Renewable Energy Engineering: Solar, Wind and Biomass Energy Systems Prof. Vaibhav V. Goud and Dr. R. Anandalakshmi Department of Mechanical Engineering Indian Institute of Technology – Guwahati

Lecture - 36 Practice Problems

Hello everyone and welcome to this lecture of wind energy. So, in this lecture, we will practice few examples on the concept covered in the wind energy. If you recall our discussion in one of the lecture of the wind energy, we describe the procedure for calculating the performance of the wind machine. So, in this context, we describe few terms in the form of power coefficient, lift coefficient, drag coefficient, then tip speed ratio as well as the solidity.

So, all these terms are essential and very much important in calculating the performance of the wind machine. So, in this lecture, we will just revise or recap few of the concepts which are covered in the wind energy.

(Refer Slide Time: 01:08)



So, wind energy if you see in case of the wind energy, the power coefficient is considered as the ratio of power extracted by the rotor to the power which is available in the free-steam wind speed. So, if you look at the equation here, here the C P is represented in the form of P suffix e that is the power which is extracted by rotor to the power which is available in the free-stream wind speed, this P e represents the power which is extracted by a rotor. So, sometimes in the literature, you may observe that this P e is also represented as P output. So, both are the same terms here, either you can represent it in the form of P suffix e or P output it represents the power which is extracted by rotor. And A is the swept frontal area of the machine that is nothing but the face area of the rotor and V infinity is the free-stream wind speed and here also if you see, it is represented as either U or V infinity.

So, in some of the literature, it is represented as capital U and some literature, it is represented as V suffix infinity. So, V suffix infinity indicates the free-stream wind speed and both this particular nomenclature carries the same meaning. So, you can use either of these representations while solving these examples. So, apart from the power coefficient, so, another important term which is used in the calculation of the performance of the wind machine is the lift coefficient.

And lift coefficient, it is represented as C suffix capital L and lift coefficient of the blade of a wind rotor is the ratio of lift force to the force of the free-stream wind speed, which is represented here in the denominator of this particular equation, where F L is the lift force on the blade. Whereas, A b represent here the projected area of the blade which is facing the wind.

So, now, if you see, there are 2 terms coming in this equation, that is lift force on the blade as well as A b that is the projected area of blade which is facing the wind. So, apart from the lift coefficient another important term which is required in calculating the performance of the wind machine is the drag coefficient.

(Refer Slide Time: 03:32)



And drag coefficient of a blade of the wind rotor is the ratio of drag force that is represented here in the form of F suffix capital D. And in the denominator, it represents the force of the free-stream wind speed again and the drag coefficient, it is represented as C suffix D. In this case, this F D is the drag force on the blade. And again A b here is the projected area of the blade which is facing the wind.

Here, the lift coefficient and the drag coefficient, if you just see these 2 equations, there is only a small change in that here in the numerator, it is F D that is a drag force on the blade and in the lift coefficient, the F L indicates the lift force on the blade of the rotor. So, these 2 equations that are the lift coefficient and the drag coefficient, these are important in calculating the performance of the wind machine.

Apart from that the tip speed ratio which is represented as lambda here is the ratio of tip speed of the blade by wind speed that is again the free-stream wind speed which is represented as V infinity. And the tip speed of the blade is calculated by omega into R. So, omega here is the angular velocity of the blade and R present here the tip radius. And this particular equation, it holds good for the horizontal axis machine.

And Apart from the tip speed ratio, so, another important parameter in the performance calculation is solidity of the blade.

(Refer Slide Time: 05:06)



So, solidity, it is the ratio of blade area to the swept frontal area of the machine and the blade area is calculated, as the A indicates here the area of the blade. And if the rotor has N number of blades, that is N maybe 2, 3, 5, 6, 10 or even 20. So, those many number of blades need to be multiplied here to get the area of the blades. And in this case also, this definition of solidity it holds good for the horizontal axis machine, where N is the number of blades.

And A is the area of the blade. Here it is the blade length into the chord length and with the help of these 2 parameter, we can easily calculate the area of the blade as well as this R is the radius of the blade of the particular rotor. So, accordingly we can calculate the solidity of the rotor as well. So, with the help of this specific term, we can calculate the performance of the wind machine easily.

So, now, after understanding the calculation part or you can say the different terms which are involved in the calculation of the performance of the wind machine.

(Refer Slide Time: 06:24)



The maximum C P that is the power coefficient can also be calculated from the following equation. So, here, if you see the equation, it is represented as C P is equal to 16 by 27 into exponential of minus 0.3538 into lambda rise to power minus 1.2946. So, this lambda here is the tip speed ratio and once we know this tip speed ratio value, we can calculate the C P that is a power coefficient value very easily.

So, the variation of this particular C P that is a power coefficient with lambda can also be tabulated accordingly by considering the range of the lambda values as well. So, you can practice this on your own just consider the lambda in the specific range. For example, say from 1 to 10 or from 1 to 15 and just find out the corresponding C P value and tabulate this particular data and obtain C P value can be plotted in the form of C P versus lambda to find out the nature of the curve.

So, you may observe that the C P value is increasing with the increasing the lambda and after a certain interval of data, you may find that the C P is almost constant and there is no much change in the value of the C P. And this equation is the empirical equation and which is widely used and it is yield an upper bound value on the C P which is regardless of the type of the wind machine.

The above equation has been modified for the propeller and the multi-blade type rotors then in that case, it needs to be modified by taking the effect of drag into the account. So, just in the above equation, we need to do small modification here that the remaining terms are more or less same. In this equation, except the addition of one particular term here which is epsilon. And it is the ratio of drag coefficient to the lift coefficient and is valid only for lambda greater than 1.

And this epsilon here, it takes into account the effect of drag. And epsilon for the propeller rotors having the blade with aerofoil cross section is in the range of 0.008 to 0.02. Whereas, epsilon for the multi-blade rotors, having curved blades, it is in the range of 0.05 to 0.1. So, after understanding this concept of the wind machine as well as the performance calculation of the wind machine, let us practice few examples on the similar line on the concept of the wind machine.

(Refer Slide Time: 09:11)



So, now, let us consider an example in which a propeller type wind machine is used, which has a rotor diameter of 70 metre and is operating at a location having wind speed of 30 kilometre per hour and rotating at 18 RPM. So, the rotational speed of the rotor is given as 18 RPM and the rotor diameter as I mentioned, it is 70 metre. So, now with the help of this data, calculate the power which machine can extract from the wind.

And in this case, there are 2 parts. In the first part, only the wake rotation is considered for calculating the power which is extracted by machine from the wind. And in the second case, the wake rotation and the effects of drag are considered and in case of part 2 also, the epsilon value is given as 0.011. So, this value we need to use in the part 2 and the density of the air is given as 1.2 kg per metre cube.

So, with the help of this given data, we need to calculate the power which is extracted by a machine from the wind by using only the wake rotation first. And then in the second part, considering both wake rotation as well as the effect of drag into consideration, we need to calculate the power which is extracted by a machine from the wind. So, now, as we know in the example, the rotating speed of the rotor is given as 18 RPM.

So, with the help of this 18 RPM, we can easily calculate the angular velocity here. So, the angular velocity can be calculated with the use of the following equation here because omega here is 18 RPM. So, simply by this conversion factor here, we can convert this 18 RPM into the radian per second. So, this is for the obvious reason, so that we can use this value for the calculation.

And once you multiplied by 2 into phi divided by 60 to 18, so it gets converted into the radian per second and the value of omega here is now 1.88 radians per second. So, once we know the angular velocity, so, with the help of the angular velocity, now, we can calculate the tip speed ratio. So, the tip speed ratio as we know, it is the ratio of tip speed of blade to the wind speed and tip speed of the blade is calculated as omega into R.

And the wind speed is that is nothing but the free-stream is speed is V infinity or you can also use another nomenclature here is that is capital U as I mentioned earlier also, you can use capital U here also. So, as in this equation, we know the omega, we also know the R and we know the V infinity as well. So, by substituting this value, we can get the value of the lambda and the lambda value comes out to be around 7.896 here.

Because if you see here, the omega is 1.88 radian per second, the radius is 35 because the diameter of the rotor is given as 70 metre. So, the diameter of rotor is given as 70 metre here. So, accordingly the radius is 35 metre. So, we have to substitute this value of R here that is 35 and the free-stream speed is given here as 30 kilometre per hour. So, this 30 kilometre per hour value, you are just converted into the metre per second.

And for that reason, this is just the multiplication factor here and this is divided by 3600 that is, we have just converted hour into the second. So, that it will match to the units and we can get the lambda in the form of 7.896. So, this is a small conversion here, I hope you can

understand this conversion here very easily. So, with the help of this conversion, now, we got the value of lambda as 7.896.

So, now, once we know the lambda that is a tip speed ratio, so, with the help of lambda, we can calculate the power coefficient of the machine.

(Refer Slide Time: 13:22)



In the part 1, considering the wake rotation, the theoretical power extracted by the machine from the wind, it can be calculated from the equation which is given here. Suppose, if you see this is the equation, which can be used only when the wake rotations are consider that is C P equal to 16 by 27 into exponential of minus 0.3538 into lambda rise to power minus 1.2946. And this equation it use only when the wake rotations are considered.

So, once we substitute the value of lambda in the equation, we will get the value of C P as 0.578 and this C P is nothing but the power coefficient. And we know the power coefficient is represented in the form of C P is equal to P e that is the nothing but the power which is extracted by rotor from the wind divided by the free-stream wind speed that is 1 by 2 rho into A that is area of the rotor into V infinity cube that is free-steam wind speed.

So, after rearranging this term, so, P e can be written as C P into 1 by 2 into rho A into V infinity to the power 3. So, in this equation, most of these values are known here, because C P we can get the value of C P from the above equation here that is 0.578 into half into 1.2 that is the density of the air. So, here, it is the density of air into the area as I said it is phi R square and here R is 35 metre and this indicates free-stream wind speed that is V infinity.

So, 30 into, we have just converted this value into metre per second here so, we can convert this entire term in the form of watt. So, that is what is the; reason to convert this value in the specific unit. So, that we; can get the entire term in the form of watt. So, now, if you do this calculation, we get the P in the form of 771.97 10 is to power 3 watt that is equivalent to 0.771 megawatt.

So, this is the power which is extracted by a machine from the wind when only the wake rotations are considered. So, in part 2 considering the; wake rotation as well as the effect of drag.

(Refer Slide Time: 16:04)



So, the theoretical power which is extracted by machine from the wind can be calculated using this particular equation here, which is represented in the form of C P is equal to 16 by 27. This part is same as that of the previous equation minus epsilon into the lambda and this epsilon here, it is the ratio of drag coefficient to the lift coefficient and this particular epsilon value, it is valid for lambda greater than 1.

So, now, once you substitute the value of lambda in this equation here, the epsilon value is 0.011. So, after calculating these, the C P comes out to be around 0.526. Similarly, we know the C P which is represented in the form of P e divided by 1 by 2 rho into V cube. So, here also just after a small rearrangement of the terms, we can calculate the power which is extracted by rotor when the wake rotation as well as the effect of drag is consider.

Now, if you see here the power extracted by the rotor comes out to be around 0.702 megawatt. So, considering the previous case, the power extracted by the rotor in the second case is lower than that of the previous case and that is mainly because of the effect of drag. And I hope, you must have studied this part also in one of the lecture of the wind energy and with the help of this particular example also.

Now, it must be clear to all of you that how this particular drag it affects the power extraction capacity of the machine. So, with this, you can see here, we solve 2 part of the one example that is considering the wake rotation of the rotor as well as considering the wake rotation as well as the effect of drag into the account. So, once we solve this particular example, so, in this example, if you see here, we are calculated the power coefficient as well as the drag coefficient as well as the lift coefficient of the rotor.

So, after studying this particular example; which is mainly focused on the power extraction capacity of the rotor, which is in terms of the electrical power. So, this particular example, it mainly discusses the power extraction capacity of the rotor, which is mainly in terms of the electrical power. So, let us try to solve another example, in which machine operates water pump which is slightly different than the example which are covered just now.

(Refer Slide Time: 18:43)



So, in case of the second example, if you see here, the main dimensions of the rotor, which is a multi-blade wind machine, need to be calculated. And here the blade length is given as 1.5 metre and the rotor has 20 numbers of blades which is mentioned here as the number of blades of the rotor are 20. And the solidity of this particular rotor is 0.5 and it is operating at a design speed of 20 kilometre per hour.

And the machine operates a water pump which is having a capacity of 5.0 metre cube per hour with a lift of around 10 metre. As if the density of water as 996 kg per metre cube, reciprocating pump efficiency as 0.5 and the transmission efficiency of the rotor to the pump to be 0.8. And in this case also, the density of the area is 1.2 kg per metre cube and the C P max value in this case is given as 0.16 and lambda is 0.8.

In case if the C P max value is not given, then the C P max value need to be calculated using either of the equation which are discussed in the previous example. So, the obtained C P value needs to be used to calculate the remaining parameter in the example. So, in case of this particular example, the pump capacity is given and that is 5.0 metre cube per hour. So, with the help of this pump capacity, we can calculate the ideal power which is required to pump the water.

So, with the help of this ideal power which is required to pump the water, we can also calculate the power which is available at the rotor. And once we know the power which is available at the rotor, we can easily calculate the remaining main dimensions of the rotor, as we know the water pump capacity is given as 5 metre cube per hour. So, the ideal power which is required to pump the water can be calculated in this particular way.

This is the water pump having the capacity of 5 and we have just converted this again into the metre cube per second. Here, it is multiplied by the density of water. And this is nothing but g that is 9.81 and 10 is the lift that is 10 metre. So, this use the power which is ideal power which is required to pump the water and it comes out to be around 135.70 watt. So, just place take a note of unit here, because here, metre cube per hour, we have converted into cube per second.

And remaining terms as you know, this is kg per metre cube, this is acceleration due to gravity that is g standard unit and this is nothing but the lift that is in metre that is the 10 metre. So, we can get the value in the form of this watt. So, once we have the ideal power which is required to pump the water, now, we can easily calculate the power which is

available at the rotor because we know the efficiency of reciprocating power which is given as 0.5 and transmission efficiency from the rotor to the pump to be around 0.8.





So, with the help of these 2 efficiency we can easily calculate the power required at the rotor because we have to simply divide it by the efficiency values. So, that you can get the power required at the rotor and it comes out to be around 339.25 watt. So, this is the power which is required at the rotor, so, that it can work effectively to pump the water. So, once we know the power which is required at the rotor. So, with the help of the C P, we can calculate the remaining dimensions of the rotor.

Because we already know the power which is required the rotor C P value is known in this particular equation here. So, V infinity is also given and the density is also known that is density of the air. So, this area is the area of the rotor that is phi R square. So, after substituting this value here in this equation and this P e is 339.25 watt, so, which you are taken from here, so, this P e is 339.25 watt.

So, once you substitute all these values here, we get this equation in this particular form. And now, in this equation, this is the only unknown term. So, by just simply doing some small calculation here, we get the R as 2.56 metre. So, this is the rotor radius and it is calculated as R is equal to 2.56 metre.

(Refer Slide Time: 23:47)



Once we know the R, so, with the help of this R, we can easily calculate the remaining dimensions of the rotor that is nothing but the mean chord length and it can be calculated from the definition of the solidity. Because solidity if you remember, it is the ratio of the blade area to the swept frontal area of the machine that is called as a face area of the machine. And the blade area is N into A divided by frontal area that is the face area of the machine is phi R square.

In this case, we know number of blades. The rotor has 20 number of blade into the area of this particular blade here, which is 1.5 into the mean chord length. So, this is a blade length into c that is called as a mean chord length of the blade. And these remaining parameters, these parameters are already known because these R is nothing but the radius of the blade. And this is phi and 0.5 is the solidity which is also given in the example.

So, by simply rearranging this term here, we get the mean chord length as 0.343 metre. So, now, with the help of these 2 equations, we could calculate the radius that is a radius of the rotor in the previous case if you remember here, this is the radius of the rotor and also the mean chord length of the blade. So, with the help of this, we could calculate these 2 terms here. So, once we know this c that is a mean chord land and radius of the rotor, ee can calculate the angular velocity of the rotor.

And it can be calculated from the following equation that is lambda is equal to omega upon V infinity where the numerator term here, it represents the tip speed of the blade and the denominator, it represents the free-stream wind speed. In this particular equation, the R is

known; the V infinity value, it is also known and lambda is also given in the example. So, with the help of these 3 terms, we can easily calculate the omega that is the angular velocity of the rotor.



(Refer Slide Time: 26:02)

So, once you substitute these values in the equation in the form of say lambda V infinity and R. So, we can calculate the omega that is the angular velocity of the rotor. So, here, if you see the lambda value is given as 0.8, this is the velocity which is the value given in the example, we are just simply converted into the metre per second again here and these R is 2.56 metre. So, now, once you do the simple calculation, the omega comes out to be around 1.736 radians per second.

So, again here; this value is given in the form of radian per second and if you need to convert it into the RPM. So, just simply multiplied by this factor, which converts the radians per second into the RPM that is 60 upon 2 into phi and omega value after conversion is comes out to be around 16.57 RPM. So, the rotor is rotating with this much RPM with the wind speed which is given as 20 kilometre per hour.

So, with the help of these equations, we could calculate the radius that is 2.56 mean chord length that is 0.343 metre and the omega as 16.57 RPM. So, with this, I hope you must have understood how to calculate the performance of the wind machine and as well as how to calculate the dimensions of the rotor. These 2 examples in the different line, so, the first example, it mainly discusses the power extraction capacity of the rotor that is mainly an electrical power.

And in the second example, machine operates a water pump and which is having a specific capacity that is a 5 metre cube per hour. So, seeing the similar line, there may be example in the assignment where the pump capacity is given in the some different range. So, we can easily calculate the main dimensions of the rotor accordingly. And similarly, in the other hand, if the some specific values of the lambdas are given, then with the help of the lambda value, we can also calculate the power coefficient.

So, as we have discussed in the first example, so, just considering the wake rotation only and with the help of lambda value, we can easily calculate the power coefficient. Whereas, in case if the wake rotation and the effect of drag both are need to be considered, so, accordingly we need to use the equation and then we can find out the power coefficient of this specific rotor. So, with this, we will end this lecture and regarding this lecture if you have any doubt, so, feel free to contact me at vvgoud@iitg.ac.in. Thank you.