

Basic Principles and Calculations in Chemical Engineering
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Lecture – 3
Unit Systems and Dimensions

Welcome to message open online course on basic principles and calculations in chemical engineering to you are discussing this introduction on the basic principles and chemical reaction engineering under module one. Now in this module we will discuss. Today about the unit systems and dimension as we are using a different do your products in our daily life.

Like that some grocery products in some transported products Pharmaceutical products so all though products produced based on in principle in process in now that process depends on different a variable different input and output also whatever it is coming after processing. So to quantify all those input and output .It is required to represent that the quantity by a certain unit or dimension.

So that unit systems and dimensions it is required to know for the analysis of their process efficiency based on certain mathematical expression. Now to express all those process efficiency by that the mathematical expression. You have to assess all those mathematical expressions by equations with their equations are dimensionally homogeneous or not. How to analyze those variables are based on the dimensionless quantity to skill of those processes to industrial scale.

So that is why you are going to discuss that the different units and dimensions are actually used for that analysis of process variables and their input and output. And also for the analysis of model equations are based on the dimensional analysis. So in this case we will discuss those entire variables. Now before going to that you have to know what the definition of that unit or dimensions.

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Unit and Dimensions

Unit	Dimensions
■ The "unit" indicates what the measured quantity represents, ■ A measured or counted quantity has a numerical value and a unit ■ Measurable units are specific values of dimensions that have been defined by convention, Example: grams for mass, seconds for time, and centimeters for length	■ The "dimension" is the measurable quantity that the unit represents. ■ Example: length, mass, time, and temperature, ■ It also be calculated by multiplying or dividing other dimensions, ■ Example: length/time = velocity, length³ = volume, and mass/length³ = density

Chemical engineers, like many other engineers, use values, units, and dimensions all the time

Now the unit actually indicates the measured quantity actually represents. Now we measure counted quantity has a numerical value and unit that measurable units are specific values of dimensions. That have been defined by convention like for mass you have to quantify based on the units like gram kg or Pound like this. For time you have to express it by unit like seconds and for the dimensions length that you have to express by a certain units like.

Centimeter meter even foot even inches all this things. And you will see that dimensions also are one of the important measurable quantities that the unit represents. Example like length mass time and temperature all are represented as a dimension. Now it also is calculated multiplying or dividing other dimension there. So whenever you are going to represent any unit by dimension.

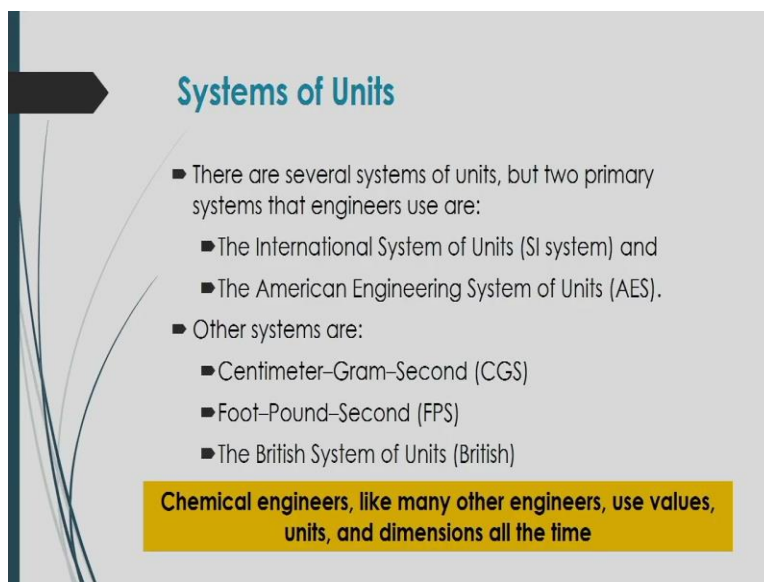
That dimension may be multiple of two dimensions or dividing of two dimensions. Combination of dimensions in such way that there will be a certain form of that unit there. Like if you are representing the velocity then you have to make its unit like what should be the length and what should be the time. Because velocity is defined by length upon time. So in that case what is the dimension of the velocity? It will be that L by T what is that L is the dimension of length and dimension of time is T.

So it will be length per time. And similarly volume what should be the dimension of volume? It will be actually length cube volume of that it will be actually L cube. And similarly density this

is basically that defined by mass per unit volume. So in that case what should be the dimension of that density? It will be like kg per meter cube like that. So In this we represent any process quantity or variable quantity by certain unit or by certain dimension there.

So any way chemical engineers like any other engineer they also use value unit and dimensions for all the time analysis of process efficiency based on at in model. For mathematical expression and what are they to see that there different systems of units are available to represent those mathematical expressions or quantity or quality of the variables which are being used for analysis.

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Systems of Units

- There are several systems of units, but two primary systems that engineers use are:
 - The International System of Units (SI system) and
 - The American Engineering System of Units (AES).
- Other systems are:
 - Centimeter–Gram–Second (CGS)
 - Foot–Pound–Second (FPS)
 - The British System of Units (British)

Chemical engineers, like many other engineers, use values, units, and dimensions all the time

So there are several systems of units but two primary system are engineer generally use those are like International System of units that is SI s system and American engineering system of unit. It is actually its short form actually it is called AES. Other systems also available like centimeter – gram – second that is CGS system. foot –pound –second system that is FPS system short form the and the British System of units like British yard and this case off course all the systems are conventionally used by engineers for analysis of Process.

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Units Associated with Systems				
System	Mass (m)	Length (l)	Time (t)	Temperature (T)
SI	Kilogram (kg)	Meter (m)	Second (s)	Kelvin (K)
AES Degree	Pound mass (lbm)	Foot (ft)	Second (s)	Fahrenheit (°F)
CGS	Gram (g)	Centimeter (cm)	Second (s)	Kelvin (K)
FPS*	Pound mass (lbm)	Foot (ft)	Second (s)	Degree Fahrenheit (°F)
British	Slug	Foot (ft)	Second (s)	Degree Celsius (°C)

***Imperial system units are sometimes referred to as FPS.**

To convert a quantity expressed in terms of one unit to its equivalent in terms of another unit, you will need to multiply the given quantity by the conversion factor (new unit/old unit)

And next that you will be see that what are units that associated with the systems in this slide in a table it shown that the different systems are the units of mass length time and temperature are given here. So please go through this the units of those the mass length and time temperature based on different system .here in SI system generally are internationally uniformly used the system for any process analysis in that case.

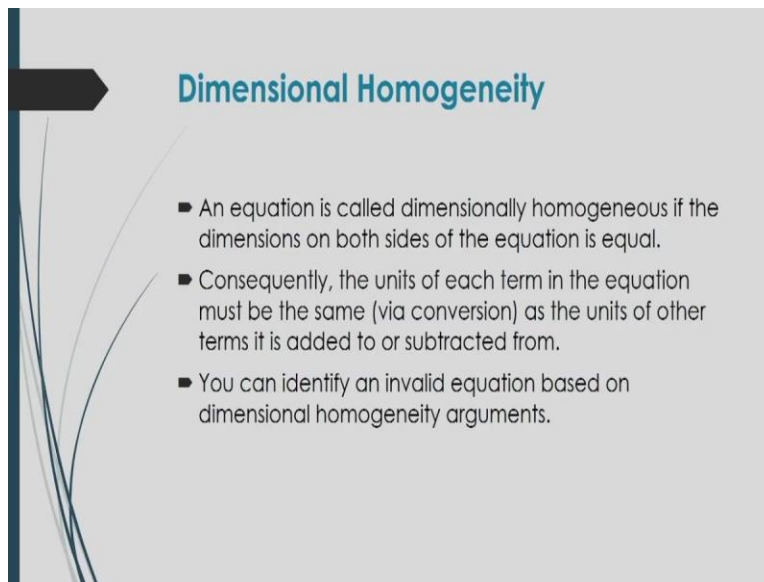
What is the unit for mass? general it is represented by kilogram and length it is represented by meter and the unit for time always it will be time second and also the temperature it would be in terms of Kelvin so SI systems are calling like this. Whereas if you are considering that AES degree systems there you will see that pound mass it would be represented as a unit for mass. Whereas foot would be a unit for length and second the same way as SI it would be taken for time and temperature it would be Fahrenheit.

Whereas CGS and FPS system respective units are given here in the table of mass length time and temperature respectively. For British system earlier they have used that slag system foot second and degree Celsius for mass length time and temperature .So for this of internationally of for uniform system that uses their actually using and that system always to be maintained as Si system. So that it will be analysis through analysis that process uniformly all over world.

And but whenever you are going to express that process variables quantity by other systems then you have to convert into one system to another system by certain factor. That factor of course will be that if you are considering other new system based on old system then you have to multiply by a factor of new unit by old unit. So convert a quantity expressed in terms of 1 unit to its equivalent in terms of another unit you will need to multiply the given quantity by the conversion factor.

So by that conversion factor you can easily calculate what should be the unit for any variables quantity and then to express all those process efficiency you sometimes need to express mathematically those proficiency by an equation. Now whenever you are going to express those equations you have to verify whether those questions are dimensionally consistent or not. So it is called homogeneousness of the dimensions of equation.

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Dimensional Homogeneity

- An equation is called dimensionally homogeneous if the dimensions on both sides of the equation is equal.
- Consequently, the units of each term in the equation must be the same (via conversion) as the units of other terms it is added to or subtracted from.
- You can identify an invalid equation based on dimensional homogeneity arguments.

So you have to identify and calculate of the units or dimensions of both sides of the equations whether the dimensions are same or not that will be actually uniform in dimensions on both sides to express that process efficiency by mathematics.

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Exceptions of Units

- **Coefficients in physical laws don't have units**
Example: $KE = \frac{1}{2}mu^2$ [=] kg m²/s² in SI; the coefficient "1/2" is unitless
- **Exponents are unitless**
Example: U^2 [=] m²/s² in SI; but the exponent "2" is unitless
- **Arguments are unitless**
Example: $\sin(x)$; while x can be in degrees or radians; $\sin(x)$ is unitless
- **Dimensionless numbers**
Example:
Reynolds number is a measure of inertial force to viscous force of a flow system

$$Re = \frac{\rho u d}{\mu}$$

We can say that some unit will not have dimensions. It will be called as exceptions of units like coefficient in physical laws that do not have units e.g. suppose energy it will be 1/2 into m into u square so in that case we will see that the coefficient half will not have any units yet similarly exponents also will be unit less like if you are expressing that u² or v² or certain other variables that to the power some value then that power or exponent will be unit less.

Similarly arguments are unit less like X is the angle that may have some unit degree but sinX that will not have any unit there similarly cosX, tanX or tan theta cos theta will not have any unit so argument are unit less similarly some quantities some expressions that are very important to express the process type or you can say that whether that process is highly turbulent or you can say that laminar like if any liquid is flowing through the pipe.

You will the velocity of the fluid would be in such way that that liquid will not be moving very laminar way or you can say that without disturbing of this molecule. It will be flowing through the pipe so if the molecule of the liquid is moving without actually displacing on its flat path. Across its path then you can say that the flow laminar where as when did all the molecules will be moving in such way that all the molecule will be randomly moving throughout the pipe.

So it will call as Starwood end now this laminar and turbulent flow now how will you access quantitative. So in that case you have to find out some dimensionless number. That is called

Reynolds number. Here Osborn Reynolds he did experiment to find out the criteria of making the flow of laminar or turbulent based on this Reynolds number and this is number because here the dimension of this quantity will not have any dimensions here of this quantity so it will be a number.

Because here you will see that it is defined by this $\rho u d / \mu$. ρ is the density its unit is kg per meter cube or its dimension is meter per L cube and u is the velocity that is meter per second and d is the unit here. Viscosity is nothing but kg per meter second. So if you substitute of that unit here for this variables $\rho u d / \mu$. Then you will see that dimension will be nullified so in that case it will be converted as a number that is why this number will be regarded as Reynolds number.

Now if Reynolds number is less than 2300 you can say that flow will be laminar whereas if it is greater than 4000 it will be highly turbulent. So by this analysis dimensionless number you can access the process it will be there. So this dimensionless group so many other dimension groups like this there are so many other dimensionless groups like this you will see in chemical process calculations there and those entire dimension is having no dimensions there.

So this is called dimensionless number so these are called exception of units there. So all are different kinds of exceptions of units. That I have found some coefficients in physical laws even exponents are unit less or dimensionless arguments are unit less dimensionless numbers are dimensionless. So like this or unit less you can say. So this you have to remember that what type of laws a dimension this or that will have some dimension.

So in that is based on which have to make the equations of course the dimensionless on both sides. Now have an example like this here make the equation dimensionally consistent.

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Make the equation dimensionally consistent

- Suppose that the composition C varies with time, t , in the following manner:

$$y = ae^{-bx}$$

where y has the units kg/L, x has the units seconds.

What are the units associated with a and b To make the equation dimensionally homogenous and consistent?

Answer: The unit of a is the same as the unit of y (i.e., kg/L). Because the argument of an exponential function must be dimensionless, the figure b must have a unit s^{-1} .

How to do this. Like if you suppose that composition C which will varies with respect to time a process that the concentration of solution respect to time express by the equation Y is equal to a minus b x square Y is concentration whose unit is kg per liter where as this x denoting for the time .so in that case the unit will be seconds Now as we know that the exponent will not have any dimension.

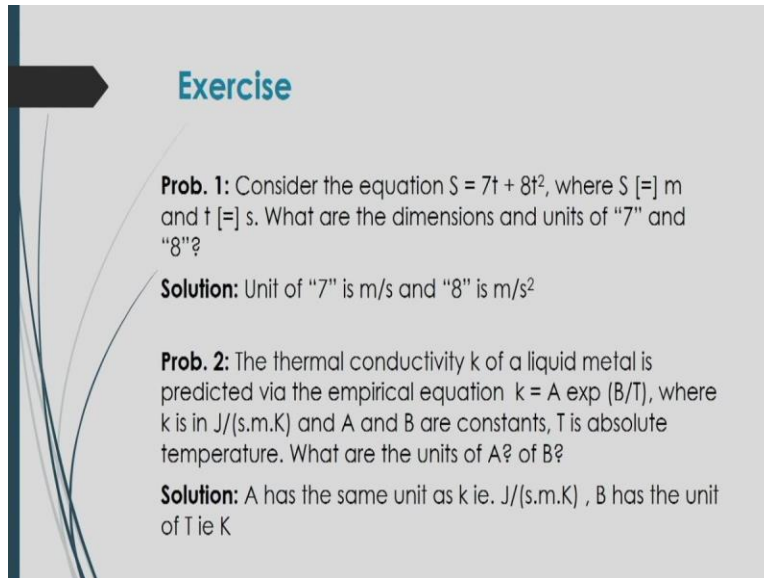
So if you are following that law here so this be the dimensions of having such a way that this exponents will not having dimensions. So how to do that so b dimensions of ' b ' will be such way that it should be nullified after multiplying of b and t . So what to do the dimension of what time is second. So the dimensions of 1 by second to so that second one by second it will be nullified to convert into 1.

So that way it should be that dimensionless so this part should dimensionless but your left hand sides have the dimension of kg per liter. So you have to make the consistence of the equation in such way that the dimension of that should be kg per liter. So that way you consider or make the consistent of the dimensions. So what are unit associated with a and b to make this educationally dimension homogeneous or consistence you have to make the unit of a as the same as the unit of y .

That is kg per liter and also exponential function must have dimensionless. So the figure b must

have a unit that is 1 by seconds or second inverse. So in this way you have to make the equation dimensionless on its both sides.

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Exercise

Prob. 1: Consider the equation $S = 7t + 8t^2$, where $S [=]$ m and $t [=]$ s. What are the dimensions and units of "7" and "8"?

Solution: Unit of "7" is m/s and "8" is m/s²

Prob. 2: The thermal conductivity k of a liquid metal is predicted via the empirical equation $k = A \exp(B/T)$, where k is in J/(s.m.K) and A and B are constants, T is absolute temperature. What are the units of A ? of B ?

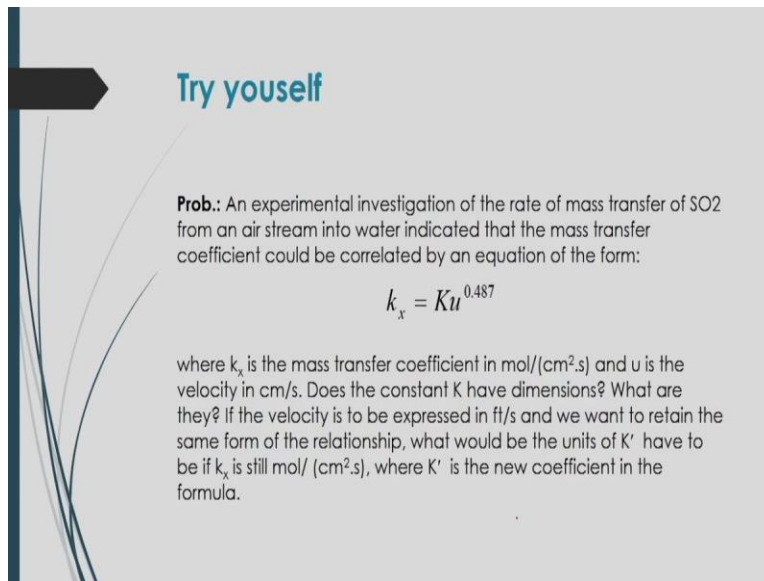
Solution: A has the same unit as k i.e. J/(s.m.K), B has the unit of T i.e. K

Like another example: consider S is a equation equal to $7t + 8t^2$ where s is referred as distance whose unit is meter and time is represented by t and unit is seconds. So what are dimension of 7 and 8 here .so you can represent that you have to just make it equivalent to dimensions on both sides of this equation. What is the unit for s ? What is the unit for t ? What is the unit for t^2 ?

Accordingly the other coefficients should have the dimensions to make it a constant for its dimension. So in that case unit of 7 will be meter per second and the unit for number 8 should be meter per Second Square. Similarly the dimension will be m by l and the also you can say m by s and also a dimensions of that will be m by square.

The thermal conductivity of k of a liquid metal is the like is predicted by other the empirical equation. like k equal to $A \exp(B/T)$ where k is jule per second meter Kelvin and A and B are constant so T is absolute Temperature so what should be the dimensions of units of A or B . so this similarly to make dimension in dimensionally homogeneous consistent. You have to make the unit of A same as that K .so it will be jule per second per meter K. Similar B has unit of T that is K.

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Try yourself

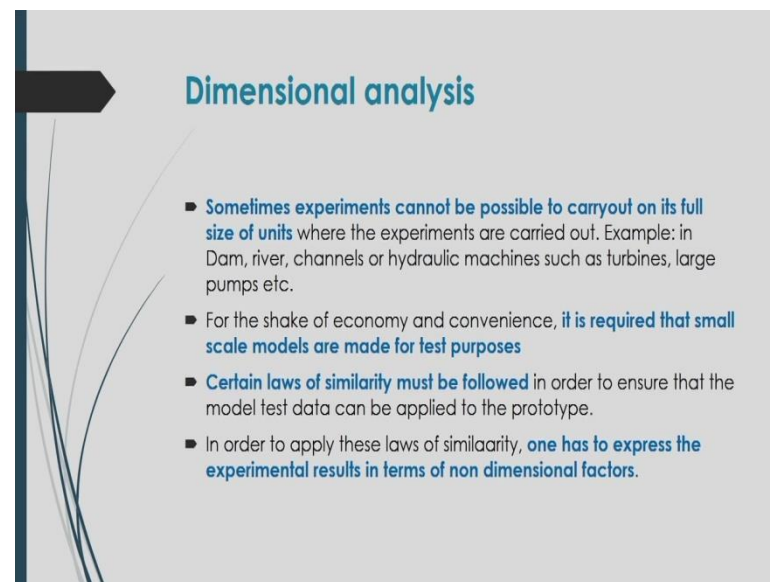
Prob.: An experimental investigation of the rate of mass transfer of SO₂ from an air stream into water indicated that the mass transfer coefficient could be correlated by an equation of the form:

$$k_x = Ku^{0.487}$$

where k_x is the mass transfer coefficient in mol/(cm².s) and u is the velocity in cm/s. Does the constant K have dimensions? What are they? If the velocity is to be expressed in ft/s and we want to retain the same form of the relationship, what would be the units of K' have to be if k_x is still mol/ (cm².s), where K' is the new coefficient in the formula.

Now you can try yourself this problem yourself to make this dimensionally consistent.

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Dimensional analysis

- Sometimes experiments cannot be possible to carryout on its full size of units where the experiments are carried out. Example: in Dam, river, channels or hydraulic machines such as turbines, large pumps etc.
- For the shake of economy and convenience, it is required that small scale models are made for test purposes
- Certain laws of similarity must be followed in order to ensure that the model test data can be applied to the prototype.
- In order to apply these laws of similaarity, one has to express the experimental results in terms of non dimensional factors.

Now another important aspect of dimension for the chemical engineering process like whenever you are doing any experiment and gating some results of the process of efficiency. Now to make this process be able to industrial scale. You have to express that mathematical expression or model to make it industrially scale label then what you have to do you have to do the dimension analysis of the equation from this lower scale to the upper scale there.

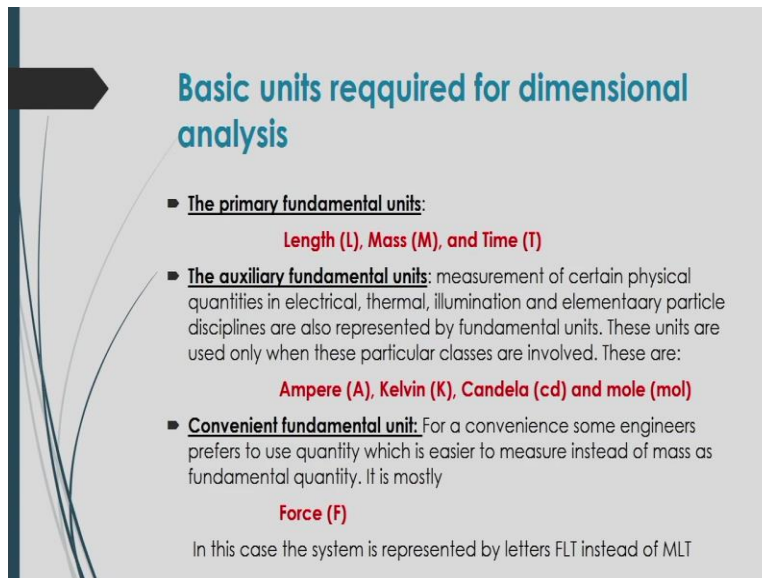
So in that case in laboratory scale sometimes you cannot do that experiment possible to carry out on its full size of unit. So there the experiments are carried out in the small scale so in that case to make it that industrial scale you have to make some model based on the experimental data. Now once you make the model equation by universal model equation or empirical model equation based on your experimental data.

You can predict those process of output quantity based on this model and way based on the process variable and you can predict at large scale just by changing the value of process variable quantity .so whenever you are making that equation. Equation to be used to calculate for the higher values of process variable. Then you can get the higher values of or projected values of what is that process output.

Now based on that industrial scale process equipment to be developed based on this experimental data and its model development so in this way whenever making any model equation. You have to make it dimensionally consistent and to make that dimensionally consistent. You have to that made this dimensionless form and that equation to be sometimes to convert it into a dimensionless groups or dimensionless number.

Now to make those are dimensionless number you have to do some dimensional analysis. There will be certain way to do that so that process will discuss yet.

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Basic units required for dimensional analysis

- **The primary fundamental units:**
Length (L), Mass (M), and Time (T)
- **The auxiliary fundamental units:** measurement of certain physical quantities in electrical, thermal, illumination and elementary particle disciplines are also represented by fundamental units. These units are used only when these particular classes are involved. These are:
Ampere (A), Kelvin (K), Candela (cd) and mole (mol)
- **Convenient fundamental unit:** For a convenience some engineers prefer to use quantity which is easier to measure instead of mass as fundamental quantity. It is mostly
Force (F)
In this case the system is represented by letters FLT instead of MLT

There are two processes to do the dimensional analysis based on the process variables. If there are N numbers of variables for analysis of that process then you have to make those entire processes variable in a dimensionless form in such way that on the left hand side and right hand side of the equations will be dimensionless. So to make that there are two processes. One is called Raleigh method and other is called Buckingham Pi theorem.

Now to analyses by these two methods are the equation dimensionally consistent or dimensionless number based equation. You have to know some primary fundamental units are based on which those dimension analysis to be done. Now that primary time fundamental units are length mass and time based on this fundamental units. You can do that dimension analysis to make that the dimensionless group. Now we will do that here one by one.

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Important Dimensions to Remember

	Quantity	Common Symbol(s)	Dimensions
Geometry	Area	A	L^2
	Volume	V	L^3
	Second moment of area	I	L^4
Kinematics	Velocity	U	$L T^{-1}$
	Acceleration	a	$L T^{-2}$
	Angle	θ	1 (i.e. dimensionless)
	Angular velocity	ω	T^{-1}
	Quantity of flow	Q	$L^3 T^{-1}$
	Mass flow rate	$\dot{m}$	$M T^{-1}$
Dynamics	Force	F	$M L T^{-2}$
	Moment, torque	T	$M L^2 T^{-2}$
	Energy, work, heat	E, W	$M L^2 T^{-2}$
	Power	P	$M L^2 T^{-3}$
	Pressure, stress	p, τ	$M L^{-1} T^{-2}$
Fluid properties	Density	ρ	$M L^{-3}$
	Viscosity	μ	$M L^{-1} T^{-1}$
	Kinematic viscosity	ν	$L^2 T^{-1}$
	Surface tension	σ	$M T^{-2}$
	Thermal conductivity	k	$M L T^{-1} \Theta^{-1}$
	Specific heat	c_p, c_v	$L^2 T^{-2} \Theta^{-1}$
	Bulk modulus	K	$M L^{-1} T^{-2}$

And before going to that you have to know important dimension also you have to remember here in this like given some variable like geometrical variable kinematics variable dynamic variable even fluid property variables and their quantities like geometry like area volume even moment area. Even you can say that kinematics like velocity acceleration angle angular velocity quantity of flow mass flow rate. Even dynamics variable like of force moment torque energy heat power pressure stress all those things.

Even some fluid property density viscosity surface tension kinematic viscosity thermal conductivity specific heat even bulk modulus of the fluid like that. So all those quantity represented by certain dimension. Here in this slide also this dimensions are given respectively here all those dimensions should be required to analyze that dimension and to make it dimensionless group. So for that this- I told that two method are there that One is Raleigh method and another is Buckingham pi- method.

So you have to remember these two methods and both the methods of dimension analysis determine only the relevant independent dimension less parameters of a problem but not the exact relation between them. So here will make it.

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Rayleigh method

The method involves the following steps:

- **Step 1:** Note all the **independent variables** that are likely to influence the **dependent variable**.
- **Step 2:** If y is a variable that depends upon independent variables $x_1, x_2, x_3, \dots, x_n$, then write the **functional equation** as

$$y = f(x_1, x_2, x_3, \dots, x_n)$$



Lord Rayleigh,
British Scientist, he
got Nobel prize in
physics in 1904

Rayleigh (1892) On the question of the stability of the flow of fluid, Phil. Mag. 34, 59

Let's discuss about that Rayleigh method first the method involves the following steps here. Step 1 notes all the independent variables that are likely to influence the dependent variable. Like any process suppose there is a flow of fluid through the pipe .OK. Now you will see there will a change in pressure whenever fluid will be flowing through the pipe.

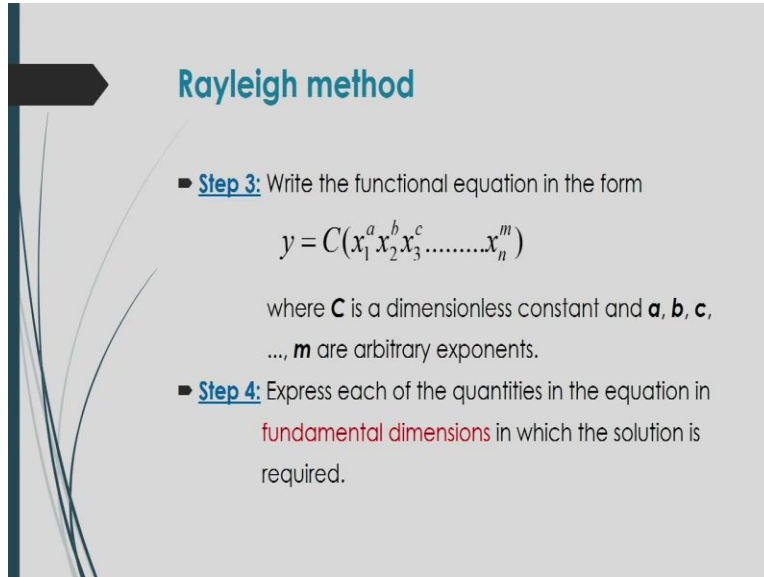
Now pressure is one dependent variable based on some other independent variables it will be change how like pressure will be changing based on the diameter of the pipe. Pressure will change if you change in velocity of liquid pressure will change if you change the type of fluid that is its physical properties.

So based on which you will see that there will be a change of pressure drop so that why here dependent variables pressure drop where independent variables physical properties of system diameter of column length of column even velocity of the fluid even there is any roughness of pipe or not .That also gives change in pressure drop. So you have to note all the independent variables that are likely to influence the process and also independent variables on which these independent variables are affected. Then what you have to do if Y is a variable.

That depends upon independent variables like X_1, X_2, X_3 and X then writes the functional equation. As Y is equal to what is that a function of $X_1, X_2, X_3 \dots X_n$. There may be n number of variables will there. It maybe now 2345678 more number of variables will be there. That would

be independent where as dependent variable will be certain variable there. OK. So in this way you have to express the functionality of dependent variables based on those independent variables.

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A presentation slide titled "Rayleigh method" in blue text. On the left side, there is a decorative graphic consisting of a dark blue vertical bar and several thin, curved lines in shades of blue and grey. The slide contains two steps: "Step 3" and "Step 4".

Rayleigh method

- **Step 3:** Write the functional equation in the form
$$y = C(x_1^a x_2^b x_3^c \dots x_n^m)$$
where **C** is a dimensionless constant and **a, b, c, ..., m** are arbitrary exponents.
- **Step 4:** Express each of the quantities in the equation in **fundamental dimensions** in which the solution is required.

After that what you have to do write the functional equation in the form of like this y is equal to like C to X1 to the power a X2 to the power b X3 to the power c and Xn to the power m .So in this case you are actually making this certain relationship of those independent and depended variables like this. Here C is the constant of dimensionless constant and a b c and m is arbitrary exponents.

Those exponents C a b c all those exponent will be obtain from that experimental data based on your process. And then after that express each of the quantities in the equation in fundamental dimension in which is solution required.

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Rayleigh method

- **Step 5:** By using **dimensional homogeneity**, obtain a set of **simultaneous equations** involving the exponents $a, b, c, \dots, m$.
- **Step 6:** Solving these equations to obtain the value of exponents $a, b, c, \dots, m$.
- **Step 7:** Formation of **dimensionless groups** by the variables with like exponents.

Now after that by using dimensional homogeneity obtain a set of simultaneous equations. That is involving the exponents a, b, c and m . after that you have to solve these equation to obtain the value of exponents a, b, c and m . Then from then find the dimensionless group by the variables with like exponents will do that example of these methods like here.

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Example: Let us consider a frictional resistance (F) when a liquid is flowing through a pipe depends on the viscosity, density of the fluid, velocity of the flow, diameter of the pipe and pipe surface roughness. Derive a rational equation for the pipe flow in terms of dimensionless groups by Rayleigh's method

Solution:

- **Step 1:** Independent variables: $\mu, \rho, v, D, \varepsilon$
Dependent variable: F

- **Step 2:** functional equation as

$$F = f(\mu, \rho, v, d, \varepsilon) \quad (\text{Eq. 1})$$

- **Step 3:** the functional equation in the form

$$F = C\mu^a \rho^b v^c D^d \varepsilon^e \quad (\text{Eq. 2})$$

Let us consider considered a frictional resistance that is represented by f . When a liquid is going through a pipe. That depends on viscosity density of the fluid velocity of the flow diameter of the pipe and the pipe roughness. Now you have to derive a rational equation for the pipe flow. In terms of dimensionless group by Rayleigh method .What you have to do same way that step1 step 2 step 3 follow like this. As a step 1 you have to fast identify what are the dependent

variable and independent variables? What is that independent variables? That is frictional resistance because this frictional resistance depending on that viscosity of the fluid density of the fluid velocity of the flow diameter of the pipe and also roughness of pipe surface. So this viscosity of the pipe is denoted by μ density is denoted by ρ velocity of the flow is v pipe diameter is d and pipe roughness is ϵ .

So this dependent variable is f that is resistance and here functional equation as you can then express as that F equal to that dependent variables should be independent as a function of independent variable like μ ρ v d and ϵ . Now next step is the functional of equation in the form there F is equal to $C\mu^a \rho^b v^c D^d \epsilon^e$ as per that step.

I have described earlier. Accordingly you have to express this after that you have to express each of the quantities in the equation in fundamental dimensions.

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► **Step 4:** Expressing each of the quantities in the equation in **fundamental dimensions** as

$$F = C \mu^a \rho^b v^c D^d \epsilon^e \quad (\text{Eq. 3})$$

$$MLT^{-2} = C(ML^{-1}T^{-1})^a (ML^{-3})^b (LT^{-1})^c (L)^d (L)^e$$

► **Step 5:** based on **dimensional homogeneity**, the **simultaneous equations** involving a, b, c, d, e

for M : $1 = a + b$ (Eq. 4)

for L : $1 = -a - 3b + c + d + e$ (Eq. 5)

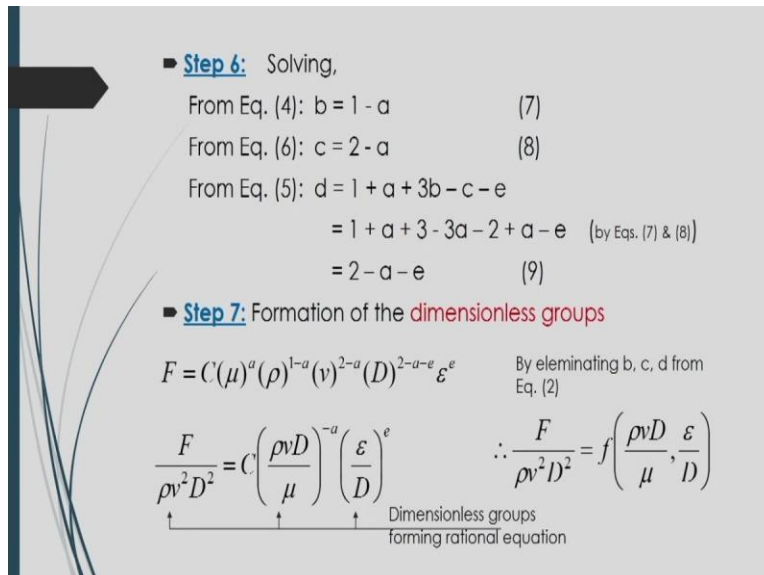
for T : $-2 = -a - c$ (Eq. 6)

Like this F is equal to $c \mu^a \rho^b v^c D^d \epsilon^e$. Now in this case dimension of F that earlier I told that you have to remember some dimensions of quantity so that will be equal to $M L T$ to the power -2 for f and c will not having any dimensions here. ML inverse and antiinverse to the dimension of viscosity and dimension of density is ML to the power -3 and dimension of v is L to the power -1 and dimension of D is L

and dimension of Epsilon is L. So based on that dimensional homogeneity concept. The simultaneous equations involving a b c d e can be represented as. If you equalize the dimension on both sides of the equation number 3 here in the slide.

You can write this are the dimensions of came first. In the left hand inside dimension of a power of this dimension M is one where the power of this dimension M on the right hand side of this equation number 3 is a and b . so it will be 1 is equal to a + b. similarly dimensions if you consider for L length then it will be a 1 is equal to -a- 3b + c+ d +e As given in question number 5. Here only for time that dimensions can be compare and you can make this equation is -2 = a - c that is given in question number 6.

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■ **Step 6:** Solving,
 From Eq. (4): $b = 1 - a$ (7)
 From Eq. (6): $c = 2 - a$ (8)
 From Eq. (5): $d = 1 + a + 3b - c - e$
 $= 1 + a + 3(1 - a) - (2 - a) - e$ (by Eqs. (7) & (8))
 $= 2 - a - e$ (9)

■ **Step 7:** Formation of the **dimensionless groups**
 $F = C(\mu)^a (\rho)^{1-a} (v)^{2-a} (D)^{2-a-e} \epsilon^e$ By eliminating b, c, d from Eq. (2)

$\frac{F}{\rho v^2 D^2} = C \left(\frac{\rho v D}{\mu} \right)^{-a} \left(\frac{\epsilon}{D} \right)^e$
 $\therefore \frac{F}{\rho v^2 D^2} = f \left(\frac{\rho v D}{\mu}, \frac{\epsilon}{D} \right)$

Dimensionless groups forming rational equation

After that you have to solve this three equations that 4 5 and 6. Like this here you have to Express that 7. This equation number 4 as b equal to 1–a and then equation 6 is equal to 2–a and equation 5 can be represented as d in terms of 2-a-e. So in this case you have to select that b c d that will be represented by some other exponent as per that equation. Now after that formation of the dimensionless group like this F you substitute that value of b c and d.

All are in terms of A and B or like this so you substitute that value of BCD there in equation number earlier that any question number 2. Then you will see that and the segregating those things and you can represent this one as left hand side and this one will be as right hand side

here. So in this you will see this one is one group and this one is one group this group. Where this group are made actually dimensionally you just check it that row $b d D b \mu$. It is a dimensionless even ϵ by D is also a dimensionless even F by $\rho v^2 D^2$ is also dimensionless.

So these 3 dimensionless groups are formed and these dimensionless groups are interrelated like this any suppose if you considering as that dependant variables are there is flow resistance. So the dimensionless group that is made by that or in terms of that of flow resistance. That is dependent variable. This group should be the dependant dimensionless groups. Whereas independent dimensionless groups are here this one and this one. So in this case we can make this functionality as like F by $\rho v^2 D^2$.

This dimensionless groups as a function of dimensionless group of right hand side here. Now this is called one equation or functionality which is expressed in terms of some dimensionless groups. So this is called rational equation where there will be no dimensions on both sides of this equation. Whereas all those variables are taking part in this your equations and the effect of all those variables on those dependent variables will be accessed by this equation there.

Now what should be that functionality exact functionality exact on an equation that you can access to by regression analysis putting that expression with experimental data? In that way you can form equation. So new jobs to make its dimensional analysis and make it the rational equation by that dimension analysis. Now this very important here but one important thing you have to remember here that how many dimensionless groups can be made based on your number of variables.

If you have 5 variable then you can make $5 - 3$ that will equal to two dimensionless groups. If you have 6 variables total dependent independent all total if you have 6 variables then you can make $6 - 3$ total three dimensionless groups. The basic rule is that if you have n number of variables then you can make the dimensionless groups or the dimensionless groups will be formed as $n - j$. What is j ? J is the number of fundamental dimensions.

Generally no. of fundamental dimensions are three. so here the number of dimensions and fundamental dimensions is three. Whereas the total variables is if it is n. Then it will be equal to n-j no. of groups will be formed. Here in this example you see.

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Example: Let us consider a frictional resistance (F) when a liquid is flowing through a pipe depends on the viscosity, density of the fluid, velocity of the flow, diameter of the pipe and pipe surface roughness. Derive a rational equation for the pipe flow in terms of dimensionless groups by Rayleigh's method

Solution:

- Step 1: Independent variables: $\mu, \rho, v, D, \varepsilon$
Dependent variable: F
- Step 2: functional equation as

$$F = f(\mu, \rho, v, d, \varepsilon) \quad (\text{Eq. 1})$$

- Step 3: the functional equation in the form

$$F = C \mu^a \rho^b v^c D^d \varepsilon^e \quad (\text{Eq. 2})$$

How many variables are there? Here μ is 1 row 2 v 3 D here 4 epsilon 5 and F 6. Total 6 numbers of variables whereas fundamental dimensions number of fundamental dimensions is 3. So how many groups will be formed? That is 6-3 that is 3 number of dimensionless group will be formed. So you will see that after dimensional analysis by this Raleigh method you got these 3 groups.

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- Step 6: Solving,

From Eq. (4): $b = 1 - a$ (7)

From Eq. (6): $c = 2 - a$ (8)

From Eq. (5): $d = 1 + a + 3b - c - e$
 $= 1 + a + 3 - 3a - 2 + a - e$ (by Eqs. (7) & (8))
 $= 2 - a - e$ (9)

- Step 7: Formation of the dimensionless groups

By eliminating b, c, d from Eq. (2)

$$F = C(\mu)^a (\rho)^{1-a} (v)^{2-a} (D)^{2-a-e} \varepsilon^e$$

$$\frac{F}{\rho v^2 D^2} = C \left(\frac{\rho v D}{\mu} \right)^{-a} \left(\frac{\varepsilon}{D} \right)^e$$

$\therefore \frac{F}{\rho v^2 D^2} = f \left(\frac{\rho v D}{\mu}, \frac{\varepsilon}{D} \right)$

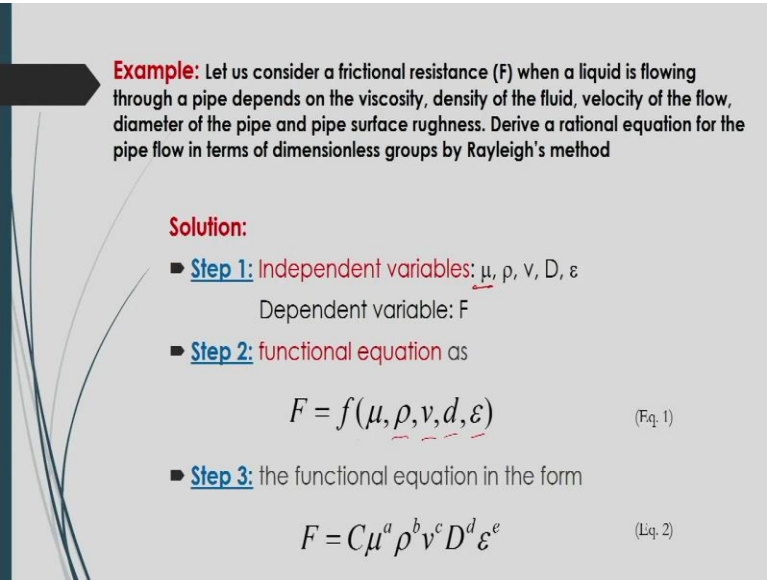
Dimensionless groups forming rational equation

Here this one this one and this one these three groups you got. Those are dimensionless groups. So this is the basic theory to get how many number will dimensionless groups based on your number of variables. Now we will discuss about that Buckingham pi method. This method to Analyze this dimension. That is dimensionally making that dimension analysis by this Buckingham method.

This also you can make that rationally equation where all the groups not having dimension there and in this case what you have to do? According to that theory the functional relationship among variable like : $X_1 X_2 \dots X_n$ can be expressed as that the function of $X_1 X_2 \dots X_n$ that will be equals to zero and here n equal to no. of variables and if j is the number of fundamental dimensions such as mettle that is length L and mass M and time as T.

Then MLT these are the fundamental dimensions then number of dimensionless groups can be formed $n - j$ that already “I told here during the analysis of dimension by Buckingham method here also saying number of dimensionless groups will gate.

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Example: Let us consider a frictional resistance (F) when a liquid is flowing through a pipe depends on the viscosity, density of the fluid, velocity of the flow, diameter of the pipe and pipe surface roughness. Derive a rational equation for the pipe flow in terms of dimensionless groups by Rayleigh's method

Solution:

- **Step 1:** Independent variables: $\mu, \rho, v, D, \varepsilon$
Dependent variable: F
- **Step 2:** functional equation as

$$F = f(\mu, \rho, v, d, \varepsilon) \quad (\text{Eq. 1})$$
- **Step 3:** the functional equation in the form

$$F = C \mu^a \rho^b v^c D^d \varepsilon^e \quad (\text{Eq. 2})$$

Now the dimensionless groups that expressed as here according to this Buckingham pi theorem that pi I is equal to X_1 to the power a I and X_2 to the power b I and X_j to the power j I and that into X_{j+1} . Here $X_1 X_2 X_j$ they are called repeating variables and X_{j+1} are called non-repeating variables. So some variable will repeating again and again it come for that analysis of

dimensionless groups.

Here like π I here π means dimensionless groups here may be n number of dimensionless groups will be formed. Here three all three dimensionless groups if it is coming then $X_1, X_2, \dots, X_j$ and then X_{j+1} . Accordingly it will come. Ok we will see here no. of repeating variables is equal to number of fundamental dimensions. You have to consider that repeating variables will be number of fundamental dimensions.

That will be three so here X_1, X_2 and $\dots, X_j$ these are called repeating variables this should be only three. Whereas X_{j+1} remaining other variables there and general j is equal to 3 here and functional relation among dimensionless group is here this should be then dimensionless groups as π I then it will be represented as function of $\pi_1, \pi_2, \dots, \pi_n$ that will equal to 0. Here this functionality in terms of what is that? In terms of 5 that is dimensionless groups.

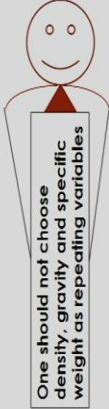
Here also we can analyze this dimension by Buckingham method by step wise. Like step 1 you to fast identify the relevant variables and function and then write down the dimensions and then establishment of the number of independent dimensions and non dimensional groups and then and then .At step 4 choose that dimensionally Independent and dependent variables generally that are repeating variables.

Generally that repeating variable should be first representing repeating variable should be fluid properties and the second is the flow characteristics and third will be geometric variables.

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Steps of Buckingham's method

- **Step 1:** Identify the relevant variables and function
- **Step 2:** Write down dimensions
- **Step 3:** Establishment of the number of independent dimensions and non-dimensional groups
- **Step 4:** Choosing j ($= 3$) dimensionally-independent repeating variables. Generally:
 - The first representing the **fluid property**
 - The second representing the **flow characteristics**
 - The third representing the **geometric characteristics**



So in that way you have to oscillate this repeating variables. So you have you cannot select that repeating variables like fluid property at three times like that. Now you have to take in such way that from the east that characteristic likes fluid property flow characteristics and geometric characteristics. You have to take one dimensions from each of this variables.

Then step 5- you have to create the pi s that is dimensionless groups by non dimensionalising the remaining variables send by solving the coefficient. And then you have to set the non dimensional relationship and then rearrange for convenience you can say this case you are free to replace any of the pi s group by a power of that pi or buy a product with the other pi s provided retaining the same number of independent dimensionless groups'

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- **Step 5:** Creating the π s by non-dimensionalising the remaining variables and by solving the coefficients
- **Step 6:** Setting the non-dimensional relationship
- **Step 7:** Rearrange (if required) for convenience. In this case, you are free to replace any of the π s by a power of that π , or by a product with the other π s, provided retaining the same number of independent dimensionless groups

Like that will do that example again are based on this Buckingham Pi theorem. Let us considered a frictional resistance again that F when a liquid is flowing through a pipe that depends on the viscosity density of the fluid velocity of the flow diameter of the pipe and pipe surface roughness. In this case is also you just arrived derived the rational equation based on this Buckingham Pi theorem in terms of dimensionless groups. Now first of all you have to consider as state one that what will be the relevant variables and the function.

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Example problem: Let us consider a frictional resistance (F) when a liquid is flowing through a pipe depends on the viscosity, density of the fluid, velocity of the flow, diameter of the pipe and pipe surface roughness. Derive a rational equation for the pipe flow in terms of dimensionless groups by Buckingham's method

Solution:

- **Step 1:** Relevant variables and function: $f(F, \mu, \rho, v, D, \varepsilon) = 0$
- **Step 2:** Dimensions:
 $F : MLT^{-2}; \mu : ML^{-1}T^{-1}; \rho : ML^{-3}; v : LT^{-1}; D : L; \varepsilon : L$
- **Step 3:**
 Number of relevant variables: $n = 6$
 Number of independent dimensions: $j = 3$ (M, L and T)
 Number of non-dimensional groups (π s): $n - j = 3$

So relevant variables that viscosity density velocity of the flow diameter of the pipe and pipe surface roughness. So out of which you can say that some will be dependent and some will be

independent but all together all the variables to be noted down and you have to make the functionality like this here function of $f, \mu, \rho, v, D, \epsilon$ that will be equal to 0 .

But here the different from that Rayleigh method is that here this f certain dependent variables will be a function of independent variables.

It is not make here general that may be μ will be function of some. In this case directly just one function to be form like this and then you have to note down what are the dimensions of each variables here. What are the dimensions of f ? What is the dimension of μ ? What is the dimension of ρ ? What is the dimension of v ? And what is the dimension of D ? And then ϵ also. So in this way note down all the dimensions for all variables.

Now in this case what will be the relevant variables number of relevant variables is 6 that n is equal to 6. No. of independent dimensions are generally 3 that is j that is M, L and T . And number of non dimensional groups will be that made based on the principle of $n-j$ that will be is equal to 3 that $6-3$ is equal to 3. Then next is that you have to choose three dimensionally independent repeating variables.

That repeating variables you have to take 1 from fluid property like density one from flow characteristic like velocity and one from geometry characteristics like diameter of the pipe. So this three repeating variables will be chooses here for the analysis. So this is very important you cannot I told that you cannot repeatedly three variables to be taken as ρ, ρ, ρ, v again ρ and you cannot do that.

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■ **Step 4:** Choosing j ($= 3$) dimensionally-independent repeating variables as:

- **fluid property:** ρ
- **flow characteristics:** v
- **geometric characteristics:** D

■ **Step 5:** Creating the π s as:

$$\pi_1 = \rho^{a_1} v^{b_1} D^{c_1} F$$

$$\pi_2 = \rho^{a_2} v^{b_2} D^{c_2} \mu$$

$$\pi_3 = \rho^{a_3} v^{b_3} D^{c_3} \epsilon$$

Remember if $n = j$, you have to consider $j = j - 1$. Example: if $n=3$, but $j=3$, then $n-j=0$, i.e., no dimensionless groups will be formed. So in this case, j should be $3-1 = 2$. In this case, you have to pick 2 repeating variables.

Separately you have to take this repeating variables one from fluid property one from flow characteristics and one from geometry characteristics. OK. And then you have to create those dimensionless groups. What is the principle that first dimensionless since you are going to from $n - j$ that is $6-3$ 3 dimensionless groups. Here that will be π_1 is equal to what that π_2 is and what is that? π_3 . So π_1 what is that will be is equal to row v and D .

These are the repeating variables and non repeating variables here one is F other non repeating variables μ ϵ there. So you can make like this with those non repeating variables with that repeating variables you can make the functionality of functional dimensionless groups like this. Similarly for 2nd dimensionless groups you can again use that repeating variable of that row v and D and other non repeating variables here μ .

And here π_3 again with those non repeating variables with those repeating variables. You have to make the equations like this so in this way you have to that dimensionless groups here. After that what you have to do you have to do that dimensionally consistency just by equating that equation. Here in this case like π_1 is equal to this from that equation here in this case π_1 .

If you considered that π_1 is equal to row to the power a_1 v to the power b_1 D to the power c_1 and F . Then if you consider that dimensions of all this variables in the left hand side and right hand side since π_1 be dimensionless group then no dimensions will be there. We can represent it

as M to the power 0 L to the power 0 and T to the power 0. What does it mean? There will be no dimensions here.

Similarly for right hand side. If you note that what is the dimension of row then it will be ML to the power -3 and there is a power also these dimensions to the power a1 similarly for v similarly for D similarly for F also you have to write down like this. After that just you equalize this dimensions power of these dimensions of this left hand side to right hand side. Then you can make an equation like this. With that exponent here like in this case.

If you are considering first M then what the dimension of what is is the power of M 0 and the right inside what are the power of this M here. Here 1 is M¹ and here is 1. So here simply a₁+1 that will be equals to zero. Similarly for L if you consider that power on both sides then it will be as 0 equal to -3a₁ + b₁ + c₁ + 1. Similarly for that T if you equalize that the exponent of this left hand side and right hand side then you can have this equation of -b₁ -2 that will be equal to 0.

Now solve this three equations you will see that the solution of this equation should give you that a₁ value will be equal to -1 b₁ will be equal to -2 and c₁ will be equal to -2 . So in this case if you substitute that a₁ b₁ and c₁. In this equation and rearrange this you will see that this pi₁ will be as F by row v square D square. You just check it and substitute the all dimensions here. You see this pi₁ will not have any dimensions.

So this is dimensionless group simply. Now this pi₁ is simply F by row v square D square. Similarly consider that pi₂ again that you have to equalize the power of all dimensions from its left-hand side and right-hand side. After substitution of dimensions of each variable there. You will see this three equations will be obtain and after solving this three equations you will see that the value of a₂ will be coming as -1 b₂ will be coming as -1 again c₂ will be coming as -1. If substitute this a₂ b₂ and c₂ value.

Here in this equation you will see that after rearrangement this π_2 will be as μ by row v D . Again you just check it that this groups will not have any dimension. That is why this π_2 will be as dimensionless groups.

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Similarly

For $\pi_2 = \rho^{a_2} v^{b_2} D^{c_2} \mu$

$$M^0 L^0 T^0 = (ML^{-3})^{a_2} (LT^{-1})^{b_2} (L)^{c_2} ML^{-1} T^{-1}$$

$0 = a_2 + 1$	$a_2 = -1$
$0 = -3a_2 + b_2 + c_2 - 1$	$b_2 = -1$
$0 = -b_2 - 1$	$c_2 = -1$

$$\therefore \pi_2 = \frac{\mu}{\rho v D}$$

Similarly for π_3 you will see that if you again substitute the dimensions of all variables here and compare the power of these dimensions on left hand side and right hand side. You can have this three equation here and after solution of these three equations. You will see that the value of a_3 will be 0 b_3 will be equal to 0 and c_3 will equals to -1 and again if you substitute this value of a_3 b_3 and c_3 in this equation 3.

You will see that it will be ϵ by D because here a_3 is equal to 0 means here row to the power 0 it will be 1 and v b_3 to the power 0 so it will simply 1 so only remaining D to the power c_3 c_3 is -1 and ϵ .

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Similarly

For $\pi_3 = \rho^{a_3} v^{b_3} D^{c_3} \varepsilon$

$$M^0 L^0 T^0 = (ML^{-3})^{a_3} (LT^{-1})^{b_3} (L)^{c_3} L$$

$0 = a_3$	$a_3 = 0$
$0 = -3a_3 + b_3 + c_3 + 1$	$b_3 = 0$
$0 = -b_3$	$c_3 = -1$

$$\therefore \pi_3 = \frac{\varepsilon}{D}$$

So epsilon there nothing there so epsilon is already so epsilon by D will be there. This is simply you know that dimensionless groups. So pi 3 another dimensionless groups. So we are gating three dimensionless groups in terms all the process variables. Now you have to make functionality based on this formed or developed or you can say that dimensionless groups that is formed that dimensional analysis.

So that will be equal to function of pi 1 pi 2 and pi 3 that will equal to 0 and after that you can make any relationship like pi 1 is a function of pi 2 and pi 3 or you can say that pi 2 is a function of pi 1 and pi 3 or you can say pi 3 is a function of pi 1 and pi 2. So that way you can form but since in your problem it is seen that that flow resistance is depending on the other variables.

That is F is depending on the other variables in that case you have to make the functionality like this. So what are the groups is coming based on that F of then you have to make it those dependent variables dependent dimensionless groups. And this will be making in terms of other dimensionless group so in this way you have to make the rational equation and then you have to rearrange it

Like this and then you have to make this as like you can say that if suppose this group 1 or pi 1 so you can pi 1 as a suppose some value of Lambda into pi 1 function like this pi 1 and this is pi 2

and this is π_3 . So in this case this π_1 will be a some function like this a into “you can say” π_2 some power b and plus here you can say π_3 .

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Step 6: Setting the non-dimensional relationship

$$f(\pi_1, \pi_2, \pi_3) = 0$$
 Or

$$\pi_1 = f(\pi_2, \pi_3) \Rightarrow \frac{F}{\rho v^2 D^2} = f\left(\frac{\mu}{\rho v D}, \frac{\varepsilon}{D}\right)$$
 Step 7: Rearrangement

$$\frac{F}{\rho v^2 D^2} = f\left(\frac{\rho v D}{\mu}, \frac{\varepsilon}{D}\right)$$

Handwritten red notes on the slide include: π_1 , π_2 , π_3 , and a diagram showing a circle with a cross and arrows pointing to it, labeled with π_1 , π_2 , and π_3 .

You can make like this type of functionality also you can make. So either way that depends on your experimental data. How this flow resistance will be changing with respect to variables. So if you have that the different data based on changing of independent variable and its result of the dependent variable and after that you have to feat this equation. Either of this equation if you feat this with that the experimental data. Then you can form or you can have this.

Value of m and lambda value there. Here a b and some c value there. So this way you can find out that Coefficient and if you know that coefficient there will be a final form of rational equation. And then you have to also check which equation will be suitable and which will be given the list error of that fitting of that experimental data with this equation. That is further step but up to this you have to know how you can make that rational equation as a function of dimensionless groups and this is called dimensional analysis.

There are two methods that “I have discussed here one is Rayleigh method and another is Buckingham pi method” which one will be easier that you have to follow. So any problem you can solve by this Ok. So here another example here flow resistance of a sphere. In this again that Buckingham pi method you can use here. Like the drag D of a sphere is influenced by sphere

diameter d flow velocity u fluid density ρ and fluid viscosity μ .

What is that whenever any a sphere is supposed fixed in a certain position and some fluid velocity is flowing over that spherical part or sphere. You see there will some drag or resistance or you can say that external force acting on that surface of the sphere with that fluid. So that drag force actually of that sphere is depending on the diameter of the that sphere and also how what is the velocity of the fluid and also density of the fluid and also viscosity of the fluid.

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Example: Flow resistance of a sphere

- The drag D of a sphere is influenced by, sphere diameter d , flow velocity U , fluid density ρ and fluid viscosity μ .
- Two dimensionless group ($n-j = 5-3 = 2$): π_1, π_2
- Obtain π_1, π_2 by Buckingham method, with ρ, v and d as repeating variables:

$$\pi_1 = D \rho^x u^y d^z$$

$$\begin{array}{l} L: 1 - 3x + y + z = 0 \\ M: 1 + x = 0 \\ T: -2 - y = 0 \end{array} \Rightarrow \begin{array}{l} x = -1 \\ y = -1 \\ z = -1 \end{array}$$

$$\pi_1 = \frac{D}{\rho u^2 d^2}$$

$$\pi_2 = \mu \rho^x u^y d^z$$

$$\begin{array}{l} L: -1 - 3x + y + z = 0 \\ M: 1 + x = 0 \\ T: -1 - y = 0 \end{array} \Rightarrow \begin{array}{l} x = -1 \\ y = -1 \\ z = -1 \end{array}$$

$$\pi_2 = \frac{\mu}{\rho u d}$$

$$\pi_1 = f(\pi_2) \Rightarrow \frac{D}{\rho u^2 d^2} = f\left(\frac{\mu}{\rho u d}\right)$$

So this drag force actually depending on this diameter of the sphere velocity of the fluid physical properties like density and the viscosity of the fluid. Now in this case how many variables are there? We can say that drag here spherical diameter flow velocity density total five. Now how many dimensionless groups can be formed by either of this method very simple $5-3$? 5 is the number of variables and 3 are here number of fundamental dimensions.

So here $5-3$ that is 2 dimensionless groups that you can make by this. So if follow that Buckingham pi method with this variables row v D as repeating variables. You can make like this π_1 will be equal to this and equalizing that power of the dimensions on both side of this equations and making this equation based on which you can solve and you can get that power of that as -1 -1 -1 and then you can form that π_1 will be equals to this.

Similarly other dimensionless groups again that repeating variables will be there and with non repeating variables of other non repeating variables here μ . Accordingly you solve these equation just by equalizing the power of those dimension on both sides of the equation then you can get solution of solution of a power of x y and z like this and then substitution of those values. You can get this value of π_2 . So here π_2 is one dimensionless group here π_1 also dimensionless group.

You just check it that there will be no dimensions on these groups and then you can make that rational equation just by functionality like that π_1 will be is equal to function of π_2 . Now how it will be related that is different issue that depend on the process variables. Now generally this suppose here π_1 that will be is equal to some m into π_2 to the power n like this. In this way also it will be varied. Now you have to find out what will the value of m ? And what will be the value of n from your experimental data just by fitting this thing with experimental data.

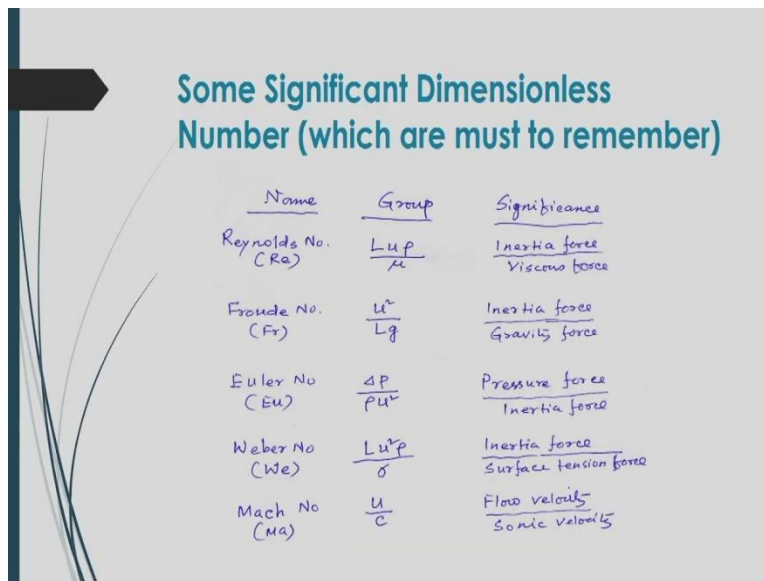
You have to find out what will be the experimental data of π_1 calculation and then π_2 . Because π_1 you can calculate simply D by row u square d square. All this things known to you because D is known to you row is density known to you velocity is you are giving that is also known to you d is also known to you. Just then what will the π_1 similarly if you change in that variable according to π_1 also change.

Similarly π_2 also will be changing accordingly if you change that viscosity density diameter of the scalar velocity of fluid. So accordingly you will see how that this π_1 will be changing based on that π_2 . So then you will get one equation based on this π_1 as a function of π_2 . Then you fit that experimental data of π_1 and π_2 and then find out by least square method to me actually discussing how to find out that all these components letter on.

Ok to find out this exponent just by fitting with experimental data that we are given in separate lecture letter on also. So this way so how to make the rational equation based on this dimension analysis by Buckingham pi theorem or Rayleigh method. You just I think glint here and you can practice also with some examples given here it. Now for chemical engineering process analysis like this forms some dimensionless groups.

You will see some dimensionless groups will some significant groups that are actually very important for the analysis of process performing there. So some dimensionless groups are given here in this slide. Here like Reynolds number that we got even Froude number Euler number Weber number Mach number. All those things and relative their significance it is given here in the slide.

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Name	Group	Significance
Reynolds No. (Re)	$\frac{L u \rho}{\mu}$	$\frac{\text{Inertia force}}{\text{Viscous force}}$
Froude No. (Fr)	$\frac{u^2}{Lg}$	$\frac{\text{Inertia force}}{\text{Gravity force}}$
Euler No. (Eu)	$\frac{\Delta P}{\rho u^2}$	$\frac{\text{Pressure force}}{\text{Inertia force}}$
Weber No. (We)	$\frac{L u^2 \rho}{\sigma}$	$\frac{\text{Inertia force}}{\text{Surface tension force}}$
Mach No. (Ma)	$\frac{u}{c}$	$\frac{\text{Flow velocity}}{\text{Sonic velocity}}$

Even in the next slide also it is given some significant groups and their respective significant their and how they are defined it is given their name group and so you have to remember this dimensionless group you check this group with their dimensionally consistent or not. You can say that whether this group is having any dimensions or not you will see that no groups have any dimensions here. So all those dimensions will be using for certain process analysis chemical engineering process.

So in future also if you do some other courses like heat transfer mass transfer; even transport phenomena fluid mechanics in there. You will see all those processes there you will see their certain dimensionless groups having for their process analysis like this here for flow generally Reynolds number Froude number Euler number even Weber number even Mach number all those group are actually related to the process where flow off course will be there or fluid mechanics related any process there these groups will gate.

Whereas for heat transfer operation you will see that Prandtl number is one dimensionless group Nusselt number is one dimensionless group even Grashof number this is one dimensionless group these three group are very important in heat transfer operations where the chemical engineering processes are related to the heat.

That will discuss different process I thing we have given previous lecture that different processes related to thermo chemical process even mechanical process there all those processes. Those processes related to the heat energy there also you will see that this dimensionless groups like Prandtl number Nusselt number Grashof number are very important.

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Some Significant Dimensionless Number (which are must to remember)		
Prandtl No. (Pr)	$\frac{C_p \mu}{K}$	$\frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}}$
Nusselt No. (Nu)	$\frac{hL}{K}$	$\frac{\text{Wall heat transfer rate}}{\text{Heat transfer by conduction}}$
Grashof No. (Gr)	$\frac{g \beta \Delta T L^3}{\mu^2}$	$\frac{\text{Buoyancy force} \times \text{inertia force}}{(\text{Viscous force})^2}$
Schmidt No. (Sc)	$\frac{\mu}{\rho D}$	$\frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of mass transfer}}$
Sherwood No. (Sh)	$\frac{k_g L}{D}$	$\frac{\text{Total mass transfer rate}}{\text{Mass transfer by molecular diffusion}}$

Even for mass transfer operation suppose if you are actually transferring one phases to another phases. Like absorption of carbon dioxide gas to the liquid. There transfer of carbon dioxide gas to the liquid so this is called mass transfer operation or diffusion you will see that diffusion process. If you suppose spread some perfume you see that perfume will be diffuse throughout the room.

So there transferring of molecules from one position to the position based on that concentration gradient. So this is called diffusion process. There also some dimensionless groups are very important like Schmidt number that number that is related to that molecular diffusion even

Sherwood number that is mass transfer operation like gas vinous molecule transferred from gaseous phase to the liquid phase. So in this way these dimensionless groups are very important. I souse you to remember this dimensionless group throughout your life; I can say that as a chemical engineering.

If you are doing the course of chemical engineering or studying that chemical engineering course that you have to remember these groups there. So Thank you for your concentration of this lecture that I think you understood that how to do the dimensions analysis and how to make their rational equation and what are the different systems of time and units that you understood so next lecture we will discuss something more about that physical and chemical properties of the material which are very important in chemical engineering process. So thank you for your attention.

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