Lec 3: Particle size & Its distribution in mixture

Hello everybody, welcome to this massive open online course on solid fluid operations. So, in the previous lecture we have discussed about the characteristics of a single particle. In this lecture we will try to describe and understand what is the particle size and its mean in a mixer and also what will be the distribution of that particle size in the mixer. So, as we know that in a powder of sample or any sample of particles where different sizes particles will be mixed and in that case we will see that the particles will be mixed in such a way that there will be a uniform mixing and there will be non-uniform mixing. In a uniform mixing you will see that all the particles will be mixed in a regular fashion whereas the non-regular mixing or you can say that heterogeneous mixing in that case the particles concentration either in terms of different sizes or the same sizes will not be the same in all the position in a mixer. Now in that case you will see that whenever we are getting the different size of the particles in a mixer what should be the size distribution of that particles and what will be the mean of that particles that we need to calculate and also we need to know how that mixing happens based on this particular particle size.

So, in this lecture we will try to understand that what should be the particle size distribution, how that particle size distribution can be explained or can be represented and also mean particle size in the mixer. This is very important to know that mean particle size in the mixer because for assessing any process yield or process performance you need to calculate what will be the particle size specially for the adsorption process, reaction engineering, reaction processes based on that catalyst particles and what will be the mean size of that catalyst particles and also what will be the distribution of that particle in the mixer which is used for that reaction or other adsorption processes. So, here then how actually that distribution can be expressed. First of all you have to know what will be the particle size distribution.

This is generally a population of particles in a particle mixer which is described by a particle size distribution and it is necessary to know the mean size of that particles in the mixer where that particle size distribution exists and also the spread of the sizes and also you will see that particle size distribution can be expressed as a frequency distribution curves or cumulative curves. That means you will see that if we have the mixture of different sizes particles within a certain range of that size let it be 1 micrometer to 100 micrometer. Now in that case you will see that if we segregate this ranges into different classes you may make this classes like 1 to 10 micrometer, 10 to 20 micrometer, 20 to 30, 30 to 40, 40 to 50 or 50 to 60 like that there are 10 classes will be there. So, in each classes you will see that there will be certain number of particles within that ranges of classes you will see that there will be certain number of particles, like 1 to 10 micrometer there will be a n1 number of number of particles. Similarly, 10 to 20 micrometer range there will be n2 number of particles.

Similarly, 20 to 30 micrometer range there will be n3 number of particles. So, similarly we can say that for all classes there will be a different number of particles. Total number of particles will be there summation of all numbers in different classes. So, in the mixture total number of classes we can have. Now, if we know that the particular number in a particular class and if we represent that within a certain classes that is within a certain range of sizes how many number of particles.

Similarly, another classes how many number of particles and similarly other classes how many number of particles in this way if we represent that particles number or particle frequency in y-axis as shown in the picture in the diagram and in the x-axis it is the particle diameter as dp. So, you will see that the number of particles will be changing with respect to the particle size, okay? Particle size class you can say that within a suppose 1 to 10 micrometer there is a class of ranges 1 to 10 micrometer and there what will be the average let it be 5 micrometer. So, 5 micrometer of that particles will have suppose n1 number of particles. So, if we represent these in a graphical form then we are having like this type of form this type of distribution, okay? Now, there are different types of distribution you may obtain. You will see that some will be that suppose in a mixer if you are having almost all the particles will be the uniform in size then you will see that the narrow size distribution will be there.

Whereas if there is a wide size distribution there will be 1 to 10, 1 to 100 micrometer in range there will be wide range of particles. So, there will be a spread of that distribution will be more and if you are having that there will be different shape particles with different type of particles mixing in a mixer which is obtained after crushing immediately crushing then you will see that there will be a different peaks of that distribution, okay? So, in this case we can represent that number versus diameter in a graphical form. So, that is actually called as that size distribution graph and in that size distribution graph we can have that size distribution that means how many numbers with respect to particle diameter. Now, sometimes it will you will see that that particle number it can be represented as a particle frequency how many numbers in a particular class, okay? In a particular range of that particle size and also it can be represented by the relative frequency that means in a particular class how many numbers out of that total numbers if I have that fraction it will be regarded as or it will be called as relative frequency. So, you can represent that distribution as a relative frequency with respect to the particle diameter, okay? So, here you can see that in this diagram it is shown that that particle relative frequency versus particle diameter and you are having different type of distribution some will be the narrow like this one, okay? And here it does mean that there will be a range will be very narrow that means almost most of the you can say that particles will be in the same or equal in size.

Whereas if you are having that widespread distribution that means here the all particles are not will be uniform in size whereas it will be the particle size will be different even in wide size range. So, in this way you can represent that population of particles in a particle mixture by a graphical form and the change of that particle frequency, particle number frequency you can say with respect to diameter that will be called as distribution. Now, this distribution maybe that frequency distribution or cumulative distribution. What is that cumulative distribution? That means here in each classes whatever number you are getting if you keep on adding for that next classes and if you represent it in a cumulative curve, we will show later on also here. So, in this case the proportion of particles either of mass fraction or volume fraction or number fraction smaller than a certain size dp is plotted against that size dp.

Now, here we can represent not only by that number frequency we can represent that in terms of volume fraction of that particles also we can represent it in mass fraction of that particle in a mixture. We can take a mass of that particles for that particular size range, another size range what will be the mass, another size range what will be the mass. Similarly, for different size ranges we can get the different masses. So, in that way we can represent in a graphical form. So, that will be as mass fraction in the y-axis and particle size will be in the x-axis.

So, this representation or this type of distribution will be called as volume fraction distribution or mass fraction distribution and if you are representing by number it will be called as number fraction distribution or frequency distribution. Now, coming to this point again here for naturally occurring materials like any material that is occurred naturally you can say suppose sodium hydroxide or that we can say that aluminum oxide, calcium oxide, calcium carbonate even some ores naturally that is obtained. So, for naturally occurring materials the curve will generally have a single peak. This you have to remember whereas for mixtures of particles there may be as many peaks as components in the mixture. For different components different sizes will be there for which you can get the many peaks.

If the particles are formed by crushing larger particles you are going to crush that means you are going to break that particles which are naturally available and if you crush it you can get it finer particles by crushing and in that case the curve may have more than one peaks, okay? Like here it is shown that you will see that the monodisperse particles, polydisperse particles and after crushing. Now, after crushing you will see that a sample of coal is obtained here. You will see that this coal will have that size in a certain range that its distribution will have more than one peaks like this. Here one peak and here another peak. And also you will see that if you are having that monodisperse particles means here in the sample all the particles are almost equal in size then you can have this type of distribution that means monodisperse particle size distribution.

And again if you are having the sample of different sizes here see larger one and smaller one in this way if you are having that mixture of the particles with different sizes then you will have this wide size distribution, okay? This wide size distribution may be having that more than one number of peaks, okay? So this can be represented. So what is that particle size distribution? I think you can understand now. Here you will see that the cumulative distribution we have represented here as a dotted line. This is basically integral of the frequency distribution. The whatever frequency distribution that is number distribution you are getting or relative frequency you are getting if you are keeping add on to the next class then you will get the cumulative way of that number.

In the first class you will get some number. In the second class then you will get the number. Now in the second class of that particle size you have to represent adding with that number 1, okay? Number 1 class. So in that case keeping on that previous classes numbers will give you the cumulative distribution. Now if the cumulative distribution is denoted as suppose capital F then the frequency distribution will be df by dd.

Dd means here, d means here particle diameter and df by dd is often written as small fd, okay? Here represented in the y axis and the distribution can be either by number or by surface of that particle or by mass or by volume. So either way you can represent. But remember that mass and volume distribution will be almost same because here volume can be converted from the mass itself or mass can be converted from the volume itself just by by multiplying the factor of that density factor, okay? So mass distribution is the same as the volume distribution where particle density does not vary with the size. So the distribution by mass, number and surface can differ significantly where particle density does not vary with size, okay? So this you have to remember that mass, number and surface all the distribution will be different. You will see that here in this figure it is shown that there are different distribution based on mass, number and surface.

You will see that here this line, this line is basically what is that? Number distribution, okay? And this line, this line is basically a surface distribution. What is the surface? Surface, how can you get the surface? If you are having the number you can easily calculate what will be the surface. If that if you are considering that particle of spherical in diameter then you can easily calculate what would be the surface. The surface will be is equal to what? Pi Dp square, okay? Or 4 pi r square it is called. So you can get that surface area.

So if there are n number of particles then n into surface area of each particle. So you can get that total surface area in that particular classes. Similarly, you can express the distribution in terms of volume. So you can calculate in terms of volume. If the number of particles and if you know the volume of each particles then what will be the total volume in that particular classes you can easily get just by calculating this way 1 by 6 pi D cube that will be your volume, okay? that what will be the diameter of that particle in that particular size range, okay? And what will be the number of particles if you multiply it by that n then you will get the total volume of that particle.

So we can represent that particle size distribution either by number or by surface or by volume or by mass, okay? And this all those cases you can represent it as a cumulative distribution also. Now let us consider here how can you convert this one distribution to the another one? Like you have to have the surface distribution, okay? From the number distribution or volume distribution from the number distribution. How to get that surface distribution? If we represent that surface function as fs, okay? So surface distribution will be this is as a function of Dp, okay? That will be is equal to what? Pi capital N, N is the number of particles and s is the surface of each particle and into D Dp square into f N Dp. So this is your that surface distribution or surface function, okay? As in terms of particle diameter and it will be as that pi by s Dp square into N into f N Dp. f N Dp is the number distribution.

Here the equation(s)

$$f_S(d_p) = \frac{\pi N}{S} d_p^2 f_N(d_p)$$

$$f_V(d_p) = \frac{(\pi/6)N}{V} d_p^3 f_N(d_p)$$

$$f_m(d_p) = \frac{(\pi/6)\rho_p N}{M(=V\rho_p)} d_p^3 f_N(d_p)$$

Similarly you can get the f that means volume distribution you can say that f B Dp that will be is equal to pi by 6 into N divided by V that means volume of that particle into Dp cube into f N into Dp, okay? So this f N Dp is the number functions and if you multiply it with that pi by 6 N by V into Dp cube then you will get that volume distribution or size distribution in terms of volume. Similarly you can represent that size distribution in terms of mass. So it can be represented f N Dp is equal to then how you can represent it the pi by 6 just you have to multiply it by density of the particle rho p into N divided by capital M mass of that particle that means is equal to V into rho p volume of the particle into density of the particle into Dp here it will be cube into number function, okay? So in this way you can have the conversion of one distribution to the another. So you can have the surface distribution, volume distribution and mass distribution in terms of number distribution in this way. So in this case you have to remember that here N is the total number of particles in the population and S is the total surface area of the population of the particles and V is the total volume of the population of particles and M is the total mass and rho p is the density of the particle.

So all the surface distribution, volume distribution or mass distribution you can obtain if you know the number distribution. So from that number distribution you can easily calculate what will be the surface or volume distribution. Then now suppose you are having that a mixture of that particles within a certain range of size like 1 to 100 micrometer or you can say that 75 micrometer to suppose 4.75 millimeter within a certain range. So in that case you can segregate that mixture of that particles into a smaller smaller sample like this here that can be done by a sheave or screen it is called screen you can see in the picture the how look that actually this is screen or sheave I think you can use or you use in your home also this type of screen for your some domestic use so for separation of that particles.

So in that case you will see that if I use that different meshes screen here you will see that different sheave numbers will give you the different openings of that sheave according to that ASTM 11 test sheave we can have different sheave numbers like 4, 10, 20, 40, 60, 100, 140, 200 like this. These are the standard sheave number and this sheave number will give you the respective openings that means what will be the size of that openings in the sheave. So for sheave number 4 it will be 4.75 millimeter so if you are having the particle of size greater than 4.75 millimeter it will be on the above on this screen whereas smaller than this opening all the particles will be going downward and it will be again separated by another sheave number which will be again giving that very smaller number of that you know opening.

So accordingly you can segregate that particle mixture into a different sample of different sizes. Here we are having just different sizes of that sample here. So here sheave number 200 it will give you the smallest sizes of this here it may be 0.075 millimeter. So you are having 75 micrometer and another one 140 105 micrometer similarly 100 sheave number it will be giving you 150 micrometer like this.

So in this way you are having different sizes particles of different samples. So particle of density suppose particular density will be there any material may be there here may be considering that 2500 kg per meter cube can be segregated based on the size by sheave or screen as shown in the table here. Now if we segregate this sample by that sheave we are getting that as per that opening of that sheave we are getting different you can say that sizes of that particles. So here the opening diameter it is given for sheave number respective sheave number in this table. Here it is the opening diameter and this is the sheave number and then you are having that sample weight here maybe you will see that this one is one sample this is one another sample this is another sample this is another sample.

So from this sample you are having that different weight that weight here it is suppose maybe in milligram in a very small amount of sample you have taken and then we are having different weight of that sample with respect to different opening diameter of that mesh. So according to that you can say that that will be your particle size and sample volume accordingly you can calculate the volume of that your sample for this sample what will be the volume. So if that weight of that sample what will be the volume very simple you can divide it by density then you will get that volume. Then particle number how can you get that particle number? Particle number if the opening diameter that will be your particle size diameter then you can calculate what will be the volume of each particle for that particular size. So you can get suppose here this is the sample of that size will be 4.

75 millimeter. So for this sizes for this sample you are having the volume of each particle as 1 by 6 into pi into dp cube that means 4.75 cube. So in this way you can convert it to volume for each particle. So once that volume of each particle and already you have calculated what will be the volume of that particle in the mixture. So if you divide that volume of that particle of that sample by individual particle volume then you will get what will be the number of particles there in the sample.

Similarly for other sample you can easily calculate what will be the number of particles based on that size of 2 millimeter and third one you can get the number of particles based on that 0.85 millimeter. Again similarly, for other cases also you can get that number of particles. So if you are having different size ranges of that particles segregating by the screen as per their opening size then you can easily calculate respective number of particles respective volume of that samples respective mass of that sample. Once that particle number then you can easily calculate what will be the surface for that particular sample of that particle.

So in that case also you have to calculate what will be the surface area of each particle. If that surface area of each particle it will be simply pi dp square. So if you multiply it by n number of particles here you will get that surface area total surface area in that particular sample. So accordingly for all classes of the materials which is segregated by the sheets you can easily calculate what will be the surface area for each size. Then you can calculate the number fraction for each classes or each sizes what will be the number then divided by total number of the particles.

Here you see that total number just summing up those all numbers you can have this total number. Also what will be the total surface area what will be the total volume what will be the total mass here. So accordingly number fraction you can calculate surface fraction you can calculate volume fraction also you can calculate. Now represent all those classes based on that diameter in the plot then you will see that in the y axis we can put it as Fn that means number fraction and Fm mass fraction Fs the surface fraction in percentage. Then with respect to diameter of that particle you can have this graph like this.

This red one you will see that this red one it will be what is that simple Fn that means number fraction distribution. Similarly this one will be this is surface area distribution. Similarly this one will be your mass or volume fraction distribution with respect to particle size. So I think you understood this one so by a crusher if you have big sizes particles you can convert it into finer size particles and you will have that some samples of that finer size particles just by crushing by this crusher here once picture of that crusher is shown here by this crusher you can convert this bigger size particles into finer size particles and if you take the samples and then segregate by screen then you will get different samples with different diameter based on the opening of the mesh or screen. And accordingly you can calculate number, number fraction, mass, mass fraction, volume fraction and then you can represent it in a graph.

So this graph will give you that what will be the number distribution, surface distribution and volume distribution or mass distribution. I think you understood this one. Next coming to the point here once you get that number distribution or size distribution or mass distribution you have to calculate what will be the mean of that particles. You will see that wide size ranges particles in the mixer. Now to assess any process you need to calculate what will be the mean size of that particles.

There are several way you can represent that mean size of that particles. I think in the previous lecture we have represent a way of finding out the mean of that particles either based on that surface, either based on the volume, either based on that that settling velocity or like that. So that is equivalent particle diameter. So equivalent particle diameter if there are more than one equivalent particle diameter you have to get a mean. Now that mean may be different way again that surface mean, volume mean in that way we can represent.

So we will come to that point here. So in most practical applications we require to describe the particle size of a population of particles okay that means millions of them will be there in the sample by a single number. Now there are many options available for this, the mode, the maiden. Generally these you know do not have special significance and then several different means including arithmetic mean, geometric mean, quadratic mean, harmonic mean, surface mean, volume mean like this. These are the different mean. Here in this slide this in the table it is shown that mean and notation respective notations are there and also in the distribution there mode, harmonic mean, geometric mean, arithmetic mean, quadratic mean, cubic mean these are shown.

So this mode maiden of course it is not that much significant so we will be representing that mean in different way and then how to then actually express that mean sizes of that particles in a volume or mass okay. Let us see that volume mean diameter dV or the mass mean diameter maybe dm that means here based on the volume okay you can represent what will be the mean of that particles diameter in a sample. In this case the mean abscissa in the figure is defined as the volume mean diameter dV here which is sometimes called as a mass mean diameter okay. So volume mean diameter and mass mean diameter both will be same. So how is it defined actually? We told that there will be a volume mean diameter.

Here the equation(s)

$$d_v = \frac{\int_{d_{p,i}}^{d_{p,j}} f(d_p)(d_p)\,dd_p}{\int_{d_{p,i}}^{d_{p,j}} f(d_p)\,dd_p}$$

$$d_v = \frac{\Sigma d_{p,i} x_i}{\Sigma x_i} = \sum x_i d_{p,i} \quad x_i = n_i(\pi/6) d_{p,i}^3 \rho_p$$

$$d_v = \frac{\rho_p(\pi/6)\Sigma(n_i d_{p,i}^4)}{\rho_p(\pi/6)\Sigma(n_i d_{p,i}^3)} = \frac{\Sigma(n_i d_{p,i}^4)}{\Sigma(n_i d_{p,i}^3)}$$

What does it mean? Volume mean diameter it is explained here this is the mathematical expression of that volume mean diameter that means if you are having that distribution of the size and if you multiply with that particle size you will see that there will be a certain range there will be that what will be the size distribution into volume diameter and divided by total number of that particles there. So here suppose number distribution here particular class you are having that dF by dV then into d you will get that number of particles divided by the total number of particles then you will get that mean. So this is one type of mean okay or you can say if you are having that mass fraction of a sample based on that particular size of that particle in a particular class then you can have this summation of dpi into xi divided by summation of xi. So dV is defined like this okay volume mean diameter it is defined as like this summation of dpi xi by summation of xi. What is that dpi? dpi means if you are considering that ith class we have shown in the previous slide that there will be some class like here you know the sample 1, sample 2, sample 3, sample 4 these are class you can consider.

So in this class it will have certain size of that particle that will be represented by dpi and what will be the mass fraction of that classes this will be xi. So for a particular class if that particle diameter and mass of that sample then you can have that total summation you can say that dpi into xi divided by total mass fraction that will be your volume mean diameter or you can represent it by summation of what is that xi into dpi. Here summation of xi is equal to 1 all the fractions addition of that fraction will be equal to 1. So what is that xi? xi this is simple if you are having the n number of particles in that ith class and if you are having that volume of that particles then ni into volume then into density you will get that mass fraction. So here dV will be then accordingly defined or mathematically expressed by this formula.

In other way also you can represent that volume in diameter here this will be summation of ni dpi to the power 4 by summation of ni dpi cube. So in this way also you can represent. In this case you need to know that number of particles in these classes as well as the diameter of the particle. Then let us have an example for this. Let us consider that size analysis was carried out by a series of sheep the data for mass fraction xi and the particle diameter dpi based on screen opening diameter dpi of the fraction is given as in the table.

What is the mass mean diameter that you have to calculate? So in this case we know that xi respective to that dpi. So xi 0.30 for a dpi value 0.

43 xi is equal to 0.40 for the diameter of particles 0.85 and xi will be 0.20 for the particle diameter of 2.0 millimeter and xi will be is equal to 0.

10 of the particle size 4.75 millimeter. So for this different sizes particles if it has different mass fractions for that respective diameter you have to calculate what will be the mass mean diameter or volume mean diameter. So we know that volume mean or mass mean diameter as we can write here mass mean diameter which is defined as mass mean diameter will be is equal to what summation of dpi xi divided by summation of xi. So it will be coming as simply summation of xi dpi. So we know that xi and dpi for this class so finally

we can get here simply we can say that based on this so we can write here from the data from the table.

So 0.3 into 0.43 that means xi into dpi plus 0.40 into 0.85 then for another class 0.20 into 2.0 plus 0.10 into 4.75 divided by total mass fraction summation of that this is xi, xi will be what? xi will be 0.

30 plus 0.40 plus 0.20 plus 0.10. So it is basically summation will be equal to 1. So finally we can get 1.

344 divided by 1 that means 1.344. So this is your mass mean diameter. So mass mean diameter will be is equal to 1.344. So from this you know mass fraction with respect to particle diameter we can calculate what will be the mass mean diameter. Another example it is given like that the diameter of sand particle in a sample range from 100 to 200 microns. The number of particles of diameter x in the sample is proportional to like this n proportional to 1 by 100 plus x.

Here the equation

$$n \propto \frac{1}{100+x}$$

mass fraction. So what is the volume or mass mean diameter of the particle? So here number of particles will be inversely proportional to the size of the particles and that function is like n is equal to some factor suppose k divided by 100 plus x. So in this case how can you then calculate what will be the volume or mass mean diameter. So in this case as per the definition of this volume in dV as per given earlier integration of what is that from the size range of initial to final we can say that it would be here dP into function of what is that dP into d dP divided by integration of f of dP this is as per definition dP. We have I think shown earlier here this equation. So as per this we can write here this f dP a function of dP into f dP and then divided by integration of f dP dP.

Now here what will be the range? Range is given what is that I think a particle 100 to 200 micrometers. So here it will be 100 here it will be 200 again here 100 here 200 micrometer. So within this size range you have to calculate. So if you substitute this function here so we can write here dP means here x and function is here simply 1 by 100 plus x to d x. X integration 100 to 200 divided by integration 100 to 200 here again what will be the function simply 1 by 100 plus x then d x.

So what will be the value ultimate after doing integration we can have this value after integration simply 245.63 after integration. So this will be your volume or mass mean diameter. Next coming to the mean sizes based on surface the surface mean diameter or volume surface mean diameter it is called surface mean diameter or simply called volume surface mean diameter.

Here the equation(s)

$$d_s = \frac{\Sigma\left[(n_i d_{p,i}) S_i\right]}{\Sigma(n_i S_i)} = \frac{\Sigma(n_i d_{p,i}^3)}{\Sigma(n_i d_{p,i}^2)}$$

$$d_s = \frac{\Sigma x_i}{\Sigma\left(\frac{x_i}{d_{p,i}}\right)} = \frac{1}{\Sigma\left(\frac{x_i}{d_{p,i}}\right)}$$

It has another norm it is called sauter mean diameter. So this sauter mean diameter or surface mean diameter represented by dS it is also represented by d32. It is also denoted by d32 that means surface mean diameter or that sauter mean diameter. How is it defined here simply summation of ni dpi into si divided by summation of ni si. What is that si is the surface area of that ith class particles or you can write it as ni into dpi cube that means what will be the volume of that particles divided by total surface area of the particles in that particular class.

So you have to know that number of particles in that particle classes. So here from this equation you can simply calculate what will be the sauter mean diameter or surface mean diameter or you can express it in terms of mass fraction. So that in terms of mass fraction it will be summation of xi divided by summation of xii by dpi or you can say 1 by summation of xi by dpi. So this definition you have to remember and d32 I told that here it is called sauter mean diameter. Why it is called d32? You will see that in this definition this it is coming 3 here it is coming 2 here.

So as per that 3 and 2 it will be called as d32. If it is suppose d10 then it will be d here it will be 0 like this. So in that way you can represent as the sauter mean diameter as per d32. Now let us do an example for this type of mean. Now in this case again the size analysis was carried out by a series of sheep the data for that mass fraction xi and the particle diameter dpi based on screen opening diameter dpi of the fraction is given in the table.

Now what will be the then volume surface mean diameter or sauter mean diameter. The same data has been given as per earlier example. So in this case that volume surface mean volume surface mean which is defined as since it is given in terms of mass fraction then we can write the definition as what is that summation of xi by dpi. This is the definition of volume surface mean diameter that means ds. So if we substitute the value here what will happen 1 by summation of xi dpi.

xi means what here for the first class that means here 0.

30 by 0.43 plus then again 0.40 by 0.85 plus 0.20 by here 2.0 then plus 0.10 by 4.75. So after simplification it will come as 0.776. So what will be the volume surface mean diameter it is coming as 0.

776. Whereas we got that volume mean diameter as 245.63 micrometer as per this example but earlier one, 1.344 millimeter. Here we are having this volume surface mean as 0.776 here millimeter. Then we can mathematically express the size distribution whatever size distribution that we are talking about that based on number, based on mass, based on volume or based on surface we can represent that particle size distribution by a some distribution function.

That distribution function can predict that experimental data of the distribution which is obtained from the sample collection after crushing. So what type of distribution that can be used to predict that size distribution of that particle in a particular mixture. Generally two types of distributions are you know to be known and it is important for you there are several distributions available but only these two distribution you have to remember. Generally these two distributions are being used to express that size distribution of the particle.

Here the equation(s)

$$f(d_p) = \frac{1}{\sigma\sqrt{2\pi}} \exp exp \left[- \frac{(d_{pi} - \bar{d}_{pi})^2}{2\sigma^2} \right]$$

$$f(d_p) = \frac{1}{\sigma_z\sqrt{2\pi}} \exp exp \left[- \frac{(z - \bar{z})^2}{2\sigma_z^2} \right]$$

$$\sigma^2 = \frac{1}{N}\sum_i \left(d_{pi} - \bar{d}_{pi}\right)^2$$

$$z = \log log\ (d_{pi})$$

$$\bar{d}_{pi} = \frac{1}{N}\sum_i d_{pi}$$

What are those? One is arithmetic normal distribution another is log normal distribution. What is that arithmetic normal distribution? I think in mathematics you have already learned different distributions. So here arithmetic normal distribution is defined as f dp that will be is equal to 1 by sigma root over 2 pi into exponent of into exponent of dpi minus dp bar this is whole square divided by 2 sigma square. This is your arithmetic normal distribution. What is that sigma and what is this dpi, what is dp bar? So this sigma is called that variance of that distribution.

It basically signifies how much spreading of that distribution will be there. Based on that wide size distribution you will see that spreading of the distribution will be there. So this sigma that means called variance will decide whether the distribution will be wide size range or not. So dpi is the particle diameter for a particular class I and dp bar is the mean diameter for that particular classes of particles. And similarly you can represent that log normal distribution in this way f dp that will be is equal to what? 1 by sigma z root over 2 pi into exponent of here it will be minus z minus z bar whole square divided by 2 sigma z square where we can write z is equal to log or ln of dpi and dp bar that means mean diameter of the particles. If number of particles in the particular class is n then you have to sum up all diameters of this particles in that particular sample and then divide it by n then you will get that mean diameter and sigma that means variance can be calculated as per definition in summation of dpi minus dp bar whole square.

So this is your sigma that is variance, sigma is called variance and dp bar is called what is that mean. So there are two distribution based on which you can express what will be the size distribution either in terms of mass, in terms of volume or in terms of surface that you can express by this distribution function. Only thing is that for that distribution you have to first calculate what will be the mean of that diameter and also variance of that distribution. Once that diameter and mean then you can easily calculate all of the variance.

So these two log normal one is called log normal distribution where here it is defined as like this. So z here 1 by sigma z root over 2 pi into exponent of minus z minus z bar whole square by 2 sigma z square. Here sigma z again will be in terms of what is that z, sigma z it will be in terms of z. Here sigma only in terms of dpi, similarly sigma z square it will be 1 by n summation of here zi minus zi bar whole square. Where zi will be defined as logarithm of particle diameter of that particular class. So I think you understood that particle size distribution and what will be the mean of that particle size in a particular sample of that particle size.

So from this lecture, you can easily then able to calculate what will be the mean of that particle size in a particular sample and also its size distribution. And the method of particle size measurement you can measure that particle size in that particular sample either by sieving or microscope or sedimentation method, permeatory method, electro zone sensing or laser diffraction. So all those methods will not be discussed in this course because it is beyond your syllabus of this UG standard. So you can have this method of that measurement of that particle size from other reference books for your further understanding.

So thank you. In the next class, we will try to understand the next module which will be regarding that particle size reduction and what will be the mechanism of particle size reduction, what will be the energy required, how to calculate that energy and what will be the efficiency of that size reducing equipment based on the surface area creation. So it will be discussed from the next lecture onward. So thank you. Have a nice day.