Basic Principles and Calculations in Chemical Engineering
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Lecture - 27
Material Balance of Transient Process

Welcome to massive open online course on Basic Principles and Calculations in Chemical Engineering. Here we will discuss about the balances on transient processes as a module 9. Now under this module 9 in this lecture we will try to discuss about the material balance of transient processes.

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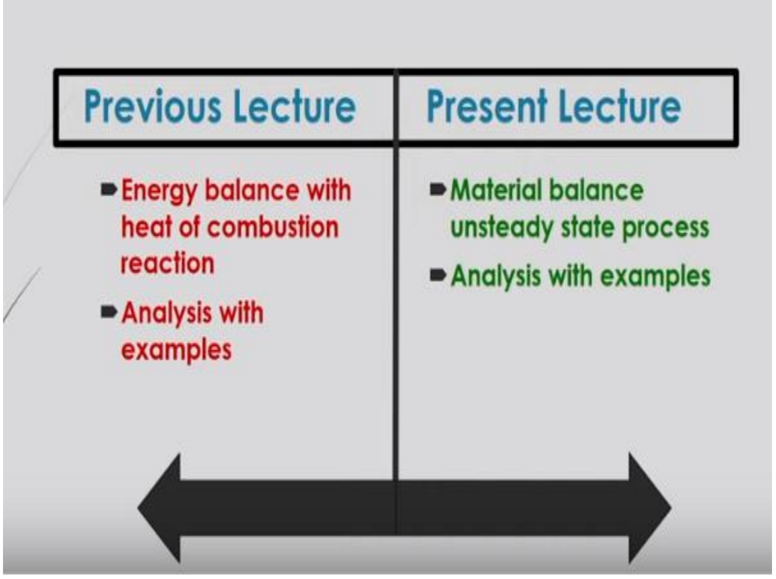

In the previous lecture we have discussed about that energy balance with heat of combustion reaction, analysis with examples. But those you know balances were you know steady state process. But here I will try to do some material balance where the process will be unsteady state and also will describe with examples here.

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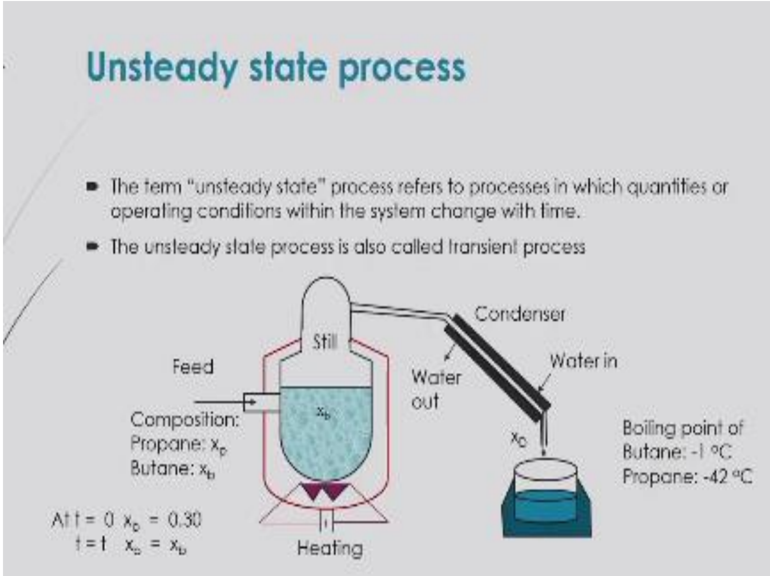

Now what is that unsteady state process? The term unsteady state process basically refers to the process in which quantities or you can say that dependent variables or operating you know variables as a dependent variables within the system changes with time and the unsteady state process is sometimes called as transient process.

So transient means here unsteady state process, you have to refer it like this. Now let us consider this chemical engineering process where we will see that the process will be based on you know time consumption like in this case here one steel at a you know that certain condition and inside this steel some amount of you know liquid mixture.

Let this liquid mixture is you know that propane and butane mixture and in this mixture certain you know amount of propane will be there and certain amount of butane will be there with a mole fractions of like X p and X b. Here X p means mole fractions of propane and X b means mole fraction of butane.

We will see that if we you know heat this steel of this mixture of propane and butane you will see that the you know higher boiling point liquid will be remaining in the steel whereas lower boiling point liquid will be you know vaporized and it will be coming out from the top portion of this you know steel.

And after that if we you know condense it through a condenser and it will be you know taken out as a you know liquid butane because here butane will be coming out as it has the boiling point is less than that is propane. And the boiling point of butane is -1 degree Celsius whereas propane boiling point is -42 degrees Celsius.

So here in this case you will see that, so, this vapor will be coming out, butane vapor will be coming out here or higher boiling point instead of you know that propane and butane mixture, any mixture of this you know different boiling point component if it is there.

So higher boiling point, you know that liquid will be coming out as a vapor whenever it will be heated and then it will be condensed out and then it will be taken out as a you know that x d that is a distillate product after separation.

Now, in this case you will see that in this steel whatever you know butane composition will be there inside this steel during that heating process, you will see that this composition of this butane will be you know changing. Now that change will be you know with respect to time.

If you boil for you know for a certain time, you will see that accordingly that composition of that, you know butane in the distillate product will be there. Now in this case that means here with respect to time, this composition in the steel will be changing. So this type of process is called that unsteady state process.

Now how long it will take to you know that condensed or you can say that separate this butane vapor from its initial concentration to a certain final concentration. So that can be obtained by this you know energy sorry material balance equation. But that material balance equation will be as per you know unsteady state condition.

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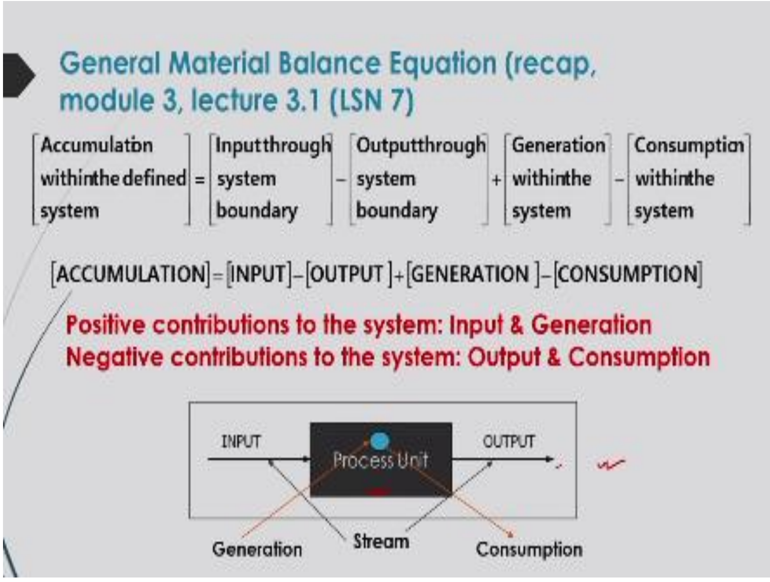

So let us look back into the general material balance equation. We have discussed it in module 3 in lecture 3.1 where lecture serial number it is 7. So there we have discussed that this general material balance equation as accumulation within a defined system.

That will be equals to input through system boundary minus output through system boundary plus generation within the system minus consumption within the system. That means, in short we can write here accumulation will be is equal to input minus output plus generation minus consumption.

Now positive contribution to this system will be input and generation and negative contribution to the system is output and consumption. So here it is seen in the figure that if this is a process unit, you will see that in this process unit input will be coming and whereas output will be like this in this stream.

And you will see that this input and output this will be you know that stream. So input stream and output stream there will be a certain amount of that you know components will be there at a certain flow rate.

And during this process in this process unit, if is there any you know generation there, like in reaction certain you know at a certain condition, there will be you know some product that will be produced with the reactants. So there will be certain generation of the components. So that components also will be coming out as a output there.

And then during that reactant you will set up during that reaction some reactants will be consumed and it will be you know that will be you know that consumed so that amount to be considered as a consumption terms. So in any process there this you know accumulation input, output, generation and consumption terms will be considered for the overall material balance there.

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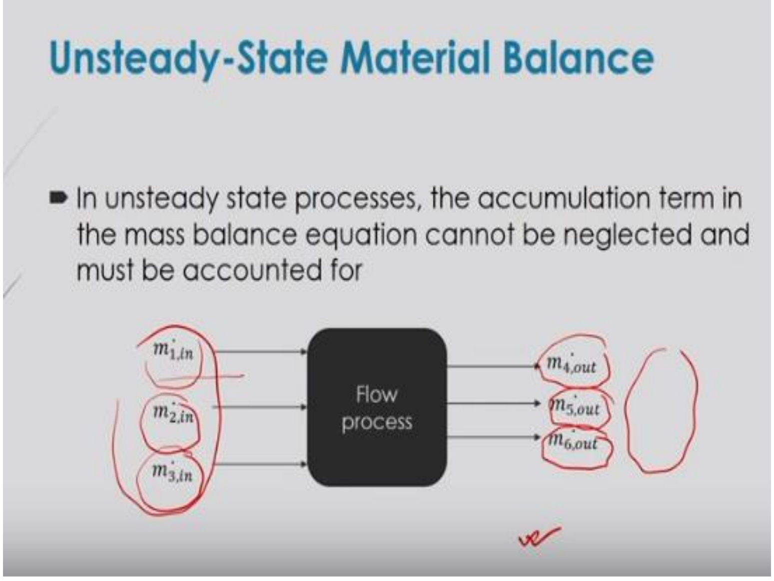

Now in this case, if I consider that, that unsteady state material balance. In unsteady state processes the accumulation term in the mass balance equation cannot be neglected and must be accounted for. So if suppose there is a steady state process that means there is no accumulation.

In that case input and output plus you know generation minus consumption will be there. Whereas, in case of unsteady state process there you will see that input rate and output rate will not be same. So in that case some amount of you know materials will be deposited or accumulated in the system with respect to time.

So that will be called as accumulation. So in that case accumulation you cannot neglect there. Here suppose in this flow process, some amount of you know that materials will be coming in as you know that mass flow rate of you know m dot 1. Similarly, another component m dot 2 and another component m dot 3.

Those are coming into this flow process unit and they are coming out here as m 4 dot and m 4 dot 5 and you know that m 6 dot there as a mass flow rate there. Now in this case you will see that overall if this output, total output is greater than this you know that input then what will happen?

You will see that there will be a you know that amount in the system will be decreasing. Whereas, if output is less than this input you will see there will be an accumulation. That means here in the system the mass will be increasing with respect to time. So in this way, this will be you know considered as unsteady state process where accumulation terms to be considered.

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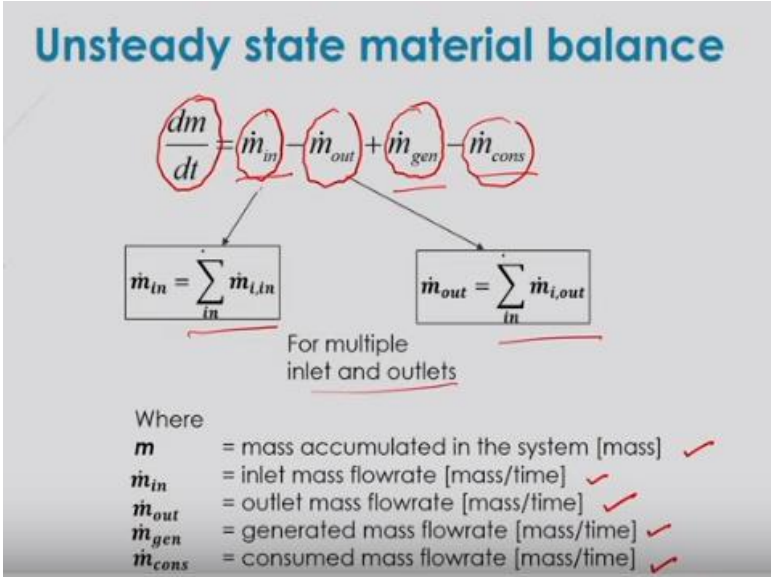

Now let us write this unsteady state material balance in symbol. Here, if we consider that the accumulation of mass in the system will be there with respect to time. So what will be the change of that accumulated amount with respect to time that can be represented by this dm by dt.

And where we will see that m dot you know in that will be your mass of you know material that is coming into the system as a flow rate here m dot in. And similarly, m dot out that is rate of you know mass of that material, which is coming out from the system at a rate that will be regarded as m dot out.

Similarly, at the same time if is there any material is generated in the system, then you have to consider that will be as you know m dot generation. Whereas if is there any mass is consumed during that processes that also you have to consider here as a rate of m dot consumption here.

Now this here m dot in there maybe you know more than one inlet. So total inlet mass flow rate will be considered as summation of that you know mass of you know several you know inputs that you have to consider as a sum. Whereas at output also you have to consider what will be the you know total amount of out based on the summation of all the output mass flow rate.

So in this way you can calculate. If there are suppose more than one components also. So separately for that individual components also you have to consider and adding up all those component's mass flow rate you will get that total amount of mass that is in will be coming into the system as an input.

And if you are considering the flow rate of that individual components, which will be coming out from the outlet that also to be considered as a total you know, mass out for that summation of all components that will be regarded as summation of m i out and it will be total output rate of that mass there. So similarly, generation and consumption to be considered.

Now this is regarded on report that multiple inlet and outlet condition. So here m is the mass accumulated in the system and m dot in is the inlet mass flow rate that is mass per unit time. Even m dot out is outlet mass flow rate that is mass per time; m dot generation it will be you know generated mass flow rate that is mass per time.

Similarly, m dot consumption it will be is equal to consumed mass flow rate that is mass per time. Now you can express in terms of moles also not only that only mass. It will be considered as a mole also as a rate. So that will be mole per time there.

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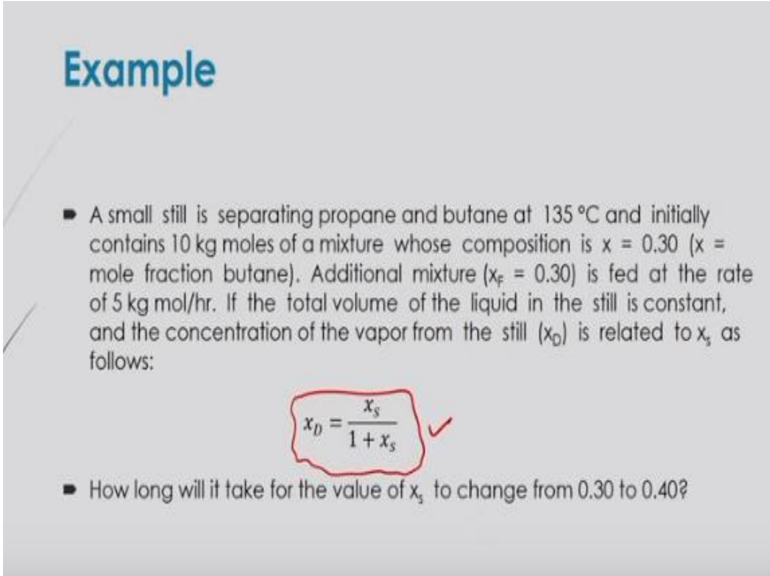

Now let us do an example for this unsteady state process here. Let us consider this small steel is you know used to separate propane and butane mixture at 135 degree Celsius and initially contains 10 kg moles of mixture in that steel whose composition is you know that x is equal to 0.3 mole fraction of butane.

And additional mixture here x F that is mole fraction of butane as 30% is fed at the rate of you know 5 kg mole per hour. Now if the total volume of the liquid in the steel is remain constant during that operation and the concentration of the vapor from the steel which is coming out as x D is related to that you know concentration of the butane in that particular steel will be you know expressed as x D will be is equal to x s by 1 + x s.

So based on this you know composition of you know distillate product, you can calculate how long will it take for the value of that concentration of that butane in the steel to change from its concentration from 30% to the 40% that you have to find out.

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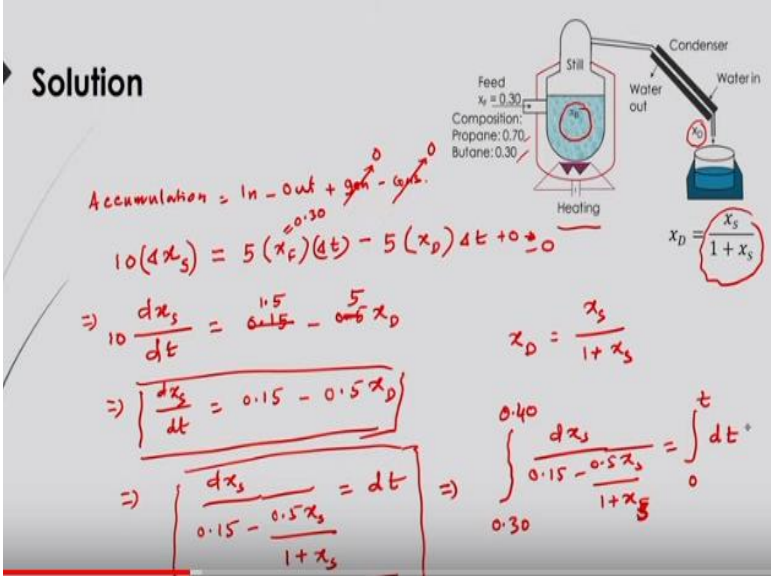

So this okay let us solve this problem here. So as for this problem we can have this steel here and in this case feed here composition is 0.30 as x F and composition of propane and butane mixture here it is given as propane it is 70% and butane as 30% and initially the steel is filled of 10 kg mole of you know propane butane mixture.

And after that, that feed will be you know supplied as a you know 5 kg moles per hour rate to this steel and then heating it and during this heating you will see that, that butane vapor will be you know coming out from that you know steel and accordingly you will see that there will be you know that concentration of that butane inside the you know steel will be changing with respect to time.

Now that you know that distillate whatever will be there inside that you know steel that will be coming out as a vapor and it will be after that condensed it will be you know taken as some D amount and where this butane concentration in that distillate you know amount would be x D.

Now this is x D change that is x with respect to you know time that will be you know changed and accordingly you can say that this x D basically how it will be related to that you know concentration of the butane in that steel. So it will be you know related by this equation here x D will be is equal to x s by 1 + x s.

What is x s here; x s is nothing but that you know concentration of that butane in the steel at a time t. So in that case how to solve this you know problem to find out that if suppose butane concentration is changing from 30% to 40% how long it will take to have such you know change of this mole fraction from 30 to 40%.

Now in this case we will do that you know material balance first, unsteady state material balance like we know that material balance as accumulation that will be is equal to inlet minus outlet plus generation minus consumption. This is your you know material balance equation.

Now in this case, since there is no reaction is going on and there is no generation of the you know components. So we can neglect it as zero and also there is no consumption of this you know any materials for reaction. So in that case we will also neglect that consumption terms. So we can neglect it.

So finally you can have this material balance equation as accumulation will be is equal to in minus out. Now what is the accumulation there? Now if we consider only that you know butane you know material, so if you do that material balance for this butane so we can right here initially butane concentration was you know, there will be some amount.

That is there 30% and 70% all the time it will be you know that they are in this steel 30% but with respect to time, you will see that it will be changing because there will be a you know that amount of butane will be changing and it will be vaporized and it will be coming out from that steel.

Now, the total volume of that steel inside that, that is steel will be remaining constant. So that is I think 10 kilo mole, sorry 10 I think it is given as per problem it is you know 10 kg mole of mixture will be you know kept constant there inside that steel. So accordingly we can say that accumulation will 1.

Now, during that you know vaporization process the concentration of that steel that is in butane concentration will be changing. Now what is that change? That we do not know. Let us have this change is delta x here s; s is the here steel concentration here butane concentration in that steel.

Now the change with respect to time, time maybe you know that with respect to time delta t, that change of that concentration will be delta x s. Now since the volume of this steel is remain constant, it is 10. So we can multiply this with 10.

Then it will be your that total amount of butane at a particular time and there will be a change of butane with respect to delta t time. It will be 10 into delta x s. So this is your accumulation and then what will be the inlet of this you know butane in this steel. Now there is a flow rate of that you know as per problem this flow rate is given, the feed flow rate is given 5 kg mole per hour.

Now in this 5 kg mole per hour flow rate what will be the amount of that is x s there or you can say that x F that is butane vapor in that feed. So we can write here then since it is 5 kg mole per hour flow rate and concentration is x F there. So we can write 5 into x F and since we are considering within a period of time that is delta t, so we have to multiply it like this.

So it will be coming out at this, this amount of you know material is in the steel and then out will be is equal to then again out at the flow rate of 5 of that x F so that, that total volume of that steel will remain constant 10. So 5 into your outlet concentration is basically x D here and the time is delta t. So in this way you can write that outlet you know material for this short period of time t.

And generation is zero and you know consumption is here zero. So finally, this equation you can get. Now dividing by t and taking the limit of you know at a process to zero. So we can write here d into x s by d t that will be is equal to 0.15 - 0.5 into x D. Why 0.15 here? Here x F is basically here 30% that is 0.30. So 5 into 0.30 it will be coming as here.

I think it is here 5 into zero point, this is 1.5 here it is 1.5 sorry this is 1.5. 1.5 – 0.5 into x D. Now after that, we will see that we can write this final equation as we can write here as what is the value here? This is yeah, what is that here 5 into x D here. So we can write this. So finally, we can write d x s by dt.

That will be is equal to here if we divide it by 10 then we can write here it will be is equal to 0.15 - 0.5 x D. So this is your final equation. This is one differential equation. From this, you know material balance that is unsteady state material balance. Now in this case, since we know that the x D will be is equal to x s by 1 + x s. So if we substitute here this x D value finally then we can write this equation as x s, d x s by 0.15 you know that -0.5 x s divided by one plus here x s.

That will be is equal to dt. So in this way we can have this equation in terms of x s and dt. Now this x s that is steel concentration will be changing with respect to time and you have to integrate this equation within a certain limit of that as per condition given in the problem.

So we can write here, integration of here concentration is changing from 0.30 to 0.40 of this d x s by 0.15 - 0.5 x s by you know that 1 + x s. So x s that will be is equal to 0 to here you can say t dt.

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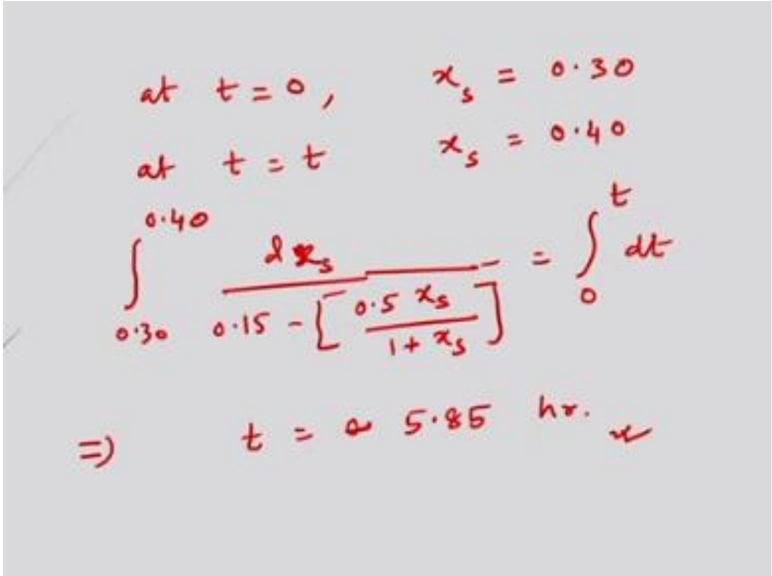

Now after that after getting integration within a limit at you know that t is equal to zero, initially steel concentration is 0.30 where you have to find out the time at t is equal to t. This x s to be 0.40. So based on this, you know limit we can have this you know integration of this 0.30 to 0.40 of d x s by 0.15 minus 0.5 x s by 1 plus x s.

That will be is equal to 0 to t here dt. So from this we can have that t will be is equal to zero point sorry, it will be as 5.85. This would be in hour. So total 5.85 hour time will be required to you know vapor this you know mixture to get this you know composition of butane from its initial to the final of 0.40 there.

So in this way, we can say that it is 5.85 hour will take to change the concentration of steel from 30% to the 40%. So this you know unsteady state material balance, I think it would be helpful to do further this material balance in unsteady state condition. Now let us do another example for this.

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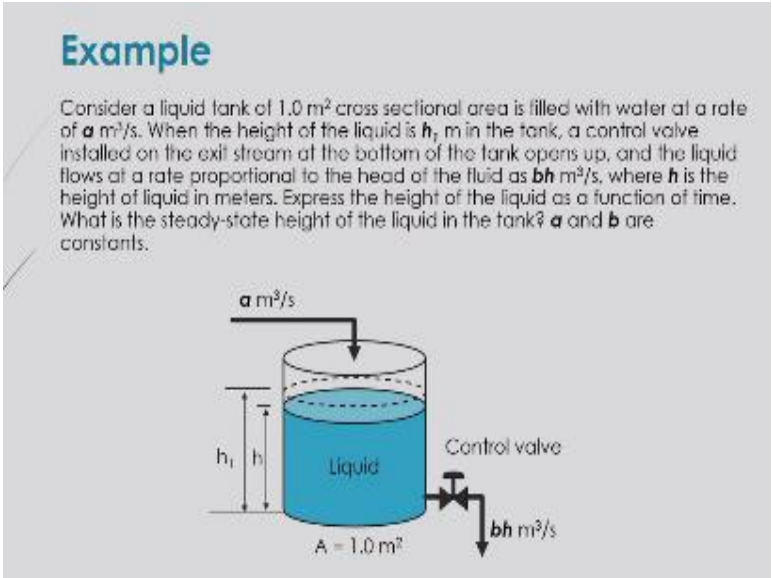

Let us consider a liquid tank of 1.0 meter square here. Cross sectional area is here 1.0 meter square and it is filled with water at a rate of you know a meter cube per second, a certain amount. Let us consider a general a meter cube per second. Now when the height of the liquid is h 1 meter in the tank a control valve installed on the exit stream at the bottom of the tank, that opens up.

And the liquid flows at a rate proportional to the head of the fluid as b into h meter cube per second, where h is the height of the liquid in meters. Now in this case, you have to express height of the liquid as a function of time, what is the steady state height of the liquid in the tank; a and b are constants here.

So basically you know that, initially there will be certain amount of liquid. Now if you allow some liquid here in this tank at a certain flow rate of a meter cube per second and you will see to keep this initial you know volume inside the liquid constant, then you have to you know allow it to you know flow out from this outlet stream.

So there you have to maintain that output rate there so that the liquid inside the you know tank will be constant. So that will be your steady state. But here in this problem, it is not there. You will see that initial volume will you know that gradually you will see that either increasing or decreasing as per you know that input and output flow rate there.

If suppose input flow rate is you know lower than this output flow rate, you will see that the liquid whatever initially was there inside the tank it will be decreasing and accordingly height of that liquid will be you know decreasing with respect to time.

If suppose inlet flow rate is higher than the output flow rate, you will see that liquid will you know that liquid level will increase in the tank with respect to time, because that volume will be increasing because inlet flow rate will be higher than the outlet flow rate. Now in this case you have to you know if suppose a is higher than this you know outlet flow rate of b into h.

So in that case liquid flow rate will increase. And again if suppose here outlet flow rate is here you will see that greater than that input flow rate what will happen that liquid level will be decreasing at a certain flow rate. Now in this case what is the steady state here what should be the you know that height of the liquid as a function of time, that you have to calculate here. You have to find out or you have to express this.

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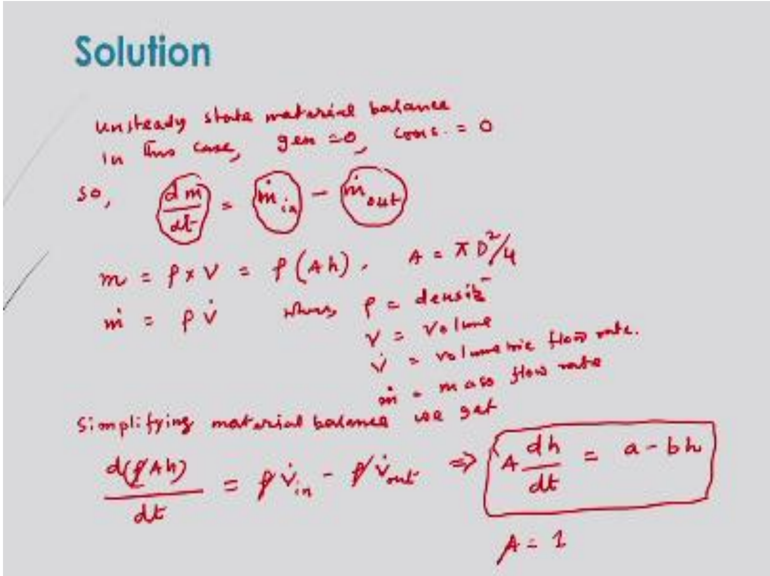

Now, this tank you know cross-sectional area is given 1 meter square, whereas the you know inlet flow rate is given a meter cube per second. Now how to solve this you know problem here. Now how to express this you know height of this liquid that is changing with respect to time there.

Now what to do here again we have to do here material balance which is unsteady state. Now if we do that unsteady state material balance, unsteady state material balance we have to consider in this case in this problem, in this case we can say that generation will be equals to zero. Consumption will be is equal to zero.

So we can write d m by dt that will be is equal to in minus out where m in as a rate minus m out that means output flow rate of that mass. So here this is accumulation and this is input rate, this is output rate. Now according to this material balance, what we can write here. In this case first of all what is m here; m is nothing but density into volume.

So it is basically rho into volume is what? Cross-sectional area into height of the liquid; h is height, A is the cross-sectional area. So here A is basically pi into D square by 4, D is the diameter of the tank and m dot that is mass flow rate here m dot will be is equal to then rho into v dot. What does it mean? That is mass that is per unit time.

What is accumulated inside the you know tank that will be based on that volume rate that is increased or decreased inside the tank. So that can be obtained just by you know rho into v dot where we can write here, rho is the density where rho is equal to density and v is equal to volume v dot is equal to volume flow rate, volumetric flow rate.

And m dot is equal to mass flow rate, mass flow rate or you can say that mass accumulation rate. Now simplifying this you know material balance, now simplifying material balance we get d rho A into h divided by dt. We have just substitute the value of m here as rho into Ah.

That will be is equal to rho into v dot in minus rho into v dot out which implies dh by dt that will be is equal to what a – bh, why? Here see rho, rho and rho. This is constant. That is here incompressible flow. So rho will be you know constant and it can be you know omitted from this left hand side and right hand side just by dividing the both sides.

And then here you will see that v in is given as a and the outlet of b is given as bh and in this case you can then write here this you know that dh by dt will be is equal to this here one important point here it will be coming Ah here so A will be remain here in this case. So we are getting this you know equation here as per this.

Now per unit cross-sectional area, we can write A is equal to 1 since it is given as per problem A is equal to 1. So we can write A will be equals to 1. Next what we have to do?

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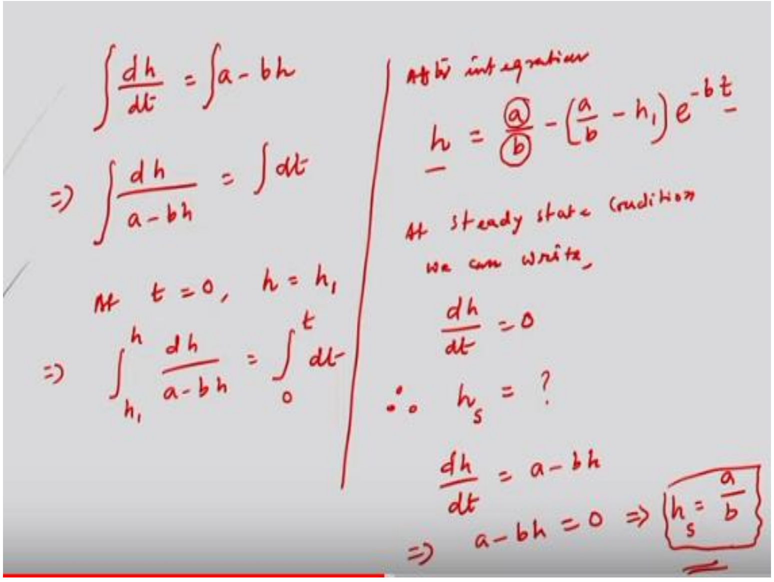

You have to that integrate this you know differential equation of this dh by dt that will be is equal to a – bh. Now integrating this equation here in this case, now how to do this here; dh by here you have to write a - bh as integration here that will be is equal to integration of dt.

Now as per you can say that you know that initial condition as at time t is equal to 0. The height of the you know liquid inside the tank, it will be as h is equal to h 1.

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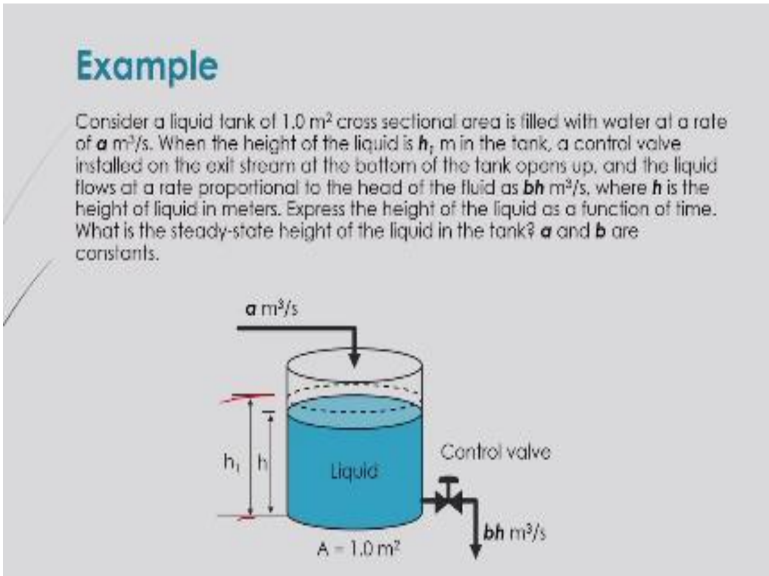

That is given in this picture here, initially up to this height was there, that is at h 1. At any time that height will be h. Now in this case we can then write here, we can write here h 1 to h as dh by a – bh. That will be is equal to 0 to t as here dt.

Now after integration, we can write after integration within this limit, we can write after simplification, h will be is equal to a by b minus a by b minus h 1 into e to the power –b into t. So this will be your solution of this integration. So this you know that height at any time t, how is it related?

So h will be is equal to a by b minus a by b minus h 1 into e to the power - bt. So this will give you that with respect to you know time that height will be changed. Now that depends on this you know input and output you know that flow rate and also that coefficient a and b here or constant a and b.

Here you know that this a is that input flow rate and b is here the coefficient at this outlet flow rate. How that output rate depends on that height of that liquid inside the tank at any time t. So that b there. So initial height is h 1. Now the last portion of this problem there, suppose what is the steady state height of the liquid in the tank a and b are constants here.

Steady state condition, at a steady state condition we can say we can write here that dh by dt that will be is equal to 0. Therefore, we can write here the steady state height as h s that will be is equal to what will be the value here because dh by dt, what we got here, this is basically dh by dt.

That will be is equal to a - bh. That means here a – bh will be is equal to 0 at steady state condition. So which implies h will be is equal to simply you know that a by b. So this is your you know at steady state condition what should be the height. That means here if suppose inlet and outlet you know flow rate will be you know same.

So in that case this steady state height will be is equal to a by b here. So if you consider that a is equal to suppose you know 1 meter cube per second and b is equal to suppose here coefficient is 2, then steady state height will be is equal to 1 by 2 here. So it will be simply you know half, that is h is half meter there.

So in this way you can calculate at steady state condition what will be the you know that steady state height of the liquid inside the column. So this is one example of this unsteady state you know material balance equation. Let us do another example here.

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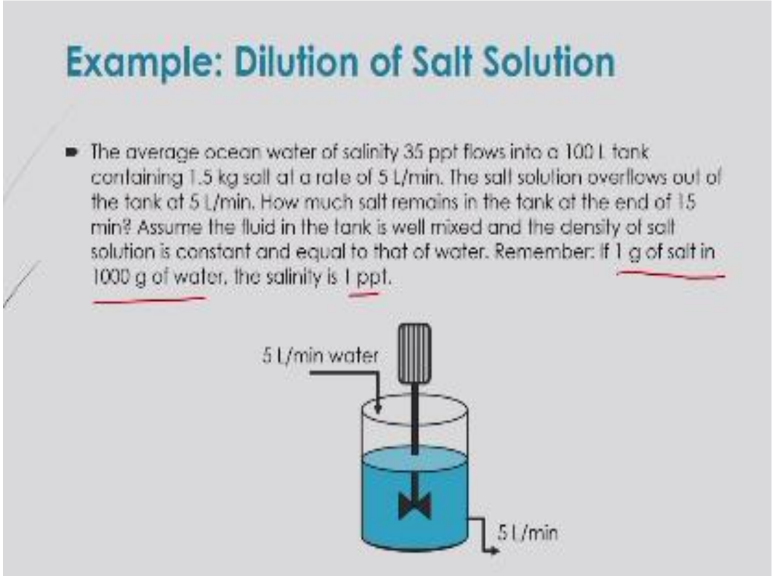

Dilution of salt solution. Now the average here ocean water of salinity is 35 ppt that flows into a 100 liter of tank that containing 1.5 kg of salt let us considered at a rate of 5 liter per minute. Now the salt solution overflows out of the tank at 5 liter per minute. Now how much salt will remain in the tank at the end of 15 minutes. That you have to find out.

Now assume the fluid in the tank is well mixed and the density of the salt solution is constant and equal to that of water. Now in this case, you have to remember if 1 kg of salt in 1000 gram of water the salinity will be is equal to 1 ppt. So 1 ppt how it is related that is given here. Now basically this is ocean water.

That salinity is 35 ppt, that is flows into a 100 liter of tank that contains 1.5 kg salt and it flows at a rate of 5 liter per minute. Now the salt solution also overflows out of the tank at a 5 liter per minute. Now in this case you will see that the salt will be you know leaving from this tank. Now how much salt will remain in the tank at the end of 15 minutes that you have to find out.

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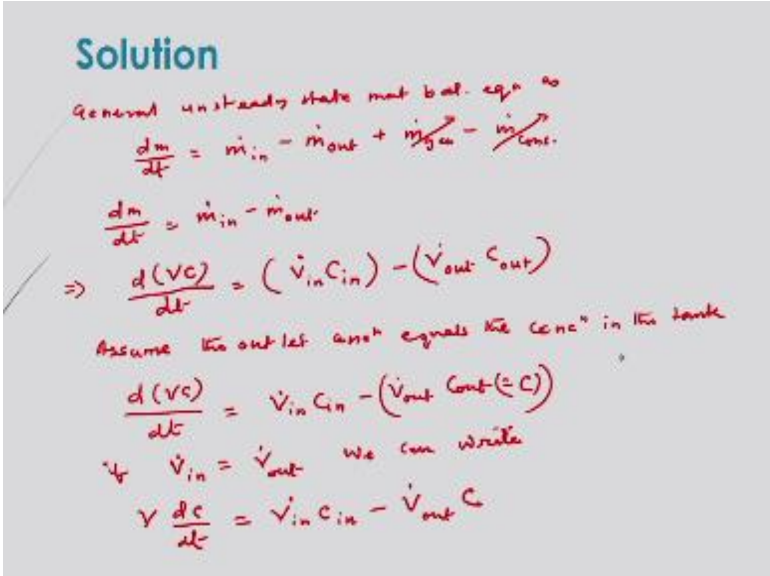

So let us do this problem here again as per you know unsteady state material balance equation. Now in this case, we can write this general unsteady state material balance, general unsteady state material balance equation as here dm by dt. That will be is equal to m dot in minus m dot out plus m dot generation minus m dot consumption.

Since there are you know no reactions, so we can neglect this generation and also you know consumption terms. So the final general material balance will come as dm by dt. That will be is equal to m dot in minus m dot out. Now if we replace this you know mass flow rate in terms of concentration we can write here d into v c, v is the volume c is the concentration by dt.

That will be is equal to then in terms of flow rate inlet condition into c in, that is concentration of the inlet stream minus again v dot out flow rate and the outlet stream with a concentration of c out there. So this you can write here in this way.

Now assuming the solution in the tank is well mixed that as per problem and the outlet salt concentration you know that equals the concentration in the tank. Assume here outlet, the outlet concentration equals the concentration in the tank. So we can write here d v c by dt.

That will be is equal to v dot in into c in minus v dot out into c out that is basically is c here; c out is basically as you know that c. So here we can write again if we consider that v in that is inlet flow rate and outlet flow rate of this stream is equal then we can write here, we can write here as, we can write v dc by dt.

That will be is equal to here we can write here what it is given that okay let it be here v here in dot c in minus b dot out that is c and in this case, then we can you know omit all those things here, here in this case, then we can omit all those you know v terms here. And after that we can get that you know integration of c inlet and c outlet.

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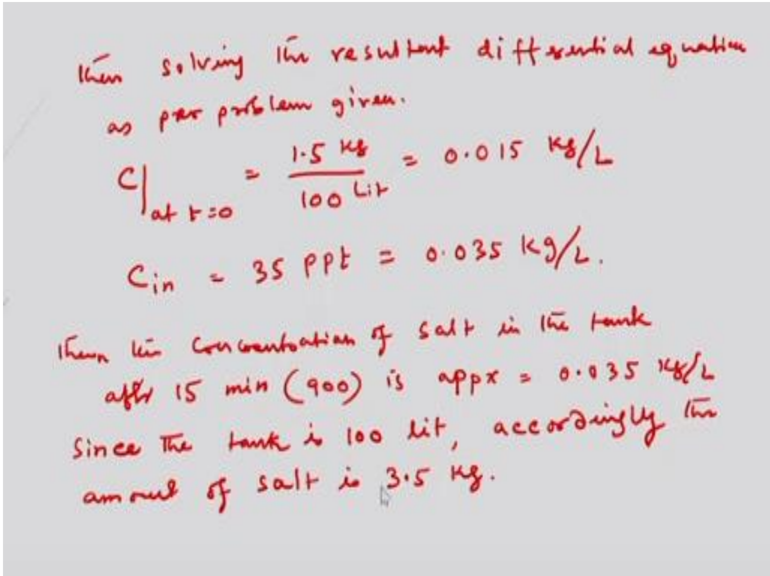

And solving that equation we can get solving the then solving the resultant differential equation as per as per problem given we can write c then at t t is equal to 0. It will be is equal to 1.5 kg per 100 liter of solution. This is 0.015 kg per liter there and c in that will be is equal to 35 ppt.

That will be coming as ultimately 0.035 kg per liter. So this is your c in there. Then after integration the concentration of the salt, okay here we can say that the concentration of the salt then as per integration then the concentration of salt in the tank after 15 minutes that is 90 second.

Let 15 minute no that is what 900 second is approximately will be is equal to 0.035 here kg per liter since the tank is 100 liter. And accordingly the amount of salt is 3.5 kg. So this will give you that after 15 minutes this amount of salt will be there inside the tank okay.

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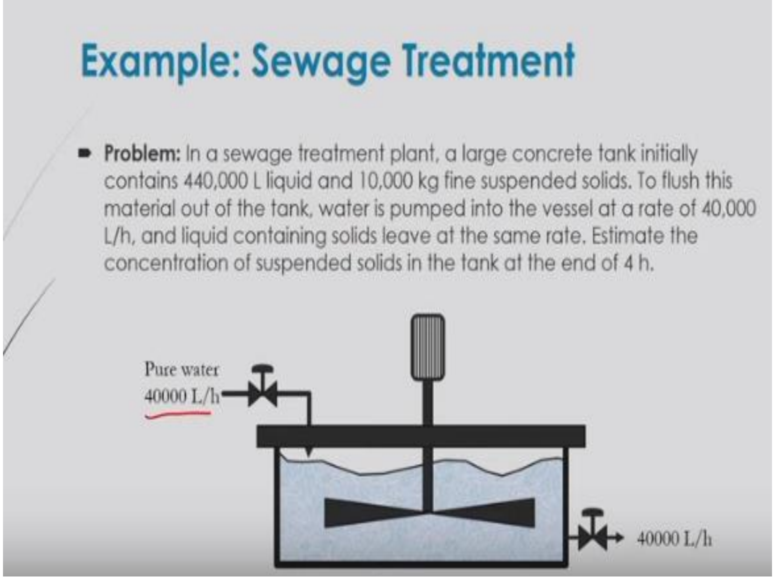

So let us do another example this type of you know problem to solve this like example for sewage treatment there. Now in a sewage treatment plant you will see that a large concrete tank initially contains here 4,40,000 liter liquid and 10,000 kg fine suspended solid.

Now to flush this material out of the tank we will see water is pumped into the vessel at a rate of you know 40,000 liter per hour and liquid containing solids leave at same rate. Now in this case estimate the concentration of suspended solid in the tank at the end of four hour. So here the input and output flow rate of this tank is 40,000 liter per hour.

Now because of this stirring and irrigation process you will see that fine suspended solids you know that it will be you know coming out from this outlet and there you will see there exchange of concentration of that suspended solids inside the sewage treatment will be there with respect to time that will change.

Now in this condition after 4 hour what should be the you know that concentration of that suspended solids that you have to find out. Now again according to this problem, if we do the general material balance here since there is no reaction here, we can also neglect here that generation and consumption terms and then finally we can write that you know, material balance equation.

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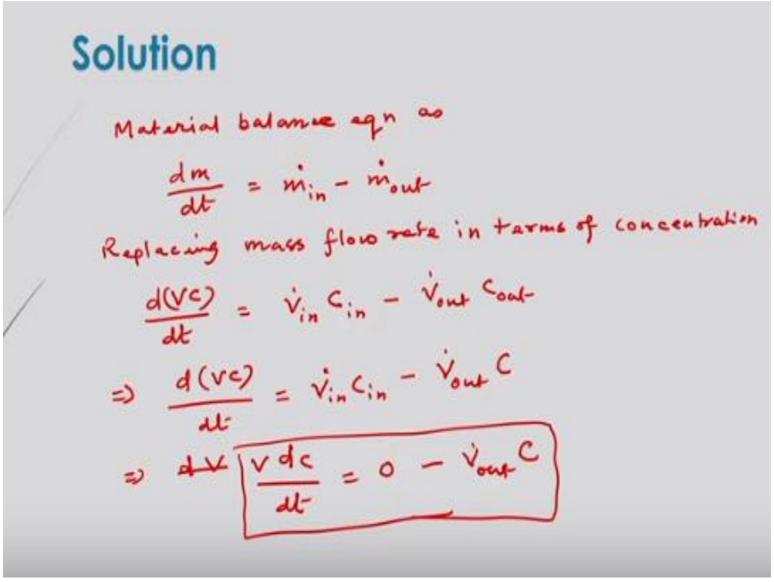

Your material balance equation as here dm by dt that will be is equal to m dot in minus m dot out. Now replacing mass flow rate, in terms of concentration we can write d v c by dt.

That will be is equal to v dot in c in minus v dot out to c out or since here v dot in and v dot out is equal 40,000 liter per hour we can write the same amount here in this equation or you just keep here and finally you can substitute that value. And in this case, the outlet concentration of the suspended solid will be considering that the concentration will be in the you know tank itself at that particular time.

So we can write here d v c by dt. That will be is equal to v dot in c in minus v dot out c out will be is equal to simply c, we can consider here. That means this concentration is at a certain time t in the tank. Now here this since water is flowing into the vessel at a rate of 40,000 liter per hour as input rate and also if we consider that only water balance accordingly we can write here this as d v, v we can keep it as constant.

So we can write here v into d c by dt and here inlet here concentration you know that of solids in the inlet pure water is 0. So we can write here 0 minus v out that is at a certain flow rate it is given there that is v dot out that is 40,000 liters per hour and in this case here we can say that this c will be c at that outlet condition.

Now what is that initial condition that time t in the you know tank that solid concentration, we have to find out first. To solve this you know resultant you know this differential equation of this concentration change with respect to time. So for this, so here we need to have this initial condition.

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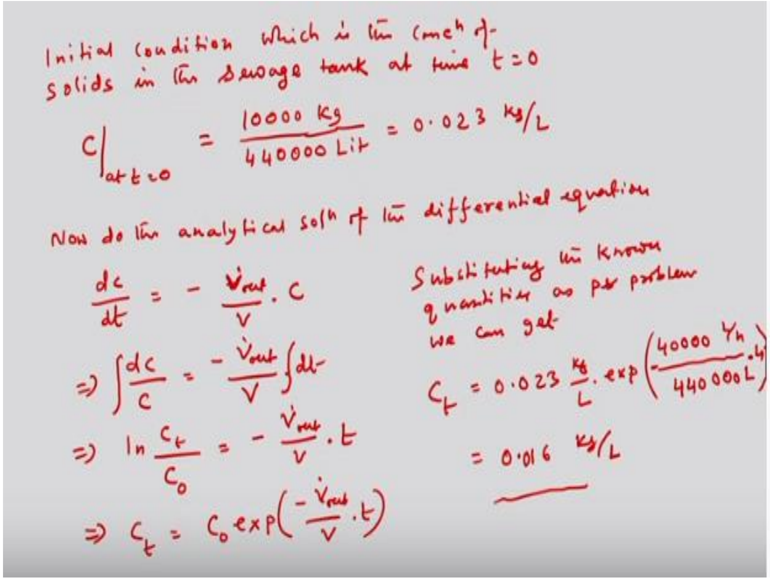

Initial condition which is the concentration, which is the concentration of solids in the sewage tank at time t is equal to 0 where you can write c at t is equal to 0. That will be is equal to what 10,000 you know kg of solid in 4, 40,000 liter of water. So it will be coming as 0.023 kg per liter. So this amount of solid will be initially in the sewage tank.

Now do the analytical solution of the differential equation that obtained as dc by dt. That will be is equal to minus here dc by dt; dc by dt will be is equal to here minus v you know that v out by v into c. So from which you can write dc by c. That will be is equal to minus v dot out by v into dt.

So from this after integration we can have the c at time t and initial condition suppose c 0 that will be is equal to minus vdot out by v into t. This will give you your after integration or we can represent it as c t will be is equal to c 0 into exponent of minus v dot out by v into t. So this is your equation or you know concentration at time t okay as a function of time t.

In this case you can find out the consideration t at any time t here, concentration c t at any time t provided c 0 and v dot out and also initial volume inside the reactor known to you. Now substitute the known quantities here. Substituting the known quantities as per problem we can get c t will be is equal to, initial concentration is 0.023 that we found.

This is kg per liter into exponent of here what is the v dot out? This is you know 40,000 liter per hour divided by that is v is 4,40,000 that is four lakh forty thousands, this is liter. But here as per equation it is minus. So finally after simplification it will come as 0.016, this is kg per liter. So this is your concentration for the time 4 hour.

Here it will be is equal to like this. So it will be zero point sorry 0.016 kg per liter. This concentration will be there in the outlet condition at time t is equal to 4 hour. So in this way we can calculate by the material balance what should be the outlet concentration of this you know solution at a time t.

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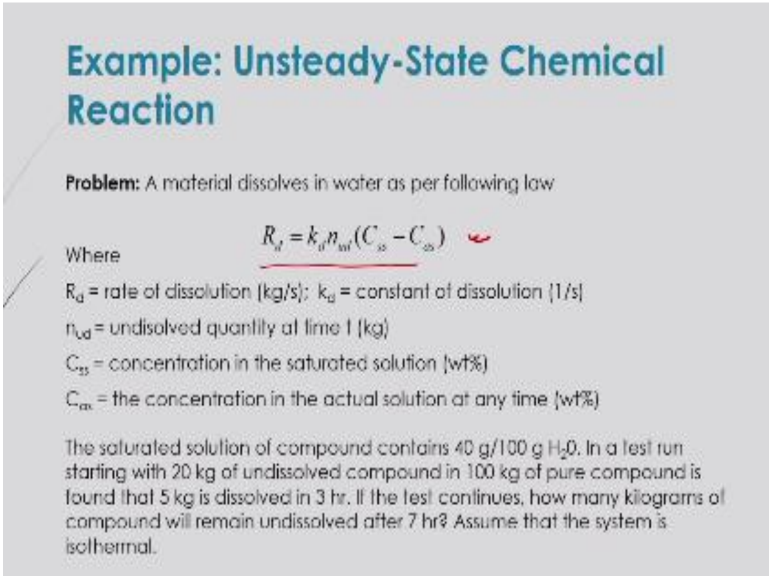

Let us do another example of unsteady state chemical reaction. There in this case a material dissolves in water as per following law it is given that is R d is equal to k d n ud into C ss minus C as where R d is called rate of dissolution. K d is equal to constant of you know dissolution, n ud is the undissolved quantity of time t.

C ss is given as concentration in the standard solution. And C as is the concentration in the actual solution at any time t. Now the saturated solution of compound contains 40 gram per 100 gram of water in a test run starting with 20 kg of undissolved compound in 100 kg of pure compound is found that 5 kg is dissolved in 3 hour.

Now if the test continues how many kilograms of compound will remain undissolved after 7 hour. Assume that the system is isothermal. So here in this case you will see that there will be a dissolution of material in water that will follow a certain laws, that is dissolution rate laws. So that dissolution rate laws is given as per this equation.

Now in this case the saturated solution of compound always contains 40 gram per 100 gram of water. Now in an experiment it is seen that for a test run starting with 20 kg of undissolved compound in 100 kg of pure compound is found that 5 kg is dissolved in 3 hour in that solution, in that liquid. If the test continuous there for a certain time.

Now you will see that after 3 hours that how many kilograms of compound will remain undissolved if you consume that time at 7 hours. So assume that the system is isothermal in this case, there will be no heat input output there at a constant temperature it will be there.

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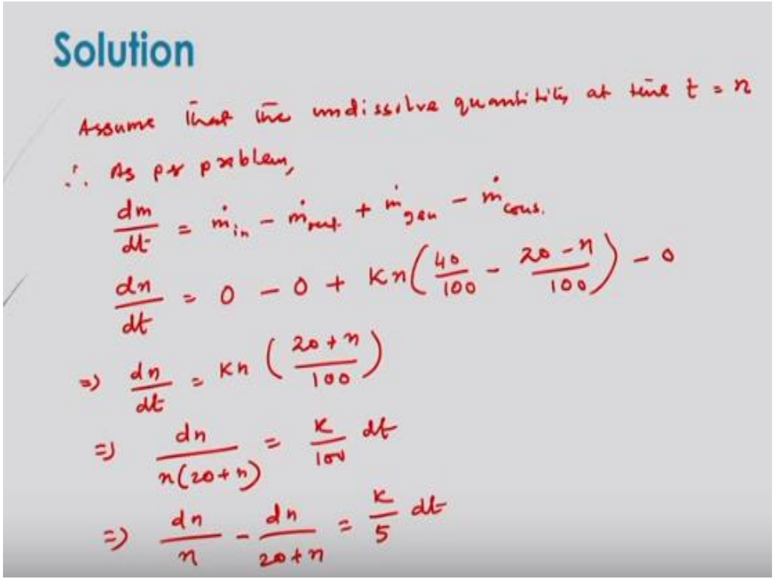

So to find out that you know kilograms of compound will remain undissolved after 7 hours that you have to calculate. Now in this case what we have to do that first of all, you have to consider that undissolved quantity at time t. So assume that the undissolved quantity at time t is equal to n.

Therefore, as per problem, we can write the material balance as dm by dt that will be is equal to m dot in minus m dot out plus m dot generation minus m dot consumption. In this case you will see that there will be a generation here with respect to time there.

So at a time t the undissolved quantity will be there and it will be generated and it will be n. So we can write here this generation time or accumulation time of that molecule here dn by dt. That will be is equal to here there will be input. There will be no output as per problem, but generation is there.

That depends on that rate laws, that rate laws as per that constant is K. And here what is that rate laws equation R d is equal to k d n ud into C ss minus C as. Based on this equation we can write then K into n into here 40 by here 100 minus 20 minus n by you know 100. So this is your generation that is initial minus final minus here consumption is zero.

So finally we can write here dn by dt, that will be is equal to Kn into 20 plus n to K in into 20 plus n by 100 after simplification, which will give you dn by here n into 20 plus n. That will be is equal to k by 100 into dt. Now we can simplify it as, or rearrange it as dn by n minus dn by 20 plus n. That will be is equal to K by 5 into dt. So in this way we can rearrange it.

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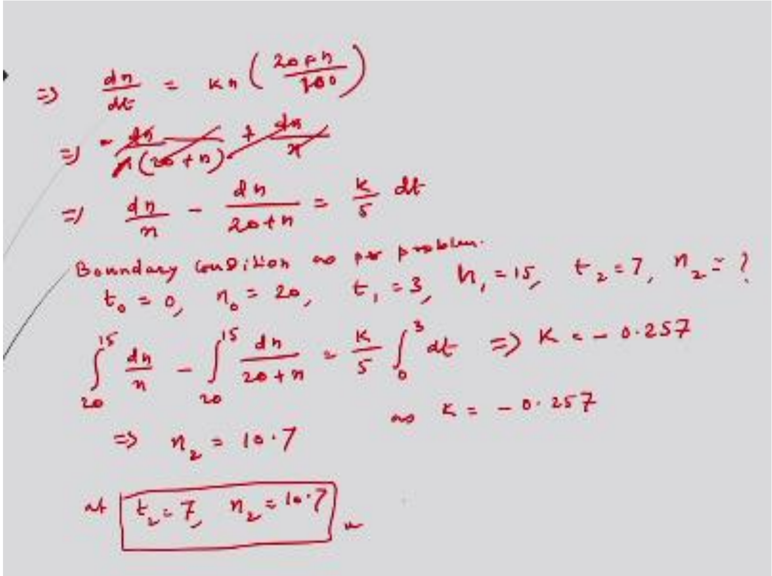

After that we can write dn by dt here that will be is equal to Kn that means here for Kn into 20 plus n by 20 sorry 100. From this dn by n into 20 plus n. That means minus dn by this is I think, no no we can write here dn by dt. That will be Kn into 20 plus n by 100 from which we can write here we can write, dn by n that is plus minus dn by 20 plus here n okay.

We can write it finally as more simplified form. That we have written earlier also dn by n minus dn by 20 plus n. That will be is equal to K by 5 into dt. Now this is your equation. Now you have to integrate this equation with boundary conditions. What is that boundary condition?

Boundary condition as per problem as t 0 that is a time t, t 0 is equal to 0, n 0 is equal to 20 at time t 0. And then t 1 is equal to 3. We can say that n 1 will be is equal to 15 at time t 2 is equal to 7 hours. Then n you know 2 will be is equal to at s 1 here what is that t 0 is equal to 0, n 0 is equal to 20 and t 1 at 3 that you know, n 1 will be is equal to 15.

At t 2 is equal to 7. Then n 2 will be is equal to what? So if we do this integration within this limit of here 20 to 15 as per you know boundary condition. So we can write dn by n minus integration 22 15 as you know dn by 20 plus n. That will be is equal to K by 5 here 0 to 3 as you know dt which will be given as here n 2 will be is equal to 10.7 as K is equal to - 0.257.

So finally we can get this at t 2 is equal to 7, n 2 will be is equal to 10.7. Why because from this here 0 to 3 in this case you can get K will be is equal to minus you know 0.257. So after substitution of this K value again here for this limit of this you know time here, 3 to 7 and within a limit of this here 22 15 you can get this you know n 2 will be is equal to here something 10.7.

Here in this case final concentration is n 2 this. So we can get this here at a you know 7 hour time this you know final concentration of this undissolved solid will be there 10.7 here kg of this amount of total n 2. In this case, the total amount of here material, we can see that here compound will remain undissolved after 7 hour will be is equal to n 2 will be is equal to 10.7 kg.

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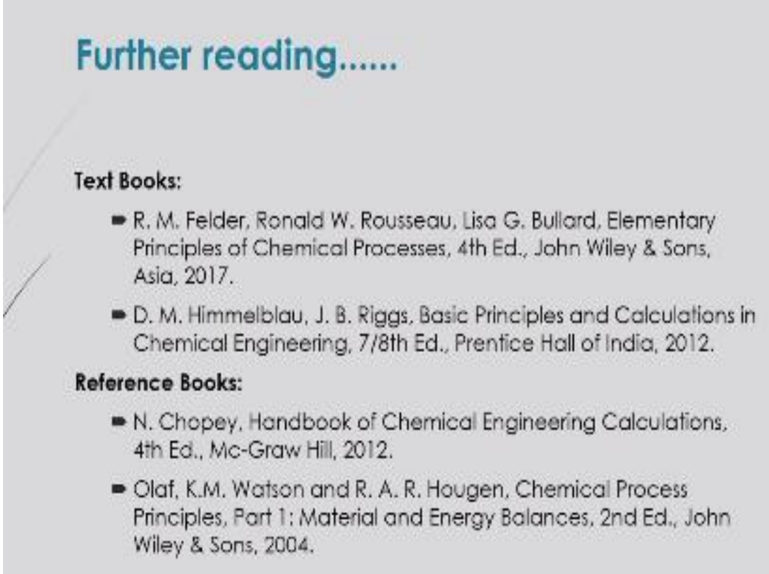

So we have solved different problems with unsteady state material balance. I would suggest you to further go through the example problems given in you know the textbook in example problem and solved by that authors. Then you will understand more you know there and also you can solve the problems with this unsteady state material balance there that is given in exercise also in the textbook and you practice it.

I think you can do. In the next lecture we will try to discuss something more about that unsteady state process, but that will be energy balance equation. So thank you for giving attention for this lecture.