

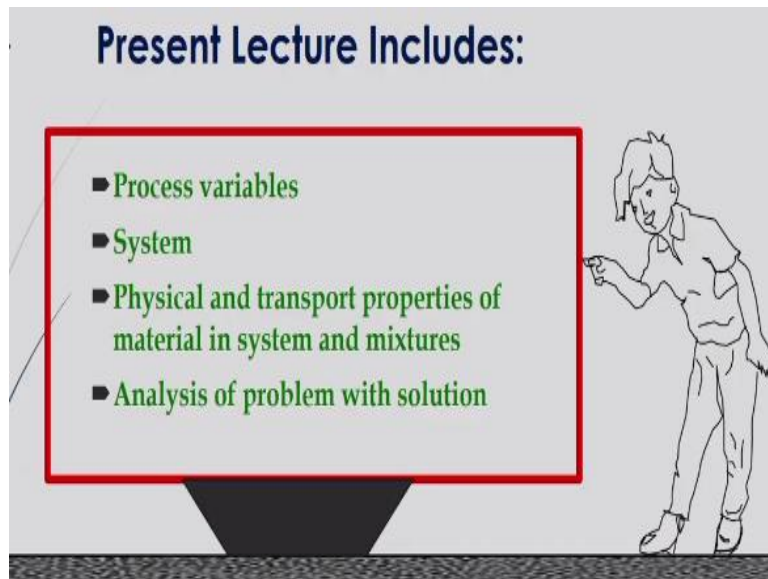
**Basic Principles and Calculations in Chemical Engineering**  
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**Module – 2: Process Variables and Rate**  
**Lecture – 2.1**  
**Variables and Properties of**  
**Material in System**

Welcome to massive open online course on Basic Principles and Calculations in Chemical Engineering. So, in the previous lectures, we have discussed introduction about that principles and calculations in chemical engineering and the role of chemical engineers in society and its history of that chemical engineering and also we have discussed that various systems and also units and dimensions how that chemical engineers can use those units for the dimensional analysis to describe the process efficiency mathematically as well as its analysis by the different dimensionless groups.

Now, in this lecture, we will start the module 2 as process variables and the process rate.

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Under this module, we will discuss about variables and properties of the material in the system where chemical engineering processes are dependent on those. Now, if we consider that what are the different processes variables are important to analyze or to you know that assess the process efficiency and in a particular systems how those process variables are that affecting the efficiency of the process system as well as what are different basic principles of

that variables like physical or transport properties of those variables that we will discuss in this lecture obsessively.

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**Process Variables**

Variables to characterize the streams entering and leaving a process unit.

- **State or Thermodynamic Variables:**
  - Temperature, Pressure of the inlet and outlet streams
- **Dynamic variables:**
  - Flow rates of inlet and outlet
- **Geometric Variables:**
  - Volume of unit, diameter, cross-sectional area, length etc.
- **Physical properties as variables**
  - Density, surface tension, viscosity of the fluid
- **Others**
  - Total amount of material in the inlet and outlet streams, composition of the inlet and outlet streams.

Now, as we know that different process variables like temperature, pressure, flow rates, even volume of unit, even that diameter of the unit or equipment, even different physical properties like density, surface tension, even you know viscosity, and other different types of variables like that is expressed by composition of the materials in a particular mixture in a particular streamlines. Now, those variables of course will be directly or indirectly affecting the process efficiency, processing yield, process you know that output, the amount based on those process variables.

Now, if we classify those variables of that process in a certain way that characterize that, you know, processes streams in the entering and leaving position of that process unit and those process variables will be useful for the design of that process equipment based on their effect on that process output or process yield. Now, in this case, we are having different types of variables, one is called state or thermodynamic variables, some are dynamic variables like geometric variables, physical properties as variables, even others also.

I think we have discussed in our previous lecture that suppose that when fluid is flowing through the pipe, there will be a certain resistance to flow of that fluid. In that case, this flow resistance actually is affected by some variables like you know that viscosity of the fluid, even diameter of the pie, even surface tension of the fluid, density of the fluid, and also you can say that velocity of the fluid. So, these variables are classified in different way like if we

classify the variables in that way, some variables would be state or thermodynamic variables, that is you know temperature.

Some process will be depending on temperature and pressure also, that you have to then calculate or assess that temperature and pressure based process efficiency based on these variables. Now, dynamic variables, that is flow rates of the inlet and outlet, you will see that that flow rates of inlet and outlet of course will be influencing the process efficiency. Now, some variables will be you know geometric variables like volume of the unit, even diameter, cross-sectional area, length and etc.

In this case, one other you know geometric variables like if you are doing any process in a particular multiphase unit like in gas liquid processing in a particular column, there you will see that gas will be distributed in the column or equipment as a dispersed phase of bubbles. So, in that case to produce that bubbles, the gas will be supplied through a uniporous media, that porous media will be regarded as spudger or you can say that gas distributor or gas diffuser.

Now, that gas diffuser will have you know some holes through which that gas will be coming out through that spudger as a dispersed pressure bubbles in the column. Now that the size of the bubbles will be depending on that hole size of that distributor. Now this, you know that if you have that hole size will be more finer, then you can get that more finer bubbles. Even if your hole size will be more coarser or larger, then you can say that more bigger bubbles you can get there.

So, accordingly you will see that there will be a formation of surface area and based on which that efficiency of the mass transfer will be depending on. So, that is why these geometric variables are very important. Another important geometric variables in a process like any gas liquid reactions you know are in presence of catalyst particles or any fluidization operation that we have earlier described that for drying for us if particles are fluidizing in a hot air medium.

So, in that case, you will see that particles sizes matter there, so this particle size if you are having more finer particle size, their efficiency of the process will be higher. So, that is why this process variables like geometric variables will you know that effect the process

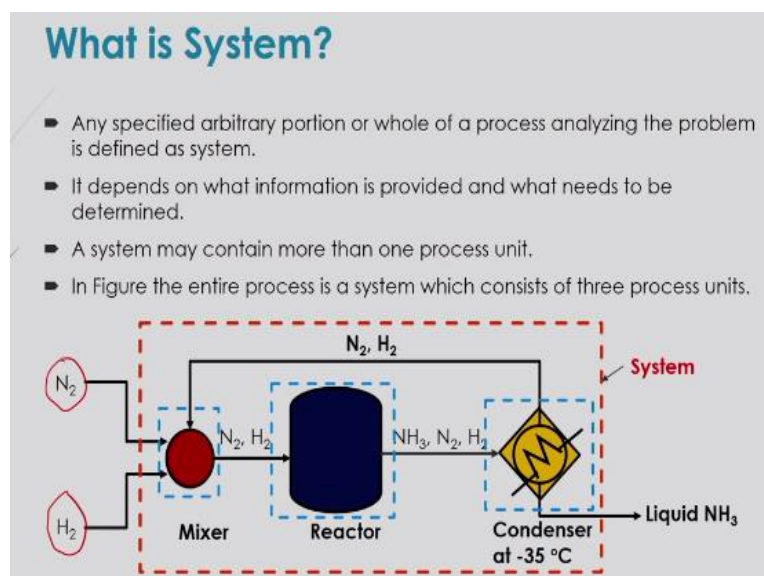
efficiency. Other variables like physical properties as the variables you will see that any process you have to analyze based on their physical properties.

If systems will have more or higher viscous fluid or liquid or you can say that system inside that you will see that there will be some resistance to flow or that resistance to give you less performance of the reactions as well as physical transfer of the materials from one phase to another phase. So, in that way you can say that the process efficiency will be depending on the viscosity of the fluid. Even other physical properties like density, surface tension, even you can say that other chemical compositions based on which that effective viscosity will be changed there.

So, that also will affect the process efficiency. Like in this case one example like surface tension. If you add some surface tension in water, you will see if you pass the gas through the spudger through this liquid of that surface tension you will see that there will be a more number of bubbles will form and there are you will see that more surface area will be formed in that, in that case you will see that there will be a more mass transfer.

In that case, more interfacial area will give you that more efficiency to transfer of molecules or more amount of molecules from one phase to another phase. So, this is one aspect of this efficiency dependence on these physical properties of the system. Like others here, total amount of material in the inlet and outlet streams, composition of the inlet and outlet streams also are very important for the assessment of process efficiency.

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Then, we have what is that particular system for a process. Now, any specified say that arbitrary portion or whole of a process analyzing the problem will be defined as system. Like you will see that if suppose if we produce ammonia gas from the mixture of hydrogen and nitrogen and it is actually allowed to react that hydrogen and nitrogen under certain catalyst particles, then you will see at a certain temperature and pressure, you will see that ammonia will be produced.

Here, on a schematic diagram of this production of ammonia is given here like in this case you will see that nitrogen and hydrogen will be passed through a mixture and you know it will be passed through a reactor and after reaction you will see that ammonia will be formed along with that unreacted ammonia and hydrogen and these outlets streams of reactor that mixture of ammonia, nitrogen and hydrogen it will be passed through a that condenser so that that ammonia will be condensed at minus 35 degree centigrade and then you can get the liquid ammonia there.

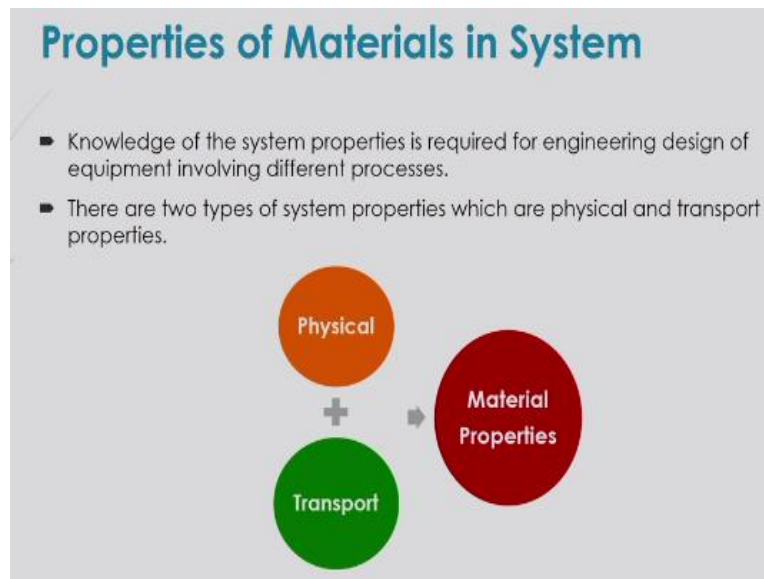
Now, here you will see that in this process, there are you know several units are required to complete the process of this. Here, briefly we are giving that main systems or equipments that are used for this process are mixture where in this case nitrogen and hydrogen gas will be mixed there in the mixture and after that it will be sent to the reactor where this nitrogen and hydrogen gas will be reacting and then producing that ammonia gas and from the outlet stream of this reactor, this ammonia gas along with unreacted hydrogen and nitrogen will come out.

After that, one condenser has been used there where minus 35 degrees centigrade temperature. This ammonia gas will be condensed down as a liquid ammonia. So, system means here you can consider for the analysis of the process individual unit as a system. Here, mixture you can consider one system. Similarly, here reactor is one system and condenser is also one system.

Here, you will see that by the dotted blue line that represented is a unit as a system whereas you can represent the system of combination of all those units also like here red line, dotted red line is shown here that considered that the whole system considering all these 3 units there. So, that is why a system may contain more than one process unit also and in this figure

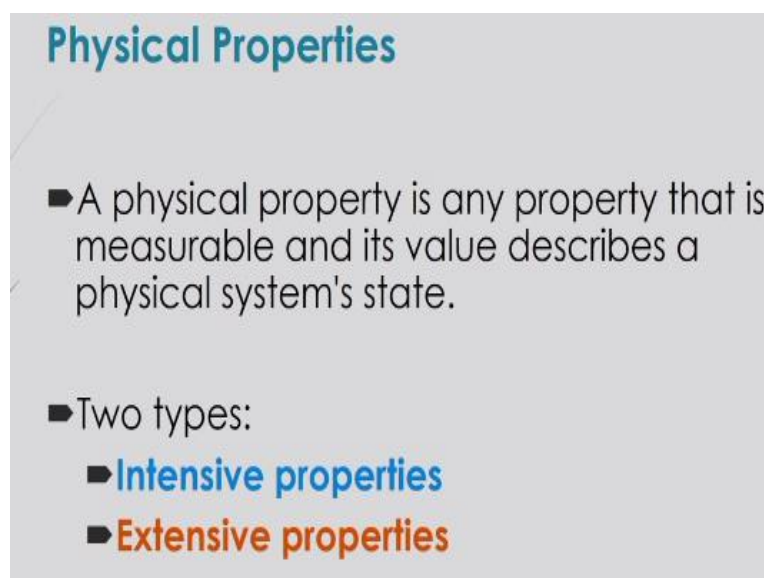
the entire processes is a system, which consist of 3 process units. It depends on what information is provided and what needs to be determined there.

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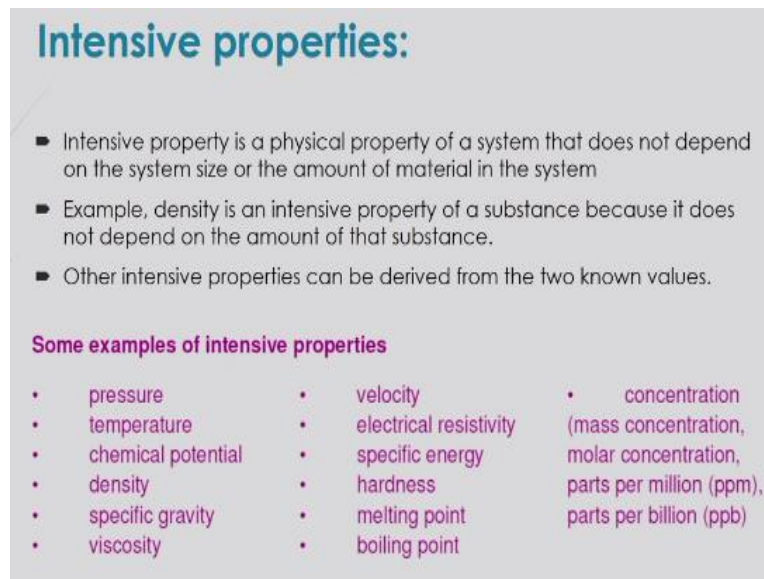
Now, we will discuss about the properties of the material in the system that are being used for particular process in a process unit. So, therefore, you know that knowledge of the system properties is required for engineering design of the equipment that involves different processes, and there are 2 types of system properties which are physical and transport properties.

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You will see that some properties will be you know of physical. In that case, that is measurable and its value describes a physical system state. Now, there are 2 types of this physical properties, one is intensive properties and another is called extensive properties.

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**Intensive properties:**

- Intensive property is a physical property of a system that does not depend on the system size or the amount of material in the system
- Example, density is an intensive property of a substance because it does not depend on the amount of that substance.
- Other intensive properties can be derived from the two known values.

**Some examples of intensive properties**

• pressure	• velocity	• concentration
• temperature	• electrical resistivity	(mass concentration,
• chemical potential	• specific energy	molar concentration,
• density	• hardness	parts per million (ppm),
• specific gravity	• melting point	parts per billion (ppb)
• viscosity	• boiling point	

Now, what is that you know intensive properties? Actually, these intensive properties are defined as a physical property of the system that does not depend on the system size or the amount of material in the system. Now, this can be, explained with examples like density is an extensive property of a substance because here it does not depend on the amount of that substance.

Other intensive properties can be derived from the known values, there some examples are given here in the slides that like pressure, temperature, chemical potential, density, specific gravity, viscosity, even you know that velocity, even electrical resistivity, specific energy, specific mass, even mass concentration, molar concentration like hardness, melting point, boiling point. So, all those examples are actually called that intensive properties.

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## Extensive properties:

- An extensive property is a physical quantity whose value is proportional to the size of the system it describes.
- Extensive properties are the counterparts of intensive properties, which are intrinsic to a particular subsystem and remain constant regardless of size.
- **Dividing one type of extensive property by a different type of extensive property will in general give an intensive value.**
- For example, mass (extensive) divided by volume (extensive) gives density (intensive).

### Some examples of extensive properties:

- |            |                   |                   |
|------------|-------------------|-------------------|
| • entropy  | • mass            | • momentum        |
| • enthalpy | • particle number | • number of moles |
| • energy   | • resistance      | • volume          |

What is that extensive properties and extensive property is a physical quantity whose value is proportional to the size of the system it describes. Now, in this case, these properties are actually counterparts of intensive properties which are intrinsic to a particular subsystem and remain constant regardless of their size. Now, if you divide one type of extensive properties by different type of other expensive properties, you will see that property as a whole will become an intensive property.

For example, like mass is extensive property, that if you divide it by volume, that is an extensive property, gives density that is intensive properties. So, that is why mass divided by volume results an intensity that is called density where intensity property that is density. Some other examples of extensive properties like entropy, enthalpy, energy, mass, particle number, resistance, momentum, number of moles, even volume. So, these are some examples of extensive properties.

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## Analog of Intensive and Extensive Properties

- There are a number of properties which have corresponding extensive and intensive analogs, many of which are thermodynamic properties.

Corresponding extensive and intensive thermodynamic properties

Extensive property	Symbol	SI units	Intensive property	Symbol	SI units
Volume			Specific volume		
Internal energy	U	J	Specific internal energy	u	J/kg
Entropy	S	J/K	Specific entropy	s	J/(kg K)
Enthalpy	H	J	Specific enthalpy	h	J/kg
Gibbs free energy	G	J	Specific Gibbs free energy	g	J/kg
Heat capacity at constant volume	C <sub>V</sub>	J/K	Specific heat capacity at constant volume	c <sub>v</sub>	J/(kg K)
Heat capacity at constant pressure	C <sub>P</sub>	J/K	Specific heat capacity at constant pressure	c <sub>p</sub>	J/(kg K)

Now, you will see some, you know, intensive properties, you will get analogs to that extensive properties. Actually, these intensive properties are made by two extensive properties or even more than two extensive properties. Like here, there are a number of properties you will see that have corresponding extensive and intensive analogs, many of which are thermodynamic properties.

In this case in the table, the slide that is given you will see that some extensive property are like volume and you will see that respective intensive property as an analogs you can say that specific volume it will be regarded as intensive property. So, here see interesting that volume is extensive property, whereas specific volume is intensive properties, because this specific volume is defined by volume per unit mass whereas internal energy is an extensive property whereas specific internal energy it is an intensive property.

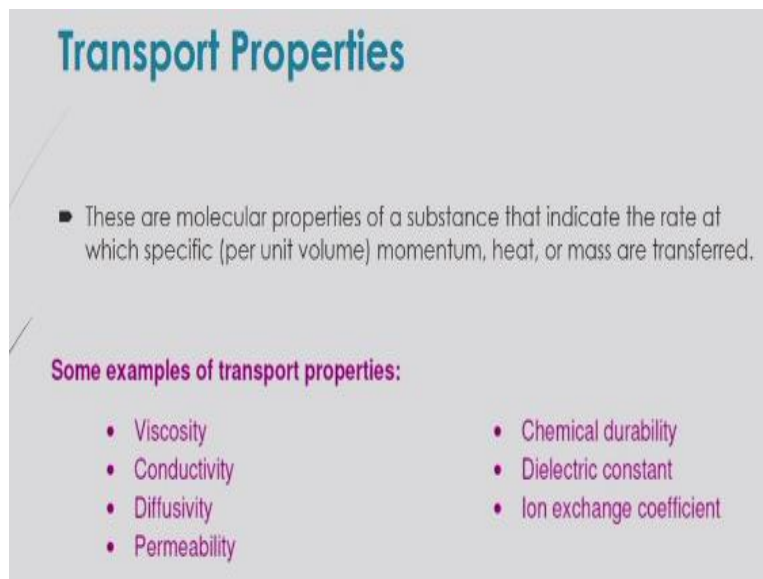
Now, whenever you are talking about that specific of course you have to think about that that should be divided by mass or by volume. So, some will be you know that mass based specific property and some would be volume based specific property. Now, if you are talking about that specific internal energy, you can say that this specific internal energy will be defined by internal energy divided by mass. So, that is why internal energy is an extensive property, whereas specific internal energy will be called as intensive property.

Similarly, entropy is an extensive property, then specific entropy will be an intensive property. Enthalpy is an extensive property, whereas, you know that is specific enthalpy will be an intensive property. Similarly, other extensive properties given in the table like Gibbs

free energy, heat capacity at constant volume, even heat capacity at constant pressure. So, respective you know intensive property will be the specific Gibbs free energy, specific heat capacity at constant volume, specific heat capacity at constant pressure they are.

In the slides in the table you will see that there are respective general symbol that is being used for the process calculation and also their respective units and international systems are given in the table. So, you have to try to remember most of these properties with their symbols as well as units.

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**Transport Properties**

- These are molecular properties of a substance that indicate the rate at which specific (per unit volume) momentum, heat, or mass are transferred.

**Some examples of transport properties:**

- Viscosity
- Conductivity
- Diffusivity
- Permeability
- Chemical durability
- Dielectric constant
- Ion exchange coefficient

Now, we have talked about that there are two properties like one is physical properties and other is transport properties. So, here transport properties actually basically it is a molecule of properties of a substance that indicate the rate at which the specific that is per unit volume momentum, heat or mass are transferred. Here in this case, the specific will be represented as per unit volume. Like here, some examples are given for transport properties, viscosity, conductivity, diffusivity, even you know that permeability, chemical durability, dielectric constant, ion exchange coefficient.

Now, this specific, you know, that momentum, heat and mass transport case that all those transport properties will be considered for process calculations there.

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## Definition of Some important properties

### Concentration

The concentration of a particular species in a multi-component mixture can be expressed in different ways.

### Mass concentration (Called Density):

For a species  $i$ , the mass concentration  $\rho_i$  is defined as the mass ( $W_i$ ) of species  $i$  per unit volume ( $V$ ) of the mixture.

$$\rho_i = \frac{W_i}{V}$$

Now, we will discuss some definition of these important properties. Those are very important for the analysis of the process and their calculations and also these are very important for the process design and the design of the process equipment. Like one is important that concentration, in this case the concentration of a particular species in a multi-component mixture that can be expressed in different ways, like some are mass concentration, some are molar concentration or some are other type of concentration like number concentration, density concentration, this is called density number of concentration.

Generally concentration, mass concentration is referred as density. Now, for the species  $i$ , the mass concentration it is denoted by that  $\rho_i$  is defined as the mass of the species  $i$  per unit volume of the mixture. Here in this slide, it is given that mathematically how that this mass concentration that is density can be expressed. Here  $\rho_i$  is for density and  $W_i$  is for mass of the species and  $V$  is the volume of the species. If it is in a mixture, then what will be the total volume of the mixture that is to be represented.

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## Mass fraction

The mass fraction of species  $i$  present in the mixture is the ratio of mass of species  $i$  to the total mass of the mixture and is given by

$$\omega_i = \frac{W_i}{\sum_{i=1}^n W_i} = \frac{\rho_i}{\rho}$$

For binary mixture of component A and B

$$\omega_A = \frac{W_A}{W_A + W_B} = \frac{\rho_A}{\rho} \quad \omega_B = \frac{W_B}{W_A + W_B} = \frac{\rho_B}{\rho}$$

Similarly, you can have the mass fraction. This is also it is required for the process calculation there. In this case, the mass fraction of the species  $i$  that is present in the mixture is the ratio of mass of the species  $i$  to the total mass of the mixture and is mathematically expressed by here on this equation given. In this case  $W_i$  that will be is equal to, sorry omega  $i$  that will be equal to  $W_i$  by summation of  $W_i$ , what does it mean.

Here omega  $i$  is the mass fraction is that is generally denoted by this symbol for a species and it will be defined by in a mixture that what is the mass of that species  $i$  and what will be the sum of mass of all species there. Then if you make an issue of this individual species mass to the mass of whole mixture, then you will get that mass fraction of that particular species. If you consider there is a binary mixture of component A and B, you see that here this mass fraction of component A will be equal to mass of A by summation of mass of A and mass of B.

So, it will be by a mass fraction of component A. Similarly, mass fraction of component B will be defined as here mass of B upon summation of mass of A and mass of B. So, from this you can easily calculate what would be the mass fraction of B. You will see interesting that summation of this mass fraction of A and B components for this binary mixture will be equal to 1. Similarly, if there are  $n$  number of components in a mixture, you will see that individual mass fraction you will calculate as per this and similarly if you make it sum up of all these mass fractions, it will become 1 there.

So, in this way, you can calculate mass fraction. Suppose here a mixture of gases like carbon dioxide, nitrogen, sulfur dioxide, even that other gaseous mixtures. So in that case, you have to first to know what would be the mass of individual gases there. So, that if it is known by moles, then you have to multiply each molecular weight, then it would become mass of that gases. So, suppose there are 5 gaseous components in a mixture.

So, individually you have to write what to be the mass fraction of component nitrogen, mass fraction of component hydrogen, mass fraction of component carbon dioxide, mass fraction of component like this. So, from there, you can easily calculate what should be the mass fraction of individual gaseous components there.

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### Molar Concentration (Called Molar Density)

- The molar concentration of species  $i$ ,  $c_i$  is defined as the number of moles of  $i$  present per unit volume of the mixture. The mass concentration and molar concentration are related by

$$c_i = \frac{\rho_i}{M_i}$$

$M_i$  is the molecular weight of species  $i$ .
- When dealing with gas phase under conditions in which ideal gas law applies, the molar concentration is given by

$$c_i = \frac{p_i}{RT}$$

$p_i$  is the partial pressure of species  $i$  in the mixture,  
 $T$  is the absolute temperature  
 $R$  is the gas constant.

Similarly, molar concentration also you can describe from that mixture, in that case it is called molar density. So, there it is actually defined that the molar concentration of species  $i$ , it is generally denoted by  $c$ , small  $c$ . So,  $c_i$  mean concentration, molar concentration of components  $i$  is defined as the number of moles of  $i$  present per unit volume of mixture. Then the mass concentration and molar concentration can be related by this equation.

Here in this case like  $c_i$  that will be equal to  $\rho_i$  by capital  $M_i$ , here capital  $M_i$  is just molecular weight of a species and  $\rho_i$  is the mass concentration that is mass per unit volume. Here interesting that, that mass per unit volume out of that, you just divide it by molecular weight, then you can say that mass by molecular weight it will become, what is that, how much molar is there, how much moles will be there. So, then it will be mole per unit volume, then simply it will be called as that molar concentration.

So, when dealing with gas phase under conditions in which ideal gas law applies, so in that case the molar concentration can be calculated from the ideal gas laws here. So, it will be defined as  $c_i$  that will be equal to  $p_i$  by  $RT$ , what is  $p_i$ ,  $p_i$  is the partial pressure of the species  $i$  in the mixture and  $T$  is the absolute temperature at which this gas is you know kept and also  $R$  is the universal gas constant there. So, this way you can represent what should be the molar concentration and also mass concentration if you know that molecular weight of this component.

Also how that molar concentration and mass concentrations are related to each other, and also for the gaseous mixtures if you know the partial pressure of each gaseous there, you can easily calculate the molar concentration at the particular temperature and partial pressure.

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### Mole Fraction

- The mole fraction of species  $i$  in a mixture is the ratio of moles of species  $i$  to the total number of moles in the mixture and is given by

For liquids & solids

$$x_i = \frac{n_i}{\sum_{i=1}^n n_i} = \frac{W_i / M_i}{\sum_{i=1}^n (W_i / M_i)} = \frac{c_i}{c}$$

$$x_A = \frac{n_A}{n_A + n_B}$$

For gases

$$y_i = \frac{n_i}{\sum_{i=1}^n n_i} = \frac{W_i / M_i}{\sum_{i=1}^n (W_i / M_i)} = \frac{c_i}{c} = \frac{p_i}{P}$$

$$y_A = \frac{n_A}{n_A + n_B} = \frac{p_A}{p_A + p_B}$$

$$\sum_{i=1}^n x_i = \sum_{i=1}^n y_i = 1$$

Now, what is the mole fraction? In the similar fashion of that mass fraction, you can also express the molar fraction or individual component of liquid or gaseous or solid components there. So, in this case the mole fraction of the species  $i$  in a mixture that will be the ratio of moles of species  $i$  to the total number of moles in the mixture and is given by especially for liquids and solids, that will be denoted by small  $x$  that is here, whereas this for gaseous mixture, the molar fraction or mole fraction you have to denote it by  $y_i$ .

So, this is in general how this to be represented. So,  $x_i$  will be what, that is mole fraction of the species  $i$  that would be is equal to moles of that individual species upon total moles of the mixture. So, here that is it is defined by this your  $x_i$  for the liquid and solid should be equal to

number of moles of  $i$  that is denoted by  $n_i$  and also what will be the total moles of components in a mixture that will be summation of all moles of the components.

So, if you divide individual moles number by the total mixture moles number, then you can easily get that mole fraction of that individual component. Similarly, for gaseous phase also, what will be the moles of individual gas and upon the total number of moles of all gaseous mixture that you can calculate here by this equation, and also you can calculate this mole fraction for the gases from its partial pressure.

If you know the partial pressure of individual gaseous components and also total pressure of the gaseous mixture, then you can easily calculate what should be the mole fraction of individual gases that will be actually the ratio of partial pressure of individual component to the total pressure of the system. So, that will be your mole fraction of gas. If you are considering that, the binary mixture of component A and component B.

Similarly you can easily calculate what should be the molar fraction of that component A that will be calculated by this equation here, simply what will be the moles of A and what will be the summation of moles of A and B and similarly  $y_i$  also you can calculate what should be the moles of A gas and what should be the total moles of gaseous mixture that is  $n_A + n_B$  in the binary mixture that will be here. Similarly, if you know that in the binary mixture like mixture of carbon dioxide and hydrogen gas, so there if you know that the moles of carbon dioxide gas is 70 moles whereas hydrogen gas is only 30 moles.

So, total moles is 100 moles. So, individual mole fractions will be what, that is one for carbon dioxide it will be what, that will be your 70 by 100, that is 0.7, whereas for hydrogen gas it will be 30 by 100 that is 0.3. So, in this way even if you know the partial pressure of carbon dioxide or hydrogen gas in the mixture, then you can easily calculate what should be the mole fraction of gaseous components just by dividing that partial pressure of each component by the total pressure there.

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## Specific gravity

- The ratio of the density (mass of a unit volume) of a substance to the density (mass of the same unit volume) of a reference substance at a specific condition.
- The reference substance is nearly always water at 4°C for liquids or air for gases. Temperature and pressure must be specified for both the sample and the reference.
- Pressure is nearly always 1 atmosphere. Temperatures for both sample and reference vary from industry to industry.

True specific gravity can be expressed mathematically as:

$$SG_{T_{ref}}^T = \frac{\rho_{\text{substance at } T}}{\rho_{\text{reference } T_{ref}}}$$

e.g.,  $SG_4^{25} = 0.67 = \frac{\rho_{\text{substance at } 25^\circ \text{C}}}{\rho_{\text{reference at } 4^\circ \text{C}}}$

Now, another important variable for the process calculation it is called specific gravity. This is actually defined as the ratio of the density, that is mass of unit volume, of a substance to the density, that is mass of the same unit volume, of a reference substance at a specific condition. Now, the reference substance is nearly always water at 4 degrees centigrade for liquids or as it is considered for gas specific gravity, the reference gas will be air for gases and temperature and pressure must be specified for both the samples and references there.

Now, pressure is nearly always 1 atmosphere to be considered or as temperature for both samples and reference that vary from industry to industry that you can give or you can mention that in pressure accordingly there. Now true specific gravity can be expressed mathematically by this equation that specific gravity at referred like T reference to T that will be equal to rho substance at temperature T and rho reference at temperature of the reference material.

So, in that case let us suppose specific gravity as 4, 25 is equal to 0.67, what does it mean that this specific gravity is nothing but that density of the substance at 25 degree centigrade relative to that reference density at 4 degrees centigrade. If you are considering that, the density of reference is water, then in this case it will be 1. So only density of that substance would be 0.67 into 1, it will be simply 0.67 at that particular temperature case as 25 degree centigrade.

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## Example

A 0.5 molar aqueous solution of sulfuric acid flows into a process unit at a rate of 1.25 m<sup>3</sup>/min. The specific gravity of the solution at 20 degree Celcius is 1.03

(a). Calculate the mass concentration of H<sub>2</sub>SO<sub>4</sub> in kg/m<sup>3</sup> (b) The mass flow rate of solution in kg/s, (c). The mass flow rate of H<sub>2</sub>SO<sub>4</sub> in kg/s, (d). The mass fraction of H<sub>2</sub>SO<sub>4</sub>

**Solution:** M.W of H<sub>2</sub>SO<sub>4</sub> is 128.

0.5 Molar Aq. Sol. Of H<sub>2</sub>SO<sub>4</sub> contains  $0.5 \times 128 \text{ g} = 64 \text{ g H}_2\text{SO}_4$

Mass concentration of H<sub>2</sub>SO<sub>4</sub> is 64 g/l = 64 kg/m<sup>3</sup> (Ans a)

Mass concentration of solution

= Density of water x Specific Gravity of solution =  $1000 \times 1.03 = 1030 \text{ kg/m}^3$

The mass flowrate of solution = Mass concentration of solution x Volumetric flowrate =  $1030 \times 1.25 = 1287.5 \text{ kg/min} = 1287.5/60 = 21.46 \text{ kg/s}$  (Ans b)

Mass flowrate of H<sub>2</sub>SO<sub>4</sub> =  $64 \times 1.25/60 = 1.33 \text{ kg/s}$  (Ans c)

Mass fraction of H<sub>2</sub>SO<sub>4</sub> = Mass concentration of H<sub>2</sub>SO<sub>4</sub>/Mass concentration of Solution =  $64/1030 = 0.062$

As an example, you can have here suppose a 0.5 molar aqueous solution of sulfuric acid that flows into a process unit at a rate of 1.25 meter cube per minute. The specific gravity of the solution at 20 degrees Celsius is 1.03. Now, in this case you have to calculate what should be the mass concentration of sulfuric acid in kg per metre cube and the mass flow rate of solution in kg per second and the mass flow rate of sulfuric acid in kg per second and also what would be mass fraction of sulfuric acid.

Now, in this case, you need to calculate first the molecular weight of the sulfuric acid, it is known to you it is 128 whereas 0.5 molar aqueous solution of sulfuric acid will contains, what will be the amount of sulfuric acid? This is actually 0.5 into 128 gram because their molar 0.5 molar means 0.5 moles of sulfuric acid in 1 liter of solution there. So, we can say that 0.5 molar means how much weight of that, so 0.5 into 128 is the molecular weight. So, just you multiply this, you will get that 64 gram of sulfuric acid.

Now in this case, what should be the mass concentration of sulfuric acid? It will be simply that since it is 64 gram as per calculation in 1 liter, so it is the mass concentration. Now, you have to convert it to you know that other unit like kg per metre cube, just simply you have to divide this gram to kg by 100 and litre to metre cube, again divided by 1000. So, simply 1000, 1000s will be cancelled out.

Then it will become 64 kg per meter cube and what from the mass concentration of the solution, that you can calculate easily just by multiplying the density of water and the specific gravity of the solution because specific gravity of the solution or specific gravity of the

materials is defined as what is that density of that particular substance upon what is that reference materials density. So, here we can say since it is a solution we have to take the reference material as water.

So, you have to multiply this specific gravity of the solution by this density of water, so you will get that what will be the density of the solution or mass concentration of the solution. It is simply that since the specific gravity of the solution is given as 20 degree Celsius is 1.03, just simply you multiply by 1000 of this 1.03 value, then you will get simply one 1030 kg per meter cube and the mass flow rate of solution it will be coming as mass concentration of solution into volumetric flow because mass into volume volumetric flow rate will give you that, what is that.

It is the mass concentration of solution and volumetric flow rate, it will give you simply what will be the mass. Now, volumetric flow rate since it is the coming with respect to time, so simply you can say that mass concentration of solution into volumetric flow rate, then will be 1030 that is your mass concentration into 1.25 that is what is that volumetric flow rate is given as per your problem. So, it will come after multiplication as 1287.5 kg per minute.

Now, if you can convert it to what should be the unit in kg per second simply you have to divide it by 60 because 1 minute is about 60 second, then you will get 21.46 kg per second. Now, next question is that what is the mass flow rate of sulfuric acid in kg per second? You know that only 0.5 molar concentration of sulfuric acid in the solution. So, you have to know what will be the exact amount of sulfuric acid in the solution.

Since you know that there will be since mass flow rate mass of that sulfuric acid in the solution to be 64 gram sulfuric acid and then if you make it mass concentration as 64 kg per meter cube and if you multiply this mass concentration by this flow rate of the solution, then you can easily calculate what should be the mass flow rate of that sulfuric acid. So, simply it is 64, the 64 kg per meter cube, that is mass concentrations of sulfuric acid, and then multiply by what is that, this is your volumetric flow rate of the solutions and then you will get that here what would be the amount that is kg of or mass of sulfuric acid.

Since it is in terms of minute, then you have to divide it by 60, then it will become kg per second, so 1.33 kg per second and what should be the mass fraction, very interesting that

mass fraction, you know, that what would be mass of the sulfuric acid in a certain volume, that is 64 kg per meter cube and also you know that mass concentration of the solution. So, from that mass concentration of sulfuric acid by mass concentration of solution, you can easily get that what is that mass fraction of that sulfuric acid.

So, it will be 64 by 1030, it will be coming as 0.062. So, these examples will give you that how you can use that specific gravity definition for calculation of that mass concentration as well as what is that mass concentration of solution and also mass concentration of components, also mass fraction of that components, and also molar flow rate, how it can be, related to that density conversion and all thing.

So, in this way, you can calculate, please go through this example, that is one example where you can understand the basic things how to convert it from mass to molar, molar to mass like this way, and then you have to go to process calculations in that direction.

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**Apparent specific gravity**

- The apparent specific gravity is simply the ratio of the weights of equal volumes of substance and reference substance (water or air for liquid or gas):

$$SG_{ap} = \frac{W_{\text{substance}}}{W_{\text{reference}}}$$

$W_{\text{substance}}$  is the weight of the substance and  
 $W_{\text{water}}$  is the weight of water or reference substance.

Another way of process you know calculation by the different process variables like apparent specific gravity. In this case, the apparent specific gravity is simply the ratio of the weights of the equal volume of substance and reference substance, water or air for liquid or gas. Similarly, here you will see that instead of earlier that a true specific gravity there it is based on density whereas in apparent specific gravity it is based on weight of the substance. So, the same way that is ratio of the weight of the substance at a particular temperature upon the weight of the reference substances and this ratio will be called as apparent specific gravity.

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## API gravity

- The American Petroleum Institute gravity, or API gravity, is a measure of how heavy or light petroleum liquid is compared to water.
  - If its API gravity is greater than 10, it is lighter and floats on water;
  - if less than 10, it is heavier and sinks.
- It is used to compare the relative densities of petroleum liquids. The formula to obtain API gravity of petroleum liquids, from specific gravity (SG), is:

$$\text{API Gravity} = \frac{141.5}{\text{SG}} - 131.5$$

Another one important process variables by which the process can be assessed it is called API gravity. The American Petroleum Institute they have actually suggested or you can say that procured one measuring systems of that liquid whether it is lighter or heavier based on certain quantity. In that case, it is called that API gravity. So, this API gravity is being used as a unit to assess that liquid whether it is heavier or lighter. So, it is a measure of how that heavy or light petroleum liquid is compared to water.

If its API gravity, suppose if any petroleum liquid if its API gravity is greater than 10, remember it, it is actually referred to as a lighter liquid and it will floats on the water. Whereas if its API gravity is less than 10, it is called as heavier liquid and it sinks. So it is used to compare the relative densities of petroleum liquids and the formula to obtain that API gravity of petroleum liquids from specific gravity, you can use this equation okay for calculation of that API gravity as a function of a specific gravity.

Once you know that a specific gravity of the substance or liquid or fluid, then you can easily calculate what should be the API gravity there.

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## Example

**Prob.:** 420 liters of 28 °API gas oil blended with 1200 liters of 15 °API fuel oil. What is the density of the mixture in kg/m<sup>3</sup>? Assume that volumes are additive. The density of water at 15 °C is 0.999 g/cm<sup>3</sup>.

### Solution

$$SG = \frac{141.5}{API + 131.5}$$

$$SG \text{ of } 28 \text{ } \circ API = 141.5 / (28 + 131.5) = 0.884.$$

$$\text{So density of gas oil} = 0.884 \times 0.999 = 0.883 \text{ g/l}$$

$$SG \text{ of } 15 \text{ } \circ API = 141.5 / (15 + 131.5) = 0.962$$

$$\text{So density of fuel oil} = 0.962 \times 0.999 = 0.961 \text{ g/l}$$

$$\text{Density of Mixture} = \text{Total mass} / \text{Total Volume} =$$

$$(420 \times 0.883 + 1200 \times 0.961) / (420 + 1200) = 0.941 \text{ g/l}$$

Now, example, let us see to calculate that specific gravity of that petroleum mixture based on their degree API. Then in that case, suppose 420 liters of 20 degree API gravity gas oil blended with 1200 liters of 15 degree API fuel oil. Now, what should be the density of the mixture in kg per meter cube? Assume that the volumes are additive. The density of the water at 15 degree centigrade is 0.999 gram per centimeter cube. So we are blending 420 liters of gas oil with the 1200 liters of fuel oil.

The gas oil has API gravity 28, whereas fuel oil has API gravity of 15. Now what should be the density of the mixture? Now, we know that relationship between API and a specific gravity, and from that relationship, we can write that specific gravity will be equal to 141.5 by API + 131.5. So from this equation, we can easily calculate what should be the specific gravity of 28 degree API gas oil. So this will be simply just we are substituting this value of degree API there in API, then we can get this value of 0.884.

Now, density of that gas oil, then it will be simply 0.884 into 0.999, that will be equal to 0.883 gram per liter. Similarly, specific gravity of 15 degree API of fuel oil, it will be as after substitution of this value, it will be 0.962. So, density of the fuel oil will be similarly 0.962 gram per liter. Now, density of mixture will be total mass per total volume, then what will be the total mass? You know that density of the gas oil and density of the fuel oil, then what should be the total mass there?

If you multiply that volume of that perspective oil to its density and then add it, then you will get the total mass of that oil mixture and what should be the volume already given that one is

420 liters and other is 1200 liters. If you add it up, then you will get the total volume. If you divide this total mass by this total volume, then you can simply get what should be the density of that mixture. So, in this way, you can calculate the density of the mixture based on degree API of the substances.

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### Volume fraction in multiphase system

- The volume fraction of phase is defined as the volume of each phase upon total volume of phases.
- The volume fraction of phase is sometime referred as void fraction or holdup of phase which is represented by

$$\alpha_P = \frac{\text{Volume of each phase}}{\text{Volume of phase mixture}}$$

$$\sum \alpha_P = 1$$

P for phase  
G for gas  
L for liquid  
S for solid

For gas-liquid mixture,

$$\alpha_G = \frac{\text{Volume of gas phase}}{\text{Volume of gas + liquid mixture}}$$

$$\alpha_L = \frac{\text{Volume of liquid phase}}{\text{Volume of gas + liquid mixture}}$$

$$\alpha_G + \alpha_L = 1$$

For gas-liquid-solid mixture:  $\alpha_G + \alpha_L + \alpha_S = 1$

Another one is volume fraction in multiphase systems. The volume fraction of phases is defined as the volume of each space upon total volume of phases. What does it mean here? Suppose there are mixture of different phases like gas, liquid, and solid. Now, individual phases like gas, liquid, or solid will have certain or will occupy certain volume in a particular mixture volume. Now, volume fraction will be defined as that the volume of individual phases upon total volume of the phase mixture.

So, that will be called as volume fraction. This volume fraction is sometimes regarded as holdup of the phases. Now, here this volume fraction of phase, it is regarded as holdup or sometimes its called as void fraction of that particular phases, which is represented by here this equation given in the slide and you will see that if there are 3 phases, then if you sum it up all void fractions or volume fraction of individual phases, then you will get that will be close to 1.

Now, if you consider that the only gas-liquid mixture, then what would the volume fraction of gases that will be denoted by alpha G, then it will be calculated by volume of gas phase by volume of gas and liquid mixture. Similarly, volume fraction of liquid, it will be volume of liquid phase by volume of gas and liquid mixture. So, in this case, you will see that the

volume fraction of gas and volume fraction of liquid if you sum it up, then you will get total will be 1.

For similarly, gas-liquid-solid mixture, you can get this  $\alpha_G + \alpha_L + \alpha_S$  that will be equal to 1. That means summation of all void fraction or volume fraction of individual phases will be equals to 1.

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**Mixture density of multiphase system**

- The sum of the bulk densities for all the phases is called the mixture density

$$\rho_m = \bar{\rho}_G + \bar{\rho}_L + \bar{\rho}_S$$

$$= \alpha_G \rho_G + \alpha_L \rho_L + \alpha_S \rho_S$$

- The bulk density is related to the true or absolute density  $\rho_p$  as:

Gas bulk density	Liquid bulk density	Solid bulk density
$\bar{\rho}_G = \alpha_G \rho_G$	$\bar{\rho}_L = \alpha_L \rho_L$	$\bar{\rho}_S = \alpha_S \rho_S$

Similarly, you can actually calculate the mixture density of the multiphase systems like the sum of the bulk density for all the phases will be called as mixture density and it will be denoted by  $\rho_m$  that will be summation of actual gas density, actual liquid density and actual solid density. This actual gas density will be represented by that what will be the volume fraction of that gas into what is the density of the gas, and similarly for density of the liquid and volume fraction of the liquid and also density of the solid and volume fraction of the solids.

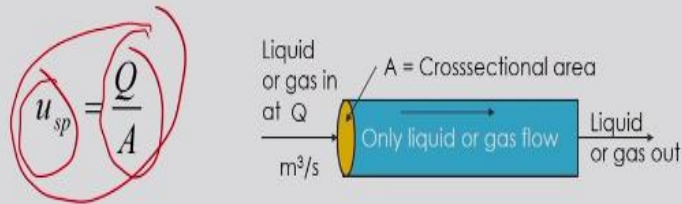
If you sum it up all these things, then you will get the mixture density. The bulk density is related to the true or absolute density that is  $\rho_p$ . Now this gas bulk density generally is denoted by what is that,  $\rho_G$  bar and it will be close to that, what will be that  $\alpha_G$  into  $\rho_G$ . So it is called gas bulk density and  $\rho_L$  bar it is similarly  $\alpha_L$  into  $\rho_L$ , similarly  $\rho_S$  bar it is the solid bulk density to be as what is that  $\alpha_S$  into  $\rho_S$ . This is basically you know that what would the fraction of that density on that particular phases.

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## Superficial velocity

- For multiphase flow in a pipe, the superficial velocity of each phase ( $u_{sp}$ ) is the volume flow rate ( $Q$ ) of that phase divided by the pipe cross sectional area ( $A$ ).
- It is the velocity of the phase if the phase occupied the whole pipe cross sectional area which can be defined by



Another important variable which should be defined is called superficial velocity. Of course, whenever any fluid streams are actually sent at a particular rate that what will be the flow rate, what will be the velocity of that fluid that you have to calculate? Now, there are two types of velocity, it is called superficial velocity and actual velocity or intrinsic velocity. Now, superficial velocities mean what? Suppose only single liquid is passing through a pipe at a particular volumetric flow rate like 5 liter per second, so in that case, this will be liter per second that is volumetric flow rate.

Now, if you divide this volumetric flow rate by its crosssectional area of that conduit or pipe, then you will get that what would be the velocity of that fluid. So, for multiphase flow system in a pipe if you are considering only single phase flows there, then it will be regarded as superficial velocity of each phases, if you are considering that crosssectional area of the empty pipe or vessel and it is the velocity of the phase.

If the phase occupied the whole pipe crosssectional area which can be defined by here like this  $u_{sp}$ , s means here superficial, p for phase, so superficial phase velocity, it may be gas, it may be liquid, it may be solid also. So, superficial gas velocity, superficial liquid velocity that will be actually calculated by this equation, it will be defined as  $Q$  by  $A$ . What is  $Q$ ,  $Q$  is nothing but volumetric flow rate and  $A$  is the crosssectional area. So, whenever you are going to calculate the superficial velocity, of course, you have to divide that volumetric flow rate by its empty crosssectional area, but that means here only one phase is flowing through that.

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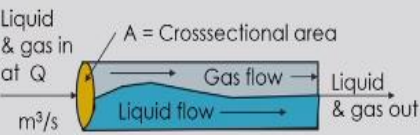


## Intrinsic or actual velocity

- If in system, more than one phase is flowing simultaneously, the intrinsic or actual velocity of each phase is

$$u_{ip} = \frac{Q}{A\alpha_p} = \frac{u_{sp}}{\alpha_p}$$

$$u_{iG} = \frac{Q}{A\alpha_G} = \frac{u_{sG}}{\alpha_G}$$

$$u_{iL} = \frac{Q}{A\alpha_L} = \frac{u_{sL}}{\alpha_L}$$


But you will see that whenever 2 phase will be there, then the quantity will be different because in that case you will see that if you supply the gas and liquid mixture simultaneously in the same pipe, what will happen, gas and liquid will not occupy whole crosssectional area by that individual phase. In that case, gas will occupy some portion of the crosssectional area and liquid also will occupy some fraction of that crosssectional area. So, in that case, the velocity will be changed.

If the velocity will be lower than the original crosssectional area, of course, that velocity will increase for the same volumetric flow rate. So, that is why the actual velocity of that phases whenever multiphase mixture will be passed through the pipe or any vessel, there you have to consider what will be the actual velocity of that particular phases or it is called intrinsic velocity of the particular phases. So, it is defined as what is that,  $u_{ip}$ , i for intrinsic, p for phase. So,  $u_{ip}$  will be equal to  $Q$  by  $A$  into  $\alpha_p$ .

In this case,  $Q$  is the volumetric flow rate and here you know that  $\alpha_p$  is the volume fraction of that, for that particular phase. So, it will be defined by  $Q$  by  $A\alpha_p$ . So, it will be simply  $Q$  by  $A$  is nothing but  $u_{sp}$  that is superficial velocity and  $\alpha_p$  is the volume fraction of that particular phases. So, this actual velocity is related to the volume fraction of the phases. If your volume fraction is more, then you will see that your actual velocity will be less, whereas if your volume fraction is less, then you will get the more actual velocity of that particular phase.

So, it will be considered for each phases like gas, liquid, if there is a mixture of gas and liquid is flowing through the pipe, then individual velocity of that phases will be higher than that superficial gas velocity or superficial liquid velocity there. So, here intrinsic velocity or actual velocity of gas will be equal to uh superficial gas velocity divided by each volume fraction.

Similarly, here intrinsic velocity of the liquid will be is equal to superficial velocity of that particular liquid here and then it will be divided by the volume fraction of the liquid. So, in this case, you can have this actual velocity for the multiphase systems how it can be calculated.

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### Relative or slip velocity

- The relative velocity between the two phases such as gas (G) and liquid (L) in case of two-phase is denoted by  $u_{r,G-L}$  which can be written as

$$u_{r,g-l} = u_{ig} \pm u_{il}$$

$$= \frac{u_{sg}}{\alpha_g} \pm \frac{u_{sl}}{1 - \alpha_g}$$

For co-current sign is positive. For countercurrent sign is negative  
The relative velocity is sometimes referred to as slip velocity

Now, another important flow rate by which you can express that process, it will be called as relative or slip velocity. If gas and liquid both are flowing through the pipe, you will see that maybe gas will be having higher velocity relative to the liquid velocity. So, there what should be the relative velocity, of course, it will be the same way in the same direction it is going, then simply you have to subtract that, that is liquid velocity from the gas velocity, then you will get that relative velocity.

If you are considering that flow is flowing opposite in direction, then your relative velocity will be just simply positive, that is summation of those two velocity. Now, what would be the actual relative velocity or slip velocity that to be calculated based on the, what is that, actual velocity of that individual phases. So, that actual velocity will be simply for the gas it will be a superficial gas velocity by volumetric flow rate of gas and superficial liquid velocity of

liquid divided by 1 minus epsilon g, that is simply volume fraction of the liquid. So, in this way, you can calculate what should be the relative velocity.

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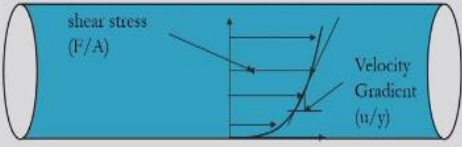
## Viscosity

- Viscosity is a measure of the resistance of a fluid which is being deformed by either shear stress or tensile stress.
- It describes a fluid's internal resistance to flow and may be thought of as a measure of fluid friction

$$\frac{F}{A} = \tau \propto \frac{du}{dy}$$

$$\frac{F}{A} = \tau = \mu \frac{du}{dy}$$

μ is the proportionality factor called viscosity



shear stress (F/A)

Velocity Gradient (u/y)

**Dynamic viscosity**, also absolute viscosity, the more usual one (typical units Pa s, Poise, P);

**Kinematic viscosity** is the dynamic viscosity divided by the density (typical units cm<sup>2</sup>/s, Stokes, St).

Now, another important variables like viscosity, so viscosity is a measure of the resistance of the fluid which is being deformed by either shear stress or tensile stress and It describes a fluid's internal resistance to flow and may be thought of as a measure of fluid friction. So, it is generally denoted by this by A, that is shear stress and it is denoted by tau and this is related to the velocity gradient.

Whenever fluid is flowing through the pipe, you will see that velocity of the fluid at the wall will be zero, whereas away from the wall you will see that velocity relatively will increase and at the center the velocity will be maximum. So, in this way, there will be some gradient of the velocity of the fluid layer in the pipe and based on which this resistance of the flow is calculated. Now, this resistance of the flow is actually measured by that what is the shear stress.

This shear stress is proportional to that velocity gradient of the liquid or fluid you can say and it is represented by mu into du by dy where du by dy is called velocity gradient in the y direction and mu is called that proportionality constant, that provisionally constant will be actually called as viscosity. So, this way, the viscosity this is basically a measure of resistance of the fluid which is being deformed by either shear stress or tensile stress. Now, there are 2 types of viscosity, it is called dynamic viscosity and another is kinematic viscosity.

Dynamic viscosity also it is called absolute viscosity, the more usual one that we are considering here that  $\mu$  and its unit typically are Pascal second, Poise, or Stokes **or** sometimes it is represented and similarly kinetic viscosity is the dynamic viscosity that is divided by the density and its unit is Stokes or you can say that simply centimeter square per second or meter square per second.

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### Viscosity of gas

- Sutherland's formula can be used to derive the dynamic viscosity of an ideal gas as a function of the temperature

$$\mu = \mu_0 \frac{T_0 + C}{T + C} \left( \frac{T}{T_0} \right)^{3/2}$$

- $\mu$  = dynamic viscosity in (Pa·s) at input temperature  $T$ ,
- $\mu_0$  = reference viscosity in (Pa·s) at reference temperature  $T_0$ ,
- $T$  = input temperature in kelvins,
- $T_0$  = reference temperature in kelvins,
- $C$  = Sutherland's constant for the gaseous material in question.

Valid for temperatures between  $0 < T < 555$  K at below 3.45 MPa. For air  $C$ ,  $T_0$  and  $\mu_0$  are 120 K, 291.15 K and 18.27  $\mu\text{Pa}\cdot\text{s}$ .  
 At 15.0 °C, the viscosity of air is  $1.78 \times 10^{-5}$  kg/(m s), 17.8  $\mu\text{Pa}\cdot\text{s}$  or  $1.78 \times 10^{-5}$  Pa.s.

Similarly, viscosity of gas, how to calculate? You can calculate the viscosity of the gas from this equation given in the slides and it is related to the temperature. If you increase the temperature, what will happen, that decrease the viscosity of the fluid. So, in this way, you can calculate if you know the temperature and also you know some constant as per this equation. Here in this case, the  $C$  is one parameter, it is called Sutherland's constant for the gaseous material.

Based on this correlation, you can calculate what should be the viscosity of the gas and basically the temperature within 555 K and the pressure at below 3.45 megapascal. You see that for air, the  $C$ ,  $T_0$  and  $\mu_0$  are the some parameters, then it will be as 120, 291.15 K, and 18.27 micro pascal per second and accordingly you can calculate what should be the viscosity of air at 15 degrees centigrade and it will be 1.78 into 10 to the power minus 5 kg per meter second, you can easily calculate or find out what should be the viscosity of the air if you know the temperature there.

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## Viscosity of dilute gas

Can be calculated by the Chapman-Enskog equation. The equation requires three empirically determined parameters: the collision diameter ( $\sigma$ ), the maximum energy of attraction divided by the Boltzmann constant ( $\epsilon/\kappa$ ) and the collision integral ( $\omega(T^*)$ ).

$$\mu_0 \times 10^6 = 2.6693 \frac{(MT)^{0.5}}{\sigma^2 \omega(T^*)}$$

where

- $T^* = \kappa T / \epsilon$  — reduced temperature (dimensionless),
- $\mu_0$  = viscosity for dilute gas ( $\mu\text{Pa}\cdot\text{s}$ ),
- $M$  = molecular mass (g/mol),
- $T$  = temperature (K),
- $\sigma$  = the collision diameter ( $\text{\AA}$ ),
- $\epsilon / \kappa$  = the maximum energy of attraction divided by the Boltzmann constant (K),
- $\omega_\mu$  = the collision integral.
- The Boltzmann constant ( $k$ ) is the physical constant relating energy at the individual particle level with temperature observed at the collective or bulk level. It is :  $1.380\,648813 \times 10^{-23} \text{ J K}^{-1}$

Similarly, viscosity of the dilute gas, it can be calculated by the Chapman-Enskog equation and the equation requires 3 empirically determined parameters like collision parameter, collision diameter, the maximum energy of attraction divided by the Boltzmann constant and the collision integral and based on which you can calculate what should be the viscosity of the dilute gases. So, from this correlation, you can easily calculate.

Here in this case, you will see that  $\mu_0$ ,  $\mu$  is the viscosity of the dilute gas,  $T^*$  is basically the reduced temperature, it is dimensionless,  $M$  is molecular mass that is gram per mole, and what is that  $T$  is the temperature in Kelvin,  $\sigma$  it is a collision diameter that is in Armstrong, and also  $\epsilon / \kappa$  is the maximum energy of attraction divided by Boltzmann constant, and  $\omega_\mu$  the collision integral.

There you will see that Boltzmann constant  $K$  is the physical constant that relates energy at the individual particle level with temperature that is observed at the collective or bulk level. So, it is generally this value is given to you  $1.380 \times 10^{-23}$  joule per Kelvin. So in this way, you can calculate viscosity of the dilute gas.

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## Viscosity of liquid

- The temperature dependence of liquid viscosity is usually expressed by the following model:

$$\mu(T) = \mu_0 \exp(-bT)$$

where  $T$  is temperature and  $\mu_0$  and  $b$  are coefficients

The dynamic viscosity of water is  $8.90 \times 10^{-4}$  Pa. s or  $8.90 \times 10^{-3}$  dyn. s/cm<sup>2</sup> or 0.890 cP at about 25 °C. Water has a viscosity of 0.0091 poise at 25 °C, or 1 centipoise at 20 °C.

Similarly, viscosity of the liquid, it is a function of temperature and this functionality given here. From this equation, you can easily calculate the viscosity of the liquid once you know that temperature and also other empirical constant as given here  $\mu_0$  and  $b$ , these are coefficients. So, you can calculate that viscosity. As per this  $\mu_0$  and  $b$  that is actually basically obtained from the experimental observation.

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## Viscosity of blends of liquids

- The viscosity of the blend of two or more liquids can be estimated using the Refutas equation

$$\nu = \exp \left( \exp \left( \frac{VBN_{blend} - 10.975}{14.534} \right) \right) - 0.8 \quad \checkmark$$

- Where  $VBN_{blend}$  is the viscosity blending number of the blend.

$$VBN_{blend} = [x_A \times VBN_A] + [x_B \times VBN_B] + \dots + [x_N \times VBN_N]$$

$$VBN_i = 14.534 \times \ln[\ln(\nu_i + 0.8)] + 10.975$$

where  $\nu$  is the kinematic viscosity in centistokes (cSt).  $\checkmark$

where  $x$  is the mass fraction of each component of the blend

Viscosity of the blends of liquids also you can calculate, that depends on actually viscosity blending number. It is generally what is the viscosity of mixture upon are dependent on the mole fractions of that individual components of that mixture. So, the viscosity of the blend of two or more liquids can be estimated using this Refutas equation. This Refutas equation basically involves with that viscosity blending number which is defined by this equation here



shown in the slides and this for individual components this VBN will be as a function of kinematic viscosity of the liquids there.

So, once you know that kinematic viscosity of that liquid in particular units as shown here in the slides, you can easily calculate what should be the viscosity of the blend of the two or more liquids.

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**Viscosity of slurry**

- The term slurry designs mixtures of a liquid and solid particles that retain some fluidity
- In the case of extremely low concentrations of fine particles, Einstein's equation is used:  
$$\mu = \mu_l(1 + 2.5\phi)$$
- In the case of very high concentrations, empirical equation proposed by Kitano et al. (1981)  
$$\mu = \mu_l \left(1 - \frac{\phi}{A}\right)^{-2}$$
  
$$A = 0.68$$

$\phi$  = volume fraction of solid particles

Kitano et al.,  
Rheol. Acta. 20  
(1981) 207 - 209.

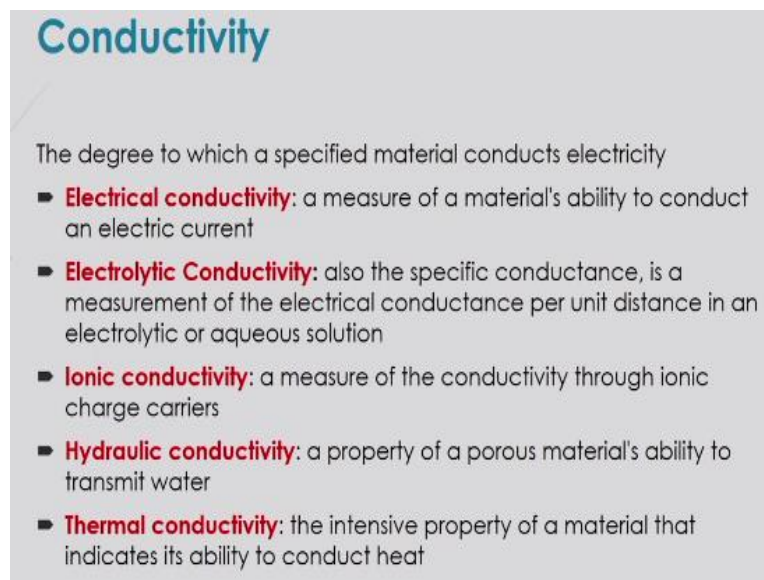
Similarly, viscosity of the slurry also you can calculate. The term slurry because sometimes some operations will be based on that mixture of liquid and solid. So, that liquid and solid will be regarded as the slurry. So, in that case you will see that if you add more particles there, you will see that viscosity will change, like powder. If any powder like we are using that maida or atta or suppose some other powder if you mix in water or make a solution you will see that if you add more powder in the water, you will see that the solution will be more thicker and thicker.

In that case, you will see that it will be very difficult to stir that solution if you add more powder there. So, that means here the solution of that solid particles and water that is slurry will be having more viscosity. So, that viscosity, that will be effective viscosity, how to calculate that effective viscosity, that depends on how much volume fraction of the solid powder will be adding into liquid. So, that volume fraction of the solids will be represented by this pie.

If you are adding a low concentration of that fine particles or powder particles there, so according to the Einstein's equation, you can easily calculate what should be effective viscosity of that slurry. So, that depends on volume fraction of the powders or solid particles and also what would be the pure water viscosity there. So according to this, you can easily calculate. If the particle concentration is very high, then you can use another empirical equation given by Kitano et al in 1981.

So according to his correlation, you can also calculate what would be the viscosity of the slurry at its high concentration of particles. In this case A is one unknown parameter that depends on what type of solid particles you are using. If you are using the simple sand particles or other fine particles, then generally this A value is coming as 0.68 there. So, in this way, you can calculate what would be the viscosity of the slurry, this is one of the important process variables where it would be required for any process in your particular process unit.

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### Conductivity

The degree to which a specified material conducts electricity

- **Electrical conductivity:** a measure of a material's ability to conduct an electric current
- **Electrolytic Conductivity:** also the specific conductance, is a measurement of the electrical conductance per unit distance in an electrolytic or aqueous solution
- **Ionic conductivity:** a measure of the conductivity through ionic charge carriers
- **Hydraulic conductivity:** a property of a porous material's ability to transmit water
- **Thermal conductivity:** the intensive property of a material that indicates its ability to conduct heat

Similarly conductivity you know the other variables, like there are several different types of conductivity which will give you that, how that is specified materials conducts electricity they are in the solution like electrical conductivity, electrolytic conductivity, ionic conductivity, hydraulic conductivity, thermal conductivity are those different conductivities are there for the solution.

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## Electrical conductivity

- **Electrical conductivity** or specific conductance is the reciprocal quantity, and measures a material's ability to conduct an electric current.
- It is commonly represented by the Greek letter  $\sigma$  (sigma), but  $\kappa$  (esp. in electrical engineering) or  $\gamma$  are also occasionally used.
- Its SI unit is siemens per metre (S/m) and CGSE unit is reciprocal of second (1/s):

$$\rho = R \frac{A}{l}$$

$R$  is the electrical resistance of a uniform specimen of the material (measured in ohms,  $\Omega$ )

$l$  is the length of the piece of material (measured in metres, m)

$A$  is the cross-sectional area of the specimen (measured in square metres,  $m^2$ ).

So, you have to calculate all those conductivity behavior of the solution there. So, electrical conductivity you can calculate by this equation given in the slides where all other parameters are given in the slides there. So, accordingly you can calculate, this is very important, it is commonly represented by the Greek letter sigma, but  $K$  is in electrical engineering or  $Y$  are also occasionally used and also you will see that this SI unit is siemens per metre and the CGSE unit is reciprocal of that second there. So, in this case, you can calculate this electrical conductivity.

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## Electrolytic conductivity

- The conductivity (or specific conductance) of an electrolyte solution is a measure of its ability to conduct electricity
- The SI unit of conductivity is Siemens per meter (S/m).
- In many cases, conductivity is linked directly to the total dissolved solids (T.D.S.). High quality deionized water has a conductivity of about  $5.5 \mu S/m$ , typical drinking water in the range of  $5-50 \text{ mS/m}$ , while sea water about  $5 \text{ S/m}$

$T$  is the temperature of the sample,

$T_{cal}$  is the calibration temperature,

$\sigma_T$  is the electrical conductivity at the temperature  $T$ ,

$\sigma_{T_{cal}}$  is the electrical conductivity at the calibration temperature  $T_{cal}$

$\alpha$  is the temperature compensation slope of the solution.

$$\sigma_T = \sigma_{T_{cal}} [1 + \alpha(T - T_{cal})]$$

Electrolytic conductivity also you can calculate based on the temperature, it is generally changed with respect to temperature. This conductivity or specific conductance of an electrolyte solution is a measure of its ability to conduct electricity and then this conductivity will be actually represented by its behavior or the ability to conduct the electricity and the SI

unit of conductivity is a siemens per metre, and in many cases, this conductivity is linked directly to the total dissolved solids in the solution.

High quality deionized water has a conductivity of about 5.5 microsiemens per metre. Typically, drinking water in the range of 5 to 50 millisiemens per metre while sea water about 5 siemens per metre there. So, if you know the temperature, what should be the electrolytic conductivity for a particular solution you can easily calculate. In this case, alpha is one parameter that is called temperature compensation slope of the solution that of course will be calculated based on your experimental observation for the process.

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**Thermal conductivity**

- Thermal conductivity,  $k$ , is the property of a material's ability to conduct heat. It appears primarily in Fourier's Law for heat conduction.
- In the International System of Units (SI), thermal conductivity is measured in watts per meter kelvin (W/(m.K)).

Diagram illustrating the relationship between Power per unit area transported, Temperature gradient, and Thermal conductivity:

$$\frac{\Delta Q}{\Delta t A} = -k \frac{\Delta T}{\Delta x}$$

A correlation for thermal conductivity of liquid n-alkanes based on the Vogel-Tammann-Fulcher-Hesse equation:

$$K = K_c \exp(A/(T - T_c))$$

For paraffin hydro carbon of Molecular weight 44 gives:-  
1.94, 561, 1630 for  $K_c$ ,  $T_c$ , and  $A$  respectively

Similarly, thermal conductivity also one of the important of variables based on which that process efficiency actually changed that basically for the heat transfer operations, it is very important. Now, this is a property of the material's ability to conduct heat. It appears primarily in Fourier's law for heat conduction. When heat is conducting to the solid material, there this conductivity, behavior of that material is actually concerned.

In that case, how to calculate the thermal conductivity of liquid for any solution especially for n-alkanes based on the Vogel-Tammann-Fulcher-Hesse equation you can easily calculate what would be the thermal conductivity of that alkanes there. For, paraffins hydrocarbon of molecular weight 44 gives its 1.94, 561, 1630 for  $K_c$ ,  $T_c$ , and  $A$  respectively there. So, based on these values, you can easily calculate what should be the thermal conductivity there.

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## Diffusion coefficient for gases

- The dependence of the diffusion coefficient on temperature for gases can be expressed using the **Chapman-Enskog theory**

$$D_{AB} = \frac{1.86 \times 10^{-3} (1/M_A + 1/M_B)^{0.5} T^{3/2}}{p \sigma_{AB}^2 \Omega}$$

$$\sigma_{AB} = \frac{1}{2} (\sigma_A + \sigma_B) \quad \text{-- the average collision diameter (\AA)}$$

<b>A &amp; B</b>	index the two kinds of molecules present in the gaseous mixture
<b>T</b>	temperature (K)
<b>M</b>	molar mass (g/mol)
<b>p</b>	pressure (atm)
<b>Ω</b>	a temperature-dependent collision integral (dimensionless).
<b>D</b>	diffusion coefficient (which is expressed in cm <sup>2</sup> /s).

Similarly, other parameters like diffusivity, this is also one of the, in this case how that material actually diffused through another materials, like diffusion of the gas through the mixture of the gases and also this degree of diffusivity of the gases can be actually assessed by the diffusion coefficient or simply it is called diffusivity. This is actually a proportionality constant between the molar flux due to the molar diffusion and the gradient in the concentration of the species.

So, whenever you are, suppose spray some perfume in the room, what will happen? You will see that there will be spread of that perfume molecules throughout the whole room, how it can be happened? This is basically there will be some concentration gradient from one position to another position and based on which you will see that they will try to you know that, based on that concentration differences, they will try to spread it out.

So, according to that, how that the concentration is spread in a particular length that will be regarded as concentration gradient and then this diffusion that is spreading of that perfume molecules or any other gases you can say that, any gaseous molecules or liquid molecule or any substance, how it will be spread or diffused based on that, that concentration gradient you can calculate. Now, that molar flux, flux means what, how much amount of molecules per unit time per unit crosssectional area, it is being spread that will be called as flux.

Now, that flux is directly proportional to that concentration gradient. Now, that proportionality constant, it will be regarded as diffusion coefficient or diffusivity. So, this is one of the important factor based on which you will see that process will be analyzed. So,

you have to know that what should be the diffusivity. So, similarly, how to calculate that diffusion coefficient for gases or liquid, you can use directly this equation for the calculation of the diffusivity, here it is given as in the slide.

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### Diffusion coefficient for solvent

- An approximate dependence of the diffusion coefficient on temperature in solvent can be found by using **semi-empirical equation of Wilke-Chang Equation**

$$D_{AB} = \frac{1.173 \times 10^{-16} (\phi M_B)^{0.5} T}{\mu_B \nu_A^{0.6}}$$

$D_{AB}$  is diffusivity of solute A in solvent B, m<sup>2</sup>/s,  
 $\phi$  is association factor  
 [for H<sub>2</sub>O =2.26; MeOH =1.9; EtOH =1.5; non-associated solvent=1.0];  
 $M_B$  is molecular weight of B;  
 $T$  is absolute temperature in K;  
 $\mu_B$  is solution viscosity, kg/m.s;  
 $\nu_A$  is solute molar volume at normal boiling point, m<sup>3</sup>/kmol  
 [ $\nu_A$ =0.0756 m<sup>3</sup>/kmol for H<sub>2</sub>O as solvent].

Similarly, diffusion of or diffusion coefficient for the solvent also you can calculate by this equation given in the slides accordingly.

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### The diffusion coefficient in solids

- The diffusion coefficient in solids at different temperatures is often found to be well predicted by Arrhenius equation:

$$D = D_0 e^{-E_A / RT}$$

$D$  is the diffusion coefficient  
 $D_0$  is the maximum diffusion coefficient (at infinite temperature)  
 $E_A$  is the activation energy (Activation energy is defined as the minimum energy required to start a chemical reaction) for diffusion in dimensions of [energy (amount of substance)<sup>-1</sup>]  
 $T$  is the temperature in units of [absolute temperature] (kelvins or degrees Rankine)  
 $R$  is the gas constant in dimensions of [energy temperature<sup>-1</sup> (amount of substance)<sup>-1</sup>]

Diffusion coefficient in the solids also you can calculate this Arrhenius type equation here. This actually basically depends on the temperature here. So, once you know this diffusion coefficient, maximum diffusion coefficient as  $D_0$  and also  $E_A$  is the activation in energy and also the temperature of the system, then you can easily calculate what is the diffusion coefficient of the solid.

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## Permeability

- It (commonly symbolized as  $\kappa$ , or  $k$ ) is a measure of the ability of a porous material to allow fluids to pass through it.
- The SI unit for permeability is  $\text{m}^2$ . ✓
- The concept of permeability is of importance in determining
  - the flow characteristics of hydrocarbons in oil and gas reservoirs, and
  - of groundwater in aquifers and
  - separation of solute by membrane.

Similarly permeability is one of important parameter based on which that you can assess process. This is a measure of the ability of a porous material to allow fluids to pass through the material. Now, the SI unit for the permeability is metre square. The concept of permeability is of importance in the determination of the flow characteristics of the hydrocarbons in oil and gas reservoirs and of groundwater in aquifers and also separation of the solute by membrane.

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- Permeability is typically determined in the lab by application of **Darcy's law** under steady state conditions.

$$Q = \frac{-kA}{\mu} \frac{(P_b - P_a)}{L}$$

- Q** = The total discharge, (units of volume per time, e.g.,  $\text{m}^3/\text{s}$ )  
**k** = the permeability of the medium, ( $\text{m}^2$ ),  
**A** = the cross-sectional area to flow, ( $\text{m}^2$ ), and  
 **$P_b - P_a$**  = the pressure drop (Pa),  
 **$\mu$**  = the viscosity, (Pa.s) and  
**L** = the length over which the pressure drop is taking place.

The negative sign is needed because fluids flows from high pressure to low pressure.  
Measurement of permeability through sandstones and shales yielded values from  $9.0 \times 10^{-19} \text{ m}^2$  to  $2.4 \times 10^{-12} \text{ m}^2$  for water and between  $1.7 \times 10^{-17} \text{ m}^2$  to  $2.6 \times 10^{-12} \text{ m}^2$  for nitrogen gas

So, this permeability you can easily calculate from the pressure difference of that medium from this side to that side, so in that case, this equation will be useful to calculate that permeability.

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## Dielectric Constant

### ► What is dielectric?

- A dielectric is a material which has poor electrical conductivity but inherits an ability to store an electrical charge (due to Dielectric polarization).
- Thus exhibiting only displacement current making it ideal to build a capacitor; to store and return electrical energy.

### ► What is Dielectric Constant?

- Dielectric Constant of a substance can be defined as:

**The ratio of the permittivity of the substance to the permittivity of the free space**

- It expresses the extent to which a material can hold electric flux in it.

Similarly, dielectric constant is also one important parameters. So, this dielectric constant is actually is a material dielectric constant that of a substance can be defined as that ratio of the permeability of the substance to the permittivity of the free space. It expresses the extent to which the material can hold electric flux.

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## Further reading.....

### Text Books:

- R. M. Felder, Ronald W. Rousseau, Lisa G. Bullard, Elementary Principles of Chemical Processes, 4th Ed., John Wiley & Sons, Asia, 2017.
- D. M. Himmelblau, J. B. Riggs, Basic Principles and Calculations in Chemical Engineering, 7/8th Ed., Prentice Hall of India, 2012.

### Reference Books:

- N. Chohey, Handbook of Chemical Engineering Calculations, 4th Ed., Mc-Graw Hill, 2012.
- Olaf, K.M. Watson and R. A. R. Hougen, Chemical Process Principles, Part 1: Material and Energy Balances, 2nd Ed., John Wiley & Sons, 2004.

So, we have discussed in this lecture different process variables which are very important for the analysis of process performance. So, thank you for your attentions for this lecture. In the next lecture, we will try to discuss about some thermodynamic variables like pressure and temperature of the flow processes.