Renewable Energy Engineering: Solar, Wind and Biomass Energy Systems
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Module No # 07
Lecture No # 33
Turbine Terms, Types and Theories: Part II

Hi everyone good morning so today's class is extension of yesterday's class of about turbine terms, types and theories. So yesterday we could probably cover the introduction about wind energy what is the source? And how to harvest wind energy in terms of electrical power and pumping power. And also we had seen brief history about wind energy and today we are going to continue the turbine terms, types and theories.

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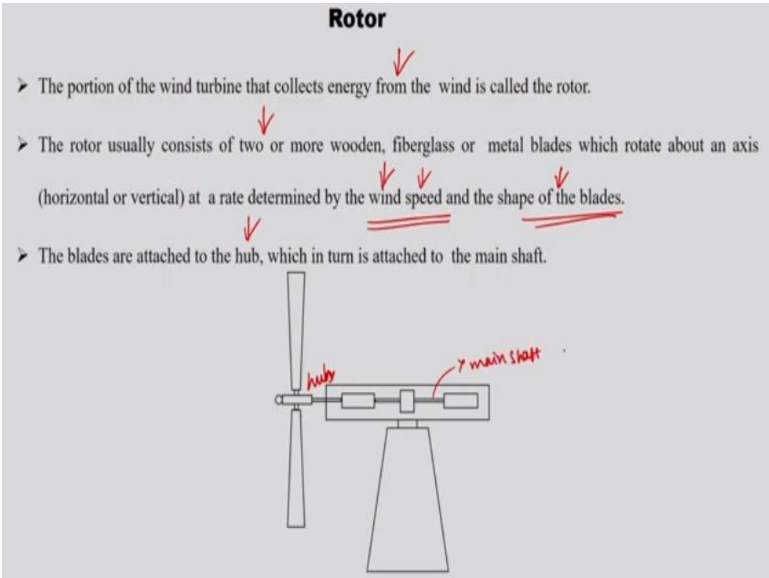

So yesterday we left here we discussed about vertical axis wind machines and horizontal axis wind machines. And wind machines can be used for converting wind energy into electrical power as well as pumping power. And we have seen extensively about components of the wind mill so which are rotor transmission system generator and tower. We supposed to start mathematical expression for governing wind power.

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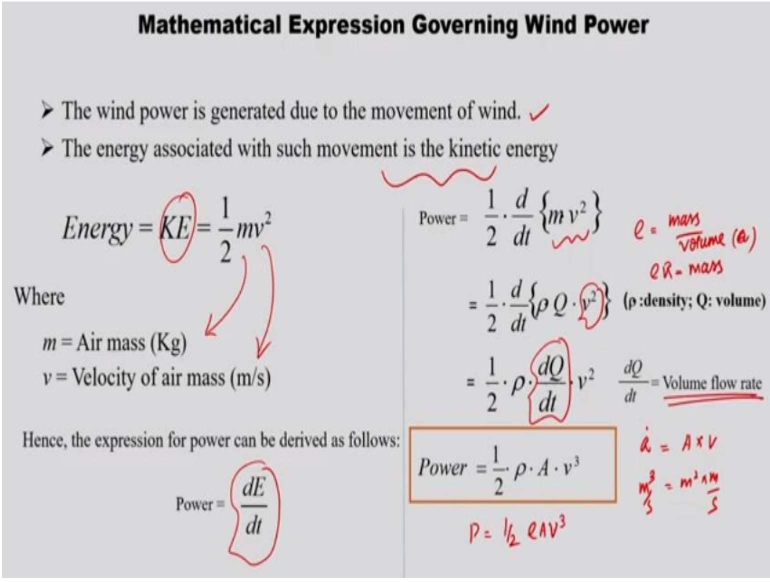

So the wind power is generated due to moment of wind so this we had already seen the energy associated with such moment is kinetic energy. So this is what we have elaborately discussed yesterday. And energy here is a kinetic energy which is nothing half mv square. So m is nothing but here the air mass and v is nothing but velocity of the air mass which is a meter per second. So and we have already seen in our introduction classes some basic definition of power which is nothing but rate of change of energy.

So which is nothing but d dt of E so we know what is that energy here is nothing but kinetic energy which is nothing but m v square which is mass and velocity of the air mass square. And here the mass can written as in terms of density is nothing but mass upon volume. So mass can be written as rho v so here since we already used the v as velocity terms so here it is represented as a Q.

So volume is here given as Q so rho Q which is equivalent to mass so rho Q v square so this d dt of Q which is nothing but a rate of change of volume which is called as volumetric flow rate. So half rho and volumetric flow rate which is; nothing but Q dot which is nothing but rate of change of volume so this can be written as area into velocity. So which is again meter square m upon s so rate this also metric cube per second. So this can be written as area into velocity.

So area into velocity into velocity square is nothing but velocity Q so power can be written as half rho A v cube.

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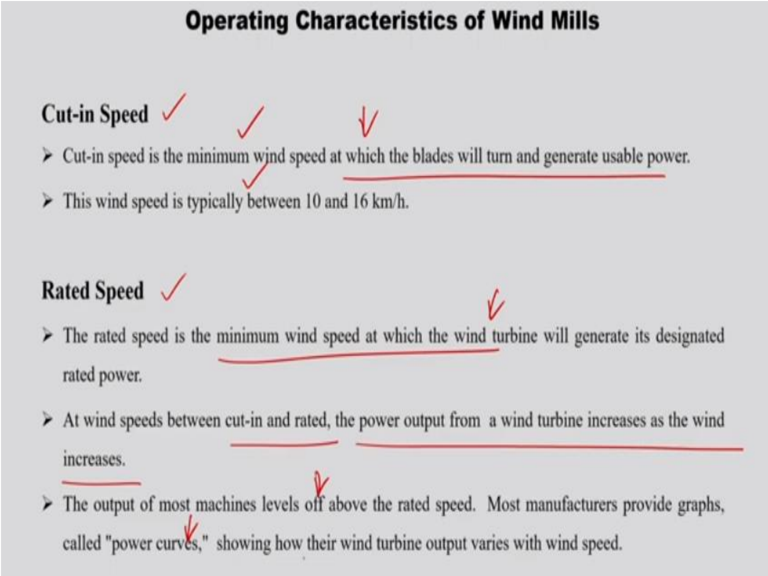

So before going into that Betz limit what is the maximum theoretical limit of harvesting wind energy? So we would review certain operating characteristic of wind mills because yesterday when I was introducing about certain wind mills I used Cut-in speed Cut-out Speed and Rated speed. So we will see what is that? So the Cut-in speed is nothing but maximum wind speed at which the blades will turn and generate useful power and this wind speed is typically between; 10 to 16 kilometer per hour.

So this is the speed at which the blades will turn and generate useful power rated speed. The rated speed is the minimum wind speed at which the wind turbine will generate its designated rated power. So this is the speed the wind mills are designed for minimum wind speed at wind speeds between Cut-in and rated the power output from a wind turbine increases as the wind increases.

So even though it is designed for a rated speed so based on your speed there may be lesser speed. So the Cut-in speed is nothing but the minimum in speed at which the blade will turn. So that is the Cut-in speed, rated speed is the speed at which the wind turbine is designated or designed. So between these 2 limits based on the wind speed, if wind speed increases the power output also increases from the speed between Cut-in speeds at rated speed.

The output of most machines levels of above the rated speed most manufacturers provide graphs called power curves which shows how their wind turbine various with the wind speed.

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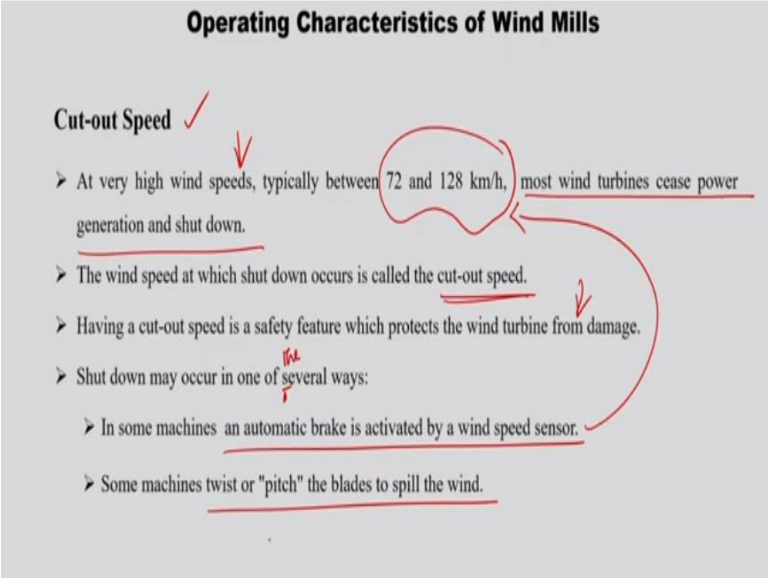

Then there is a speed called Cut-out speed so at very high wind speed typically between 72 to 128 kilometer per hour most wind turbines cease power generation and shut down. Because the designed wind mills will not be able to take it up that high or heavy speed. So this wind speed at which shut down occurs is called Cut-out speed having a Cut-out speed is a safety feature which protects the wind turbine from damage.

So shut down may occur in one of the several ways. In some machines an automatic break is activated by a wind speed sensor. So this wind speed sensors if the wind speed is greater than 72 kilometer per hour or not or whatever is the Cut-out speed and then it automatically tells the automatic break to Cut-down. And some machines twist or pitch the blades to spill the wind so instead of taking the wind it is spills out the wind.

So that is also some machines as the twist or pitch moment so this is about Cut-in speed and rated speed and Cut-out speed.

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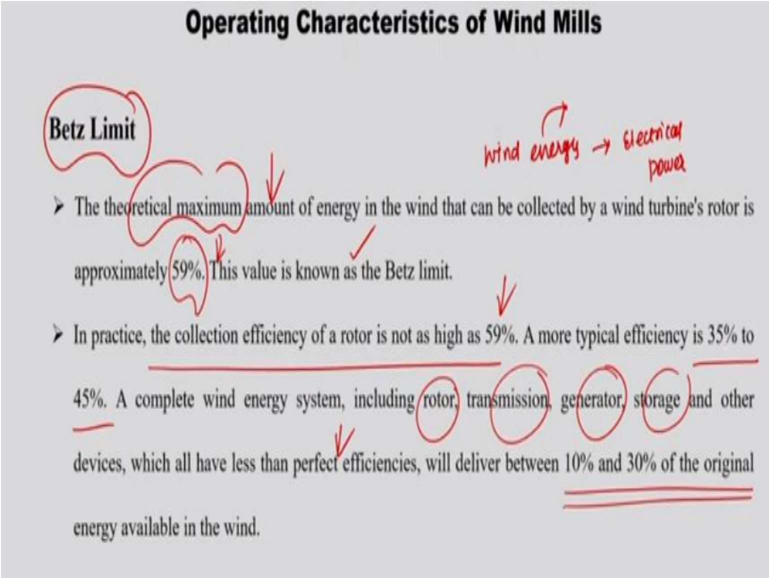

And something called Betz limit this is what yesterday I was telling about. So the theoretical maximum amount of energy in the wind that can be collected by a wind turbine’s rotor as approximately for 59%. So this value is known as the Betz limit so our ultimate aim is to convert wind energy into electrical power. So in case how much wind energy can be converted how much is the maximum conversion possible?

So this theoretical maximum itself is a 59% we already discussed about this as well because kinetic energy is half mc square. So when you make the wind speed or when it touches the rotor it becomes 0 the wind speed becomes 0 then you are converting almost all energy into useful power which is nothing but electrical power here. But that should not happen because when you ceases the wind to flow then it stays over there it will not give a rule for further air to come in and extract the power.

So because of which so there should be some maximum theoretical limit so with which the velocity of the air is to be reduced. So that theoretical maximum is about 59% but in practice the collection efficiency of the rotor is not high as 59%. Because we; always theoretical is nothing but ideal limit but in practice you would be having another loss as well. So a more typical efficiency for the wind turbine is 35 to 45%.

So a complete wind energy system including rotor, transmission, generator and storage and other devices which all have less than perfect efficiency will deliver between 10 to 30% of its original energy available in the wind. So because of which all these losses related to rotor transmission system generator, storage and other devices. So you will not be able to get the maximum of 60% around 59% but you get around 35 to 45%. Anyway this we will derive today after finishing this design function requirements.

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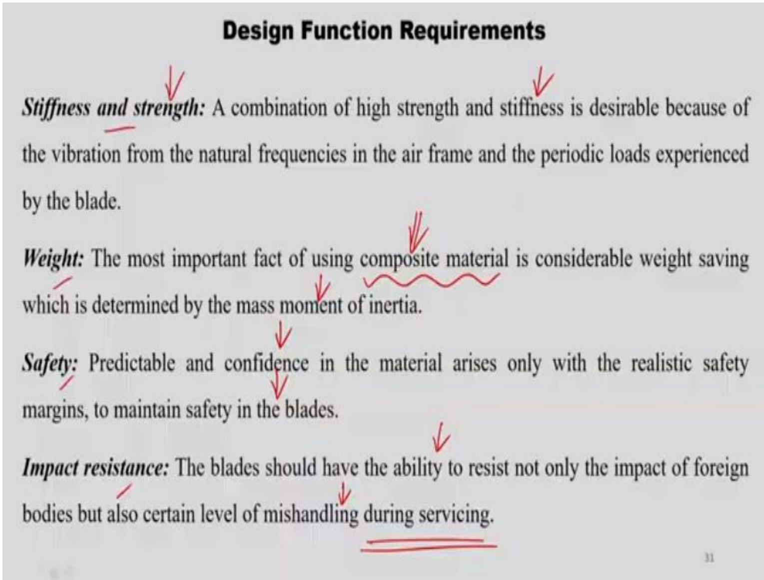

So for design function requirements other than this calculation of efficiency so you need to have certain properties at its best.

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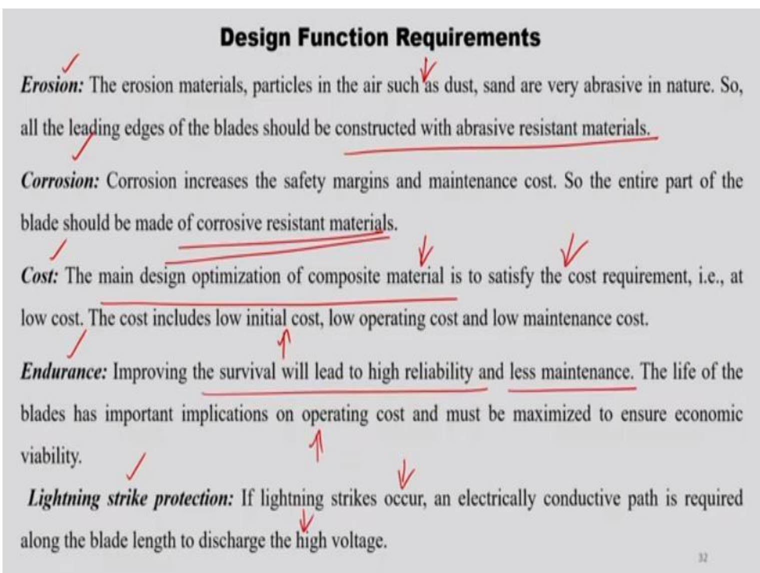

So those properties are stiffness and strength weight, safety, impact resistance, erosion, corrosion, cost, endurance, lightning strike protection. So if you see stiffness and strength that combination of high strength and stiffness desirable because of the vibration from the natural frequencies in the air frame. And periodic loads experiences by the blade so your blade should have good stiffness and strength and then weight this is most important factor alright.

So nowadays the composite material, are being used to have a better qualities is considerable weight saving which is determined by the mass moment of inertia. For example one material may be good in stiffness strength etc., but weight of the material may be large so to compensate all the properties with less weight or the weight required by the mass moment of inertia. So you might use composite material. Composite material is nothing but more than 1 material is composited together to deliver all the required properties.

For example 1 particular material may be good at one particular property or it may be bad at another property. But composite material is something so I will sandwich 2 or more material to deliver all the properties in a desirable limit. So that is why nowadays the interest is towards composite material to deliver all the properties at a reasonable limit. The safety predictable and confidence in the material arises only with the realistic safety margins to maintain safety in the blades.

Safety is also important. So impact resistance the blade should have the ability to resist not only the impact of foreign bodies but also certain levels of mishandling during servicing alright. So we may not expect only air comes and hits the blade alright for power generation. So there may be other dust particles etc., whatever comes along with the air also there. So the blades whatever we are using so that should have the ability to resist not only the air but the impact of foreign bodies which comes along with the air.

And also during servicing there may be mishandling of the blades then the blades should have impact assistance over there as well. The erosion materials particles in air such as dust, sand are very abrasive in nature this I have already told. So, all leading edges of the blade should be constructed with abrasive resistant materials so same thing with corrosion. So it should also have corrosive resistive materials because the corrosion increases the maintenance cost.

And cost the main design optimization of composite material is to satisfy the cost requirement so I already told composite material is designed in such a way that to satisfy all the requirements.

So that means to give all the properties at a reasonable rate but there is another factor which also to be considered is cost. Even when we were discussing solar energy we discussed about net present value. So it is not only the design is important it is also economically viable design is important.

So in that case the main design optimization of composite material if at all we intend to use composite material for the design then it should satisfy the cost requirement as well. So that means at low cost the low cost includes initial cost, operating cost as well as maintenance cost. And then endurance improving the survival will lead to high reliability less maintenance. The life of the blades as important implications on operating cost and must be maximized to ensure economic viability.

So the long term survival of the blades also important so lighting strike protection if lightning strikes occur the electrically conductive path is required along the blade length to discharge high voltage during lightning. So these are all certain design function requirements this also to be considered while designing a wind turbine it is not only design to grab maximum wind energy to generate electrical power or pumping power. So these are all also additional design constraints which to be taken care by the designer.

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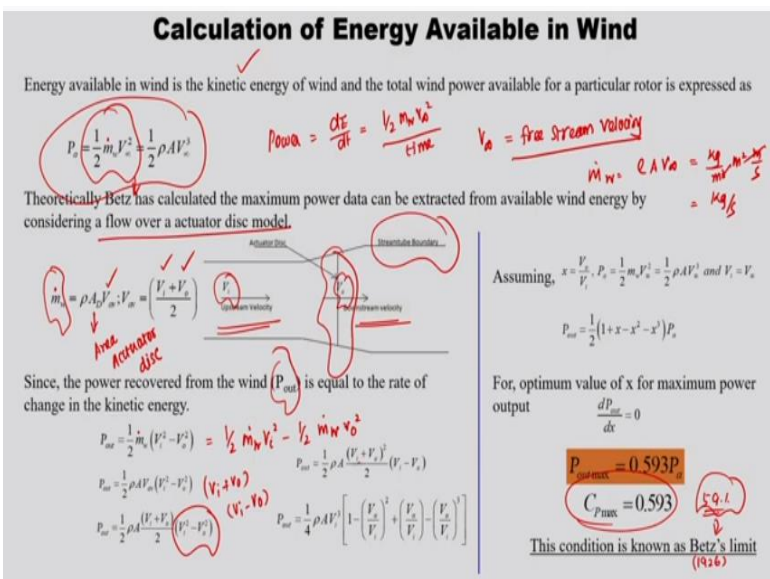

So now we are here to calculate how that 59% is told by Betz, so probably in 1926. So he derived this maximum theoretical limit of 59% power extracted from the wind energy. So energy available in the wind is the kinetic energy of the wind and total wind power available for a particular rotor is expresses as half m w V infinity square. So this is nothing but here so it is a mass flow rate because the power is nothing but rate of energy.

So energy per time so energy is half m w V infinity square upon time so which is given here as half m dot w V infinity square. So V infinity here is nothing but free stream velocity so m w is nothing but mass of the air m dot w is mass flow rate of the air. So mass flow rate of the air we can write it as rho A V infinity so what is rho? Rho is density of the air A is the rotor area and V infinity is the free stream velocity. So kg per meter cube meter square meter per second so which is nothing but kg per second so which is nothing but mass flow rate.

So half rho A V infinity square into V infinity is V infinity cube. So this is nothing but the power available in the wind as a kinetic energy of wind alright. So now Betz so he calculated the maximum power data that can be extracted from available wind energy by considering the flow over a, actuator disc model alright. So he considered the rotor as a, actuated disc so if you see this particular arrangement yesterday we have also discussed about the Bernoulli's equation the same arrangement.

So here is your upstream velocity and here is your downstream velocity so the rotor component is replaced as a, actuated disc. So upstream velocity is here as a V i and downstream velocity is V naught. And total arrangement is considered as a stream tube. So from here he is calculating the mass flow of the bend which is nothing but Rho A D. A D is nothing but an area of the actuated disc and V average is nothing but initial velocity that is nothing but upstream velocity and V naught is downstream velocity upon 2.

So we know what is the mass flow rate of the wind? So since the power is recovered from the wind which is nothing but P out is nothing but the rate of change of kinetic energy. So half m w V i square – V naught square so which is equivalent to half m dot w V i square so that is due to upstream velocity and due to downstream velocity is nothing but V out square. So when you are doing so half m w dot which is nothing but mass flow rate V i square – V naught square.

So P out is nothing but m dot w we know already rho A D V average into V i square – V naught square. So rho A D so here the area of the actuated disc A D is represented as A only and V average is nothing but V i + V naught upon 2 and V i square – V naught square. So now this V i square – V naught square can be written as V i + V naught into V i – V naught. So if you substitute here of rho A V i + V naught whole square upon 2 into V i – V naught alright. So this 2 comes here 1 upon 4 rho A V i cube we are taking out.

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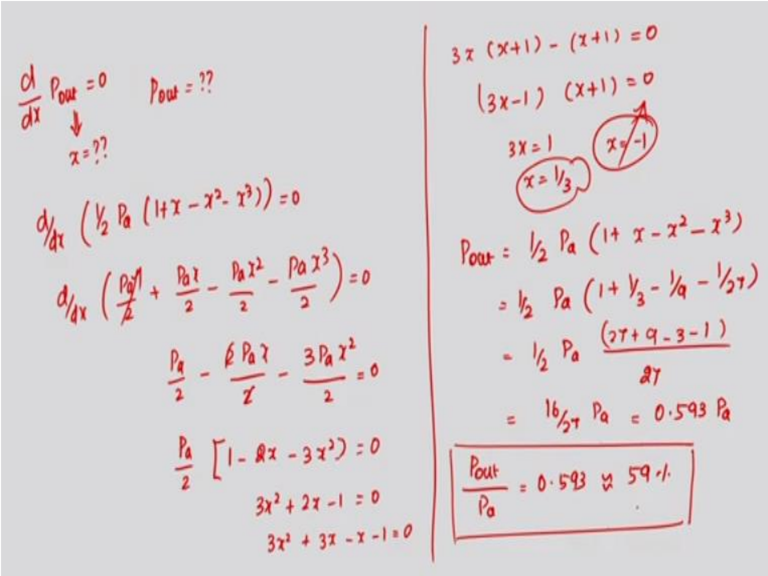

So T out here is half m dot w V i square –V naught square. So we already know half rho A D V average into V i square – V naught square. Remember this V average is taken as a, upstream velocity plus downstream velocity upon 2 that is nothing average between upstream velocity and downstream velocity. So if you ask me what is the reason behind it? So we will do momentum analysis also for the same actuated disc model and then we will prove this.

So why he has taken V i + V naught upon 2, so there are certain assumptions that we will discuss later. So here if you see this can be written as V i + V naught V i – V naught so this is 2 so half rho A D V i + V naught whole square V i – V naught upon 2. So if you take 1 upon 4 rho A D V i + V naught whole square V i –V naught. So if you write 1 upon 4 rho A D V i square + 2 V i V naught + V naught square into V i – V naught which is equivalent to 1 upon 4 rho A D V i cube I am doing multiplication +2 V i square V naught + V i V naught square.

For V naught V i square – 2 V i V naught square – V naught cube. So 2 V i square V naught and V i square V naught. So if you remove then 1 V i square V naught would come and V i V naught square so if you remove then 1 – V naught square would come. So which is equivalent to 1 upon

4 rho A D V i cube + V i square V naught – V i V naught square – V naught cube. So if you take V i cube out it should be 1+ upon V i because you have to multiply and divide by V i whole cube.

So V i cube if you divided by V i cube this is –V i upon V naught upon V i whole square – V naught upon V i whole cube. So we are writing half into half rho A D V i cube into 1 + V naught upon V i – V naught upon V i whole square –V naught upon V i whole cube. So what is this? Here we are making certain assumptions the free stream velocity is nothing but equivalent to upstream velocity.

And V naught upon V i we will take as a x just for simplification and half rho A D V infinity square this is nothing but actual power available with the wind of V A 1 + x – x square – x cube. So this is nothing but P out alright so what is that we require? We require maximum P out happen at what x? So what we supposed to do d upon d x P out is equivalent to 0. So from here we will determine what is x?

And then substitute in the P out equation and say that is the maximum power output available. So what is P out? P out is nothing but d dx of half P a 1+ x – x square – x cube which is equivalent to 0. So first we will work on those one P a by 2 + x by 2 – P a x square by 2 – P a x cube by 2. Which is equivalent to 0 so if you differentiate with respect to x so this is constant so this will go the second term would remain as P a upon 2 third term would remain as 2 P a x upon 2, 2 gets cancelled.

And then 3 P a x square upon 2 which is equivalent to 0 so if you take P a upon by 2 out it should be 1 by because 2 you have taken. So you would be multiplying with 2 so P a, upon 2 1 – 2x – 3x square which is equivalent to 0. So, 3x square +2x– 1 which is equivalent to 0. So 3x square + 3x –x – 1 = 0 so 3x, x + 1 – x + 1 so 3x + 1 – x + 1 which; is equivalent to 0. So either it is 3x – 1 or x + 1 equivalent to 0 so 3x = 1 or x = 1 by 3 or x = -1 so the minus cannot happen so x = 1 by 3.

So we know P out is nothing but half P a 1 + x – x square – x cube so half P a 1 + 1 by 3 – 1 upon 9 – 1 upon 27. So if you take LCM 27 + 9 – 3 – 1 so which is nothing but 16 by 27 P a so

16 upon 27 is nothing but 0.593 P a. So the resistor output power so this is the actual power so which is equivalent to 0.593 so that means so you would be able to extract only 59% of the wind power for conversion.

So this is what the maximum theoretical limit which is nothing but Betz limit. As I already said he considered the wind turbine the rotor then wind comes and hits and converted into other form of energy. So he considered that arrangement as the actuator risk so if such analogy is made so what are all the assumptions made here. And also we said that it is the theoretical possible limit.

So if it is the theoretical possible limit then when it goes to practice so what should be the assumptions considered or relaxed. So, that we will discuss in terms of moment analysis so that is what done here.

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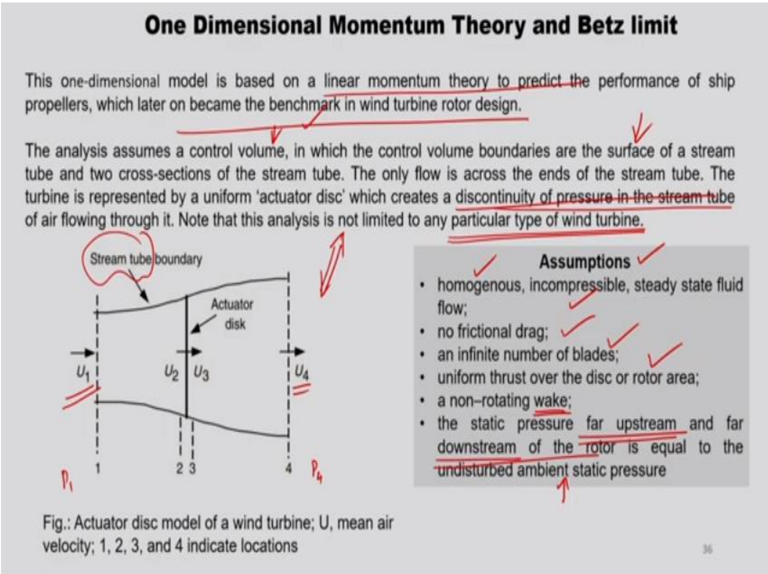

So it is done based on 1 dimensional momentum theory and Betz limit so this one dimensional model is based on the linear momentum theory to predict the performance of ship propellers. So, that he; used as a bench mark in wind turbine rotor design. So the analysis assume as a control volume in which control volume boundary are the surface of the stream tube. So this we already told that the boundaries considered as a straight stream boundaries.

And cross section of the stream tube the flow occurs only in the ends of the stream tube so that is nothing but entering is here outgoing is here. And the turbine is presented as a uniform actuated disc which creates the discontinuity in the pressure. So the pressure drop occurs here so that is why we have given 2 and 3 so this is U 2 is just face of the actuated disc and behind the actuated disc it is U 3.

So that is why pressure drop occurs and note that this analysis is not limited to any particular type of wind turbine. So you can use it as a common one and the assumptions made so without these assumptions those, whatever we derived as a 59% may not be valid. So one is homogenous incompressible steady state fluid flow and remember, here it is assumed that you know what is called linear momentum analysis.

And also what is called control volume and stream tube etc., because this, itself is a separate study because here in most of the time so there is a, analogy between aerodynamic principles and wind turbine design. So we used certain principles from aerodynamics to wind turbine design yesterday you have seen the chord length to thickness ratio should be high to have high lift force etc.

So one should keep in mind that as well because here we are not discussing in depth you supposed to be comfortable with what is control volume analysis? What is control mass analysis? What is stream tube, stream lines and momentum theory etc.? But I am here putting as simple as possible but still if we go in depth about each term then it would be difficult finish this particular topic within the required number of lectures. And also it is not a part of this particular course.

And assumptions here homogeneous so that means similar there is no different phases involved incompressible where you are density changes with temperature and pressure is moderate. And steady state analysis there is no change with the time and no friction drag is involved the blades are infinite number of blades. If you have infinite number of blades then there would be a, tip losses but we have not taken into account those losses while deriving the Betz limit.

And uniform thrust to over a disc or rotor area and non-rotating wake. So if there is a wake formation some of the energy lost over there as well when you employ this wind turbine in practice. And the static pressure far upstream and far downstream of the rotor is equivalent to the undisturbed ambient static pressure. So that means the P 1 and P 4 are equivalent to ambient static pressure.

So the assumption of your wind turbine rotor into actuates disc model is a simple model which as all these assumptions and derived the maximum theoretical possible format is 59% is that one should keep in mind.

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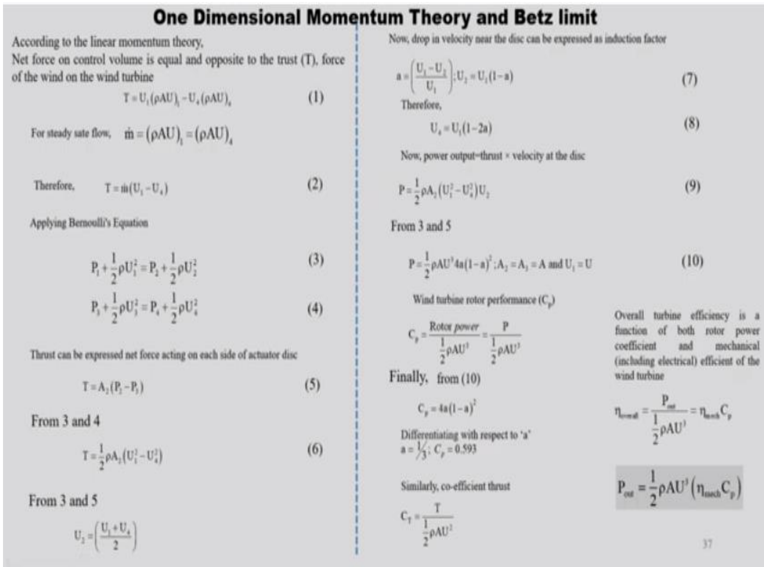

So the derivation here is given in one slide for you to have crisp of all the analysis but however I will derive on my own. So here we are going to derive 1 dimensional linear momentum theory to calculate maximum available power.

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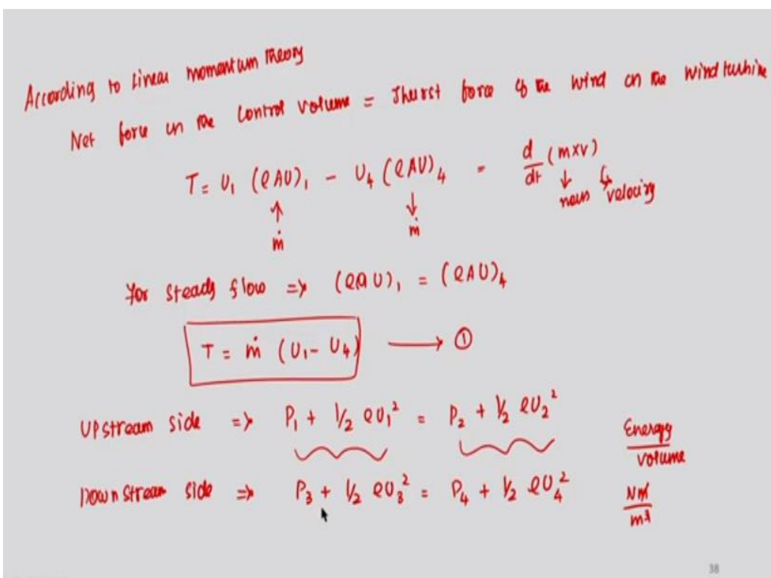

So; according to linear momentum theory the net force on the control volume equal to the thrust force of the wind on the wind turbine. So thrust force is nothing but U 1 which is upstream velocity rho A U at 1 so which is nothing but mass flow rate. And –U 4 so if you remember here in this U 1 is upstream U 4 is downstream and actuated disc this side is U 2 that side is U 3. All are linear velocities so U 4 rho A U 4 so this is also mass flow rate.

So if you remember the force is nothing but rate of change of momentum so d by dt of m into v. So m is mass v is velocity so that is what written here velocity is nothing but thrust force velocity is nothing but U 1 rho A U at 1 or 4 is nothing but mass flow rate. So for steady state flow so that is one of the assumption for steady flow so what you get is rho a U at 1 which is equivalent to rho U at 4. If you substitute here your T should be mass flow rate at because rho a u 1 or 4 does not matter mass flow rate into U 1 – U 4.

So this is I will consider equation 1 as a thrust force we calculated. And if you remember this discussion on a Bernoulli's equation where upstream side if we write so which is nothing but P 1 + half rho U 1 square which is equivalent to P 2 + half rho U 2 square. So what we are doing here? We are equating the energy per unit volume which is nothing but energy density. So we are not here considering here the pressure energy as well as kinetic energy upon volume because there is no potential energy term because it is at a same height.

So this is for upstream side if you write same for downstream side. So what you get is rho 3 + half rho U 3 square + P 4 half rho U 4 square. So this unit consistency everything we have discussed yesterday I am not going into detail because it is nothing but an energy per volume. So energy is nothing but joules, which is nothing but newton meter upon 1 meter cube so Newton per meter square.

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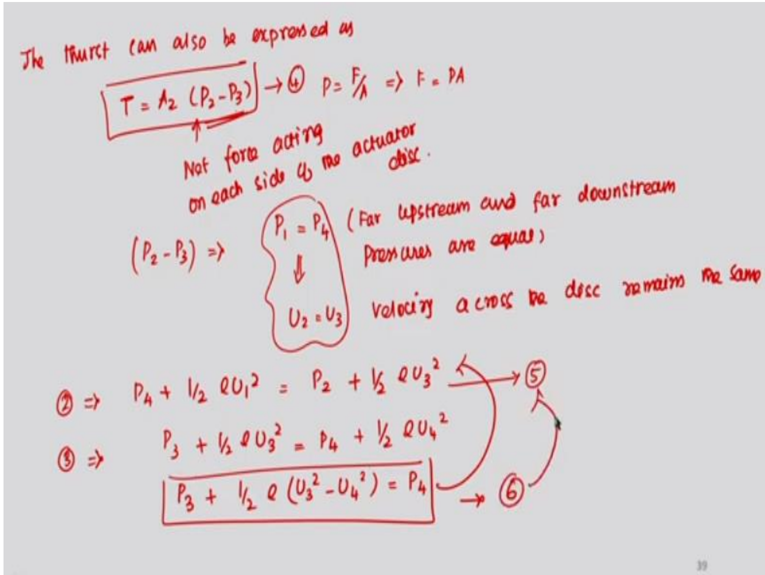

So which is, nothing but a pressure energy so then the thrust can also be expressed as thrust is nothing but A 2 into P 2 – P 3. Remember the locations 2 and 3 are in the disc location alright so that you supposed to remember so why it is written? Because we know the pressure is nothing but force upon area. So force can be written as P into A that is what we have written here? Because the rotor or disc area is same but the upstream side and downstream side of the actuated disc will have different pressure.

So that is why P 2 – P 3 net thrust because there should not be any misunderstanding it is nothing but net force acting on each side of the actuated disc. So now to get this pressure difference which is nothing but P2 – P 3 we are going to use another assumption so what is that assumption? That for upstream for downstream so that particular assumption we are going to use the assumption is P 1 = P 4 alright.

So for upstream and for downstream pressures are equal and same way so we can say U 2 is also U 3 so which is nothing but velocity across the disc remains the same. So using these assumptions going back to the Bernoulli’s equation so, this I will put as equation number 2 and equation number 3. So in the equation number 2 I am going to replace that P 1 as a P 4 as well as U 2 as U 3 based on our assumptions. So if I write then my equation 2 becomes P 4 + half rho U 1 square which is equivalent to P 2 + half rho U 3 square.

And the downstream we already we have P 3 + half rho U 3 square which is equivalent to P 4 + half rho U 4 square. So if you do certain mathematical manipulations if we write P 3 + half rho U 3 square – U 4 square which is equivalent to P 4 so we got one more equation. So this P 4 we are going to substitute here so this I will call it as a, equation number so this we will put it as equation number 4 and this I will put it as 5 this is 6.

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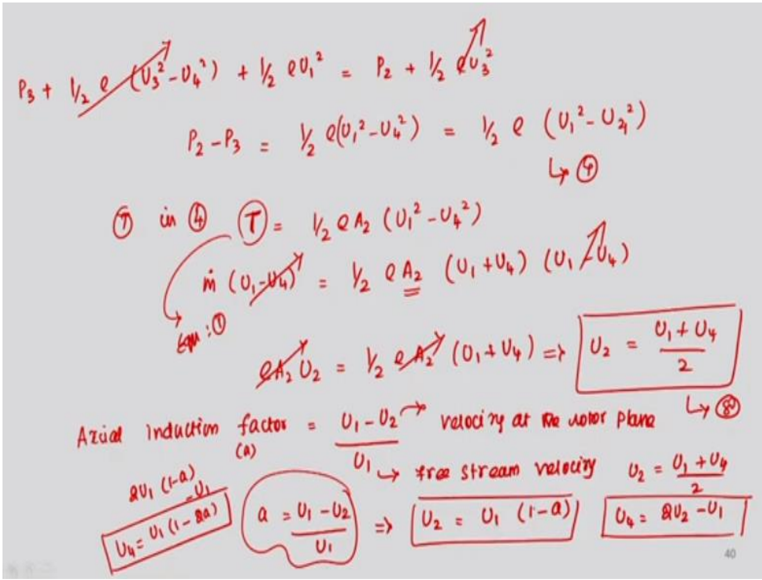

So I am going to substitute 6 in 5 so if we do that P 3 + half rho U 3 square – U 4 square which is nothing but P 4 and + half rho U 1 square is already there which is equivalent to P 2 + half rho. So it is P 4, P 2 so P 4 we have substituted in terms of P 3 and rho U 1 square is there. So here it should be P 1 which is again converted into P 4. P 4 we have substituted and what we have is? Half rho U 1 square and then P 2 + half rho U 3 square which is also assumption was being applied.

So half rho U 3 square so this and this first term goes away and what you get is P 2 – P 3 so if this goes that side and what you get is half already you have rho U 1 square – U 4 square so we derive P 2 – P 3 which is half rho U 1 square – U 4 square. So remember here this rho so rho is density and this is I will put it as equation number 7 and I will substitute 7 in 4 to calculate the thrust. So which nothing but A 2 half rho A 2 P 2 – P 3 is nothing but U 1 square – U 4 square and then if we do some manipulation.

So what is that? So the thrust so thrust I can write it as mass flow rate into velocity because already we have 1 thrust equation number 1. So from equation number 1 I am substituting here U 1 – U 4 which is equivalent to half rho A 2 U 1 + U 4 into U 1 – U 4. So this U 1 square - U 4 square I have expanded so U 1 – U 4 will get cancelled and mass flow rate how we can write.

So rho because we are considering with A 2 so we will also do it with rho A 2 U 2 which is equivalent to half rho A 2 into U 1 + U 4. So rho A 2 goes so we are telling that U 2 is nothing but U 1 + U 4 upon 2. So which is nothing but upstream velocity plus downstream velocity upon 2 it is average. So that is what if you have seen here I already told so the momentum analysis we will prove that so V i + V naught upon V 2.

So that is nothing but average velocity already we have done alright so we proved as a U 2 and then we are introducing here something called axial induction factor so which is nothing but U 1 – U 2 upon U 1. So U 1 is upstream velocity. So U 2 is nothing but the velocity at the rotor plane. So upstream velocity we can call it as a free stream velocity as well. So this probably we will put it as equation number 8, free stream velocity U 2 is nothing but velocity at the rotor plane.

So this is called as a, this a, is nothing but U 1 – U2 upon U 1 so if you want to write U 2 in terms of a so this can be written as U 1 into 1- because 1- U 2. So this is 1- U 2 upon U 1 so if you take U 1 this side so it is nothing but U 1 into 1- a. So if you want to write U 2 so because U 2 is nothing but U 1 + U 4 upon 2 so U 4 is nothing but 2 U 2 – U 1. So 2 U 2 is there 2 U 1 – 1- a – U 1 so if you take U 1 out so what you get is 1 – 2a, because this is 2 U 1 so you will get U 1 – 2 U 1 a.

So U 1 take it out so U 4 is nothing but U 1 into 1 – 2a so what we have done we have defined axial induction factor. And we defined our downstream velocity as well as the U 2 which is nothing but the velocity at the rotor plane in terms of free stream velocity and axis induction factor.

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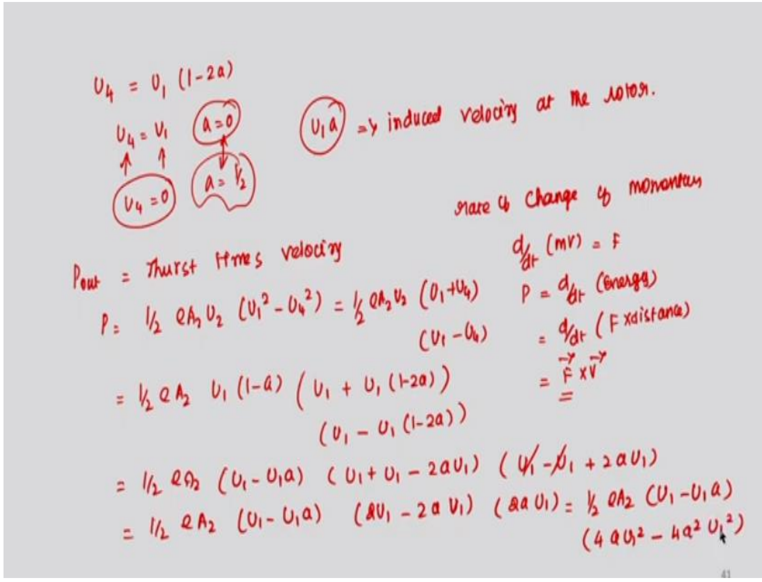

So if you see here the U 4 is nothing but downstream velocity so that is after leaving it static energy to that turbine how much is the velocity of the air stream going out. So which is nothing but U 1 into 1 – 2a so if a = 0 then U 4 becomes U 1 at a = 0. So that means what your downstream velocity as well as your free stream velocity is same so the no energy has been harvested.

And if you put a = 1 upon 2 so this becomes U 4 becomes 0 so that also cannot happen that is a maximum thing but we have also seen thermodynamics loss. So you cannot any energy fully there should be some losses when you are introduction system like this. So the practical components when you have components there would be losses and you cannot get 100% efficiency.

So in that case your, a, should be lie in 0 to 1 by 2 so that is the implication so here this U 1 into a so this particular product component is called induced velocity at the rotor. So this we will see later where here using it so now we are ready with the, a factor but now we supposed to calculate P out. So this is also thrust times velocity so P is nothing but half rho A 2 U 2 and that is nothing but thrust mass flow rate into U 1 square – U 4 square.

if you see here thrust we define here thrust is half rho A 2 U 1 square – U 4 square but here what we are calculating here P out which is nothing but thrust times velocity. So rho A 2 U 1 square – U 4 square and it should be multiplied by U 2. For example if you see a rate of change of momentum is nothing but force m v which is nothing but force if you see power that is nothing but d, dt of energy rate of change of energy.

So energy can also be written as force into distance so distance upon time is nothing but velocity. So force into velocity so force is nothing but thrust here and multiplied with velocity is nothing but your P out. So this can be written as rho A 2 into U 2 U 1 + U 4 U 1 – U 4. So now we have for U 1 or what to substitute you have U 2 defined as U 1 into 1 – A and U 4 is defined as U 1 into 1 – 2a. So half rho A 2 U 2 I can write as U 1 into 1 – a and U 1 + U 4 can be written as U 1 into 1 – 2a and U 1 – U 1 into 1 – 2a.

So if we write half rho A 2 U 1 – U 1 a which is equivalent to U 1 + U 1 – 2a U 1 into U 1 – U 1 + 2a U 1. So this gets cancelled so what you get is 2a 1 only in this terms so half rho A 2 U 1 – U 1 a which is equivalent to 2 U 1 – 2a U 1 which is equivalent to so 2a U 1 equal to half rho A 2 U 1 – U 1 a, into 2 U 1 – 2a U 1 into 2a U 1. So if you simply further half rho A 2 U 1 – U 1 a so this term I am multiplying so 4a U 1 square – 4 a square U 1 square.

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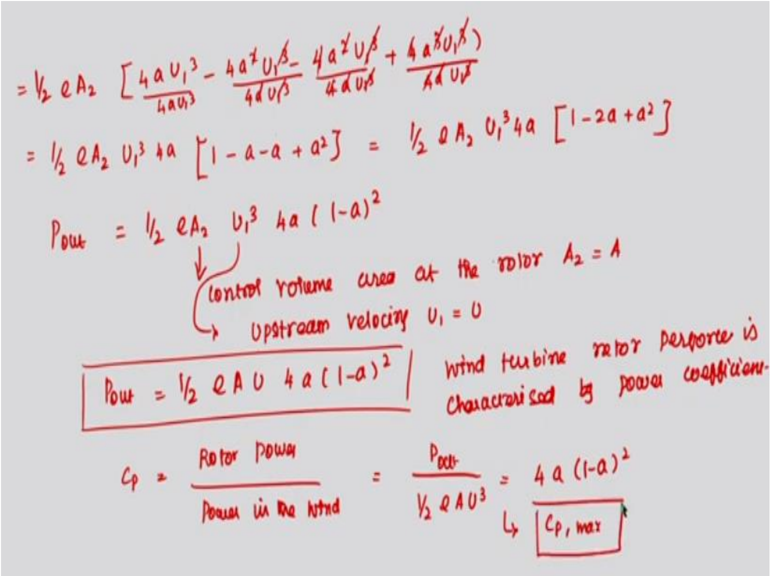

So simply further rho A 2 so you have 2 terms here and this is multiplied with this is another term. So what you would get as 4a U 1 cube – 4a square U 1 cube – 4a square U 1 cube + 4a cube U 1 cube. So rho A square so you take it out U 1 cube 4 a and what you would get as 1 – 4a U 1 cube so what you get is 1- this is a 4a U 1 cube half rho A 2 so you are taking out U 1 cube into 4a so this is 1.

If you do it 1 – a U 1, U 1 cube get cancelled and 4, 4 gets cancelled another a U 1 cube, U 1 cube gets cancelled what you get is a square + a square so which is nothing but half rho A 2 U 1 cube 4a into 1 – 2a + a square. So if you write so P out which is nothing but; half a 2 rho U 1 cube 4a, into 1 – a whole square. So this is control volume area at the rotor so this we write it as normal A and this is nothing but upstream velocity. So this we can write it as U 1 as U which is nothing but free stream velocity.

So if we write then P out should be half rho A U 4a, into 1 – a whole square because 1 – a squared – 2a, is nothing but 1 – a whole squared. So this is the output power available so this wind turbine rotor performance is characterized by power coefficient. So what is this power coefficient? Which is nothing but rotor power divided by power in the wind so rotor power is P whatever we derived which is nothing but P out the actual power in the wind is half rho A U cube right this we already knew.

So if you compare this then you what you get is 4 a, into 1 – a whole squared so this is what your C p but what we intend to derive is C p max what should be my maximum power coefficient for that what you supposed to do?

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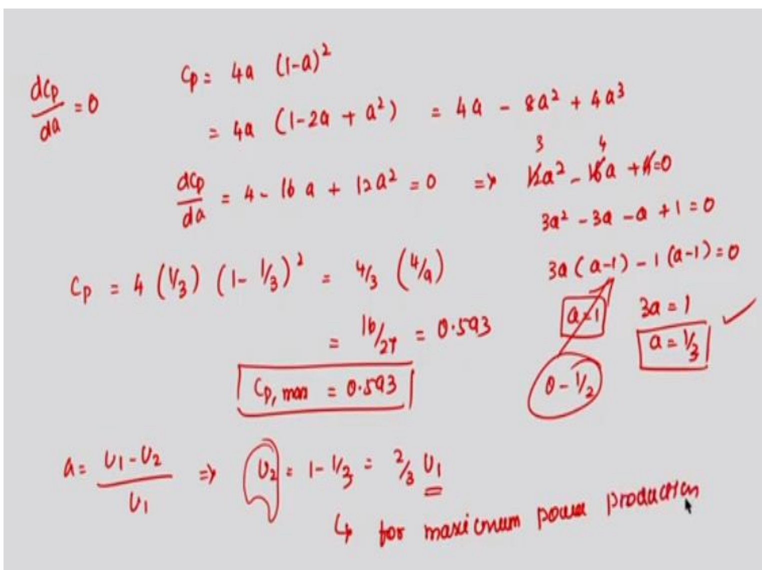

You supposed to differentiate with respective x equal to 0 alright so what is C p? C p is nothing but 4a, into 1 – a whole squared so 4a 1 – 2a + a square which is nothing but 4a – 8a square + 4 a cube. So if you take C p upon d x which is not d x it should be differentiated with respected to a. Because we are working with axial induction factor so what you get is 4 – 16 a 4a 8 – 8a square + 4a cube + 12a square alright so we should be equivalent to 0 so which is 12a square – 16a + 4 = 0.

So 4, 3 so 3a square – 3a – a + 1 = 0 so 3a, a-1 – 1 of a – 1 so which is equivalent to 0 a = 1 or 3a, which is equal to 1a = 1 by 3. So if you remember a, cannot be 1 because the limit we are already is 0 to 1 by 2. So it cannot be 1 so this as to be taken so if you substitute in C p 4 into 1 upon 3 into 1 – 1 upon 3 whole square 4 by 3 into 3 – 1 2 by 3 whole square so which is nothing but 4 upon 9 so which is nothing but 16 upon 27 again it is 0.593.

So this is nothing C p max which is nothing but 0.593 so this is what we already derived as well and if you want to again substitute it in your axial induction factor so which is nothing but U – U 2 upon U 1 which is equivalent to we already know a = 1 by 3. So U 2 is nothing but 1 – 1 by 3 so which is nothing but 2 by 3 U 1. So your rotor velocity so this one as to be 2 by third of free stream velocity for maximum power protection.

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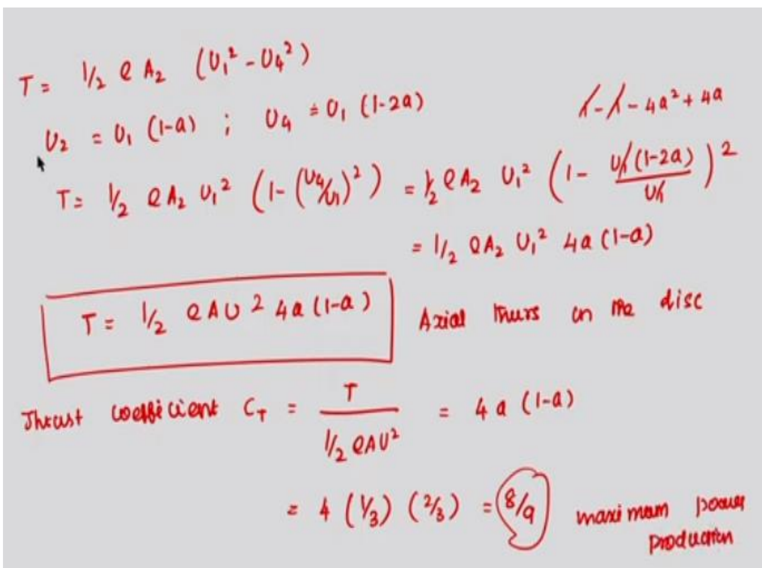

So this is another implication so we already know thrust is nothing but half rho A 2 U 1 square – U 4 square. We also know U 2 is nothing but U 1 into 1 – a and U 4 is nothing but U 1 into 1 – 2 a. So if you substitute here half rho A 2 U 1 square I am taking it out 1 – U 4 upon U 1 whole square so which is nothing but rho A 2 U 1 square into 1 – U 4 is U 1 into 1 – 2 a, I am keeping U 1 as U 1 whole square.

So which is equivalent to half rho A 2 U 1 square so this is nothing but 1 – 4 a square 2 a which is nothing but -4 a minus of minus is plus + 4a 1, 1 get cancelled. It is nothing but 4a into 1 – a so thrust is nothing half rho A because I am taking A 2 as A U 1 as U already we told. So U square 4 a into 1 – a so this is nothing but is a axial thrust on the disc because we substituted this in terms of a.

So there is something called thrust coefficient as well so which is nothing but C T already we know power coefficient this is thrust so which is nothing but thrust upon half rho A U square. So this is half rho A U 2 square 4 a, into 1 – a, upon of rho U A square so what you get is 4 a, into 1 – a. So what is that? 4 into a, already we know for maximum theoretical power protection so 1 by 3 1 – a, is 2 by 3 so which is nothing but 8 upon 9. So your thrust coefficient should be 8 upon 9 for maximum power protection.

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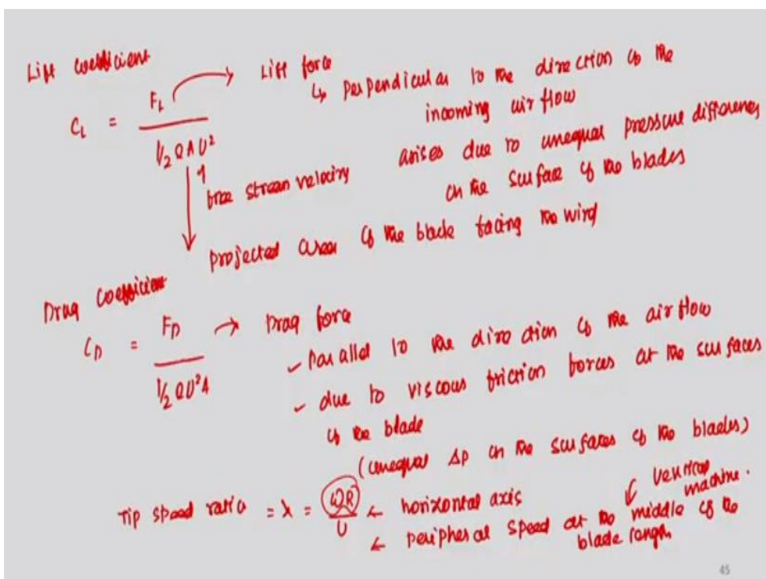

So apart from that you will have lift coefficient so which is C L which is nothing but F L upon half rho A U square so this is nothing but free stream velocity. So what is this? This is nothing but lift force yesterday we have discussed in depth how this is been created so this is perpendicular to the direction of the incoming air flow also it arises due to unequal pressure differences on the surface of the blades.

So what is this area? Area is nothing but projected area of the blade facing the wind so another one is drag coefficient which is nothing but F D upon half rho U square A. So this drag force so this also we discussed so which is nothing but parallel to the direction of the air flow. So it is created due to Viscous friction forces at the surfaces of the blade as well as this is also due to the unequal pressure difference unequal del p on the surface of the blades.

So other than that there is a tip speed ratio so which; is nothing but lambda which is defined as the speed of the blade tip upon free stream pin speed. So omega R is rotational velocity upon U which is nothing free stream velocity and; remember here it is only applicable for horizontal axis machine horizontal axis wind mill. So if you are taking for vertical axis machine then the speed of the blade tip so that should be taken as peripheral speed at the middle of the blade length.

So this is for vertical machine this is drag coefficient so today we have discussed about what should be the theoretical maximum possible wind power we can extract. And we have done 1 dimensional momentum analysis in depth having considered the assumptions being made and we also proved that C p maximum coefficient as 59%. Apart from that we have also seen how to calculate lift coefficient and drag coefficient which is nothing but using the formula given.

And tip speed ratio as well so if one goes and apply wind turbine in the location so what we would require probably is here we are talking about the free stream velocity. So we would need the wind velocity or wind data or wind speed. So what kind of data available for calculating the wind speed? And also this is the theoretical maximum possible limit we have derived but 1 goes and applies this in the particular location.

So what should be the losses and how it affects that also we will see in next class and we will do some practice problem.

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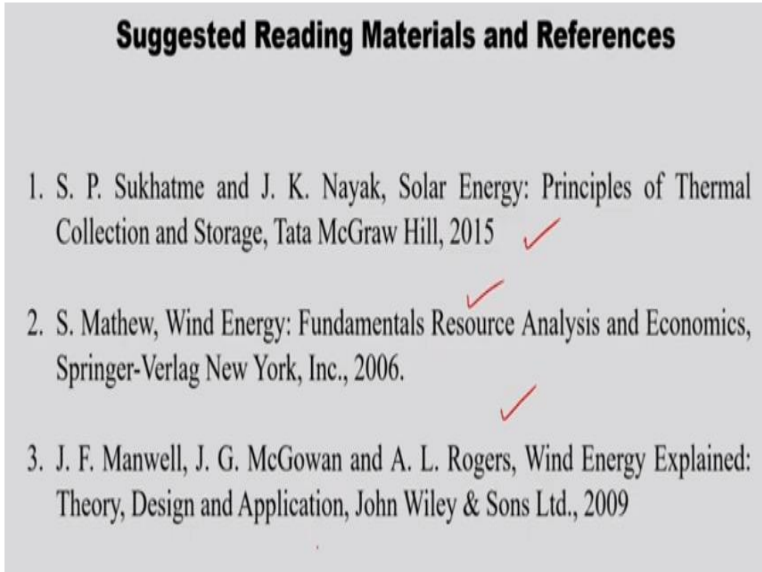

So for this lecture again the same reference material whatever we discussed yesterday Sukhatme solar energy book and S. Mathew and Manwell wind energy related books thank you.