# Basic Principles and Calculations in Chemical Engineering

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## Lecture # 20

## The Mechanical Energy Balance

Welcome to massive open online course on basic principles and calculations in chemical engineering. So, we are discussing about basic laws of energy equations and basic equations how to calculate that, you know energy balance for a particular system in this lecture under the module of energy balanced on a nonreactive processes will discuss the problem based on that mechanical energy balances.

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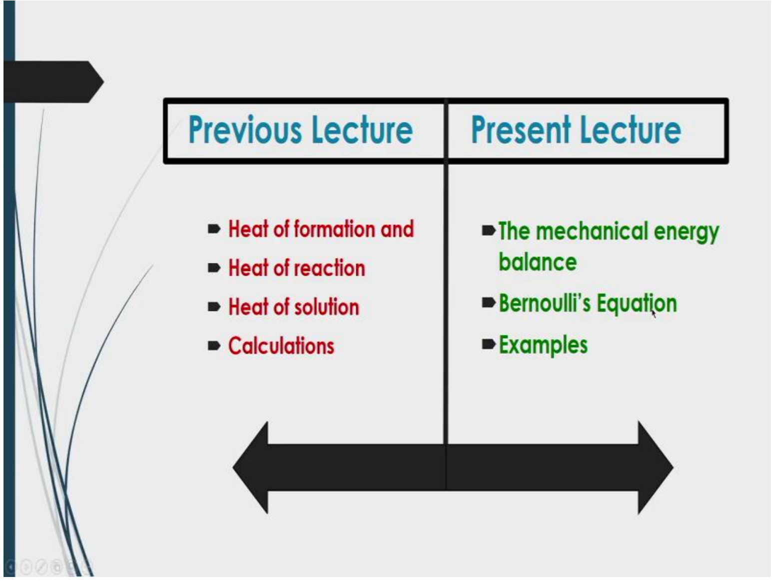

| Previous Lecture | Present Lecture |
|---|---|
| - Heat of formation and<br>- Heat of reaction<br>- Heat of solution<br>- Calculations | - The mechanical energy balance<br>- Bernoulli's Equation<br>- Examples |

So, we have discussed that basic equation for that you know energy balance and from which you will be calculating that mechanical energy also we have discuss about the heat of formation and the heat of reaction heat up solution all those things, you know previous lectures in this case will try to you know that derived that mechanical energy balance and from whose will also try to understand that model is equation with some examples.

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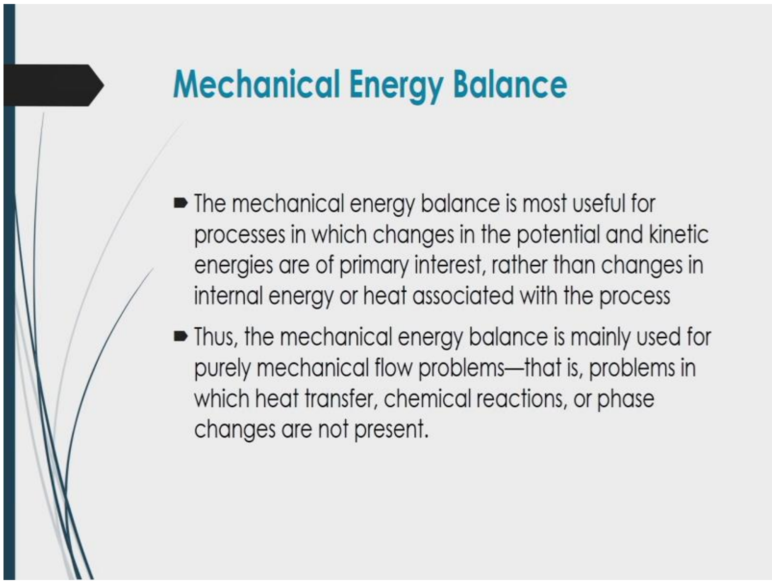

## Mechanical Energy Balance

- The mechanical energy balance is most useful for processes in which changes in the potential and kinetic energies are of primary interest, rather than changes in internal energy or heat associated with the process
- Thus, the mechanical energy balance is mainly used for purely mechanical flow problems—that is, problems in which heat transfer, chemical reactions, or phase changes are not present.

So, what is that mechanical energy balance this is a basically, you know, useful for processing which changes in the potential and kinetic energies which are primarily interest or rather than changes you in you know enthalpy or internal energy or heat associated with the you know our processes that is why this mechanical energy balance will be mainly used for purely mechanical problems that is some problems increase our heat transport chemical reactions or free change are not present in that particular processes, we have already discussed.

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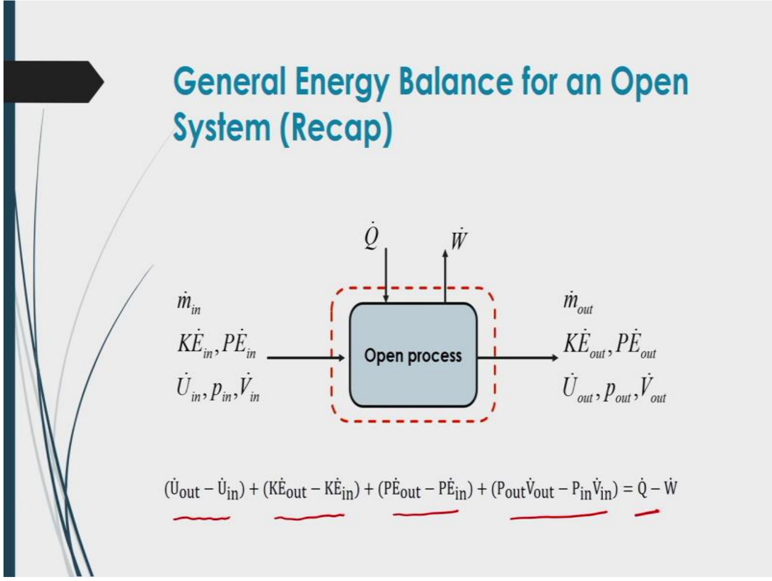

## General Energy Balance for an Open System (Recap)

$$\underline{(\dot{U}_{out} - \dot{U}_{in})} + \underline{(\dot{KE}_{out} - \dot{KE}_{in})} + \underline{(\dot{PE}_{out} - \dot{PE}_{in})} + \underline{(P_{out}\dot{V}_{out} - P_{in}\dot{V}_{in})} = \underline{\dot{Q} - \dot{W}}$$

that is generally how that energy balance for the open system can be you know represented here in the slides, we have given that for an open system, there will be you know heat energy to be supplied to the system for us by this system some work will be done and in this system also there will be you know some mass flow rate and also you will see that there will be a change of you know velocity because at least there will be a change of kinetic energy.

And also the you know position of that system in a particular you know, label based on which there will be a certain change of you know energy that will be regarded as you know potential energy and also due to the seat energy supply to the open process and based on which you will see that there will be a certain change of energy in the system that will be regarded as you know internal energy and also you will see that there will be some pro and it will be working on the system.

Because of that change of volume of the system or because of the change of pressure in the inlet and outlet top position. So, based on whose, you will see there will be a certain change of energy that is called that pore energy So, upon all those you know, tums considering we can you know that right, that energy balance equation for the open process like you will know that part of the change of internal magic what will be the change of kinetic energy, what will be the change of potential energy.

What will be that change of you know pore energy that of course, will be equal to you know that what will be the net energy you know supply to the system that means, you know what the heat energy supply to the system and also what will be the amount of work done by the system if you subtract these 2 quantities, then you will get that need energy supply to the system and this net energy supply to the system will be close to summation of all changes of internal energy kinetic energy potential energy even improved energy out there.

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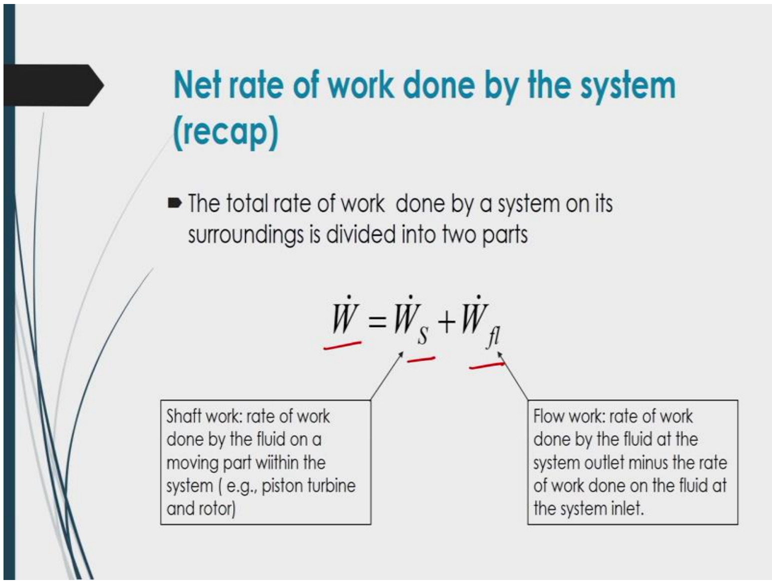

## Net rate of work done by the system (recap)

- The total rate of work done by a system on its surroundings is divided into two parts

$$\underline{\dot{W}} = \dot{W}_{\underline{S}} + \dot{W}_{\underline{fl}}$$

Shaft work: rate of work done by the fluid on a moving part wiithin the system ( e.g., piston turbine and rotor)

Flow work: rate of work done by the fluid at the system outlet minus the rate of work done on the fluid at the system inlet.

So, in that case when you work done by the system will be working you know that the net of work done by that system. Actually, we be you know summation of 2 works done that will be you know that piano So, what is called the shaft work and another is called flow work. So, here we can write that the flow of work done by the system that will be equal to summation of this shaft work and flow work.

What is the shaft work this is basically the rate of work done by the fluid all a moving part within the system like piston turbine and to talk us through work this is basically rate of work done by the fluid at the system outlet minus the rate of work done on the fluid at the system inlet. So, this will be called us flow work.

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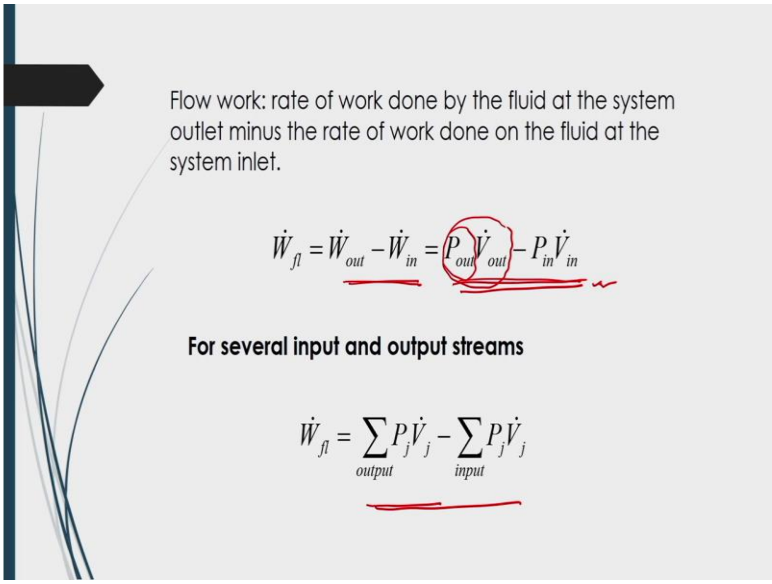

Flow work: rate of work done by the fluid at the system outlet minus the rate of work done on the fluid at the system inlet.

$$\dot{W}_{fl} = \underline{\dot{W}_{out}} - \dot{W}_{in} = \underline{\boxed{P_{out}\dot{V}_{out}} - P_{in}\dot{V}_{in}}$$

**For several input and output streams**

$$\underline{\dot{W}_{fl} = \sum_{output} P_j\dot{V}_j - \sum_{input} P_j\dot{V}_j}$$

So, this flow work is basically rate of work done by the fluid at the system that will be at the outlet and if you subtract that the rate of work done in the fluid at the system inlet you will get that flow work and this will be regarded as actually P out into V dot out - P into V dot in what does it mean? What is the pressure at that outlet condition and what will be the volume at that outlet condition and what is the pressure in it inlet condition and what the volume in the inlet condition.

So, if you are considering as a rate P you cannot regard it as rate. So, in that case only volumetric flow rate you can you know consider their outlet and also volumetric product in the inlet. So, finally, you can say that V. into P – P dot into P in inlet condition if you have this then you will see that total work will be defined by this quantity. Now, if you have that several input and output streams then flow what can be you know regarded as this.

What would be the summation of that, you know flow work at this you know outlet condition and what should be the upload work at the you know outlet condition and input condition there. So, subtracting the summation of that input flow work and output flow work, we can get that what will be that total you know flow work for this several input and output streams.

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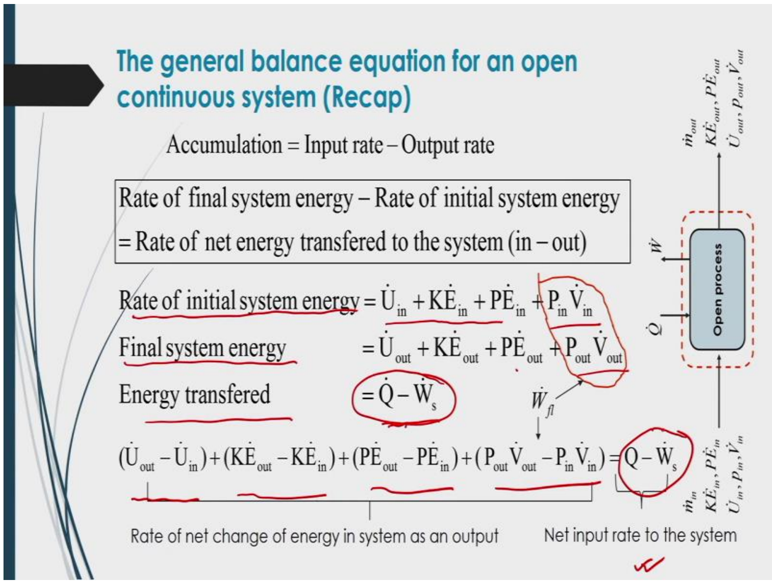

## The general balance equation for an open continuous system (Recap)

Accumulation = Input rate − Output rate

Rate of final system energy − Rate of initial system energy
= Rate of net energy transfered to the system (in − out)

Rate of initial system energy $= \dot{U}_{in} + \dot{KE}_{in} + \dot{PE}_{in} + P_{in}\dot{V}_{in}$

Final system energy $= \dot{U}_{out} + \dot{KE}_{out} + \dot{PE}_{out} + P_{out}\dot{V}_{out}$

Energy transfered $= \dot{Q} - \dot{W}_s$

$\dot{W}_{fl}$

$$\underbrace{(\dot{U}_{out} - \dot{U}_{in}) + (\dot{KE}_{out} - \dot{KE}_{in}) + (\dot{PE}_{out} - \dot{PE}_{in}) + (P_{out}\dot{V}_{out} - P_{in}\dot{V}_{in})}_{\text{Rate of net change of energy in system as an output}} = \underbrace{\dot{Q} - \dot{W}_s}_{\text{Net input rate to the system}}$$

Now, from this you know terms we can make the general energy balance equation for an open continuous system, in terms of rate just by here in this case, we do that accumulation will visible to you input - output. And if it is you know continuous then you can see that accumulation will be equal to input rate - output rate and the rate of final system energy – rate of initial system energy that is equal to rate of net energy transport to the system that is in - out.

So, based on whose we can ride that you know rate of initial system energy that will be you know that summation of internal energy, kinetic energy potential energy and flow and are the work at that inlet condition. Similarly, for the final system energy, we can have the summation of internal energy, kinetic energy potential energy and you know that flow energy upload work at that outlet condition.

So, energy transport actually basically, it will be what will be the heat energy transport and what will be the work done by the system and if you subtract this you will get that net energy you know transferred to the system. Now this net energy transfer to the system will be equal to here that energy that is not just by rate of final and final system energy - rate of initial system energy can be you know equated.

And finally, you can get that that you are you know that difference in unit you know that internal energy rate difference in kinetic energy rate difference in you know potential energy rate and difference in flow work rate there are so, summation of all these you know, differences of these you know, internal energy kinetic energy potential energy and pore work, then it will be equal to you know that net you know energy transfer to the system that is inlet and outlet condition that.

So, this way we can have this you know, general form of you know, this general form of this, you know, energy balance equation for an open continuous system.

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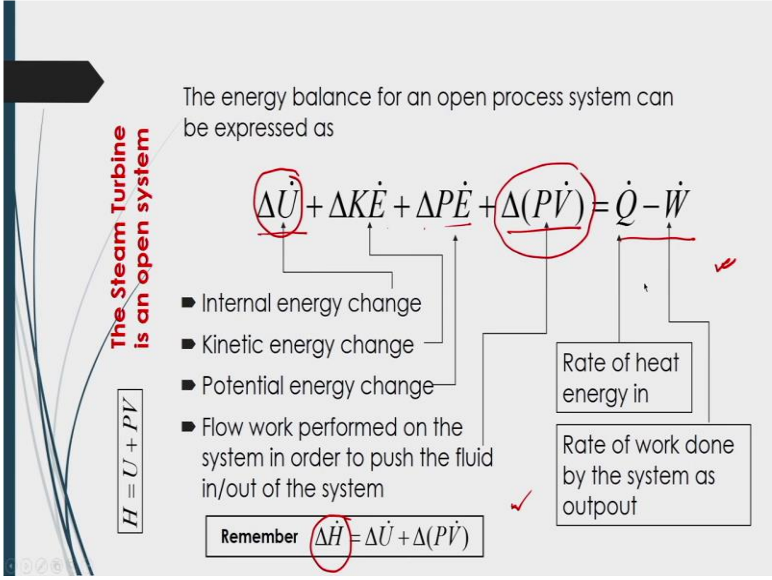

The Steam Turbine is an open system

$H = U + PV$

The energy balance for an open process system can be expressed as

$$\Delta\dot{U} + \Delta K\dot{E} + \Delta P\dot{E} + \Delta(P\dot{V}) = \dot{Q} - \dot{W}$$

- Internal energy change
- Kinetic energy change
- Potential energy change
- Flow work performed on the system in order to push the fluid in/out of the system

Rate of heat energy in

Rate of work done by the system as outpout

**Remember** $\Delta\dot{H} = \Delta\dot{U} + \Delta(P\dot{V})$

And from these, you can worried this here more concisely like that you know that rate of internal energy change it will be = delta u dot rate of kinetic energy send we will Delta kinetic energy dot here delta key here similarly, potential energy rate change it to be delta PE and a flow work you know change rate will be equal to you know that delta PV dot that will be = P dot - W dot here in this case, this is given is term what do you know the definition of that is terms here given in this slides.

So, remember this here this internal energy change + this you know flow energy change will be equals to that are will be denoted by are will be you know defined as are will be referred as are will be you know known as enthalpy change there. So, this enthalpy basically the summation of internal energy + you know full energy there. So, rate of change of that enthalpy will be equal to rate of summation of internal energy + rate open you know pure energy change there.

So, these are you know equation will give you that the generally general energy balance equation for the open conditioner at continuous more.

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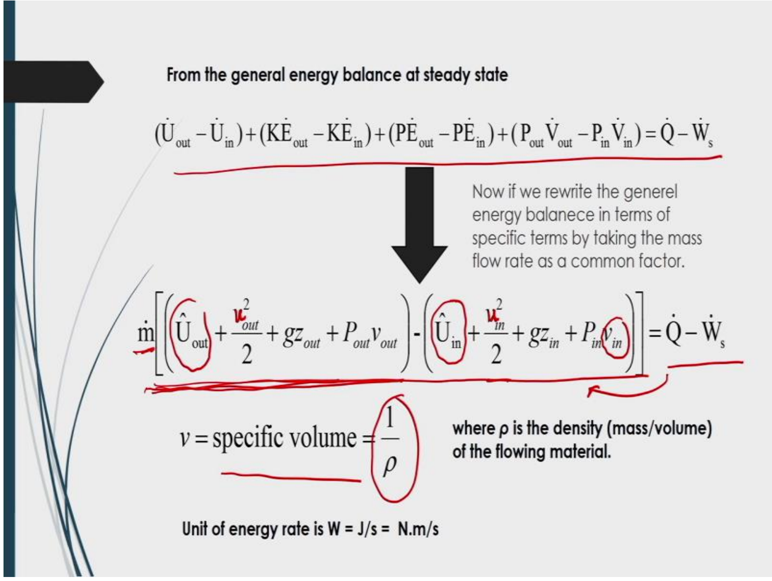

From the general energy balance at steady state

$$(\dot{U}_{out} - \dot{U}_{in}) + (\dot{KE}_{out} - \dot{KE}_{in}) + (\dot{PE}_{out} - \dot{PE}_{in}) + (P_{out}\dot{V}_{out} - P_{in}\dot{V}_{in}) = \dot{Q} - \dot{W}_s$$

Now if we rewrite the generel energy balanece in terms of specific terms by taking the mass flow rate as a common factor.

$$\dot{m}\left[\left(\hat{U}_{out} + \frac{u_{out}^2}{2} + gz_{out} + P_{out}v_{out}\right) - \left(\hat{U}_{in} + \frac{u_{in}^2}{2} + gz_{in} + P_{in}v_{in}\right)\right] = \dot{Q} - \dot{W}_s$$

$$v = \text{specific volume} = \frac{1}{\rho}$$

where ρ is the density (mass/volume) of the flowing material.

Unit of energy rate is W = J/s = N.m/s

And from this general energy balance equation, at a steady state we can write these you know, equation here from this you can write, you know this internal energy kinetic energy potential energy and flow energy in terms of its specific energy rate there. So, in that case a specific energy can be represented by this you know term that you had similarly here in the inlet condition to be you, hat in.

So, we can right here, this you know, at inlet and outlet condition what will be the change of you know that energy there we can write this equation in terms of specific terms by taking the you know mass production as a common factor here. So, from this we can write this equation and we can then express this equation that will be = Q dot - W dot, here in this case pore energy has 2 components 1 components is going to these sites that is why we can write these you know in terms of that specific you know, terms there.

In this case here v a small v What about represented here in this case, this will be specific volume that is 1 by rho, whereas here, this is v we can write here E will. So, this your velocity as U, so, this is U and so, what will be the ability to change in the inlet and outlet condition that will be regarded as what is that you know kinetic energy change and since kinetic energy is defined by that half m u squared, so, here m is common factor m dot so, it would be you know kinetic energy change.

So, half into m dot into U squared at the outlet similarly, half into you know m dot into U in square that will be in your inlet condition that will be the kinetic energy there and so, we saw the density of the material. If it is constant for incompressible fluid we can simply then consider that here to be you know that v you know that it will be you know that constant there.

So only that because of pressure change that you can get that you know flow work there. So, in this way we can write this general form equation in terms of specific you know terms by taking the mass flow rate as a common factor.

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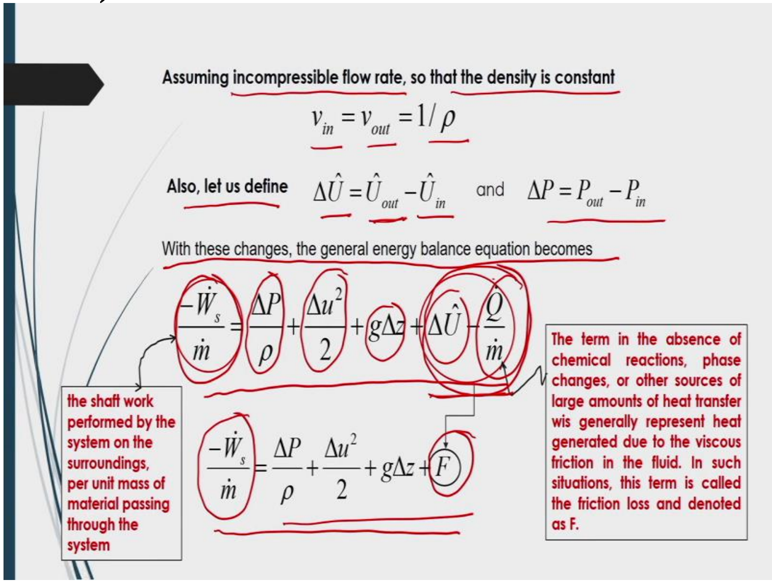

Assuming incompressible flow rate, so that the density is constant

$$v_{in} = v_{out} = 1/\rho$$

Also, let us define $\Delta\hat{U} = \hat{U}_{out} - \hat{U}_{in}$ and $\Delta P = P_{out} - P_{in}$

With these changes, the general energy balance equation becomes

$$\frac{-\dot{W}_s}{\dot{m}} = \frac{\Delta P}{\rho} + \frac{\Delta u^2}{2} + g\Delta z + \Delta\hat{U} - \frac{\dot{Q}}{\dot{m}}$$

$$\frac{-\dot{W}_s}{\dot{m}} = \frac{\Delta P}{\rho} + \frac{\Delta u^2}{2} + g\Delta z + F$$

the shaft work performed by the system on the surroundings, per unit mass of material passing through the system

The term in the absence of chemical reactions, phase changes, or other sources of large amounts of heat transfer wis generally represent heat generated due to the viscous friction in the fluid. In such situations, this term is called the friction loss and denoted as F.

Now, after you know considering that incompressible flow rate, so, that the density is constant, so, you can right here V in that will be equal to V out that will be 1 by rho. Similarly, also let us define here that delta you had that will be able to here U you know out hat - you add in that means specific internal energy difference of this outlet condition from which inlet condition and data P will be regarded as P out – P in there.

These changes the general energy balance equation can be written as here like this after simplification like this in this case here - W are .s by m dot this is basically a shock to our prepared by the system on the surroundings per unit mass of material that is passing to the system. Similarly, this is your you know pressure work and this is your kinetic energy and this is your potential work and this is what is that internal energy and this is your you know that heat energy per unit mass.

So, the term this case in the absence of chemical reactions, the change or other sources large amount of heat transfer wise generally we can represented heat generated due to the viscous friction in the fluid in such situations this term, you know as regarded as you know, that loss and denoted as F. So, what we can right here, these total terms that delta U hat - Q dot by m dot that would be regarded as you know that F this F is called friction loss.

So, finally, you can have this equation of energy balance equation in terms of you know that frictions this you know total and that you will be equal to here, what will be the you know, the shaft work referred by the system on the surroundings.

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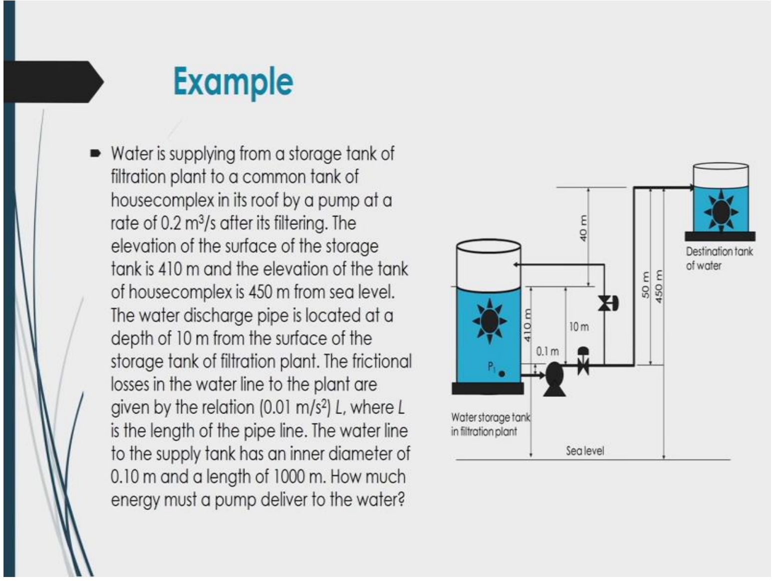

## Example

- Water is supplying from a storage tank of filtration plant to a common tank of housecomplex in its roof by a pump at a rate of 0.2 $m^3/s$ after its filtering. The elevation of the surface of the storage tank is 410 m and the elevation of the tank of housecomplex is 450 m from sea level. The water discharge pipe is located at a depth of 10 m from the surface of the storage tank of filtration plant. The frictional losses in the water line to the plant are given by the relation (0.01 $m/s^2$) L, where L is the length of the pipe line. The water line to the supply tank has an inner diameter of 0.10 m and a length of 1000 m. How much energy must a pump deliver to the water?

Now, let us do an example with this you know, mechanical energy balance equation. So, this is basically mechanical energy balance equation. So, in this case, if we consider that the water is supplying from a storage tank or filtering plant of a you know common tank of house complex in its roof by a pump at a rate of 0.2 meter cube per second after this filtering and the elevation of the surface of the storage tank is a 410 meter.

The elevation of the tank of house complex is 450 you know meter from sea level by water distressed pipe is located at a depth of 10 meter from the source of the storage tank of filtration plant. The frictional losses in the water line to the plant are given by a relation 0.01 meters per second is squared into L are L is the length of the pipeline. The water line to the supply tank has an inner diameter of that 0.1 meter and linked up 1000 meter.

Now, how much energy must a pump deliver to the water destination tank there in the house complex and this case basically what happened that Water to be supplied by a pump from the water storage tank which is kept in filtration plant from whose that you know filtered water to be supplied to house complex by this pump. Now, in this case that the distress line of this pump is a certain distance from that you know water level of that you know storage tank it is given it is 10 meter you know that about this you know this line.

And also this tank this storage tank is you know it is in a you know level of 410 meter high from the you know sea level for as the destination tank of water in the house complex it is 450 meter above that sea level and also it does seem that the you know based on this you know level difference you can say that the distance line from the pipe this you know, saturated you know, 50 meter below that.

You know, destination tank or you can say that the storage tank water level is 40 meter below that, you know this tension my tank of that water in the house complex. Now, at this condition, there are certain you know fictional losses will be there whenever water will be flowing to the pipe, the rate of frictions is given here as 0.01 meter per second is square then you have to multiply by the length of this you know pipe whatever required total length to get to the frictional loss as 0.01 meter per second is current to length of the pipe there.

Now in this case, by diameter is given 0.1 meter inner diameter whereas length is given 1000 meter total length. So, at this condition what are we the energy master pump delivered to the water so, that it will reach to that you know destination water tank there in the house complex.

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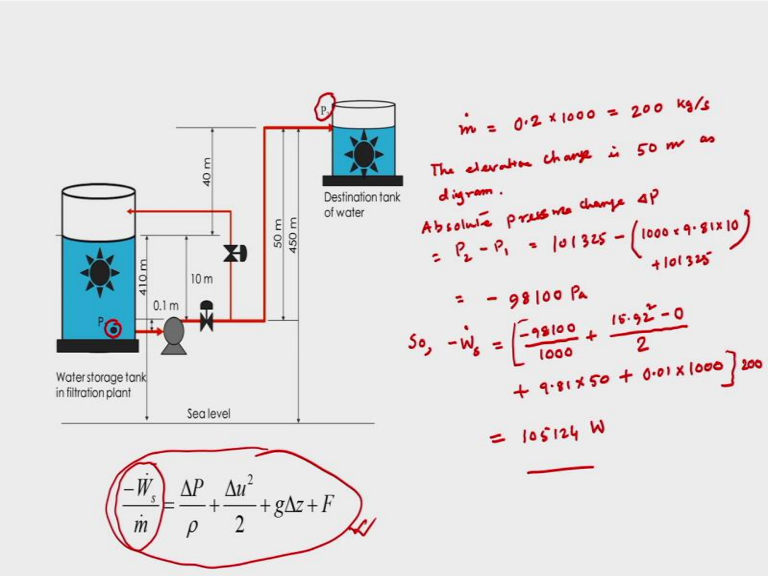

$\dot{m} = 0.2 \times 1000 = 200$ kg/s

The elevation change is 50 m as digram.

Absolute pressure change $\Delta P$

$= P_2 - P_1 = 101325 - \left(1000 \times 9.81 \times 10 + 101325\right)$

$= -98100$ Pa

So, $-\dot{W}_s = \left[\frac{-98100}{1000} + \frac{15.92^2 - 0}{2} + 9.81 \times 50 + 0.01 \times 1000\right] 200$

$= 105124$ W

$$\frac{-\dot{W}_s}{\dot{m}} = \frac{\Delta P}{\rho} + \frac{\Delta u^2}{2} + g\Delta z + F$$

Now, in this case how to solve this problem, we know that mechanical energy balance by this equation here we have already described. So, based on this mechanical energy balance you have to calculate what should be the you know energy required to you know supply that water from that you know storage tank to the you know destination tank here. So, this – W .s by m dot that will be equal to delta p by rho + delta U squared by 2 + g delta z + F.

In this case first of all you have to find out what will be the mass flow rate. So, what is the mass flow rate here, let us calculate here. So, m dot who can write m dot will be = mass flow rate. So, this is basically it is given 0.2 into 1000 so, that will be equal to 200 kg per second. So, this is you are mass flow rate and the elevation change is 50 meter, elevation changes 50 meter as for diagram given as for diagram given here.

So, we know that this mass flow rate next we have to calculate what will be the you know pressure change. So, what should be the absolute pressure change, we can calculate as delta P here so, that will be equal to simply P 2 - P 1 as per diagram this is you are P 2 at that you know level of water at the destination tank and this P 1 from where this you know water is sucking by this pump or delivering that pump.

So, this will be = delta p will be = here P 2 - P 1. So, it will be is equal to what is P 2 here sensitives atmospheric pressure p 2 101 325 - what to be the P 1 here, P 1 is here 1000 into 9.81

into 10 this is 10 meter height this is you are you know that pressure there of this water pressure, what is this water pressure at this point P 1 by this level of this water in this storage tank + this is + atmospheric pressure 101325 this is your atmospheric pressure this is Pascal.

So, finally, it is coming as what is that - 98100 Pascal, this is your absolute pressure change there. So, we can write this as part equation here, we can write - then W s dot, that will be is equal to, here, we can right here delta P - still here delta P by rho what is that delta p here, this is 90 - 98100 Pascal divided by rho is the density of water here + kinetic energy change for to the, you know velocity there it is given this velocity we can have this velocity from this flow rate, we can calculate here.

This velocity here 15.92 that will be square - here initial velocity 0 divided by 4 is that 2. So, this is your velocity difference so, based on which you can calculate what the kinetic energy will come and then + potential energy, what is the potential energy term here, g is 9.81 into what is that delta z what is the elevation change there it is I think 50 meter, it is given here. Frictional loss as you do that, as part problem that frictional loss is you know depends on the distillation of the water.

And the length of the pipe their solution is given 0.01 meter per second squared into length of this pipe is given 1000 So, finally you can get this you know that delta P by rho and what is the kinetic energy change and also potential energy than frictional energy. So, after that you have to multiply it by mass flow rate that is here what is that mass flow rate here m dot here it is given as, what is that 1000, 200 kg per second.

So, we can then finally, have this as what is that 105 here 124 watt that means here, joule per second, we can right here, watt there. So, in way we can calculate what should be the you know energy supply by the pump to deliver this water to the destination tank.

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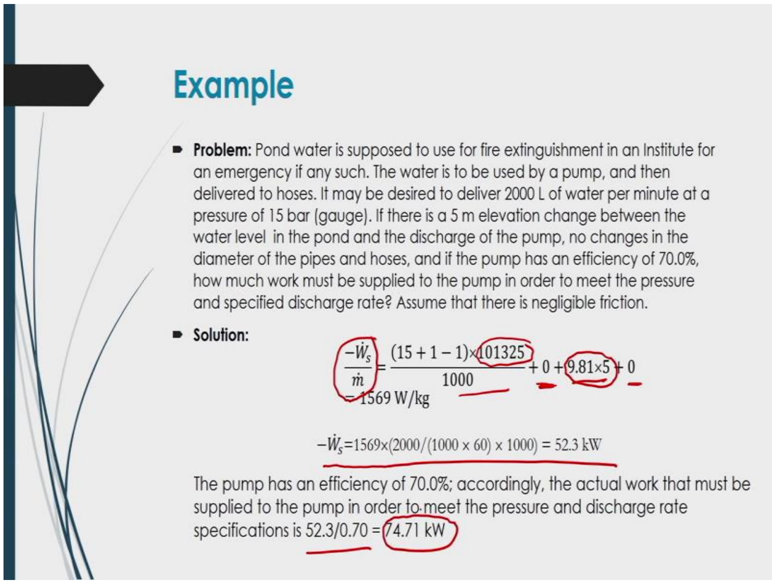

## Example

- **Problem:** Pond water is supposed to use for fire extinguishment in an Institute for an emergency if any such. The water is to be used by a pump, and then delivered to hoses. It may be desired to deliver 2000 L of water per minute at a pressure of 15 bar (gauge). If there is a 5 m elevation change between the water level in the pond and the discharge of the pump, no changes in the diameter of the pipes and hoses, and if the pump has an efficiency of 70.0%, how much work must be supplied to the pump in order to meet the pressure and specified discharge rate? Assume that there is negligible friction.
- **Solution:**

$$\boxed{\frac{-\dot{W}_s}{\dot{m}}} = \frac{(15+1-1)\times\boxed{101325}}{\underline{1000}} + \underline{0} + \boxed{9.81\times5} + \underline{0}$$

$$= 1569\ \mathrm{W/kg}$$

$$\underline{-\dot{W}_s = 1569\times(2000/(1000\times60)\times1000) = 52.3\ \mathrm{kW}}$$

The pump has an efficiency of 70.0%; accordingly, the actual work that must be supplied to the pump in order to meet the pressure and discharge rate specifications is $\underline{52.3/0.70} = \boxed{74.71\ \mathrm{kW}}$

Let us do another example here like suppose, a pond water is supposed to use for fire extinguishment in an you know Institute for an emergency if any sauce case, that is the water is to be used by a pump and then delivered to hoses it may be designed to deliver 2000 liter of water per minute at a pressure of 15 bar gauge if there is a 5 meter elevation change between the water level in the pond and the discharge of the pump that case no change in the diameter of the pipes and hoses.

And if the pump has an efficiency of you know 70% there in this case how must work must be supply to the pump in order to meet that pressure and specified discharge rate to you know, utilize for the per extinguishment. So, assume that there is a negligible friction. So, based on which we can then calculate again by this you know that general energy balance equation where you can write here you know W s dot by m dot that will be equal to what is that pressure energy change there you have pressure is 15 bar.

So, you have to make it an absolute since it has given us gas pressure. So, you have 15 + 1 bar - 1 that would be you know atmospheric condition. So, this is your pressure change in terms of bar. So, what from the you know pressure change in terms of you know that here in Newton per meter square then you have to multiply it by you know that 101325 then finally, you will get this 1 after that you have to calculate the kinetic energy.

Since, here this velocity at the inlet and outlet conditions are you know negligible or kinetic energy is negligible in that case, we can say that this difference will be you know 0 or you can see the same velocity of that you know inlet and outlet of this pump there of the system, then you can see that there will be you know kinetic energy change will be 0 for has potential energy it will come because of that elevation. So, you can calculate simply that you know g into here delta z and then friction is 0 here it is neglected.

So, finally, we can have this calculation of this you know this W s dot that will be is equal to here after substitution of all those values, then you can get this here like this 52.3 kilo watt. Now, the pump has an efficiency of 70% accordingly the extra work that must be supplied to the pump in order to meet the pressure and research rate specification, then you have to you know divide this you know 52.3 by its efficiency factor, that is point 70 then finally, you can get the actual work to be done here 74.71 kilowatt, so, in this way you can calculate.

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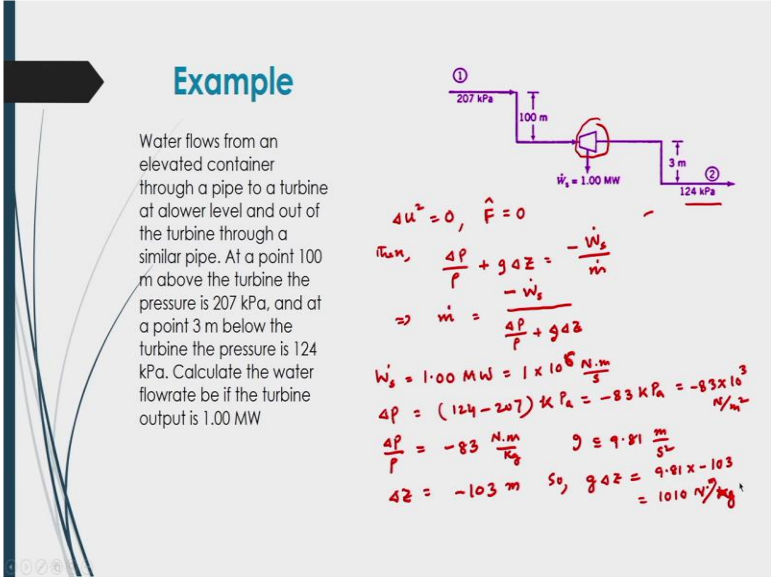

## Example

Water flows from an elevated container through a pipe to a turbine at alower level and out of the turbine through a similar pipe. At a point 100 m above the turbine the pressure is 207 kPa, and at a point 3 m below the turbine the pressure is 124 kPa. Calculate the water flowrate be if the turbine output is 1.00 MW

$\Delta u^2 = 0, \quad \hat{F} = 0$

Then, $\dfrac{\Delta P}{\rho} + g\Delta z = -\dfrac{\dot{W}_s}{\dot{m}}$

$\Rightarrow \dot{m} = \dfrac{-\dot{W}_s}{\dfrac{\Delta P}{\rho} + g\Delta z}$

$\dot{W}_s = 1.00\ MW = 1 \times 10^6 \dfrac{N.m}{s}$

$\Delta P = (124 - 207)\ kPa = -83\ kPa = -83 \times 10^3\ N/m^2$

$\dfrac{\Delta P}{\rho} = -83 \dfrac{N.m}{kg} \qquad g = 9.81 \dfrac{m}{s^2}$

$\Delta z = -103\ m \qquad So,\ g\Delta z = 9.81 \times -103 = 1010\ N.m/kg$

Now, let us do another example, like water flows from any elevated container through a pipe to a turbine at a level at a lower level and out of the turbine to a similar pipe here has given in the picture in the slide and you will see that at a point 100 meters above from this you know turbine the pressure is 207 kilo Pascal and at a point 3 meter below the turbine the pressure is 124 kilo Pascal.

Now, in this case you have to calculate the water flow rate that must be if the turbine output is 1 megawatt there. Now in discuss how to calculate these things. So, again here you can apply that you know, mechanical you know energy balance equation to calculate this water flow rate here. So, in this case what you have to do fast as for the problem you have to see whether that is there any kinetic energies they are not here velocity you know that will be same since we are using that the same cross sectional area of the pipe.

So, delta u squared here that will be equal to 0 and friction loss we can you know neglect here you know a specific fiction energy you can neglect here So, F dot it a hat that would be = 0 then, we can write from that energy balance equation delta P by rho + g into delta z that will be = - W s by you know that m dot here like this and from who is who can right here m dot that will be = - W s dot by here delta P by rho + g delta z.

So, based on this equation, we can calculate what will be the you know water flow rate that is V if the turbine output is a certain amount there. So, in this case, what would be the W s dot here, this is given 1.00 megawatt. So, this is basically we can write 1 into 10 to the power 5 Newton meter per second that will be = what is that this is you know there and similarly you can write here it is 1 megawatts to do I think this is you are 10 to the power 6 1 into 10 to the power 6 Newton per Newton meter per second and delta P will be = here 124 - 207 kilo Pascal.

So, it will be a circle to - 83 kilo Pascal and here that is basically - 83 into 10 to the power 3, here newton per meter is square you can write delta P by rho is your 1000 that is density. So, it will be coming finally, here - 83 then Newton meter per kg and then what is that delta z accordingly to be coming as what is that? – 103 meter and g is given here 9.81 meter per second is square. So, we can calculate g into delta z as here, finally to come here what is that 9.81 into - 103 this will be is equal to you know 1010 neutron per Newton meter per kg.

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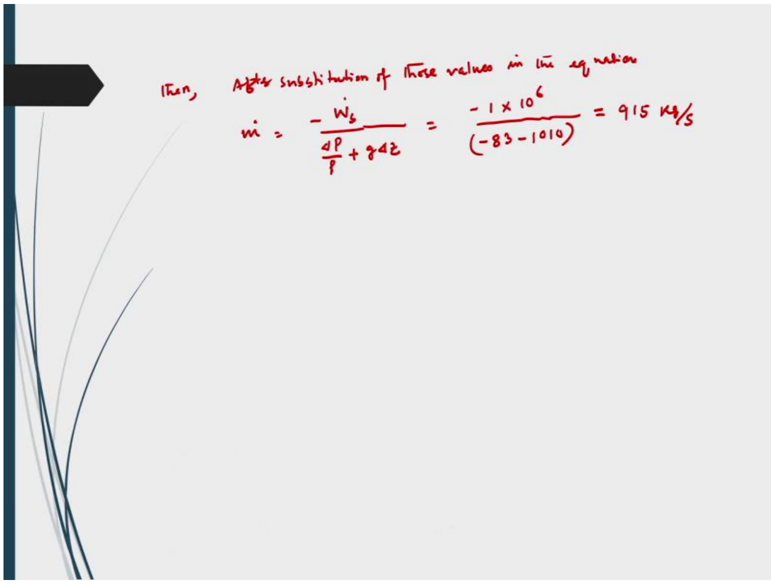

Then, After substitution of these values in the equation

$$\dot{m} = \frac{-\dot{W}_s}{\frac{\Delta P}{\rho} + g\Delta z} = \frac{-1 \times 10^6}{(-83-1010)} = 915 \text{ kg/s}$$

And then after substitution of those values in the equation m dot that will be = - W s dot by delta P by rho + g delta z it will be coming has - 1 into 10 to the power 6 divided by this is - 83 here - 1010 So, it will be coming as finally 915 that is kg per second. So, in this way we can calculate what should be the mass flow rate of water that is flowing through the, you know 5 and to the turbine and based on which there will be a you know energy supplied by the turbine there.

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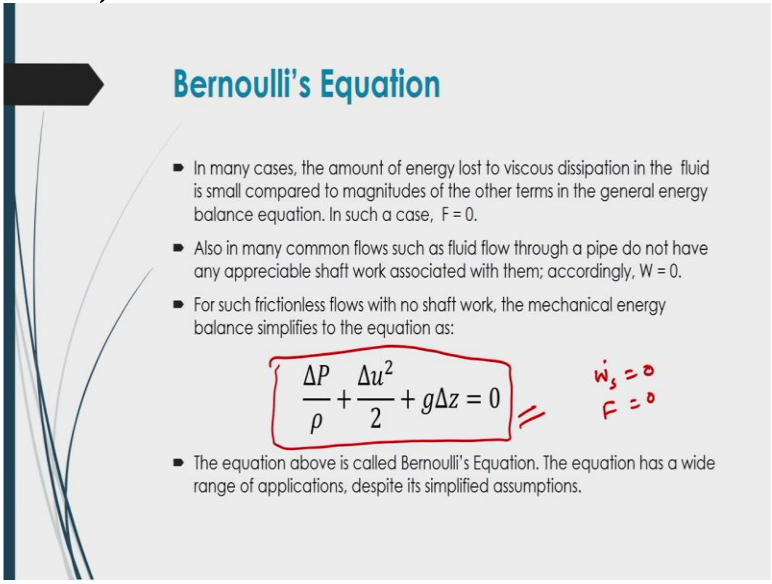

## Bernoulli's Equation

- In many cases, the amount of energy lost to viscous dissipation in the fluid is small compared to magnitudes of the other terms in the general energy balance equation. In such a case, F = 0.
- Also in many common flows such as fluid flow through a pipe do not have any appreciable shaft work associated with them; accordingly, W = 0.
- For such frictionless flows with no shaft work, the mechanical energy balance simplifies to the equation as:

$$\boxed{\frac{\Delta P}{\rho} + \frac{\Delta u^2}{2} + g\Delta z = 0}$$

$\dot{W}_s = 0$

$F = 0$

- The equation above is called Bernoulli's Equation. The equation has a wide range of applications, despite its simplified assumptions.

Now, let us you know consider another you know principle of this mechanical energy this is called Bernoulli is principles that case in many cases you will see that the amount of energy lost due to his cost dissipation in the fluid is a small that will be compared to magnitudes of the other terms in the general energy balance equation. In such cases, you will see there will be a you know loss of friction.

So, we can simply F can be, you know, regarded as you know 0. So, we can simply you know negate that frictional energy. So, also are in very common processes fluid flow through a pipe that do not have any appreciable shaft work that is associated to them accordingly we can write that w because 20 suppose as frictionless flow per unit frictional you know a contribution will be negligible that we have considered that for that amount of energy loss to discuss distribution in the fluid will be small compared to the other terms in the energy supplied or consideration.

So, in that case we can say that the flow is that frictionless flow without you know shaft work then we can simplify that mechanical energy balance into this you know equation here. So, we can simply that neglect that Ws that will be = 0 and also friction, because to F will = 0. So, here Ws that will because to 0 and frictional force that will = 0. So, after you know neglecting these, 2 terms who can then write this equation here.

So, this equation is called Bernoulli’s equation, and the equation has a wide range of application despite its simplified assumptions. So, remember this when you bar this Bernoulli’s equation to be applied to any system that initially you have to that you know assess whether that system is ideal or not that means frictionless or not is there any you know shaft work is working on that or not that also to be you know considered.

So, simplify form of that mechanical energy will give you this Bernoulli’s equation, what is this you know that summation of this pressure energy kinetic energy and that potential energy that will be equal to 0, what does it mean that change of these 3 energy summation of these 3 energy will be you know constant there. So, based on these principles, we can you know solve some problems are you know some process.

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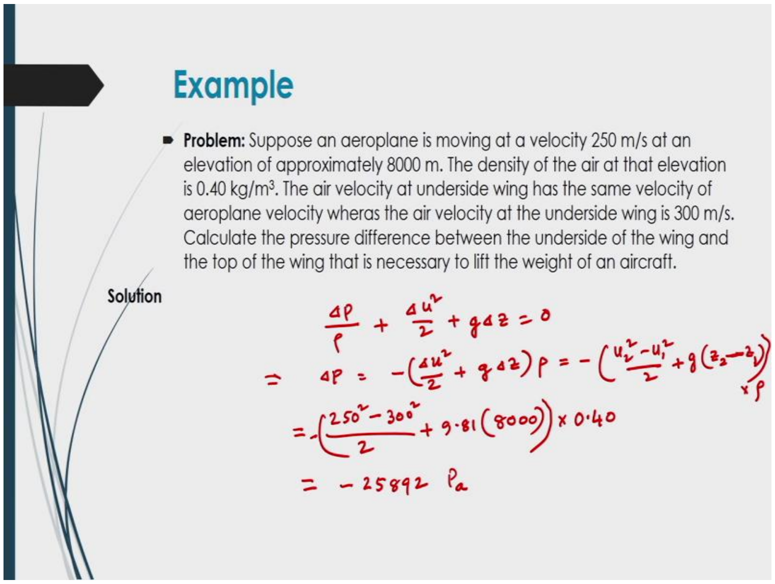

## Example

- **Problem:** Suppose an aeroplane is moving at a velocity 250 m/s at an elevation of approximately 8000 m. The density of the air at that elevation is 0.40 kg/m³. The air velocity at underside wing has the same velocity of aeroplane velocity wheras the air velocity at the underside wing is 300 m/s. Calculate the pressure difference between the underside of the wing and the top of the wing that is necessary to lift the weight of an aircraft.

Solution

$$\frac{\Delta p}{\rho} + \frac{\Delta u^2}{2} + g\Delta z = 0$$

$$\Rightarrow \Delta p = -\left(\frac{\Delta u^2}{2} + g\Delta z\right)\rho = -\left(\frac{u_2^2 - u_1^2}{2} + g(z_2 - z_1)\right) \times \rho$$

$$= -\left(\frac{250^2 - 300^2}{2} + 9.81(8000)\right) \times 0.40$$

$$= -25892 \text{ Pa}$$

Like this let us do an example here. Suppose an aero plane is moving at a velocity 250 meter per second at an elevation of approximately 8000 meters Now, the density of the air at that elevation is considered as 0.4 kg per meter cube the air velocity at under side wing here of that aero plane has the same velocity of the aeroplane velocity whereas, the air velocity at the underside wing is 300 meter per second.

In this case, calculate the pressure difference between the underside of the wing and the top side of the wing that is necessary to lift that weight of that aero plane to a height. So, here basically we are having that, the airplane is moving at a velocity of 200 meter per second at an elevation of 8000 meter. The density of course will be changing along to that height is given that the height is .40 kg per meter cube whereas the air velocity, what is the air in the underside of the wing.

This is basically that same as that wing velocity there it is basically 300 meters per second there. Now, based on the you know velocity change that elevated height you have to calculate that what would the pressure difference there in this case we are considering that there will be a frictionless movement of that you know aero plane and also there will be no you know shaft work done on us done by that system and also there will be no heat energy supply to the you know, system there.

So, based on this we can apply, that does mechanical energy balance equation here what is that equation as per Bernoulli’s equation this is delta P by rho + delta u squared by 2 + g into delta z that will be = 0. So, according to this equation, we can write delta p that would be lower to - delta u square by 2 + g into delta z whole bracket - so, we can make it common here into rho are you can write it here as - u 2 squared – u1 squared by 2 that is velocity difference there.

And then accordingly kinetic energy difference will be there + g into here elevation height here z2 - z1 we can right into density after that, then we can right after substitution of respect the value of here velocity here u2 is given 250 Square u1 is here for 300 and then divided by 2 + here + g is 9.81 into z2 is what is that z2 - z1 is given z2 – z1 it is basically here the elevated height is 8000 meter. So, into whole bracket into density of the fluid here 0.40 after simplification or calculation, we can have this as - 25892.

So, this is you know that pressure difference as you know pascal. So, this mass pressure difference will be there according to you are problem. So, in this case we are applying that Bernoulli’s equation and how to solve this equation based on this Bernoulli’s equation like this.

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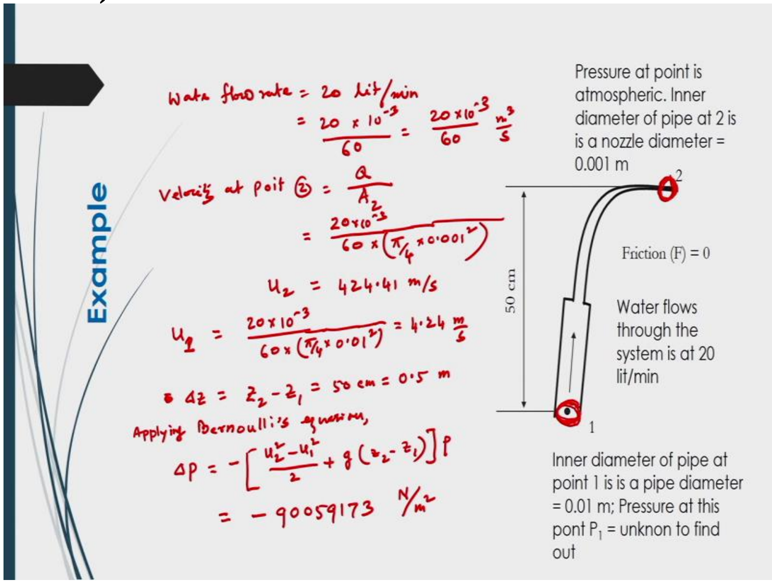

Water flow rate = 20 lit/min

$$= \frac{20 \times 10^{-3}}{60} = \frac{20 \times 10^{-3}}{60} \frac{m^3}{s}$$

Velocity at poit ② $= \frac{Q}{A_2}$

$$= \frac{20 \times 10^{-3}}{60 \times \left(\frac{\pi}{4} \times 0.001^2\right)}$$

$$u_2 = 424.41 \ m/s$$

$$u_1 = \frac{20 \times 10^{-3}}{60 \times \left(\frac{\pi}{4} \times 0.01^2\right)} = 4.24 \ \frac{m}{s}$$

$$\Delta z = z_2 - z_1 = 50 \ cm = 0.5 \ m$$

Applying Bernoulli's equation,

$$\Delta p = -\left[\frac{u_2^2 - u_1^2}{2} + g(z_2 - z_1)\right]\rho$$

$$= -90059173 \ N/m^2$$

Pressure at point is atmospheric. Inner diameter of pipe at 2 is is a nozzle diameter = 0.001 m

Friction (F) = 0

Water flows through the system is at 20 lit/min

Inner diameter of pipe at point 1 is is a pipe diameter = 0.01 m; Pressure at this pont $P_1$ = unknon to find out

Let us do another example of this Bernoulli’s equation like here one you know that nozzle is shown in the slide here and water is flowing through a nozzle before coming to that or before playing to that nozzle here, this is 1 nozzle at this point 2 and before flowing through that nozzle, the fluid is coming from a pipe at point 1. Now, the pipe diameter at point 1 it is given a certain value whereas at this nozzle the nozzle diameter will be something different from this pipe. Generally nozzle diameter will be very smaller than compared to this you know pipe diameter.

Now, pressure at point is atmospheric here at this point to inner diameter of the pipe or you can see that nozzle here at 2 is 0.001 meter there and at this point in our diameter of the pipe is given 0.01 meter this is 10 times higher than you know that nozzle diameter pressure at this point is unknown to you, have to find out whereas pressure at this point 2 is atmosphere. So, there will be a pressure at this 2 point and there is the elevation of this 2 point is 50 centimeters there is no friction which is considered water is flowing through this you know pipe and then nozzles at the rate of 20 liter per minute.

Now, at that situation, you have to find out what should be the pressure at this point 1 at its 5 here. So, let us again apply that Bernoulli's equation. So, before applying that, you have to first calculate what would be the water flow rate in terms of you know, volumetric flow rate in you know SI system cost meter per second then only we can calculate. So, water flow rate what is the water flow rate here water flow rate is given here 20 liter per minute.

this will be equal to 20 into liter you have to convert it to meter cube into 10 to the power - 3 that is 1000 liter that will equal to 1 meter cube divided by here 60 to be converted to second then it will come as here to empty into 10 to the 4 - 3 by 60 here it will be meter cube per second and velocity at point 2 this is will be equal to this is your flow rate if you consider flow rate is cube in meter 2 per second the velocity you can calculate just by dividing it by its cross sectional area at this point 2.

So, it will be coming as for that velocity flow rate is 20 into 10 to the power - 3 divided by 60 then you have to divide it by you know that cross sectional area what is that 5 by 4 into you know diameter at this point is 0.001 into that is square. So, this is your cross section then it will come finally, as u2 = 424.41 meter per second. Similarly, you can calculate the velocity at point 1 similarly, for the same product is going so, you can have 20 into 10 to the power - 3 by 60 divided by its cross sectional area, it is 5 by 4 into 0.01 here the diameter.

So, it will be as here 4.24 meter per second and what is the elevation here? Elevation is delta z that will be = z2 - z1. This is your 50 centimeter, converting to meter to 0.5 meters. then applying Bernoulli's equation, we can write, applying Bernoulli's equation Bernoulli's equation we can write delta P that will be equal to - u2 squared - u 1 squared by 2 + g into z2 – z1 into rho.

So, based on this, we can calculate after substitution all the you know values of u1 u2 z1 z2 and g all these things and then density finally, we can have this value of pressure differences, - 90059173 this is in terms of unit as neutron per meter squared. So, this is basically pressure difference.

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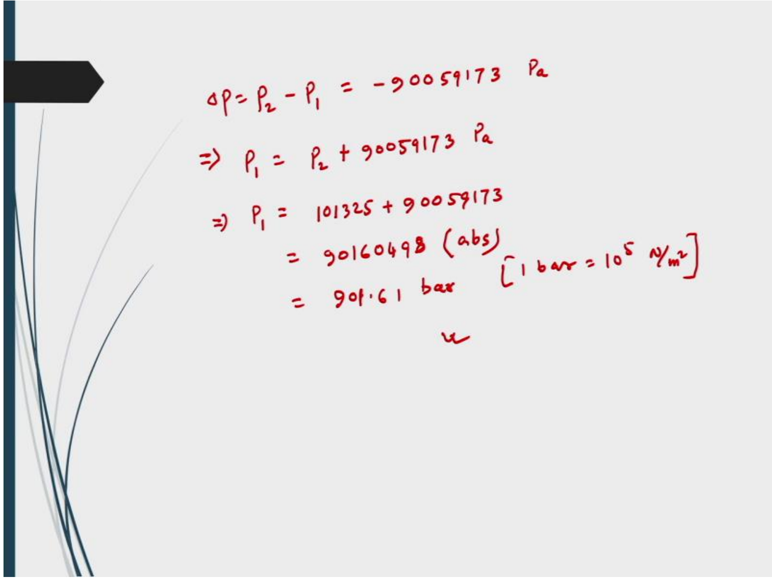

$$\Delta P = P_2 - P_1 = -90059173 \text{ Pa}$$

$$\Rightarrow P_1 = P_2 + 90059173 \text{ Pa}$$

$$\Rightarrow P_1 = 101325 + 90059173$$

$$= 90160498 \text{ (abs)}$$

$$= 901.61 \text{ bar} \qquad [1 \text{ bar} = 10^5 \text{ N/m}^2]$$

Now, as you know that this pressure difference is basically P 2 - P 1 that is basically is - 90059173 newton per meter squared or you can say that it is Pascal which implies that P 1 will be = P 2 + 90059173 Pascal which implies P 1 that will be = what is the P 2 value this is atmospheric pressure as per atmospheric pressure you can say that it will be here 101325 then + 90059173 that will be coming as 90160493 0493 is it  is 930 or  it is getting to be here 8.

And this will be considering as absolute why absolute because we are adding here with this values as you know, atmospheric pressure here. So, finally to be coming as an absolute pleasure as a P 1. So, this would be = you know patient at that point 1 there. So, finally we can write it as a bar as 90 901.61 bar, because 1 bar that will be = 10 to the power 5 newton per meter squared.

So, in this way, by applying the Bernoulli's equation we can solve of for pressure we can solve for velocity we can solve for even elevation height also provided other terms are they are as per your you know problem.

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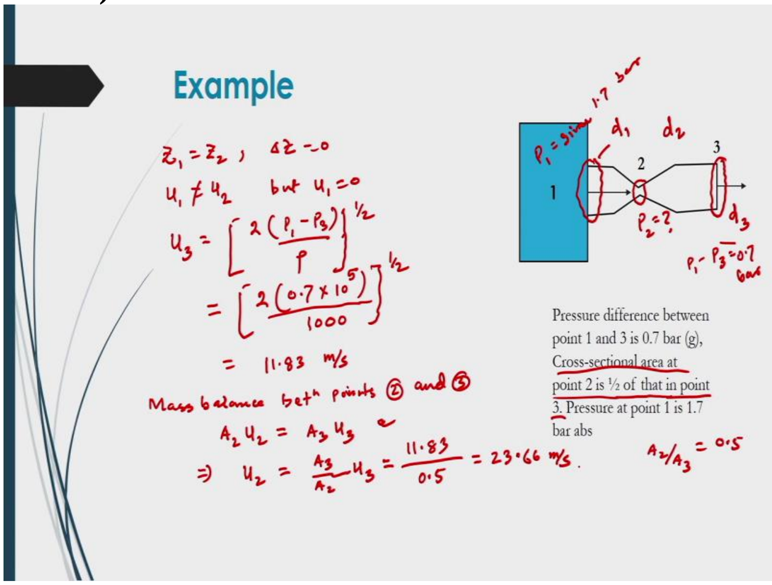

**Example**

$z_1 = z_2\,,\quad \Delta z = 0$

$u_1 \neq u_2 \quad \text{but } u_1 = 0$

$$u_3 = \left[\frac{2(P_1 - P_3)}{\rho}\right]^{1/2}$$

$$= \left[\frac{2(0.7 \times 10^5)}{1000}\right]^{1/2}$$

$$= 11.83 \text{ m/s}$$

Mass balance betn points ② and ③

$A_2 u_2 = A_3 u_3$

$$\Rightarrow u_2 = \frac{A_3}{A_2} u_3 = \frac{11.83}{0.5} = 23.66 \text{ m/s}.$$

Pressure difference between point 1 and 3 is 0.7 bar (g), Cross-sectional area at point 2 is ½ of that in point 3. Pressure at point 1 is 1.7 bar abs

$A_2/A_3 = 0.5$

Let us do another examples here see 1 figure is given in the slides this case water is flowing from a tank to a you know that converging diverging, you know 5 this called you know venturing 5 and you will see that when you are this venturing pipe is let us to that tank at this location. The diameter of this part of this density is here at this study one for us at this you know converting section of this venturing at point 2 its diameter is d2.

Whereas again from this you know conversion section, it will be converted to diverging as for that principle of density for getting that lower pressure at this point 3, you will see that the diameter is d3 here at this 3 sections, you will see the diameter are you know d1 d2 and d3 whereas at this point 2 you will see that d2 is very smaller compared to that d1 and d3. This d1 and d3 will be the same diameter whereas d2 will be the difference from this you know 2 and 1 and 2 you know point.

Now, when you got a fluid to be flowing to the dispensary meter, you will see there will change of pressure between point 1 and 2 again you can get the pressure between 1 and 3 also again the pressure difference between 2 and 3 will be there, because of that velocity change, whenever fluid to be flowing through that lower cross sectional area, the velocity will be higher relative to that higher cross sectional area.

So, based on these velocity sayings you will see there will be change your pressure for a constant elevated height, even if you change that elevation there you have to consider that how much energy will be changing. So, based on this, what will be the pressure change or velocity change that you can calculate based on that Bernoulli's equation. So, pressure difference between this point 1 and 3 is given here. 0.7 bar by this manometer you can measure it cross sectional area at point 2 is half of that in point 3 here, pressure at point 1 is 1.7 bar absolute.

So, according to these saw you no problem you have to find out what should be the pressure at point 2 here, pressure at point 2, you have to find out pressure at .1 is given to a here you that is here 1.7 bar pressure at .3 P3 is given that is, you know that pressure difference that you know p 1 - p 3 that is given here 0.7 bar. So, accordingly you have to find out what the P velocity also you have to calculate their based on this cross sectional area.

So, let us apply that Bernoulli's equation what is that Bernoulli's equation you know that in this case what would be the z elevation here since z 1 that will be equal to z2. So, we can right here delta z it will be = 0, you want to not be equal to you know that u2 of course, since the cross sectional area is different, but u1 = 0 because the liquid is coming from that stream condition from this tank and then we can calculate u3 as what is that from that Bernoulli's equation as 2 into what is that P 1 - P 3 by P3 by rho between this point 1 and 3 then it will be equal to what half.

So, if we apply this Bernoulli's equation between .1 and .3 we can have this here z delta z because to 0 and you know that u3 that can be calculated from this pressure difference and u1 is then you know it is you know 0. So, u3 can be calculated, if you are having that cross sectional area at this point 1 and 3 are not same and then what is that even if there is a cross sectional area same.

You see that if the velocity at this cross sectional 0. Then you have to of course calculate what is the velocity at this point 3 there because of that by cross sectional area. So, after substitution of this value that means here 2 into what is the pressure difference between 1 and 3 that is 0.7 bar you have to multiply by 10 to the power 5, then you will get the pressure in terms of Newton per meter squared divided by density is here water then it will be density of water is 1000 after substitution.

And then calculation we can get these value as 11 point you know that 83 here meter per second after that if you do the mass balance between this .2 and 3 we can have this A2 into u2 that will be = A3 into u3 what is that A2 is the cross sectional area u2 is the velocity at that point 2 that means A2 into u2 we it will give you that volumetric flow rate at that 2 and A3 into u3 will give you the volumetric flow rate at that point 3 since these volumetric flow rate are constant, so you can write this equation here.

And from these we can write you know u2 = A3 by A2 into u3 that will be = what here A2 by A3 is given to you A2 by A3 is given. You know that 0.5 according to this problem we are because cross sectional area at point 2 is half of that in point 3 So, we can write A2 by A3 will equal to 2.5 So, finally we can right here after substitution of A2 by A3 and you see value here as u3 11.83 divided by then A2 by A3 it is simply it is 0.5 then we can have this value as 23.66 this is meter per second.

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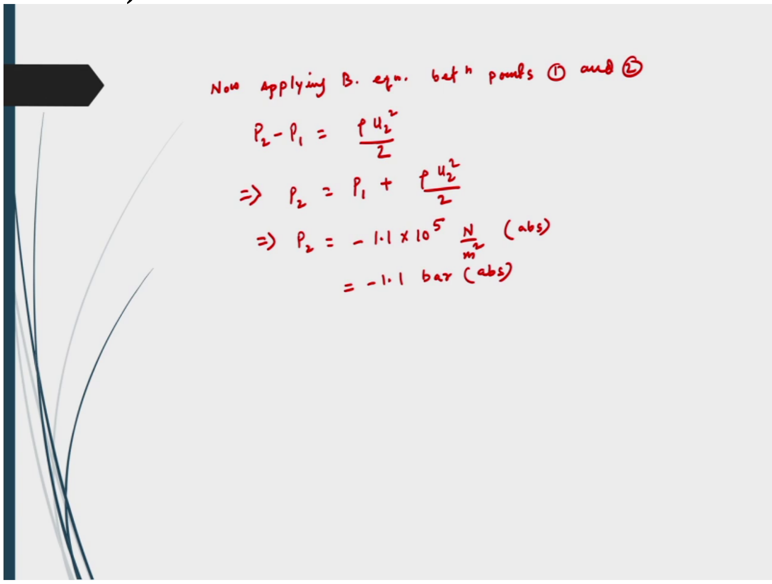

Now applying B. eqn. betn points ① and ②

$$P_2 - P_1 = \frac{\rho u_2^2}{2}$$

$$\Rightarrow P_2 = P_1 + \frac{\rho u_2^2}{2}$$

$$\Rightarrow P_2 = -1.1 \times 10^5 \ \frac{N}{m^2} \ (abs)$$

$$= -1.1 \text{ bar } (abs)$$

Now, applying that Bernoulli's equation 1 and 2, we can get P2 - P1 as rho u2 square by 2 whose implies P2 = P1 + rho u2 square by 2 that implies P2 = here after substitution of those values - 1.1 into 10 to the power 5 neutron per meter squared. So, that will be your absolute value this is simply you can write 1.1 bar absolute. So, in this way we can apply this bottle is equation for the different problems.

Whereas we know this you know, either of the values of these you know kinetic energy terms and potential energy terms or you know pressure different stops there. So, we are having this Bernoulli's equation and solving those problems based on that general mechanical energy balance equation.

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## Further reading......

**Text Books:**

- R. M. Felder, Ronald W. Rousseau, Lisa G. Bullard, Elementary Principles of Chemical Processes, 4th Ed., John Wiley & Sons, Asia, 2017.
- D. M. Himmelblau, J. B. Riggs, Basic Principles and Calculations in Chemical Engineering, 7/8th Ed., Prentice Hall of India, 2012.

**Reference Books:**

- N. Chopey, Handbook of Chemical Engineering Calculations, 4th Ed., Mc-Graw Hill, 2012.
- Olaf, K.M. Watson and R. A. R. Hougen, Chemical Process Principles, Part 1: Material and Energy Balances, 2nd Ed., John Wiley & Sons, 2004.

I would suggest to you know read more about this you know gentle energy balance equation and solving the problems as you know suggested this textbook here in the slides for this course. So, I think you understood here in this lecture How to solve the problem based on this mechanical energy balance equation and based on which you can solve more problems from this textbook, in the next lecture. We will try to discuss more about this you know energy balance equation, higher entropy balance will be considered without reaction and solving some problems. Thank you.