

Renewable Energy Engineering: Solar, Wind and Biomass Energy Systems
Dr. R. Anandalakshmi
Department of Chemical Engineering
Indian Institute of Technology – Guwahati

Lecture - 35
Characteristics and Power Generation from Wind Energy - Part II

Hi everyone, today in renewable energy engineering: solar, wind and biomass energy systems. We are going to see last lecture of wind energy system.

(Refer Slide Time: 00:39)

Wind Energy: Basics and Concepts
Lecture 1: Turbine Terms, Types and Theories

- Introduction
- Wind Speed Vs. Height
- History of Wind Turbine
- Wind Energy Regions in India
- Classification of Wind Turbine
- Main Components of a Wind Turbine
- Operating Characteristics of Wind Turbine
- Mathematical Expression Governing Wind Power
- Design Function Requirements
- Calculation of Energy Available in Wind

In last week, we had wind energy basics and concepts in that we spent almost lecture 1 and 2 on turbine terms, types and theories, which included all these topics.

(Refer Slide Time: 00:53)

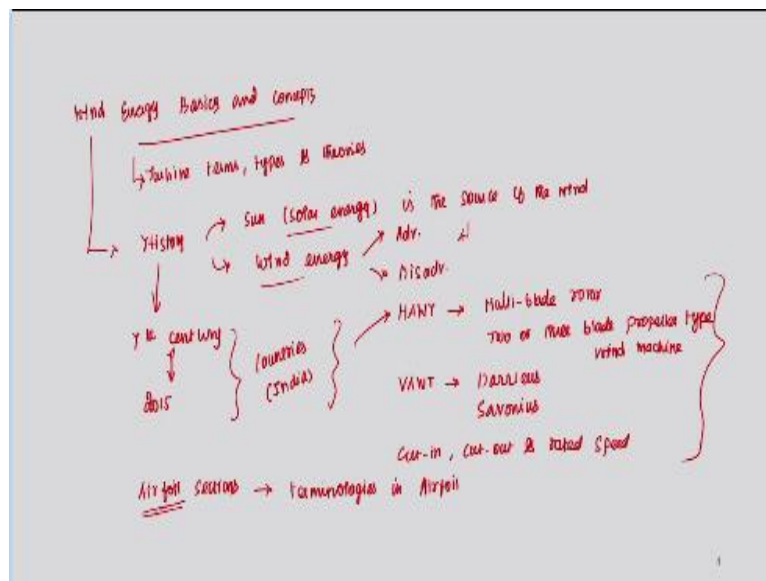
Wind Energy
Lecture 3: Characteristics and Power Generation from Wind Energy

- Wind Data ✓
- Airfoils ✓
- Airfoils and General Concepts of Aerodynamics ✓
- General Terminology Used in Wind Speed Estimation ✓
- Formula for Velocity Probability Distribution ✓
- Torque at Maximum Efficiency ✓
- Wind Tunnel Performance Parameters ✓
- Practice Problems

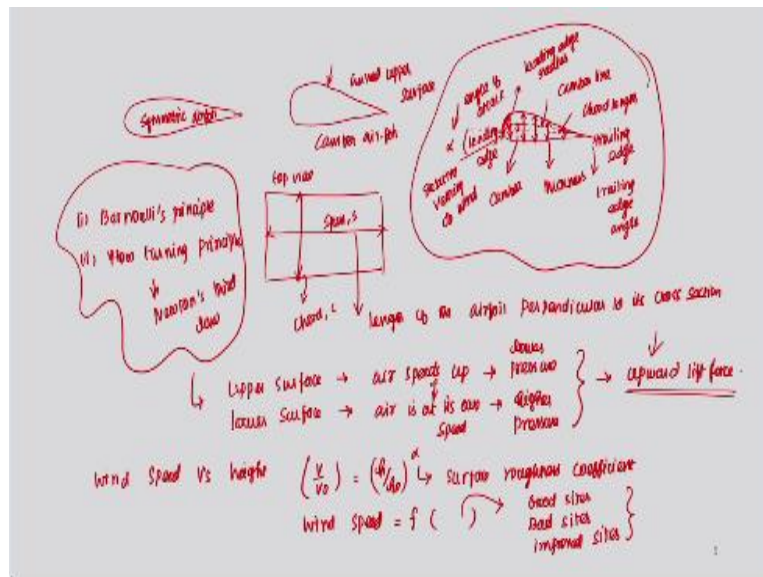
In this week of characteristics and power generation from wind energy, first lecture, we could cover till wind data, because we took a bit of time to recap whatever we have done in last week with some additional information. So, today in this continuation of characteristics and power generation from wind energy, so, we would be covering airfoils and general concepts of aerodynamics, general terminology used in wind speed estimation, formula for velocity probability distribution, torque at maximum efficiency, wind tunnel performance parameters.

In all these things to explain that concept better, we would be doing parallelly practice problems. Apart from these practice problems, Professor Goud who is my co-instructor of this course, would also be covering a few practice problems on same characteristics and power generation from wind energy. With these 2 lectures, we would be concluding this week, this is probably 8th week of this course and then you would be having almost 2 weeks for your final exam.

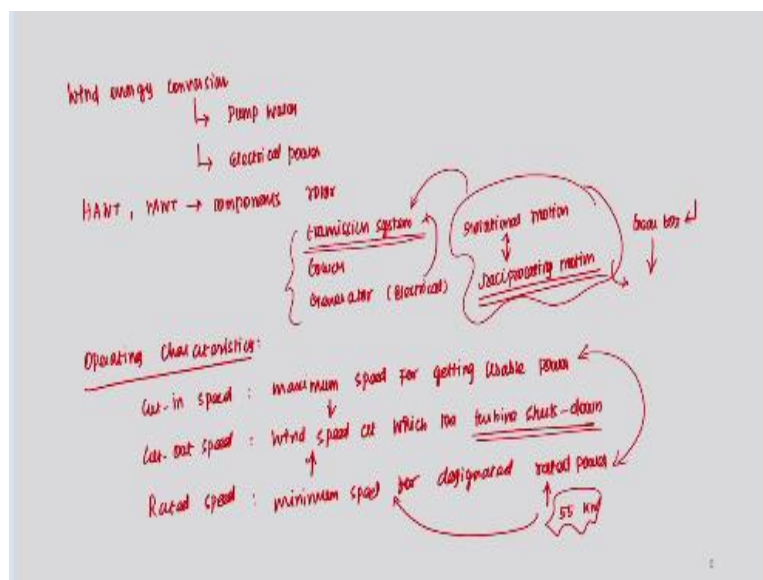
(Refer Slide Time: 02:11)



(Refer Slide Time: 02:14)



(Refer Slide Time: 02:16)



(Refer Slide Time: 02:17)

$$C_p = \frac{\text{Rotor Power}}{\text{Power in wind}} = \frac{\frac{1}{2} \rho A U^3 4a(1-a)^2}{\frac{1}{2} \rho A U^3} = 4a(1-a)^2 = \frac{16}{27} = 0.593$$

$a = 0.33$ $C_p \text{ max} \rightarrow a = 1/3$

$$C_T = \frac{\text{Thrust force}}{\text{Dynamic force}} = \frac{\frac{1}{2} \rho A U^2 4a(1-a)}{\frac{1}{2} \rho A U^2} = 4a(1-a) = \frac{8}{9} = 0.89$$

Wind rotor:
Lift coefficient = $C_L = \frac{F_L}{\frac{1}{2} \rho A U^2}$ → Lift force: perpendicular to the direction of the incoming airflow
arises due to unequal pressure on the surfaces of the blades
= $\frac{\text{Lift force}}{\text{Area of the free stream}}$ → projected area of the blade facing the wind

(Refer Slide Time: 02:20)

Solidity (σ)

$$\sigma = \frac{\text{Blade area}}{\text{Swept frontal area}} = \frac{\frac{Nc}{\pi R}}{\pi R^2} = \frac{Nc}{\pi R^2}$$

↑ HAWT
VANT $\frac{Nc}{R} \rightarrow$ variable type

Number of blades
mean chord of the blades
mean radius

Assumption
blade length = tip radius

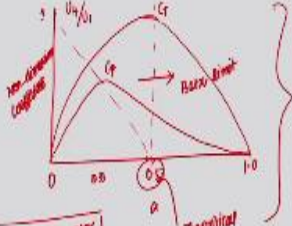
$C_{T, \text{max}} = 59.3$ → theoretical maximum
↓ Assumption
Practical case

$$\frac{Nc}{\pi R} = \frac{N(c \times \phi)}{\pi (R \times \phi)} = \frac{Nc}{\pi R}$$

So, if we see whatever we have done in the last lecture, we reviewed extensively what we have done last week and then we introduced certain extra additional information which is nothing but solidity. And here, we had seen the formula for solidity which is nothing but blade area upon swept frontal area. So, you might be wondering how it came us and C upon phi R. So, this is nothing but blade area upon frontal area.

So, even if you see the blade area this N is number of blades, so, C into span chord like into span and if you are seeing this one, this is phi R into R; phi R square. So, since you have assumed that blade length is nothing but the tip radius. So, this both are same. So, what you finally get is N C upon phi R. So, this I just wanted to tell you like how it derived. As I

Since it is an introductory course, we are taking directly the formulas and calculating certain parameters, but if you are exclusively taking wind energy course, you would be knowing what is the basis of all these things.

$C_{f, \max} = 59.1$
 $\alpha = 1/3$ → maximum power production
 $\alpha = 1/2$ → Betz limit

 non-dimensional coefficient
 Betz limit
 Theoretical Limit
 $\alpha > 0.5$ complicated flow pattern
 $\hookrightarrow C_f > 2$
 rotor → Actuator disc model
 (i) Rotation of wake formation behind the rotor
 (ii) Finite no. of blades → associated tip losses
 (iii) Non-zero aerodynamic drag

$$\eta_{\text{rotor}} = \frac{P_{\text{out}}}{1/2 \rho A U^3} = \eta_{\text{act}} C_p$$

$$P_{\text{out}} = (\eta_{\text{act}} C_p) \frac{1}{2} \rho A U^3$$

$C_p = \frac{16}{\lambda^2} \exp(-0.3538 \lambda^{-1.2946})$ ← $C_{T, \max}$
 ELP speed ratio
 $\lambda \rightarrow \infty$
 $C_p = f(\lambda, \xi)$
 $\hookrightarrow \xi = \frac{C_p}{C_L} \quad \boxed{\lambda > 1}$
 $= \left(\frac{16}{\lambda^2}\right) \left[\exp(-0.3538 \lambda^{-1.2946}) - \xi \lambda \right]$
 $\left\{ \begin{array}{l} \xi = 0.005 - 0.02 \text{ propeller type motors having blades with airfoil section} \\ \xi = 0.05 - 0.1 \text{ multi-blade rotors having curved plates as blades} \end{array} \right.$

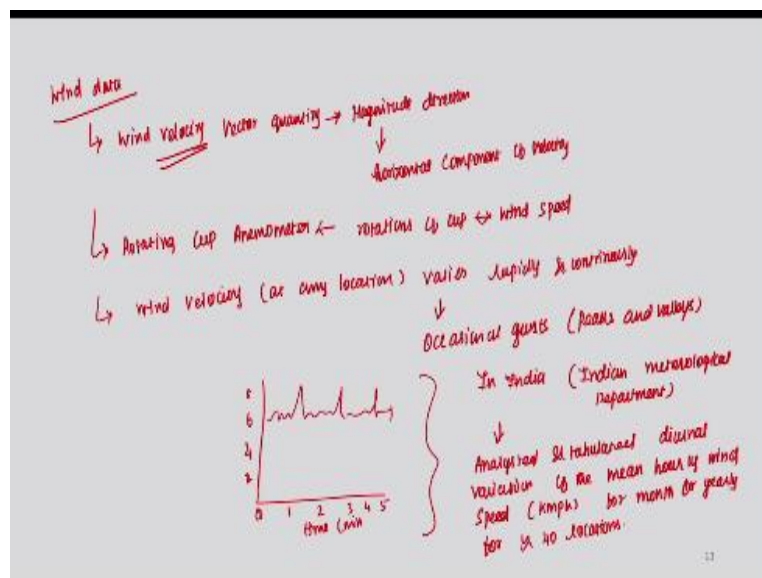
And also I might probably told you that C_P which is nothing but power coefficient. So, that also can be done from certain correlations. So, in that one thing I forgot to mention here, when you were explaining this epsilon, which is nothing but C_D upon C_L drag coefficient

upon lift coefficient. So, this is valid when only λ is greater than one. So, that limit is also there.

Anyway, these 2 formulas would be covered or would be used for certain practice problems by Professor Goud, he also would explain you over there, but this also one should remember like what is the applicability because, these kind of information I would have told you in my very first lecture of solar energy. When you are using a certain correlation, so, you should be making sure that so, those constants, so, those were fitted constants.

So, you should be very careful about the applicability of those correlations and whether any certain assumptions being made. For example, here, if you want to do it for propeller type rotors, or if you want to do it for multi-plates rotor types, then your ϵ may not be similar. So, such kind of information one should be careful about before using any certain type of correlations.

(Refer Slide Time: 05:13)



(Refer Slide Time: 05:19)

New location
 ↳ Result wind speed of the location on some representative days over 1 year

$$\bar{V} = \frac{1}{N} \sum_{i=1}^N V_i$$
 ↳ average speed

$$\sigma^2 = \frac{1}{N-1} \sum_{i=1}^N (V_i - \bar{V})^2$$
 ↳ standard deviation

$$\bar{V} = c \Gamma \left(1 + \frac{1}{k} \right)$$

$$\sigma^2 = \bar{V}^2 \left[\frac{\Gamma \left(1 + \frac{3}{k} \right)}{\Gamma^2 \left(1 + \frac{1}{k} \right)} - 1 \right]$$
 ↳ Weibull distribution
 1 sec

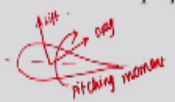
(Refer Slide Time: 05:27)

Airfoils

Lift force – defined to be perpendicular to direction of the oncoming air flow. The lift force is a consequence of the unequal pressure on the upper and lower airfoil surfaces.

Drag force – defined to be parallel to the direction of the oncoming air flow. The drag force is due both to viscous friction forces at the surface of the airfoil and to unequal pressure on the airfoil surfaces facing toward and away from the oncoming flow.

Pitching moment – defined to be about an axis perpendicular to the airfoil cross-section



So, today we will start with airfoils. In airfoils, lift force and drag force, I already explained you, So, lift forces nothing but perpendicular to the direction of the oncoming airflow and this is also consequences of unequal pressure on the upper and lower airfoils surfaces. Opposite to that drag forces nothing but parallel to the direction of the incoming airflow.

And drag forces due to both viscous friction forces at the surface of the airfoil as well as due to unequal pressure on the airfoil surfaces facing toward and away from the oncoming flow. Pitching moment something like defined to be an axis perpendicular to the airfoil cross section. So, if you have a airfoil something like this, so, you would be having the lift force in this direction and this would be your drag. So, pitching moment is nothing but this way. This is pitching moment which is about an axis perpendicular to the airfoil cross section.

(Refer Slide Time: 06:45)

Airfoils and General Concepts of Aerodynamics

Lift coefficient

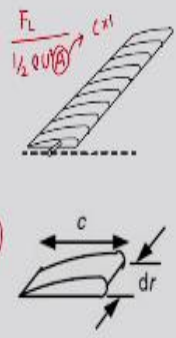
Drag coefficient

Pitching moment coefficient

$$C_l = \frac{F_l / l}{\frac{1}{2} \rho U^2 C} = \frac{\text{Lift force / unit length}}{\text{Dynamic force / unit length}} = \frac{F_l}{\frac{1}{2} \rho U^2 C}$$

$$C_d = \frac{F_d / l}{\frac{1}{2} \rho U^2 C} = \frac{\text{Drag force / unit length}}{\text{Dynamic force / unit length}}$$

$$C_m = \frac{M}{\frac{1}{2} \rho U^2 A C} = \frac{\text{Pitching moment}}{\text{Dynamic moment}}$$



where ρ is the density of air, U is the velocity of undisturbed air flow, A is the projected airfoil area (chord, $C \times \text{span}$, S), C is the airfoil chord length, and l is the airfoil span.

And then this lift coefficient, drag coefficient, pitching moment coefficient, everything we already defined, but however, if you want to consider them in terms of chord length and span then you can define and that way as well. So, I probably might have told you F_L upon half $\rho U^2 A$, so, this A is replaced by C into L . So, what is C ? C is nothing but airfoil chord length. L is nothing but airfoil span.

So, this airfoil span also I explained you here; span and chord, I explained you. So, if you substitute area as C into L , so, the C is given here and L is taken over there, so, that is why it is just like the way of representing it. So, lift force per unit length upon dynamic force upon unit length. So, $\rho U^2 C$ that is by chord length and this is span. So, the same way a drag force upon unit length total divided by dynamic force upon unit length.

So, this is again $\rho U^2 C$. The L is taken above pitching moment, the pitching moment upon dynamic moment, so, $\rho U^2 A$ into C and ρ is nothing but density of air, U is velocity of undisturbed air flow or free-stream velocity, I have already told you some author use U in reference already told Manwell uses U and in Sukhatme's book, it is represented as V_∞ . Both represents the free-stream velocity.

A is the projector airfoil area which is chord into span and C is the airfoil chord length and L is the airfoil span.

(Refer Slide Time: 08:34)

General Terminology Used in Wind Speed Estimation

Air density

1 atm = 1.01325×10^5 Pa

$$\rho = \frac{P}{R T}$$

$$\rho = \frac{1.01325 \times 10^5}{287 \times 288} = 1.226 \text{ kg/m}^3$$

Handwritten: $T = 15^\circ\text{C} = 288\text{K}$

Logarithmic wind profile

$$U(z) = \frac{U^*}{k} \ln \left(\frac{z}{z_0} \right)$$

Handwritten: $U = \text{velocity}$

$U^* = \text{Frictional velocity}$
 $z_0 = \text{surface roughness length}$
 $\tau_0 = \text{Surface shear stress value}$
 $k = \text{Von Karman's constant } (\sim 0.4)$

$$\ln(z) = \left(\frac{k}{U^*} \right) U(z) + \ln z_0$$

Power law profile

The power law represents a simple model for the vertical wind speed profile. Its basic form is:

$$\frac{U(z)}{U(z_r)} = \left(\frac{z}{z_r} \right)^\alpha$$

$U(z_r)$ is the reference wind speed at height z_r
 α is the power law exponent

Handwritten: $\ln \frac{z}{z_0} = \frac{U(z) - U^}{U^*} \Rightarrow \ln z = \left(\frac{U(z) - U^*}{U^*} \right) + \ln(z_0)$*

Handwritten: $\frac{U}{U^} = \left(\frac{z}{z_0} \right)^\alpha$*

Table: Values (approximate) of surface roughness length for various types of terrain

Terrain description	z_0 (mm)
Very smooth, ice or mud	0.01
Calm open sea	0.20
Blown sea	0.50
Snow surface	3.00
Level grass	8.00
Rough pasture	10.00
Fallow field	30.00
Crops	50.00
Few trees	100.00
Many trees, hedges, few buildings	250.00
Forest and woodlands	500.00
Suburbs	1500.00
Centres of cities with tall buildings	3000.00

And then the general terminology used in wind speed estimation. So, in the introductory classes, I have told you about this power law profile. The power law represents a simple model for the metrical wind speed profile. The basic form is what I might have told you is V upon V_0 which is equal to h upon h_0 power α . So, the α is nothing but power law exponent or we can say it as a surface roughness coefficient and U at z_r .

z_r is equivalent to h and h_0 ; z is the particular height with which we want wind speed and z_r is equivalent to h_0 which is nothing but some reference height at which wind speed is known. And U at z_r already known at reference height. U at z is supposed to be found out. So, this is simple power law profile and apart from that there is a logarithmic wind profile so, which is nothing but U of z is equal to U^* upon $k \ln$ of z upon z_0 .

U is here the velocity which is to be found out. U^* is nothing but root of τ_0 upon ρ ; τ_0 is nothing but surface shear stress value and ρ is nothing but the density of the air. So, if you substitute U^* , then what is left with k . k is Von Karman's constant. So, these are all comes from fluid dynamics or fluid mechanics you are requested to refer any standard fluid mechanics book as I told you here, we are defining as well as using certain formulas to find out parameters required.

But, we are not going in depth and how to derive such formula since, it is a introductory 4 lectures of 2 weeks course on wind energy, because, if some of you are not working in mechanical or chemical engineering or energy engineering, so, you might not have this

particular course on fluid mechanics, it would be very difficult for you to get these terminologies.

So, you can take any good online help or you can refer certain books on fluid mechanics to get to know in depth about these shear stress value, Von Karman's constant etcetera. So, k is Von Karman's constant; z is nothing but that at particular height and z_0 is nothing but surface roughness length for value of surface roughness length for various types of terrain is given here z_0 which is mm.

So, based on the terrain types, you can have z_0 . So, using z_0 , k and then U^* one can find out U of z . So, that is nothing but this is $\ln$ of natural logarithm of z upon z_0 which is nothing but U of z k upon U^* . So, this is $\ln z$ upon z_0 which is nothing but $\ln z$ minus $\ln z_0$. so, if $\ln z_0$, you want to calculate so, this particular term plus $\ln$ of z_0 equal to mx plus C form. So, from that also, one can find out z by graphical as well as by pen and paper.

Apart from that there is a air density calculation at one atmosphere and temperature of 15 degrees centigrade so, which is equal to 288 Kelvin. So, we already know the ideal gas law $PV = nRT$.

(Refer Slide Time: 12:19)

Handwritten derivation of air density using the ideal gas law:

$$PV = nRT$$

$$\frac{P}{RT} = \frac{n}{V} = \frac{\text{moles}}{\text{volume}} = \text{molar density}$$

$$\text{Molar} = \frac{\text{Mass}}{\text{Mol wt}} \Rightarrow \text{mass} = \text{molar} \times \text{Mol wt}$$

$$\rho = \frac{PM}{RT}$$

$$= \frac{1.01325 \times 10^5 \frac{\text{N}}{\text{m}^2}}{8.314 \frac{\text{J}}{\text{mol K}} \times 288 \text{ K}} \times \frac{29 \frac{\text{g}}{\text{mol}}}{1000 \frac{\text{kg}}{\text{g}}} = 1.226 \frac{\text{kg}}{\text{m}^3}$$

Additional values shown: $R = 287 \frac{\text{J}}{\text{kg K}}$

So, $PV = nRT$; P by RT is equivalent to n upon v . So, n is nothing but number of moles per volume. So, we call this as molar density. So, again the mole is defined this mass upon molecular weight. So, if you want to convert into mass, then moles should be multiplied with

molecular weight of the air. Then number of moles into molecular weight is mass. Mass upon volume is density so, that is PM upon RT.

And another one here is, the P is given. So, P is given 1.01325 into 10 to the power of 5 Pascal which is nothing but Newton per metre square into M that is 28.97. The unit is kg upon kilo mole or r if I substitute 8.314 kilo joules per kilo mole Kelvin. For T, if I substitute 288 Kelvin, so, Kelvin, Kelvin get it cancel kilo mole, kilo mole. So, here we have kilo joules and here we have kilogram. So, this Newton metre, one Newton metre is one Joule, the force into distance.

So, if I multiply with metre above and below, so, this will go. So, this is nothing but kilogram per metre cube and one k is here. So, if you do this, then what you would be getting is 1.226 kilograms per metre cube. Or if you do not want, then here itself you can substitute that 314 kilojoules per kilo mole Kelvin. So, if you want to convert mole into mass kilojoules per kg Kelvin because that whatever we substituted here, I said 287 is in kilojoules per kg Kelvin.

So, one can directly substitute this way or you can substitute and do the manipulations and get rho as 1.226 kg per metre cube. So, that is all about the air density at reference pressure of 1.01325 into 10 to the power of 5 Pascal at T at 288 Kelvin, which is nothing but 15 degree centigrade.

(Refer Slide Time: 15:31)

Formula for velocity Probability distribution

a) Rayleigh Distribution

This is the simplest velocity probability distribution to represent the wind resource since it requires only a knowledge of the mean wind speed, $\bar{U}$.

Then the **probability density function** are :

$$P(U) = \frac{\pi}{2} \left(\frac{U}{\bar{U}^2} \right) \exp \left[-\frac{\pi}{4} \left(\frac{U}{\bar{U}} \right)^2 \right]$$

the **cumulative distribution function**

$$F(U) = 1 - \exp \left[-\frac{\pi}{4} \left(\frac{U}{\bar{U}} \right)^2 \right]$$

So, that next one is formula for velocity probability distribution. So, this is the Rayleigh distribution is the simplest velocity probability distribution to represent the wind resource,

since, it requires only the knowledge of mean wind speed. So, if you see probability distribution function ϕ upon $2 U$ upon U bar square exponential minus ϕ by $4 U$ upon U bar whole square. So, this some books refer as small f V so, the U and V can be interchangeably used.

So, this probability density function and cumulative distribution function is capital F of U or F of V which is equal to $1 - \exp$ minus ϕ by $4 U$ upon U bar whole square.

(Refer Slide Time: 16:17)

Formula for velocity Probability distribution

b) Weibull Distribution ✓

Use of the Weibull probability density function requires knowledge of two parameters: k (shape factor), and c (scale factor). The Weibull probability density function and the cumulative distribution function are given by:

$$P(U) = \frac{k}{c} \left(\frac{U}{c} \right)^{k-1} \exp \left[- \left(\frac{U}{c} \right)^k \right] \quad \checkmark$$

k = Shape factor
 c = Scale factor ✓

$$F(U) = 1 - \exp \left[- \left(\frac{U}{c} \right)^k \right] \quad U = V \checkmark$$

22

And variable distribution I already told so, based on certain comparison between experimental data and other statistical models. Weibull distribution function is almost accurately matching with the existing experimental data. Use of probability density function requires knowledge of 2 parameters. One is shape factor and scale factor. k is Weibull shape factor. c is Weibull scale factor.

So, this is also P of U probability density function is nothing but k upon c U upon c power $k - 1$ exponential minus U upon c to the power k and the cumulative distribution function F of U is nothing but $1 - \exp$ minus U upon c k I probably used instead of U, v . So, this is followed by Sukhatme's book.

(Refer Slide Time: 17:12)

Annual Energy Production by Wind Turbine

The average wind machine power $\bar{P}_w$ can be expressed as

$$\bar{P}_w = \rho \left(\frac{2}{3} D \right)^2 \bar{U}^3$$

$\bar{P}_w$ = average wind machine power

$\bar{U}$ = average wind velocity

Multiplying above equation by 8760 hr will yield an expected annual production.

$\bar{P}_w = f(P_w(U), P_w(U))$
 $\bar{P}_w = \int_0^\infty P_w(U) P(U) dU$
 $\bar{P}_w = \frac{1}{2} \rho A \int_0^\infty U^3 C_p(U) P(U) dU$
 $\bar{P}_w = \frac{1}{2} \rho A \int_0^\infty U^3 C_p(U) P(U) dU$
 $\bar{P}_w = \frac{1}{2} \rho A \int_0^\infty U^3 C_p(U) P(U) dU$

Estimate the annual energy production from a horizontal axis wind turbine with a 10 m diameter operating in a wind regime with an average wind speed of 7 m/s. Assume that the wind turbine is operating under standard atmospheric conditions ($\rho = 1.225 \text{ kg/m}^3$) and turbine efficiency of 0.35.

Solution:

Average power produce

$$\bar{P}_w = \rho \left(\frac{2}{3} D \right)^2 \bar{U}^3$$

Annual production (P_A) = $\bar{P}_w \times 8760 \text{ h/yr}$

Power output (Annually)

$$P_{\text{output}} = \eta_{\text{turbine}} \times P_A \times 8760 \text{ h/yr}$$

Handwritten notes in red:
 One-line power equation
 Betz limit (59.3%)
 $D = 10 \text{ m}$
 $\bar{U} = 7 \text{ m/s}$
 $\rho = 1.225 \text{ kg/m}^3$
 $\eta = 0.35$
 $P_A = 1.225 \times \left(\frac{2}{3} \times 10 \right)^2 \times 7^3 = 18.67 \text{ kW}$
 $P_{\text{output}} = 0.35 \times 8760 \times 18.67 \text{ kW}$
 $= 57.25 \text{ MWh}$

And then the average wind machine power $\bar{P}_w$ can be expressed as $\bar{P}_w$ which is equivalent to $\rho \left(\frac{2}{3} D \right)^2 \bar{U}^3$. So, $\bar{P}_w$ is nothing but average wind machine power and $\bar{U}$ is average wind velocity. So, if you multiply this with 8760 hours will yield an expected annual production. So, how we arrived at 8760? I already explained you.

So, where from this particular formula comes is nothing but that $\bar{P}_w$ is nothing but function of $P_w(U)$ that is probability distribution function PDF and then it is a function of P_w at U . So, this is nothing but the known turbine characteristic, known turbine power curve. So, if you want to write $\bar{P}_w$ which is 0 to infinity P_w at U that is characteristic curve, power characteristic curve and P_w of U into dU .

So, we already know that $P_w(U)$ or P_w suppose I will use U only so, that you would not get confused. So, P_w at U can be written as $P_A \eta$ and then C_P . C_P is power coefficient. η is efficiency and P_A is nothing but actual power. So, this can be written as $\frac{1}{2} \rho A U^3 C_P \eta$. So, if you go and substitute here in the $\bar{P}_w$, so, you can take it out $\frac{1}{2} \rho A$ can be referred as ϕr^2 and η and then C_P because as a function of λ I already given U correlations formula.

So, then U^3 and then P_w of U dU . So, if you want to calculate at maximum, then you can substitute C_P as Betz limit. So, then $\bar{P}_w$ is nothing but $\frac{1}{2} \rho \phi r^2$ and if you take η as 1 for ideal, because we are considering C_P Betz limit. C_P Betz then what you would get is U^3 P_w of U dU . If this integral between 0 to infinity is calculated, then what

you would be finally arriving is $\rho \bar{w}$, which is equivalent to $\rho \frac{2}{3} D^3$ whole square into $\bar{U}$ cube. So, this is called Rayleigh Betz limit. Why Rayleigh Betz limit?

So, we already know this probability distribution function, you can substitute it either from Weibull distribution or from Rayleigh distribution. So, if you substitute this p of $\bar{U}$ probability distribution function from Rayleigh, then you would be arriving this formula Rayleigh Betz limit. So, this was done by the researcher called Carlin in 1997. He call it as a One-two-three equation because the density is stressed to power 1 and $\frac{2}{3} D$ is rise to power 2 and $\bar{U}$ bar is rise to power 3.

So, that is the equation we would be using to calculate the average power produce. Otherwise, this is when you substitute this probability distribution function and then you are calculating otherwise if you are given with average wind speed, you can directly substitute instead of $\bar{U}$ as well. So, we will see both the way how it is been calculated. So, to explain you this particular concept, this practice problem is taken.

Estimate the annual energy production from a horizontal axis when a turbine with a 10 metre diameter operating in a wind regime with an average wind speed of 7 metre per second. So, D is given as 10 metre. $\bar{U}$ bar is given as average wind speed 7 metre per second. Assume that the wind is operating at under standard atmospheric conditions. ρ is given as 1.225 kg per metre cube and efficiency is given as 0.35.

So, if you substitute here in $\rho \bar{w}$, so, what you would be getting is 1.225 into $\frac{2}{3} D$ is 10 into 10 whole square into $\bar{U}$ bar. $\bar{U}$ bar is nothing but 7 cube. So, this particular value turned out to be 18.67 kilowatt. So, if you want to do it in annual production, then you supposed to multiply that with 8760 hour per year as well. Instead of second step, I did totally power output annually. So, turbine efficiency is 0.35 and annual is 8760 and then your P_A is 18.67 kilowatt.

So, if you calculate, it is turned out to be 57.25 megawatt hour because we are calculating in terms of hours. So, this can be done or if you want to calculate directly using the formula of the actual power, actual power we know. So, how we define P_A ?

(Refer Slide Time: 23:34)

Based on Average wind speed

$$\bar{P}_A = \frac{1}{2} \rho A \bar{U}^3$$

$$= \frac{1}{2} \times (1.225) \times \left(\frac{\pi \times 10^2}{4} \right) \times 7^3$$

$$= 16.5 \text{ kW}$$

$$\bar{P}_W = 0.35 \times 8760 \times 16.5 \text{ kW}$$

$$\bar{P}_W = 50.58 \text{ MWR}$$

So, your P_A is nothing but this is based on average wind speed. $\bar{P}_A$ is nothing but half $\rho A \bar{U}^3$; half ρ is 1.225 they have already given into A is $\pi \times 10^2$ because 10 is the diameter, πD^2 upon 4 into 7 to the power of cube. So, if you calculate, so, this has turned out to be 16.5 kilowatt. So, if you want to calculate power output, average power output, then efficiency 0.35. And the annual production in hours 8760 hours into whatever you got here that is 16.5 kilowatt.

So, it is coming around to be 50.58 megawatt. So, one can calculate this way as well but you need to mention based on which you calculated the annual power production. So, if you have already probability distribution function, you can substitute this formula or if you are making the assumption that this itself average wind speed, you can calculate in this way also but remember assumptions or any assumptions being made to be clearly told in for the way you are calculating the average power production.

(Refer Slide Time: 25:10)

Torque at Maximum Efficiency

There are two types of force which are acting on the blades:

- ✓ Circumferential force acting in the direction of wheel rotation that provides the torque.
- ✓ Axial force acting in the direction of the wind stream that provides an axial thrust, must be counteracted by proper mechanical design.

The circumference force or torque T can be obtained from

$$T = \frac{P}{\omega} = \frac{P}{\pi D N}$$

The actual efficiency $\eta = \frac{P}{P_{\text{total}}}$

$$P = \eta P_{\text{total}} = \eta \left(\frac{1}{2} \rho A V^3 \right)$$

$$T = \eta \frac{1}{2} \rho D V^3 N$$

As maximum efficiency = 0.593 = 59.3%

$$T_{\text{max}} = \frac{16}{27} \left(\frac{1}{2} \rho D V^3 N \right)$$

$$T_{\text{me}} = \frac{2}{27} \rho D V^3 N$$

Solution:

Determine the power in the wind if the wind speed is 18 m/s and rotor radius is 45 m. (Wind speed, $v = 18$ m/s, Air density, $\rho = 1.23$ kg/m³)

$V = 18 \text{ m/s}$
 $R = 45 \text{ m}$
 $\rho = 1.23 \text{ kg/m}^3$

Area: $A = \pi \times 2025 = 6362 \text{ m}^2$
 $\pi \times 45^2 = \pi \times 2025$

Wind power = $\frac{1}{2} \rho A V^3$

$$\frac{1}{2} \times 1.23 \times 6362 \times 18^3 = 22817 \text{ kW}$$

T	Torque
P	Power
ω	Angular velocity
D	Diameter of rotor
N	Shaft speed
P_{total}	Total power
A	Area
V	Velocity
ρ	Density
η	Efficiency

And then torque at maximum efficiency. So, there are 2 types of forces which are acting on the blades. One is circumferential force which acts in the direction of wheel rotation that provides the torque. The second one is axial force in the direction of wind stream. So, that will be contracted by proper mechanical design. So, the circumference force or torque can be obtained from T equal to P upon ω .

So, what is P ? P is power ω is $\phi D N$. D is the diameter of the rotor. N is nothing but shaft speed. P is power and the actual efficiency η is nothing but P upon P_{total} . P_{total} is nothing but total power. So, from this, if you want to calculate P , P_{total} into efficiency. Efficiency and the P_{total} power available as half $\rho A V^3$ or U^3 . Then T , T is nothing but torque which is equivalent to η of 1 upon 8 because here T is told as P upon $\phi D N$.

So, we already calculated P is nothing but η 1 upon 2 ρ . This A , I will write it as ϕD^2 upon 4 into V^3 divided by $\phi D N$. So, ϕ , ϕ get it cancelled 1 D , 1 D . So, N is, N would be there. So, 1 upon 2 1 upon 4 into 1 upon 8 η 1 upon 8 $\rho D V^3$ upon N . At maximum efficiency then we already know it is 59.3% or your C_P is 16 upon 27. So, this T should be multiplied with 16 upon 27 to get torque at maximum efficiency. So, which is 16 upon 27 into 1 upon 8 $\rho D V^3$ upon N .

So, if you calculate, this is 2 upon 27 $\rho D V^3$ upon N . So, if you want to determine simple power, then what is the formula to be used half $\rho A V^3$. So, here, the wind speed is given that is V is given 18 metre per second and then the rotor radius is directly

given which is 45 metre and the air density is given in this problem 1.23 kg per metre cube. This is simple calculation.

So, if you want to calculate the area which is πr^2 , which is π into 45 whole square, so, you would get 6362 metre square and wind power is half $\rho A V^3$. Half ρ is given as 1.23. A is 6362 and 18 to the power 3, then what you would be getting is 22817 kilowatt.

(Refer Slide Time: 28:20)

Torque at Maximum Efficiency

Wind at 1 standard atmospheric pressure and 15°C has velocity of 10 m/s calculate :

- 1) The total power density in the wind stream
- 2) The maximum obtained power density
- 3) Torque at maximum efficiency

Consider turbine diameter of 100 m, and turbine operating speed of 42 rpm for a propeller type wind turbine

Solution

Wind power ($P_{\text{total}} = \frac{1}{2} \rho A V^3$)

1) Power density = $\frac{P_{\text{total}}}{A} = \frac{1}{2} \rho V^3$

Kinetic energy flux = $\frac{1}{2} \times 1.226 \times 10^3 = 613 \frac{\text{N}}{\text{m}^2}$

2) Maximum obtained power density

$\Rightarrow \text{Max PD} = \frac{\text{Max. P}}{A} = \frac{8}{27} \times \frac{1}{2} \rho V^3$

$= \frac{8}{27} \times 1.226 \times 10^3$

$= 363.25 \frac{\text{W}}{\text{m}^2}$

3) Torque at maximum efficiency

$T_{\text{max}} = \frac{2}{27} \times \frac{\rho D^3 V^3}{N}$

$= \frac{2}{27} \times \frac{1.226 \times 100 \times 10^3}{40/60}$

$T_{\text{max}} = 12974 \text{ N}$

So, if you want to calculate the torque at maximum efficiency whatever we have derived as a formula, so, this problem is taken to explain that, so, wind at 1 standard atmospheric pressure 15 degree. So, that means ρ is 1.226 kg per metre cube and velocities is $V = 10$ metre per second. We supposed to calculate the total power density in the wind stream and then the maximum obtained the power density the torque at maximum efficiency for considering the turbine diameter of D is nothing but 100 metre.

And then turbine operating speed is given as 42 RPM revolutions per minute for the propeller type wind turbine. So, before going into calculating the torque at maximum efficiency, there are 2 things were told to calculate the total power density in the wind stream as well as the maximum obtained power density. So, the wind power is half $\rho A V^3$ that we know. The power density here is the P total upon A .

So, you might be wondering the energy density is nothing but energy upon volume. Power density is nothing but power upon volume. It is not the convention followed in the same way for even here, in wind energy calculations as well as in membrane applications. So, the power

density is defined as the total power upon area. So, what happens when you do that? Half rho A V cube upon A, so, this A get cancelled.

What you get is Half rho V cube. So, that means your power density is proportional to the velocity to the power cube. So, irrespective of your the size of the rotor and number and other characteristics, you will be able to compare the systems that is why power density here define as the kinetic energy flux. Why it is flux? It is divided by area and also the power is nothing but rate of change of kinetic energy. This we have already seen.

So, because of this reason to compare systems irrespective of the rotor diameter, so, here power density is defined as the wind power upon area. So, then from this, we would get to know the power is proportional to the wind speed rise to the power 3. So, if you use to calculate power density, then half rho is given as 1.226 and V is given as 10 metres per second. So, what you would be getting is 613 watt per metre square.

So, what is the maximum obtained the power? So, the maximum obtained the power density is nothing but maximum power obtained maximum P upon A. So, then we already have that is half rho V cube. So, if you multiplied with 16 upon 27, then you would be getting. So, this is 8. 8 divided by 27 into rho is 1.226 into velocities 10, 10 to the power of cube. So, what you will be getting is 363.25 watt per metre square.

So, this is the maximum power density; one would be obtaining using this rotor diameter and not rotor diameter because it is independent of rotor diameter using the 10 metre per second velocity and air density of 1.226 kg per metre cube and then torque at maximum efficiency, we have formula $\frac{2}{27} \times 1.226 \times \text{rotor diameter}^3$ is nothing but 100 and V is 10 to the power of 3 so, N, N is rotations per minute, remember, here minute is given but other data the V is in metre per second.

So, you are supposed to convert minute into seconds. So, that is why we are converting 60 second as one minute. So, 40 upon 60. So, if you calculate this, then what you would be getting is 12974 Newton because torque is nothing but a force. So, what you get is in terms of Newton.

(Refer Slide Time: 33:03)

Wind Power Vs Height

A 25 m diameter wind turbine is placed on a 40 m tower in terrain with a power law coefficient (α) of 0.18. Find the ratio of available power in the wind at the highest point the rotor reaches to that at its lowest point.

Handwritten solution:

$D = 25 \text{ m}$
 hub height = 40 m
 $\alpha = 0.18$
 $h_{\text{low}} = 40 - 12.5 = 27.5$
 $h_{\text{high}} = 40 + 12.5 = 52.5$

$\frac{V}{V_0} = \left(\frac{h}{h_0}\right)^\alpha$
 $\frac{V}{27.5} = \left(\frac{52.5}{27.5}\right)^{0.18} = 1.12$
 $\left(\frac{P}{P_0}\right) = \left(\frac{V}{V_0}\right)^3 = (1.12)^3 = 1.417$
 $P = 1.417 P_0$

Find the size of a wind turbine rotor that will generate 120 kW of electrical power in a steady wind of 10 m/s. (Assume: air density is 1.225 kg/m^3 , $C_p = 16/27$ and $\eta = 1$).

Handwritten solution:

$P_m = \frac{1}{2} \rho A U^3 C_p$
 $120 \times 10^3 \text{ W} = \frac{1}{2} \times 1.225 \times \pi \times R^2 \times 10^3 \times \frac{16}{27}$
 $R^2 = \frac{120 \times 10^3 \times 27}{1.225 \times \pi \times 10^3 \times 16}$
 $R = 10.2 \text{ m}$

So, then next practice problem would be on power versus height. So, here the wind turbine diameter is given as 25 metre and is placed on a 40 metre tower in terrain with the power law coefficient is given alpha which is 0.18. Find the ratio of available power in the wind at the highest point of the rotor and lowest point. So, D is 25 and the hub height are the tower height is given as 40 metre. Alpha is given as 0.18.

So, here we need to use the formula V upon V_0 is nothing but h upon h_0 power alpha. So, the height we supposed to know here. So, what is height? Height low and h_{low} is nothing but if you imagine, so, this is how you have the rotor of like this. So, the rotor total diameter is given as 25. So, the maximum is here 12.5 and here 12.5. So, this height is given for us 40.

So, the lowest height is this 40 minus 12.5 because this part comes down, so, which is nothing but 27.5. The height at high is nothing but 40 plus 12.5 which is nothing but 52.5. So, if we want to calculate the V upon V_0 , then it would be h upon h_0 . So, this is 52.5 upon 27.5. Low is, we have taken as a reference. So, to the power alpha, it is 0.18, which is coming as 1.12 but that is not here asked. Here, it is asked to find the ratio of available power.

So, we already know if we want to calculate the power that is equivalent to the velocity to the power because power is half $A \rho U^3$. So, if both are same for 2 situations, if area and ρ is same, then it is proportional to U^3 only. So, P upon P_0 is nothing but V upon V_0 whole cube, so, which is equivalent to 1.12 to the power cube, what do you get is 1.417. So, how the power is nothing but 1.417 of the power at reference.

This ratio P/P_0 which is equivalent to 1.417 is the ratio of available power in the wind at the highest point that rotor reaches to that at its lowest point. Then the second one is sun. How to calculate output power? Find the size of a wind turbine rotor that will generate 120 kilowatt of electrical power in a steady wind of 10 metre per second. Assume air density is 1.225 kg per metre cube and the power coefficient is given as 16 upon 27. η is given as 1.

So, η is given as 1. C_P is given as 16/27 and velocity is given as 10 metre per second. We supposed to calculate wind turbine rotor. So, then A is nothing but πr^2 , but we do not know what is r . So, that is what we supposed to find it out and then the P_{out} is given. P_{out} which is nothing but 120 kilowatt. Remember the formula the P is nothing but $\frac{1}{2} \rho A U^3$ kg per metre cube metre square into metre squared upon second square. This is coming as kg metre.

ρ is kg per metre cube metre squared upon second square. So, what do you get is cube metre cube upon second cube. So, cube, cube get it cancelled, so, kg metre square upon second cube, so, that means kg metre second square into metre per second. So, this is nothing but force. Force into distance which is nothing but the r . This is force into metre Newton metre upon second, so, joules per second that is power.

So, remember, this is given us joules per second. Joules per second is nothing but watt. So, your power is given as kilowatt that one should be careful when you are substituting. So, P_{out} is given 120 into 10 to the power of 3 watts half ρ is given 1.225, area we do not know, π into r^2 and U is given 10 to the power of cube, mechanical efficiency is given as 1 and C_P is given as 16 upon 27.

So, if you want to calculate r^2 which is nothing but 120, so, this 10 to the power of 3, 10 to the power of 3, we can cancel it out. So, 120 into 2 that is in denominator, so, divided by 1.225 into π r 16 upon 27 is nothing but 0.593. So, this is nothing but r^2 . So, from this, if you calculate r , r should be 10.2 m. So, this is nothing but rotor diameter.

(Refer Slide Time: 39:16)

Wind Tunnel Performance Parameters

Power coefficient (C_p)

Power coefficient = $\frac{\text{Power of wind rotor}}{\text{Power available in the wind}}$

$$C_p = \frac{P_{\text{rotor}}}{\frac{1}{2} \rho A U^3}$$

Tip speed ratio (λ)

$$\lambda = \frac{\text{Blade tip speed}}{\text{Wind speed}} = \frac{R\Omega}{U}$$

R = rotor radius
 Ω = rotor angular velocity

Reynolds number

$$Re = \frac{\rho L u}{\mu} = \frac{u L}{\nu}$$

ρ is the density of the fluid (kg/m^3)
 u is the flow speed (m/s)
 L is a characteristic linear dimension (m)
 μ is the dynamic viscosity of the fluid (Pa·s)
 ν is the kinematic viscosity of the fluid (m^2/s)

A 30 m diameter, three-bladed wind turbine produces 600 kW at a wind speed (hub height) of 12 m/s. The air density is 1.225 kg/m^3 .

Find:

(a) The rotational speed (rpm) of the rotor at a tip speed ratio of 4.

(b) What is the tip speed (m/s)?

Solution

a) $\lambda = \frac{R\Omega}{U}$; $\Omega = \frac{\lambda U}{R} = \frac{4 \times 12}{15} = 3.2 \text{ rad/s}$

$\frac{2\pi N}{60} = \frac{\lambda U}{R}$

$N = \frac{\lambda U}{2\pi R} \times 60$

$\frac{2\pi N}{60} = 3.2 \text{ rad/s}$

$N = \frac{3.2 \times 60}{2\pi} = 30.55 \text{ rpm}$

b) $\lambda = \frac{R\Omega}{U}$

$\lambda U = R\Omega = 48 \text{ m/s}$

$N = \frac{3.2 \times 60}{2\pi} = 30.55 \text{ rpm}$

And then the next and last problem of this course is, calculating the rotational speed and then tip speed, the power coefficient already be defined and tip speed ratio also we defined already. So, this is sometimes you might require that Reynolds number that is $\rho u L$ upon μ . Then μ upon ρ is nothing but ν which is nothing but kinematic viscosity of the fluid. μ is dynamic viscosity. L is the characteristic linear dimension and u is nothing but flow speed and ρ is nothing but density of the fluid.

So, here the problem given is 30 metre diameter, 3 bladed wind turbine produces 600 kilowatts at wind speed of 12 metre per second. Air density is given. We supposed to find out rotational speed of the rotor at the tip speed ratio of 4 and then what is the tip speed at in metre per second. So, a tip speed ratio λ is nothing but $R\Omega$ upon U . So, this Ω can be used as this way as well.

So, again the notation the different books follow. You need not get confused. So, this also can be used. So, here the λ is given as 4 and R is also given because here 30 metre diameter, so, then R is nothing but 15 metre and U is given at the wind speed of 12 metre per second. So, what do you suppose to find out is Ω . So, 4 into 12 upon 15, so, this will come as 3.2 radians per second.

The second one, what we supposed to find it out is what is the tip speed. So, this particular thing is equivalent to $2\pi N$ upon 60. So, $2\pi N$ upon 60 which is Ω is nothing but 3.2 radians per second and N is nothing but 3.2 radians per second upon 2 into π . So, this is radians upon revolutions because they asked in RPM. So, what we suppose to do it in minute,

so, then we say 60 seconds upon one minute, so, radian-radian get cancelled, second-second get cancelled, so, revolutions per minute.

So, N is nothing but 3.2 into 60 divided by 2 into 3.14 . So, what you would be getting is 30.55 revolutions per minute. So, this is first question the rotational speed of the rotor at the tip speed ratio 4 . The second one is what is the tip speed. So, tip speed is nothing but λU which is equivalent to $R \omega$. So, R is what? R is 15 metre and ω is 3.2 radians per second. So, if you multiply 15 into 3.2 , so, it is 48 metre per second.

So, that is all about wind tunnel performance parameters. So, the suggested reading materials and references are these for all these 4 lectures whatever I have done in wind energy.

(Refer Slide Time: 42:52)

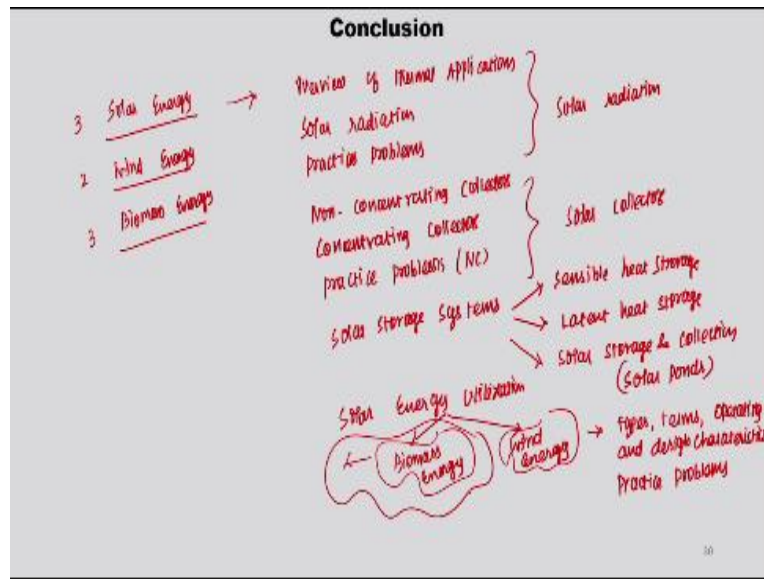
Suggested Reading Materials and References

1. S. P. Sukhatme and J. K. Nayak, Solar Energy: Principles of Thermal Collection and Storage, Tata McGraw Hill, 2015 ✓
2. S. Mathew, Wind Energy: Fundamentals Resource Analysis and Economics, Springer-Verlag New York, Inc., 2006. ✓
3. J. F. Manwell, J. G. McGowan and A. L. Rogers, Wind Energy Explained: Theory, Design and Application, John Wiley & Sons Ltd., 2009 ✓

29

So, the first week, we had basics and concepts. Second week, we had characteristics and the power extraction from wind energy. Apart from that the wind energy can be used to pump the water as well. So, there would be few problems that will be solved by Professor Goud in continuation to practice problems whatever we have done today. So, with that I would like to conclude the course. So, this is basically a 8 week course.

(Refer Slide Time: 43:29)



So, in that we covered solar energy and then wind and then biomass. So, we divided each module as 3 and then here 3 and then here 2. I started this course with solar energy. So, we had seen overview of thermal applications, then solar radiation and then we did practice problems. So, these are all about the solar radiation. Then second week of the course, we have done the solar collectors, non-concentrating collectors and concentrating collectors and then we did practice problems. So, this is we have done for non-concentrating collectors.

So, this concluded solar collectors for the second week and then in the third week, we started with solar storage system. So, in this, we have done the sensible heat storage and then latent heat storage and then we have seen special case of solar storage and collection, which is nothing but solar ponds and then in this particular week, we also had solar energy utilisation. So, in that we had 2 major category. One is on biomass energy systems and then wind energy system. So, here I introduced you the both concepts how it is derived from solar energy.

And then it was taken up by Professor Goud to explain the biomass energy rule and then we both came back and started the wind energy. So, in wind energy, we had types, terms, operating and design characteristics and we both done practice problems. And this part was conducted by Professor Goud also, I do not have much idea about that. So, this is what we wanted to execute. We hope, we could meet your expectations regarding this course.

So, with this conclusion slide, I would like to thank Centre for Education Technology, head and office team, project team and whoever involved with us in recording as well as editing for their help and support throughout the course. And then I would like to thank my co-

instructor of this course Professor Goud for all the discussions and support. And I would also like to thank my 2 TAs, Mr Brunel and Miss Verna for their help and uploading the assignments and preparing assignments and helping me in this course for all the extended help and support.

So, with that I would like to thank you for your passions throughout this course. And then if you have any further doubts apart from the discussion forum, you can write to both of us for any clarification, etcetera or if you are comfortable with TAs, if you have some doubts, you can as well discuss with them. Thank you. Thanks a lot.