Lec 6: Laws of Energy for Size Reduction3

Hello everybody. Welcome to this massive open online course on solid-fluid operations. So here we will today discuss about laws of energy for size reduction as lecture 6. In previous lecture we have discussed about the different common machines, those are used for reduction of size of the particle. So this is the last lecture for this module for size reduction. So in this lecture we will try to discuss about the different laws of energy for size reduction based on which you can calculate what will be the energy or power required for reducing size of particular size to get the desired size.

So here we will see that whenever you are going to reduce the size there will be some efficiency of that reduction. That efficiency depends on how much surface area will be created whenever the size will be reduced. That we have already discussed in the first lecture of this module. There you will see that as per mechanism of compression, attrition or impact based on those mechanism whenever size will be reduced there will be formation of surface area just by reducing its size and to reduce size we are applying that some stress and when stress is applied you will see that material are distorted and strained.

And in that case you need some work to get the strain which is stored as a mechanical energy till the ultimate strength is reached for that particular material and breakup into pieces beyond this ultimate strength is applied and whenever this materials will be breaks up into pieces it will give you more surface area. So energy will goes to increase in surface area and if there is excess energy which is not being utilized for the size reduction will be liberated as heat. Now how to calculate that size reduction efficiency that we have already discussed. Still in this slide we are giving this equation that size reduction efficiency will be based on the surface energy created by a certain mechanism upon energy absorbed by the material. So it is defined by this equation here given that this will be As into Asp minus Asp by Ws where Es is equal to surface energy per unit area and As is the specific surface area per that weight of the material and Ws is the energy absorbed by unit mass of the material or solid.

And then you will see that whenever energy will be absorbed by the solid that cannot be utilized 100 percent for reducing its size. So some energy will be lost and you will see that that lost energy will be emitted as heat energy. Hence there is some mechanical efficiency of that shaft energy which is absorbed by the solid and that actually will be utilized to get its size reduction. So there will be certain efficiency of that mechanical energy which is supplied to utilize to reduce the size of the material. So in that case that mechanical efficiency will be defined by that how much energy will be absorbed or utilized by the material denoted by Ws and by the energy input to the machine as W.

So we can write the W will be is equal to Ws by eta m, eta m is basically the mechanical efficiency. So after substitution of Ws from the earlier equation we are having this equation as S into ASP minus ASF divided by eta m into eta c.

$$\text{Mechanical efficiency } (\eta_m) = \frac{\text{Energy absorbed or utilized by the material}}{\text{Energy input to the machine}} = \frac{W_s}{W}$$

$$W = \frac{W_s}{\eta_m} = \frac{e_s(a_{s,p} - a_{s,f})}{\eta_m \eta_c}$$

$$P = W\dot{m} = \frac{e_s(a_{s,p} - a_{s,f})\dot{m}}{\eta_m \eta_c}$$

Now what will be the power requirement? Now if you are continuously supplied the material at a certain rate then what will be the rate of the mass which is to be reduced into a particular size then we can have the power requirement as what will be the mechanical energy that is absorbed or energy input to the machine into mass flow rate of the solid from which you can easily calculate what will be the power requirement. Now you will see that it may be used to create particle of a certain size and shape to increase the surface area that is available for chemical reaction or to liberate valuable minerals held within particles. Now in this case the energy used utilized as follows like in producing elastic deformation of the particle before it fractures.

In producing that elastic deformation which is result in size reduction in you know that causing elastic distortion of that equipment you will see that that energy will be utilized even you will see the in friction between particles and between particles and the machine to recover to or withstand that friction that energy will be utilized and because of that friction that energy will be lost some fraction of the energy will be lost as a heat energy. Also the energy will be utilized when there will be formation of noise or heat or there will be vibration in the plant so some energy will go to them there also and also as a frictional loss during the operation that energy will be utilized. So all the energy will not be utilized for that size reduction some fraction of that energy will be utilized for that size reduction and remaining energy will be converted as a heat energy or some creation of vibration or noise or other form of energy. Now coming to the main part of this lecture here this lecture is basically that will follow some laws to calculate the energy utilization to reduce the size. So there are basically three laws based on which the energy consumption by the machine to reduce the size per unit mass are calculated.

Those are one is called Rittinger's law another is called Kick's law and then Bond's law. These three laws are applicable to calculate the energy utilization there or the energy consumption by the machine to reduce the size per unit mass. Now what is that Rittinger's law? The Rittinger's law states that the energy required for reduction in particle size of a solid that will be directly proportional to the increase in surface area. Mathematically it can be expressed like this though we will derive it in the later slide you will see that. And then Kick's law what is that? This Kick's law states that the work required for crushing a given mass of material that will be constant for a given reduction ratio irrespective of the initial size of the material.

What is that reduction ratio? The reduction ratio is the ratio of the initial particle size to final particle size. And then another one is called Bond's law. In this case this law states that the work required to form particles of size from a very large particle size is proportional to the square root of the surface to volume ratio of the product. So these are three laws that you have to remember based on which you can assess or calculate the energy consumption by the machine to reduce. Now let us understand these three laws in more details.

Let us first discuss about the Rittinger's law. If we consider that some energy, a small amount of energy is required to affect a small change dL of its characteristics length. dL is basically the small change of characteristics length. So this characteristics length may be a diameter of the particle and or other length or in terms of other characteristics dimension. So basically whenever the particles size will be reduced its characteristics length will be considered as a diameter.

So you will see that in the feed material which is to be reduced in size may not be uniform in size, may not be the same shape of those particles. So in that case you have to consider the equivalent diameter of the particle. So here in this case we will consider that some energy let it be dE energy which will be required to affect a small change of that characteristics length in the size of unit mass of material and which will be related to the characteristics length. Now what will be the change of that energy per unit mass per unit length that will be proportional to the characteristics length of the material. Now how then it will be related? What is the proportionality constant? How exactly whether it will be linear relation or non-linear relation will be there? So let us consider it is a first non-linear relationship.

There will be certain relationship like that can be expressed like this dE by dL that will be is equal to minus C into L to the power P.

$$\frac{dE}{dL} = -\ CL^{p}$$

Here C is a constant and P is the power coefficient. So this power will give you that value which is as per the laws of that either Ritinger's law or Kick's law or Bond's law. So let us have this first in this case from this equation we are having that C is as C as a constant, P is a some coefficient.

Now what is that P value? Let us first consider that P is equal to minus 2 and after considering that minus 2 then integration will give you as E will be is equal to C into 1 by L2 minus 1 by L1.

$$E = C\left(\frac{1}{L_2} - \frac{1}{L_1}\right)$$

Now we are getting here 1 by L2 minus 1 by L1. What is this L2? L2 is the product characteristics length and L1 is the feed particles characteristics length. That means integration within a limit of L1 to L2. So L2 is the product size characteristics and L1 is the feed size characteristics length. Now important point here that the Ritinger's law states that whatever energy consumption will be there that will be proportional to the production of the surface area.

Now you will see that since the surface of unit mass of material is proportional to 1 by L, the surface is proportional to 1 by L. The energy required for size reduction is then directly proportional to the increase in surface. So here this is basically this is okay represent the surface area. This is the surface area. So what will be the surface area change that is obtained based on the energy consumption.

So that energy consumption it will be based on the energy supplied by the machine. So that will be represented by E. Now this E is a per unit mass of the material which is being used to reduce its size. So we can say simply E will be is equal to C is equal to 1 by L2 minus 1 by L1 which ultimately says that E will be is equal to C into S2 minus S1 because this 1 by L2 is proportional to S. So simply we can say that E is proportional to surface area.

So that actually given by this Rittinger's as per Rittinger's law it states like this. So we are having as per this Rittinger's law that E will be is equal to C into 1 by L2 minus 1 by L1 where C is constant that C is constant is represented by kR into fC. What is kR? kR is called that Rittinger's constant and fC is called the crushing strength of the material. So this Rittinger's constant depends on the crushing strength of the material. So finally we can write this equation as E is equal to kR fC into 1 by L2 minus 1 by L1.

$$E = K_R f_c\left(\frac{1}{L_2} - \frac{1}{L_1}\right)$$

So here we are getting that as per Rittinger's law what should be the equation to calculate the energy required to effect a small change of that particle from its feed size to the product size per unit mass of that material. Now in this case you have to remember one thing is that the surface area is proportional to 1 by L that means surface area is inversely proportional to the characteristics length. Of course it will be there because if characteristics area is a diameter then for spherical or any irregular shape particles which has the equivalent diameter as D. So if you reduce the size of course its surface area will increase. So we can say that for bigger size particles it will be less surface area whereas for smaller size particle it will have larger surface area.

So if you can reduce the size to a smaller size then you have more surface area that means here energy which is utilized is proportional to the surface area creation which is said by that Rittinger's. So as per Rittinger's law we can say that energy requirement will be proportional to the surface area or inversely proportional to the size of the material. Here you have to remember that L is here a general term of size and it is basically mean diameter of the particle. Then coming to the next law which is called Kick's law. The Kick's law says that the energy required is directly related to the reduction ratio.

What is that reduction ratio? This is the ratio of feed size to the product size which means that energy required to crush a given amount of material from a certain size to another size which will be the same as required to reduce from a proportionality from another size to another size. Like this here one important point here suppose that energy required to reduce from its size 150 millimeter to 75 millimeter. In this case what is the energy required? It will be the same energy would be required if you are reducing the size of 100 to 50 millimeter that means here reduction ratio earlier one is what is that 150 by 75 that is 2 and here also the second case the same energy will be utilized to reduce the size from 100 millimeter to 50 millimeter that is also reduction ratio is 2. So whatever reduction ratio if you are maintaining the same reduction ratio energy will also be same. So this is the statement of that Kick's law.

Now we are having this law to derive that mathematical expression by which you can calculate what will be the energy will be required to reduce that particle size to have a certain ratio of reduction. Now in this case if you consider that P is equal to minus 1 then we can say after integration from this basic equation dE by dL is equal to C minus C into L to the power P.

$$\frac{dE}{dL} = - CL^{p}$$

So in this case E will be is equal to C into Ln L1 by L2 after integration

$$E = Cln\frac{L_1}{L_2}$$

where C here is again constant it is denoted by KK into fC here KK is the Kick's constant and fC is the strength of the material or you can say that crushing strength of the material. So we can have this E will be is equal to KK into fC into Ln L1 by L2. So this is known as Kick's law this is as per statements of the Kick's law.

$$E = K_K f_c ln\frac{L_1}{L_2}$$

So energy will be required the same if you keep the reduction ratio that is L1 by L2 fixed. So this is as per Kick's law. So once that Kick's law constant and the material strength and product and feed size of that material you will be able to calculate what will be the energy requirement for getting that reduction ratio. So this law is based on stress analysis if plastic deformation within elastic limit and more accurate than Rittinger's law for coarse crushing which will be applicable for feed size greater than 50 millimeter.

So you have to remember this. Coming to the third one that is called Bond's law. You will see that Bond's law suggested that there will be certain energy requirement from the infinite size to a fixed size. So in that case the energy requirement will be inversely proportional to the size of the material or it is actually inversely proportional to the square root of the surface area formation. So how to actually assess this laws to find out the mathematical expression based on this Bond's law. So Bond's has suggested law intermediate between Rittinger's and Kick's law by putting here P is equal to minus 3 by 2 in that basic equation of dE by dL is equal to minus C into L to the power P from which by taking this P is equal to minus 3 by 2 and integration which will give you that E is equal to 2C into 1 by root over L2 minus 1 by root over L1.

$$E = 2C\left(\frac{1}{\sqrt{L_2}} - \frac{1}{\sqrt{L_1}}\right) = K_B\left(\frac{1}{\sqrt{L_2}} - \frac{1}{\sqrt{L_1}}\right)$$
$$= K_B\sqrt{\left(\frac{1}{L_2}\right)}\left(1 - \frac{1}{q^{1/2}}\right)$$

So that will be equal to KB into 1 by root over L2 minus root over L1. L2 is the product size characteristics length and L1 is the feed materials characteristics length and that will be is equal to what KB into root over 1 by L2 into 1 minus 1 by Q to the power 1 by 2. What is Q? So Q is basically that L1 by L2. Now this L1 by L2 that is Q is called reduction ratio. Writing this KB here as a Bond's law constant.

So how that KB will be related to that material characteristics. In this case one important terms to be noted down here the Bond terms as WI which is called as work index which is defined as what will be the amount of energy required to reduce unit mass of material from a large infinite particle size that means L1 is equal to infinity to a size of 100 micrometer that means 0.1 millimeter size. That means here Q will be is equal to infinity here.

So this property is called work index. So it is basically what is the energy required to reduce the size of infinite size to the fixed size of 100 micrometer. That means 0.1 millimeter. So as per this if you apply this energy requirement here as WI, here in this equation where E will be is equal to WI and KB is the constant. Here 1 by L2 is basically what that 100 micrometer that means 0.

1 millimeter if you substitute it here 0.1 millimeter into 1 minus 1 by Q. Q is basically the reduction ratio. Since it is reducing from infinite size to the finite size its reduction ratio will be is equal to infinity. So 1 by Q to the power half that means 1 by infinity to the power half is basically 0. So we are having that only E is equal to KB into 1 by L2 to the power half.

So that means WI will be is equal to KB divided by root over 0.1 which will be as KB is equal to 0.3162 WI. What is WI? So this is basically the energy required for that unit mass of material from an infinite size to the 100 micrometer size.

So that is the WI. So what is the relation between WI and KB? So KB will be is equal to 0.3162 WI. So after substitution of that this KB is equal to 0.3162 WI in this equation then we are having final form of equation as E is equal to 0.3612 WI into 1 by root over L2 minus 1 by root over L1.

$$E = 0.3612W_i\left(\frac{1}{\sqrt{L_2}} - \frac{1}{\sqrt{L_1}}\right) = 0.3612W_i\sqrt{\left(\frac{1}{L_2}\right)\left(1-\frac{1}{q^{1/2}}\right)}$$

So it will be is equal to 0.3612 WI into root over 1 by L2 into 1 minus 1 by Q to the power half. So this is another way you can express. So this is your actually bonds equation based on which you can easily calculate what will be the energy will be required for reducing a size from a fixed to another product size. So in this case one important property of the materials to be considered that is called work index. The work index will be different for different material because its ductility, strength will be different and for which you will see that energy required from infinite size to the 100 micrometer size it will be different for different material that is why the work index will be different.

So work index is a material property. So in this case you have to remember that what will be the work index for particular material and based on which what will be the energy required. Another important point you have to remember in this case the size of the material is to be taken as the size of the square hole through which 80 percent of the material will pass through a sheave or screen. So this is your main important point here to get that or to consider that particle size or average particle size or mean particle size of that material. Now here some work index value is given for different materials like for bauxite this work index value is 8.78 and for cement or raw material it will be as 10.

51 where if some rock it is 6.73 for limestone 12.74 for phosphate rock it is 9.92 even for clay it will be 6.30 some iron ore you can say it will be 12.84. So these are the some materials for which you can easily have this work index value based on which you can calculate what will be the energy will be required to reduce its size.

Then here one slide it is shown that some comparison of that specific energy requirement in kilowatt hour per unit ton of materials as per the particle size if we reduce the particle size from this feed size then you will see that Bond's law will give you the energy loss like this and Rittinger's law the specific energy profile and for Kick's law specific energy profile. For Kick's law you will see that it will be applicable if your particle size is beyond 10 to the power minus 3 meter and the specific energy requirement will be almost constant whereas Bond's law will give you the energy will be required less if your particle size will increase. Then you will see that your Rittinger's law as per that the specific energy requirement will be less for higher particle size there. Now where that Rittinger's law, Kick's law and Bond's law can be applicable one important notes here that you have to take care to apply this Rittinger's law, Kick's law or Bond's law. Now Rittinger's law it is applicable where surface area created is significant fine grinding specially for fine grinding.

Energy input is not very high in this case applicable to brittle materials undergoing fine milling and in this case this theory ignore deformation before fracture and it applies to sizes less than 100 micrometer. Whereas Kick's law it will be applied if the plastic deformation within the elastic limit and it is suitable for coarse grindings in which there is a relatively small increase in surface area per unit mass. For compression of large particles the Kick's theory is very useful and it generally applies if your particle size is greater than 10 millimeter and in case of Bond's law generally used for rough mill sizing and it is used for generally hard materials and the work index is useful for comparing efficiency of the milling operation and this Bond's law can be applied to calculate the energy requirement if the particle size will be within a range of 100 micrometer to 10 millimeter. Let us do some example here based on this laws. Where a material is crushed in a black jaw crusher in such that the average size of particle is reduced from 60 millimeter to 10 millimeter with the consumption of energy of 10.

0 kilowatt per kg second what would be the consumption of energy needed to crush the same material of average size of 75 millimeter to an average size of 20 millimeter. Now in this case you can assume the Rittinger's law you can assume also Kick's law. So both the laws you can apply here and let us see what will be the result coming out. So as per Rittinger's law if we use the equation of that Rittinger's that E is equal to Kr fc into 1 by L2 minus 1 by L1. Here E is given to you in the initial condition that to reduce the size from 60 millimeter to 10 millimeter the energy consumption is 10 kilowatt by kg per second.

So if you substitute here E as 10 and Kr and fc it is not known to you let it be like this Kr into fc here and substitute the value of L2 which is given as 10 millimeter that means 0.010 meter we can write and L1 is the feed size that is 60 millimeter you can write at a 0.060. So ultimately it will be coming after solving this equation it will come as Kr into fc that will be equal to 0.

12 kilowatt meter per kg. So this is your constant value now this constant value will be same for other material crushing also but that means same material but converting another size to into its lower size. So thus the energy required to crush again 75 millimeter of the same material to 20 millimeter will be then again if you substitute those values here in this equation here E will be equal to Kr fc here now it is 0.12 the same material so we can use that Kr fc which is obtained in the earlier equation earlier condition and then here 1 by L2 here in this case the size will be here 0.20 millimeter in terms of meter to be 0.

$$E = K_R f_c \left(\frac{1}{L_2} - \frac{1}{L_1} \right)$$

020 meter and then L1 will be 0.075 meter where feed size is given 75 millimeter. So after

calculation you will get to be as 4.40. So what will be the energy required to reduce the size of 75 millimeter to 20 millimeter we are having this 4.

40 kilowatt per kg. Now if we apply the Kick's law the Kick's law as per Kick's law the equation is E will be is equal to KK fc ln L1 by L2.

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$$E = K_K f_c \ln \frac{L_1}{L_2}$$

So here again K is given to you initial condition and from this initial condition you can find it out what will be the KK fc here it will be as 5.58 kilowatt per kg per second. Here then for the second condition the energy required to crush 75 millimeter material to the 20 millimeter then you can have again that E will be is equal to KK fc ln L1 by that same equation that you have to use here and then after substitution of this value of KK fc from this here you will get 5.

58 into ln now into L1 by L2, L1 is here 0.075 that means your feed size and the product size is 0.020 meter. So after calculation it will come as 7.38 kilojoule per kg. So as per Kick's law you are getting this value whereas the retention law the less energy is given as per calculation.

So it may be that you have to suitably use this law for that particular application. As per discussed in the previous slide that what will be the applicability of that particular laws for that which material what will be the size that you have to consider. Then another important example which will be calculated based on the Bond's law here. In this case the energy required per unit mass to grind bauxite particle of very large size to 150 micrometer is 258.

78 kilowatt hour per ton. Now in this case what will be the energy required to grind the particle from a very large size to 50 micrometer by using the Bond's law. So you can apply here Bond's law. What is the Bond's law equation here that is here E is equal to 0.3612 Wi to 1 by root over L2 minus 1 by root over L1.

$$E = 0.3612 W_i \left(\frac{1}{\sqrt{L_2}} - \frac{1}{\sqrt{L_1}} \right)$$

So if you substitute here the value of E is given to you, that it will be 258 it is given initial condition and then here L1 is that infinite size they told so it will be infinity and L2 it is given to you.

So ultimately it is coming what will be the work index value here. This is coming at 8.78 kilowatt per hour as per this initial condition. Later on it is told that you have to reduce the size to its 50 micrometer. Now again if you use that equation as E is equal to 0.3612 into here work index as already obtained by this equation the previous condition then it is 8.

78 and then 1 by root over L2 is what L2 product size is what 50 micrometer. So 50 into 10 to the power minus 6 this will be converted into meter and then 1 by root over infinity that means the feed size. So that will be here 1 by root over infinity means 0. So ultimately it is coming as after calculation 448.25 kilowatt hour per ton. So this equation will give you that what will be the power consumption based on the Bond's law as per this statement of example.

Another example it is given here like a material consisting originally of size 100 millimeter is crushed to an average size of 50 millimeter which requires 10 kilojoule per kg for this size reduction. Determine the work index of the material and the energy required to crush the materials from 10 millimeter to 5 millimeter. The same way it is initially what will be the as per Bond's law what is the reduction ratio it is 2. So E after substitution of this equation here you will get that work index value as 361.

73. So once that work index value again you substitute there for the second condition here feed size and product size and then reduction ratio that will be there too. So then after substitution and calculation you will get you will be is equal to 31.6227 kilojoule per kg. Now let us have an another example here like let it be 100 ton per hour of iron ore feed of which 80 percent passed through a mesh size of 2.54 millimeter where it is reduced in size such as that 80 percent of that crushed product passed through a mesh size of 1.

27 millimeter. The power consumption was 100 kilowatt if 100 ton per hour of the same of 5.08 millimeter of mesh size of 2.54 millimeter what would be the power consumption in this case you can use Bond's law. So what is that Bond's law here E is equal to k B into 1 by root over L2 minus 1 by root over L1.

So here E1 as per first condition E1 is equal to k B into 1 by root over 1.27 it is given to you minus 1 by root over 2.54 that is L1 it is given to you then finally we can say that k B by root over 1.27 into 1 minus 1 by root over 2. As per second condition you will see that E1 is equal to k B into again here L2 is given to you 2.

54 and L1 is given 5.08 and you will see that finally it will be coming as k B by root over 2.54 into 1 minus 1 by root over 2. Now this equation 1 and 2 if you divide you will see that E2 by E1 will be equal to 1 by root over 2. So what will be the E2 value? So 1 by root over 2 into E1 then you will see that 1 by root over 2 into E1 E1 value is given to you 100 kilowatt so into 100 so it will be coming as 70.

71 ton per hour. So this type of example you can practice and also you can understand and how to calculate the energy required for crushing a material from a feed size to the final product size and what will be the different way that you can calculate that energy based on

different laws but you have to apply as per applicability of the laws there. So I think I understood that three different basic laws based on which how to calculate the energy requirement for reducing the size. In the next lecture we will try to discuss about more of that solid fluid operation with the new module it will be as size enlargement process, okay that module will be started from the next lecture onward. So what will be the different techniques to increase the size of the material that is also another aspect to apply this enlarge size in different processes so that will be discussed. Thank you for your attention. Have a nice day!