to travel from PWF to PHF and we call this is WPR or TPR. WPR stand for the Valbourn performance relationship, TPR is the tubular performance relationship.

Again the same relationship we do the flow rate is related to the pressure difference PWF to PHF. At the surface at the Christmas tree a device called the choke that is actually control the production is again characterized by the relationship we call the CPR, Choke Performance relationship that is again relate the  $Q$  versus pressure difference. In this case the pressure difference is PHF and the outlet pressure or the atmospheric pressure or whatever the pressure we are producing at this condition. So, this is the conventional way of producing the hydrocarbon reservoir fluid from the reservoir to the surface. To reach the reservoir as I mentioned several step is required first is the exploration identifying the site where to drill it perform the drilling operation that itself is a very complex process because different geological terrain are faced during the drilling process once everything is done well completion part is performed.

After the well completion means installing the Christmas tree it is hand over to the production engineer and the fluid start producing. When the fluid produced to the surface, we need to characterize this reservoir. The reservoir can be characterized primarily based on the GOR that is the gas oil ratio. So, at the separator we can separate oil gas and water we calculate the volume of the oil and the gas produced accordingly we classify the reservoir. Another parameter is degree API.

Degree API is the density of the fluid produced based on that also the reservoir can be classified like it is a heavy oil, light oil or medium oil reservoir and the color is also one of the indications that is what type of the reservoir is. There are various ways to classify the reservoir we will discuss further during this course. The production from the reservoir to the surface happens in three steps. First is called the primary recovery. This primary recovery happens because of the natural driving force that is the pressure

already existing in the reservoir when we complete the drilling well completions upon production started the reservoir is at very high pressure like several thousand order of magnitude and from where the fluid start producing to the surface and because of this naturally pressure difference the fluid is producing to the surface the production is happening under the primary recovery.

There are various dry mechanism those exist during this production we also discuss various dry mechanism that is responsible for maintaining the pressure naturally and producing the fluid to the surface. Over the time pressure declines and because of that the fluid is not able to produce to the surface and to maintain the pressure underneath the surface or in the reservoir domain we inject the external fluid that could be the gas or could be water. The pressure maintenance mechanism to produce the oil by external fluid is called the secondary recovery process. Even after the secondary recovery process the fluid is not completely coming out to the surface because of the interaction of rock and fluid or the fluid characteristic or the reservoir characteristic additional means need to be implemented to recover that remaining oil and the process called the tertiary process or enhanced oil recovery process. There are various means to recover that oil by tertiary processes we will discuss those also during this course.

Tertiary recovery in general account for 5 to 15% of the original oil in place that is called OOIP. Secondary recovery is pressure maintenance mechanism that is continued to production up to 30 to 40% of OOIP. The remaining oil is targeted with the tertiary processes. So, the role of reservoir engineer is to choose the recovery methods when to produce under primary condition when to switch from primary to secondary conditions and when to target the depleted reservoir abundant reservoir after the secondary recovery for various types of the EOR methods. Forecasting the production at what rate the fluid should be produced or at what rate the fluid will be producing in the future and control the production depending on the supply and the demand each.

All these can be done by setting up the balance equation that is the mass conservation equation understanding the flow behavior within the reservoir fluid phase behavior because the fluid those we are dealing are oil gas and the water their phase behavior depends on the temperature and pressure they are subjected to and production decline curve analysis is one of the techniques that is characterized the reservoir performance. All these things are performed by the reservoir engineer periodically and make the assessment for the future actions. So, as I was mentioning the reservoir is not a swimming pool kind of the things where the hydrocarbon fluid are just stored we can drill and get it out it is a complex mixture where the fluid are stored in a porous and permeable path. So this picture depict that thing because of the density difference the fluid are settled in that porous and permeable path at the bottom because of the density the water is

settled first on top of that oil and the gas being the lighter settle on the top. The reservoir engineering job or the production engineering job is not a one man job a team work is required to complete the production and from the identifying the site for the exploration to the end users a team of different caliber skilled people are required to perform the reservoir management that is start from the land acquisition taking the legal advices management team is required to make the decisions geology and geophysicist are required to understand the rock formation identifying the drilling sites and then reservoir engineers of course for the production purpose economics team drilling engineering that itself is like a subject to study those team member are also required as a part of the reservoir management team other than those service engineer environmental consideration.

So it is a team work that is actually perform the operation of taking the fluid from the reservoir domain to the surface. As I was mentioning the GOR is one of the indications that is characterized which type of the oil well it is because it is required to identify the oil well the oil well or the gas well because the surface facilities installed at the site itself or far away from the original site depends on the type of the oil well. So GOR is one of the parameters that characterize the type of the well that is drill it is going to be the oil well gas well or condensate well if the GOR value is greater than 1 lakh SCF per STB the well should be characterized as the gas well if the value of GOR is lesser than 5000 it is dominantly the oil well if the value is in between gas and oil well for example less than 1 lakh and greater than 5000 SCF per STB the well is characterized at the condensate well. It is pretty much required to identify the well because the facilities to be installed at the surface are determined by the type of the well like in case of the gas well this after the separator the processing of the gas is done at the installation site itself while in case of the crude oil the crude oil is transported to the refinery for the further processing. Based on the degree API if the degree API it is the measure of the density if the value is 0 to 20degree API it is heavy crude oil if the value is between 20 to 30 this is medium crude oil and if the value is over 30-degree API the oil is considered as the light crude oil.

Another important aspect of petroleum production system is the thermodynamic behavior of the fluid that is getting produced. As we know the temperature and pressure is continuously changing from the reservoir to the surface the phase behavior of the fluid will also change. At the reservoir condition if we identify what is the reservoir temperature and pressure with the help of the thermodynamics behavior, we can characterize the reservoir further in different classes like for the oil reservoir under saturated oil reservoir saturated oil reservoir or the gas cap reservoir. Similarly for the gas reservoir it could be the dry gas reservoir means the reservoir is predominantly producing the gas only wet gas reservoir which is producing some amount of the liquid along with the gas and condensate gas well reservoir that is the special case of

hydrocarbon fluid where the gas changes the phase from gas to the liquid depending on the temperature and pressure condition. We will discuss the thermodynamic behavior of the hydrocarbon fluid in detail in other lecture.

With this background I would like to brief you about how these petroleum fluids are produced underneath the surface. So, the petroleum products like the natural gas and the crude oil they come under the fossil energy. Fossil energy include natural gas crude oil and the coal. The basic theory says the organic matter on the geological time scale gradually become coal oil or natural gas. The picture which I am showing here is the surface picture that is taken somewhere Colorado in the United States by myself where the geological formation is depicted.

You can see layer wise formation is there and these layer wise organic and mineral deposition is called the oil shell. The core sample taken out from this region is having certain lines if we can see those lines are actually the organic matter that is laminated with the minerals. This oil shell is exposes to different time temperature history to get the crude oil out of it. The similar kind of the setup is expected to be arranged underneath the surface even similar or more complex where the organic material went through geological time scale at a different temperature and pressure under chemical, thermal chemical, biochemical or biological processes and got converted into oil and gas. So, if plot temperature versus time we can reduce the time to get this organic matter converted into oil and gas if the temperature can be increased.

But naturally underneath the surface the temperature is low around most of the reservoir are around 90 to 70 to 120 degree Celsius while if we expose this material at the surface to 400 degree Celsius temperature the time can be reduced to few hours. But at that temperature naturally underneath the surface takes geological time scale means millions of years to convert this organic matter into oil and gas and that organic matters is having the tendency to migrate from one place to the other place because of the permeability of the rock it is trapped in. We will discuss these aspects in detail in the petroleum reservoir section. So, the reservoir fluid those are produced from the reservoir are crude oil, natural gas and the formation water. The reserve underneath the surface are characterized into three sections proven reservoir, potential reservoir and the probable reservoir where there is a proven reservoir those are producing with the non quantity or the information is available we can forecast the production.

Potential reservoir those are having the potential to produce the fluid at the predicted rate and probable reservoir those are still under the development more information are required to characterize those kind of the reservoir. So when it comes to the temperature pressure history underneath the surface it depends ,on the ease of reservoir what type of

the fluid means crude oil and the gas got produced. Deposition history and the history of the temperature and pressure both increases as we go deeper underneath the surface. In deposition history we see if we are around 1000 feet depth the porosity that is one of the characteristics of the rock is around 30%. If we are going deeper the porosity decreases and around 20,000 feet the porosity is just 10%.

For the permeability similar analogies can be said as we go down because of the compaction of the rock the permeability also decreases at 3000 feet the permeability could be 10,000 milli Darcy when we go 10,000 feet depth the permeability reduces and it is in the range of 10 to 100 milli Darcy around 20,000 feet depth the permeability is 0.1 milli Darcy means in this order. So as we go down the porosity and permeability both are supposed to be decreased but the exact value or the exact nature of the porosity and permeability depends on many factor geological setting age of the rock and other factor. So these are the typical values those we can relate if we are going deeper temperature is increasing we are going deeper pressure is increasing permeability is decreasing porosity is also decreasing. Similar as we are going deeper it is expected the organic material gone through the higher temperature and higher pressure history.

So at 3000 feet depth if the reservoir is drilled at 3000 feet depth and the crude oil is found that is in the nature of heavy oil or some light oil where the gas is dissolved. If we going deeper the light oil is expected to be produced and if we are going further deeper it is expected the oil is producing the natural gas predominantly. This is another typical picture of petroleum reservoir where I have shown like three types of the rocks those are important for any petroleum reservoir fluid to be stored in the reservoir domain. First one is the source rock the rock that is carrying the hydrocarbon expulsion nature where the organic material is there that is over the geological time scale got converted into oil and gas. So the source rock from where the hydrocarbon are generating and expulsion is happening.

Second is the reservoir rock a rock that is actually holding these reservoir fluid it is kind of a pool where the fluid produced from the source rock are migrating primarily migration and secondary migration is happening they are migrating to a place where they are stored that is called the reservoir rock. In the picture you can see this is the reason where the fluid are stored this is the reservoir rock and this is the source rock from where the fluid or hydrocarbon fluids are produced. When they reach to the reservoir rock there should be a rock that is trapping this hydrocarbon reservoir at the location not allowing them to come out by its own to the surface and that is called the cap rock. Cap rock is very highly impermeable rock that is not allowing any seepage of gas and oil out of this reservoir formation. So these three type of the structure are required or the rock assembly is required to get the hydrocarbon reservoir pool formation underneath the surface.

The fluids are oil, gas and water as I mentioned water comes to the bottom because of the density then the oil and then on the top we are having the gas. When we drill the well to reach this pool or reservoir section from where the hydrocarbon fluid will be produced along with the water the location of the drilling well is very important. Where we are drilling and the production string attached to this production tube at the bottom which is perforated means created the holes through those holes the fluid is travelling and coming to the surface it is very important. It is reaching to the gas well only it is reaching to the oil section or it is below the oil section. Based on that there are different types of the drive mechanism those are responsible for the production of the primary recovery of this hydrocarbon fluid like depletion drive, rock and fluid expansion drive, water drive, gravity drive and many more.

We will also discuss those in detail when we will be setting up the material balance equation. So, in general this is the setup of the hydrocarbon reservoir the shape here I created is one of the representative shape there could be very complex very irregular like faulty or dome shape or many other kind of the structure are possible underneath the surface. Actually, it is a very complex setting that is several thousand feet depth or like the deepest valley trail up to 15 km 49000 feet depth. So, you can understand at that condition what kind of the reservoir setting is there it is very complex to predict. Some of the structure are predicted those are like dome shape and the fault shape and in general it is as we can assume that the source rock is producing the fluid they are going to the reservoir rock and seal cap is trapping those fluid underneath the surface at the reservoir rock location from where we are producing.

The different production wells can be installed in the same location to produce the reservoir fluid at a higher rate. There is a possibility that the nearby water body is also encroaching to this reservoir fluid it is called the aquifer. So the water aquifer is naturally maintaining the pressure and when this is happening the drive mechanism is called the water drive. We will discuss those different type of the drive mechanism later on. So, the seal or cap rock it is an unit with low permeability that impedes the escape of hydrocarbon current from the reservoir rock.

The maturation is the technique or the analysis of assessing the thermal history of the rock or the source rock in order to make prediction of amount and timing of hydrocarbon generation and expulsion. The source rock is important because it is actually producing the hydrocarbon fluid. So, in this course we are not going to discuss about the separator we are not going to discuss about the choke we are not going to discuss about the WPR. Our focus will be on this reservoir domain only where the reservoir pressure is PE



average reservoir pressure the pressure at the bottom of the valve is the PWF how the fluid means hydrocarbon fluid along with the water are getting produced from reservoir pressure to PWF. As I mentioned there is a possibility that different shape of the hydrocarbon reservoir rock is possible one of the interclinal shape I am showing here where the fluid are stored in a source rock from the source where the fluid are stored from the source rock to the reservoir rock based on the density gas cap is on the top oil and the water.

So if we drill the well for example here at this location where production string is the gas is also pushing the oil to get produced this called the gas cap drive the water will also push from the bottom section this called the water drive that could be combined drive mechanism also. So, when we understand how the fluid is producing from reservoir to the wellbore we need to understand certain aspects that is the types of the fluid in the reservoir the flow regime that is existing in the reservoir, reservoir geometry and the number of flowing fluid in the reservoir and on top of that the flow equation that is actually characterized or the performance of the reservoir how the fluid is flowing through porous media. So, the types of the fluid in the reservoir the isothermal compressibility coefficient is one of the important parameter that is actually determined how compressible the fluid is. Broadly we can classify the fluid into three category incompressible fluid slightly compressible fluid or compressible fluid. So when we apply the pressure in a constant volume system or in a container we see the volume of the fluid decreases correct.

So when this is happening we can classify the fluid at what length this is happening we can classify the fluid to incompressible slightly compressible and compressible fluid. So let us see the fluid density how it is changing with respect to pressure if the fluid density is not changing when we implement the pressure the fluid is called the incompressible fluid. In this case the density of the fluid is constant. Second aspects when the density of the fluid is changing or other way this is the definition of isothermal compressibility at a constant temperature, we assume it when the C value is changing with pressure but not changing linearly, we call the fluid is compressible in the nature. There is a situation where the fluid density is changing but it is changing linearly with respect to pressure we called the fluid is slightly compressible.

Actually in nature there is no fluid that is completely incompressible but within the study domain or the pressure domain we are going to deal with the fluid we can characterize the fluid as incompressible. If there is no appreciable change in the density of the fluid and for example we can consider water as incompressible fluid natural gas is highly compressible we can reduce significant volume by implementing the pressure on the natural gas and that is follow the compressible pathway. So, natural gas is actually

highly compressible and the crude oil can be characterized as slightly compressible because its density changes while we apply the pressure on to it. So, in our system reservoir engineering we are dealing with all three systems crude oil, water and the natural gas. Pressure diagram can be drawn for the volume pressure relationship.

So, the volume is not changing when we are implementing the pressure it is changing linearly or it is changing non-linearly accordingly, we can classify incompressible slightly compressible or the compressible fluid. Second important aspect is flow regime. So, there could be a possibility how this fluid is traveling under what condition it is traveling. It means how the pressure decline is happening over the time can be characterized into three parts that is steady state, unsteady state and pseudo steady state case. If we plot pressure versus time relationship in at a particular location in the reservoir if we are measuring the pressure how the pressure is changing over the time pressure will change because the flow is happening because of the pressure energy.

So, over the time when we are producing the pressure should change. So, let us say at a particular location I we are measuring the pressure if the pressure is not changing with respect to time at that location the system should be characterized as the steady state flow. There is no condition in the reservoir fluid production system where the steady state flow condition exists but for the simplicity purpose or solving the complex equation or understanding the reservoir performance the situation can be hypothesized where we say the pressure is not changing with respect to time means the derivative of pressure with respect to time at a particular location,  $I$  is 0 the system is steady state flow condition. Then the pressure with respect to time is changing but changing with a constant rate and that situation is called the semi steady state or the pseudo steady state conditions. The third condition where the pressure is changing with respect to time at a particular location  $I$  non-linearly and it is changing with respect to location also and time also that situation called the unsteady state flow condition.

Most of the time reservoir produced under the semi steady state flow condition only the conditions when we start producing we are having the unsteady state flow condition or when we are setting the well that time also unsteady state condition could be present. Once the production started it passed through the transient phase or the unsteady state phase the remaining time the reservoir is mostly producing under the pseudo steady state flow condition. We will discuss some of the case when we will set up the radial diffusivity equation for our reservoir case. The third important aspect is the reservoir geometry actually this is the shape of the reservoir how the fluid is traveling from the reservoir domain to the well bore. So the flow behavior of the reservoir fluid can be characterized by the reservoir geometry in actual there could be many complex geometry but we can classify them in four standard flow regimes of flow geometries that is the

radial flow, linear flow, spherical flow, hemispherical flow.

So in radial flow as we can say this is our production stream and this is the reservoir boundaries. The reservoir boundary could be very wide here just one with the boundary as a diagram here I am just showing as a circle but it could be very large boundaries. In the radial flow the fluid is traveling to the production stream from all the direction and you can see the fluid is coming to the production stream from all the direction and getting produced through this well bore. If this kind of the flow is existing, we can classify that as a radial flow. In the side view of radial flow you will see this is the well bore this is the perforated region of the well bore through which the fluid will enter inside the well bore and the flow line is coming from all the direction towards this production stream.

In linear flow what happens the fluid is just traveling only one direction and where we are having the production stream or the well bore. So not from all the direction only one linear direction the fluid is traveling towards the production stream. This kind of the situation could be there depending on how precisely we are drilling the well and how the production stream is installed. This is the isometric view of the linear flow there could be a possibility where the perforated region or the holes in the production stream are less than the page on thickness. So this called the page on thickness through which the fluid is produced or the fluid is stored in the reservoir and if the perforated region is not enough to accommodate all the page on thickness the fluid will try to travel in such a manner to get the spherical shape around this production stream and this kind of the flow behavior is called the spherical flow.

There could be a situation when the fluid is not produced from the perforated region but produced from the bottom of the production stream and the fluid from all the direction coming to the bottom section and just traveling through this well bore or production stream. This kind of the situation is called the hemispherical flow situation. As I mentioned most reservoir are having very irregular geometries or irregular boundaries very rigorous mathematical descriptions of their geometry is often required and that is only possible with the numerical simulator. In general, when we are setting up the analytical methods or basic balance equation to characterize the reservoir we mostly deal with the radial flow and the linear flow system. We will be doing the radial diffusivity and the linear flow equation for our system later on when we will discuss the flow behavior of the reservoir fluid.

The fourth important aspects of the reservoir fluid production is number of flowing fluid present in the reservoir. In general most of the reservoir are having the oil, gas and the water but as I mentioned based on the GR we can classify the reservoir is just having only the gas predominantly ,only the oil that we can classify it as the gas well or the oil

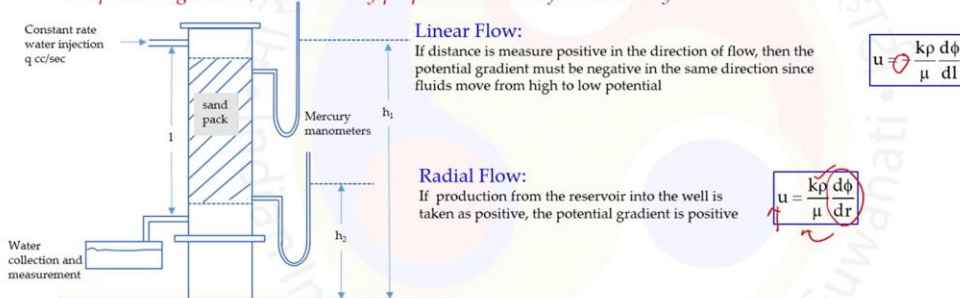
well respectively. So, the form and the complexity of the problem increases as the number of fluid is increasing in the system. So, there could be a situation where only single-phase flow is present that is oil, water and gas at a time either only the oil is present or the gas is present or the water is present. There could be a two phase flow system where the oil water is there, oil gas is there or gas water is there.

In three phase system oil, water and gas all three are present and they are traveling towards the production string from the reservoir domain. As the number of fluid is increases the complexity is increases so the description of the fluid flow and subsequent analysis of pressure data becomes more difficult as the number of mobile fluids increases in the reservoir domain. We will discuss the setting of the equations for the single-phase flow system. For the two phase flow system we will try to understand how the saturation and the other parameters are estimated or the residual oil recovery are estimated when the two phase flow are present and for the three phase flow system we will discuss in the reservoir simulation domain as it becomes more complex problem when we are dealing more than one fluid or three fluid of three different phases like the liquid as the oil, gas as the natural gas and the water as the aqueous phase the problem becomes very complex. After this understanding we need one flow equation that is actually characterize the flow of the reservoir fluid from this porous permeable domain and that is done with the help of the Darcy law.

## Fluid Flow Equations

### Darcy's Law and Applications

- The fundamental law of fluid motion in **porous media** ✓
- Developed by Darcy in 1856 states that the **velocity of a homogeneous fluid** in a porous medium is **proportional to the pressure gradient, and inversely proportional to the fluid viscosity**.



Darcy law is a fundamental law for the fluid motion in porous media. There are several models are developed later on based on the Darcy law but the Darcy law is still the fundamental law to characterize the flow through porous media. The non-Darcy behavior of the fluid flow is characterized either by modifying the Darcy law or implementing some correction factor. This Darcy law was developed by Darcy in 1856 states that the velocity of a homogeneous fluid in a porous medium is proportional to pressure gradient and inversely proportional to fluid viscosity. He developed this relationship when he was

doing the sand pack experiment for the water purification system where the constant flow rate water was injected through a sand pack of a different packing which is having the different permeability. So constant flow rate where the pressure across the sand pack was measured and the water of different viscosity was flown through this and the relationship was established.

If the system is linear then the Darcy law comes with the negative sign because the potential gradient in our case it is the pressure gradient that is in the reverse direction of the length while the radial flow pressure is declining in the same direction where the radius is also declining and in that case the expression is not having the negative sign. If you see this expression the velocity of the fluid that is passing through the porous region depends on its density, viscosity and the potential gradient and another parameter that appears here is actually the characteristic of the sand pack or the reservoir rock or the formation is called the permeability. Based on his name the unit of permeability is also given as Milley Darcy or the Darcy unit. The Darcy law was established under certain assumptions like the following condition must exist when we are implementing the Darcy law like the laminar or viscous flow is happening, the fluid is incompressible in the nature, steady state flow is happening and the homogeneous formation. The formation is having constant porosity, permeability and isentropic in the nature.

So we will be implementing this Darcy law in our material balance equation for different cases where the fluid is considered as compressible, incompressible, slightly incompressible, different flow regime, different flow geometry and when single phase fluid is present. Another important aspect of the reservoir engineering is the unit system. The units are used in the petroleum industry is called the US field unit system. It is different than the CGS, SI or MKS unit system that we study for the other courses. In US field unit system it is actually the hybrid unit system where the parameters are measured in a combination of the unit system.

For example the flow rate in the CGS unit system which is cc per second while in the US field unit system it is standard tank barrel per day for the liquid and standard cubic feet per day for the gas. Similarly, the pressure is measured in the PSI, viscosity is measured in the centipoise and the length is measured in feet. The important one is the permeability. The unit in the SI unit system is meter square while in the US field unit system it is mostly given in the unit of Darcy or the milli Darcy.

When we deal with the different unit system unit conversion becomes essential. Some of the unit those need to be converted from one unit to the other unit. For example, from SI unit to the US field unit system the conversion factor I listed out. For example, the value of conversion for the area in US field unit system the area is measured in acre and

the conversion factor is required. Similarly for the flow rate for the standard tank barrel per day or MSCA per day for the oil and gas respectively the conversion factor needs to be implemented if the data are given in the SI unit system or any other unit system. So this is very much required to make sure you are dealing with the correct unit system when you are solving the numerical problems.

So when we set up the mathematical equation we got sometimes very complex equation set and we need to have the knowledge of one simple optimization technique or one numerical integration techniques that can help us to get the solution of the expression those are complex otherwise. So, Newton Raphson method it is a simple technique based on the linear approximation where the root of the equations or the system where the parameter is a function of itself can be solved. So, this is based on the simple idea of linear approximation recursive algorithm is required means iterations are required to be performed. In this case what we do the function is arranged in the form of  $fx$  is equal to 0.

### Newton Raphson Method: Example

Step 1:  $f(x) = 0$

Step 2:  $(x_n = x_1)$

Step 5:  $(x_n = x_{n+1})$

$$f(x) = 5x + x^4 - 10 = 0$$

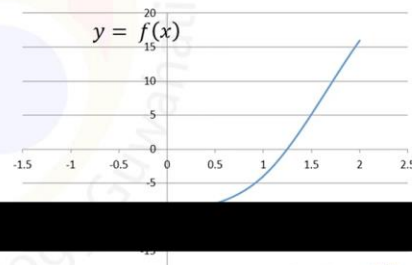
$$f'(x) = 5 + 4x^3$$

$$x = \frac{10}{5 + x^3}$$

Step 3:  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Step 4:  $|\epsilon_a| = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100$

Iteration	$x_n$	$f(x)$	$f'(x)$	$x_{n+1}$	$ \epsilon_a $
1	2.000000	16.000000	37.000000	1.567568	27.586207
2	1.567568	3.876004	20.407735	1.377639	13.786496
3	1.377639	0.490184	15.458433	1.345930	2.355980
4	1.345930	0.000006	14.736142	1.345165	0.000032
5	1.345165	0.000006	14.736142	1.345165	0.000032



So, take everything on one side make it is equal to 0. So, this is regressive algorithm means the iteration need to be performed repeatedly to get the exact solution or the more accurate solution. There are certain steps need to be performed so for example rearrange the equation in the form of  $fx$  is equal to 0 this is the first step. Second is choose a suitable initial gas for example you choose  $x_n$  is equal to  $x_1$  some initial value of the parameter calculates the new value  $n$  plus 1 using this formula that says the linearly the tangent is drawn at that point and the new value is equal to old value minus the function value at the old value and the function derivative value at the old value in the denominator. So with the help of this expression we can find out the new value and the new value should be compare with the old value in the form of the error if the error is

acceptable we can consider this as the solution if it is not acceptable we have to perform the steps again. So if accepted then end otherwise next step what is next step replace the old value with the new value and go to step 3.

So you have to again perform the operation function value derivative function value at the new value find out the new value again and the iterative procedure should be done till we get the desired solution this can be seen on this diagram. So let us see initially we assume the  $x_1$  value we calculate the function value here that is not the solution so go to the process find out the new value  $x_2$  replacing  $x_1$  by  $x_2$  and find out the  $x$  function value here similarly we do for the  $x_3$  and by doing the iterative procedure we can get the value that is actually the solution of that function or that is actually the root of that function. We can take the example where  $x$  is equal to  $a$  plus  $bx^q$  here the  $x$  is a function of  $x$  itself we can use Newton Raphson method to find out the value of  $x$  that satisfy this equation. So let us say in the numerical term this is  $x$  is equal to 10 divided by 5 plus  $x^q$  so the first step is arranged in this form so that we did like this  $fx$  is equal to  $5x$  plus  $x$  to the power 4 minus 10 is equal to 0 the derivative of this will be 5 from here  $4x^3$  from here. So we are having now  $fx$  value  $f$  prime value the next step  $x_2$  assume some value so let us say we assume the value of  $x_n$  is equal to 2 that value is placed in this formula in step 3 to find out the new value of  $x_n$  plus 1.

So we calculated  $fx$   $f$  prime  $x$  got the new value of  $x_n$  plus 1 you see  $x_n$  was 2 the new value is  $x_n$  plus 1 is 1.56 the error is 27 by step 4. So, the error is still high we cannot accept this value so now the value of  $x_n$  will be replaced by  $x_n$  plus 1 value now the step 3 and 4 should be repeated again to calculate the new value as well as the error value and we see as we are keep doing the repeated experiment we reach to a point where the error is so low 0.00032, we can terminate the process and can accept the  $x_n$  plus 1 value as the root or the value of  $x$  that satisfy this equation. You can do this kind of the operation in MATLAB, Polymath or even in the excel if you use the excel there is a option data in the ribbon of the excel file there you will find what if analysis within that you will get the goal seek.

So, if you click on the data, you will get what if analysis and within that what if analysis you will get goal seek where you will set up your cell the cell that you are going to optimize or you are going to solve what is the target value that you can mention there and iterative procedure will be done and give you the solution of the equation. The advantage of using the excel sheet is that you do not need to have the  $fx$  arrange in a manner  $fx$  is equal to 0 you can solve any equation where the equation will get change and target value will be changed. So now your target value become 10 now the same procedure can be implemented go to data what if analysis goal seek set the set cell target value and

changing which cell you will get the solution for your set of the equation you will get the solution for the equation. For the integration trapezoidal rule is considered as one of the simplest methods, for integrating the function technique for approximating the definite integral we can use the trapezoidal rule you can use any integral approach I am going to explain one of the methods that is the trapezoidal rule as a part of the course. This is the integration the process of measuring the area under a function plotted on a graph.

## Trapezoidal Rule

### A Technique for Approximating the Definite Integral

**Integration:** The process of measuring the area under a function plotted on a graph.

$$I = \int_a^b f(x) dx$$

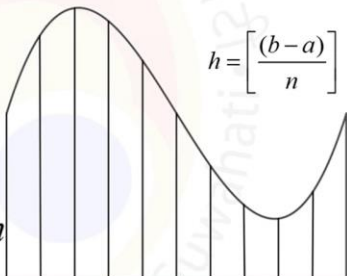
$f(x)$  is the integrand  
 $a$  = lower limit of integration  
 $b$  = upper limit of integration

The area under the curve is a trapezoid.

$$h = \left[ \frac{(b-a)}{n} \right]$$

$$A = (b-a) \left[ \frac{f(a) + f(b)}{2} \right]$$

$$y_n = f(x_n)$$

$$x_{n+1} = x_n + h$$


So, we want to calculate the area under this curve that is actually the integration so integrant I is equal to you want to integrate from A to B the function fx so this is function fx we can relate this is equal to y for that purpose what we can do the area under the curve can be segmented in a equal segments and as many as segments we are having means more finer B are classifying the more accuracy of the system will be there. So for example if we are not dividing this we just calculating the function value at FA, FB taking the average of it and multiplying this B – A we will get the area under the curve but that is not actual or that is away from the accuracy for accurate purpose we should divide this area under the curve in different segment and use this formula where h is B – A divided by n so this is the as many as segment you are dividing the h value will be accordingly this obtained h by 2 function value at 1 function value at the last and 2 times of the remaining value of the function all those intermediate segment put in this formula you will get the area under the curve that is actually the integral part of that fx from A to B. So, with this background would like to take you to Slavers again where this course is divided for 8 weeks 20 hours lectures, I put up the lecture plan like week 1 we will be starting the petroleum reserve overview of the petroleum geology briefly discussing about the drilling methods some of the segment like the basic reservoir engineering migration and trap reservoir drive may be appropriately placed where they are required. So there might be some changes in the plan of this MOOCs course as we move further



but what I did the broadly I classified this slavers into 3 parts section 1, section 2 and section 3. In section 1 we will understand the petroleum reserves, reservoir origin, its compositions, energy need and supply aspects, petroleum geology how these reservoir are formed, drilling method how to reach to those reservoir and along with that the important aspect is reservoir rock and fluid properties because all the mathematical aspects will be based on the reservoir properties.

So the section 1 we will try to finish let us say 2 and half weeks time and the section 2 will be the more focused on mathematically setting up the balance equation for our oil and gas reservoir that is fundamental of oil and gas flow in the porous media will be discussed. Oil diffusivity equation will be set up once the equations are set up we will go for testing the oil and gas well and predicting the reservoir performance. So, this will take around 4 to 5 weeks time for that purpose and the third section the enhanced well recovery the advanced techniques those are getting implemented in the reservoir to recover the remaining oil. Those are water flooding, polymer, caustic flooding, surfactant flooding, thermal we can classify broadly thermal and non thermal processes those will be discussed along with I will give you the exposure of introduction to reservoir simulation. As I mentioned reservoir is very complex system to understand better about the reservoir, we have to do some computational simulations and if time permits, we will also discuss about unconventional natural gas production specifically gas hydrates.

The acknowledgement is for the material or the books that I used to prepare the course content specifically Tarik Ahmad's Reservanching Handbook its 5th edition is in the market that is published in 2019. Terry, LP deck, Katz and Lee so these are the books which I use other than the books other sources from where the material information and graphs have been extracted are also acknowledged. So, in summary we discussed about conventional petroleum production system US field units and the conversion that is very much required as we move further in this course. For example the stock tank oil or the free gas oil after the separator when the gas is separated from this if we measure in different units let us say one barrel that is equal to 31.

5 US gallon and in litre it is 1119.24 litre. Even the gallon there are two types of the gallon one is US gallon that is 3.78 litre one imperial gallon or the UK gallon that is 4.546 litre. So, the unit conversion is also very much required or you must be familiar when you are attempting the numerical solution for system. Other than US field units and conversion we also discussed the numerical method those are required so we discussed about the integral method that is trapezoidal rule and the Newton Raphson method for finding out the root of the equation or simply optimizing the system.

In this week we will continue our discussion with the petroleum reserve petroleum geology and well drilling methods. I will not go in detail about each of these three but give you the brief idea how the reserves are stored what kind of the geological properties are important and how the drilling methods are performed to reach to the reservoir fluid those are stored in the reservoir domain. With this I would like to thank you for your attention I can be accessed at the email id reservoir.

iitg. gmail.com this is the separate email id other than the portal where you can place your questions if you write on this email id also, I would try my best to answer your queries. With this I would like to thank you again. Thank you very much..