Computer Aided Applied Single Objective Optimization
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Lecture - 08
Teaching Learning Based Optimization

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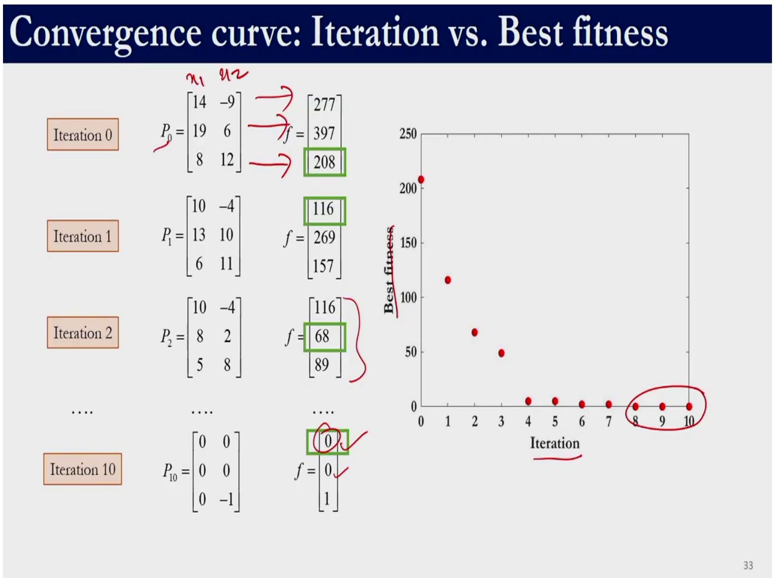

Welcome we have various types of convergence curve. The one that is probably the most widely used is curve between the number of iteration and the best fitness functions value. So, how do we plot it? Let us say the beginning my population was P 0 right. These were the so, this value of decision variable x 1 and x 2 right and these are their corresponding fitness function value or objective function value.

So, what we can do is make a plot the x axis iteration and the y axis is the best fitness function value right. So, among these three solutions if we see 208 is the best solution. So, I am going to plot that particular value alone, I am not going to plot 277 or three 387 only the best particular best value in the fitness function is plotted right

So, then subsequent to this we perform the teacher phase, the learner phase and at the end of iteration one let us say this is our population and these are their corresponding fitness function value right. So, in this at the end of iteration one, the least value is 116 in this right. So, I retain the first point because it corresponds to my iteration 0 and in at the end of iteration one, I had 116. So, we have plotted that 116 and we continued doing so.

So, at the end of the iteration 2, let us say this is the fitness function value the best value here is 68. So, we plot is at 68 over here and then we keep doing it, then say at the end of iteration 10 have this fitness function value right 001. In this the least one is 0, it does not matter whether you take the 0 or this 0 right. We are not plotting the solution the solutions, we are plotting only the fitness function value right.

So, this will look like something like this. So, this is called as the convergence curve. So, this curve shows you that as iterations progress how much improvement, we were able to do in the objective function value. In this case, it seems to have converge to the global optima right. So, there are there is not much change or at least feasible change in those three values. So, this curve tells us the performance of the algorithm as iteration proceeds how we are improving in terms of the best solution that we have. So, if you think about it in TLBO, we never eliminate a solution until we have found a better solution than that right.

So, even in teacher phase and learner phase whenever a solution was to be included in the population, it had to meet the criteria that it is better than the solution which was used to generate the new solution right. So, that way if you see our this curve will also be monotonically decreasing; for minimization problem, it will be monotonically decreasing right because we are never losing out on a good solution in favor of a bad solution.

So, over a period of iteration, we may not be improving from one iteration to other iteration the fitness function is definitely not going to deteriorate right. So, that is called as a monotonic convergence and TLBO exhibits this monotonic conversions.

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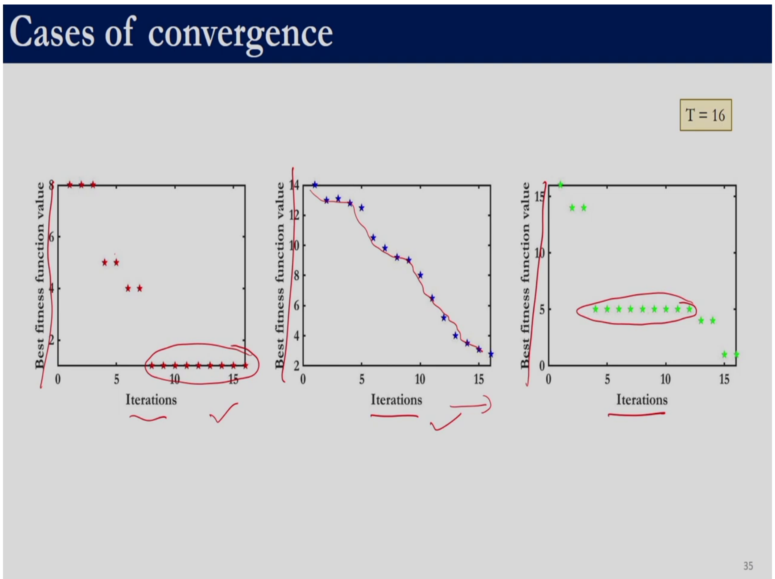

These are three different cases of convergence curve right. So, in all the three cases, the x axis is the iterations right. So, and the y axis is best fitness function value obtained at that particular iteration right.

So, here as I just discussed for iteration 5, the best value obtained is this point. So, in the first case if you see right, the values have more or less stabilize right. So, for a large number of iteration, they does not seem to be significant change right. However, if you see in the second plot; this plot, over here if you see it is continuously decreasing; it has not stabilized. So, if you come across this situation; that means, that if you perform few more iterations there is a likely hood again remember, there is no guaranty there is a likely hood that you will get a better solution right.

So, here we do not say that the algorithm has converged right. So, as you see it is still trying to improve the fitness function value right. So, these two cases are pretty much clear right. So, but you can also end up in cases like this right where in if you see over here for a substantial number of iteration there as there is no improvement in the fitness function value right, no observable improvement in fitness function value, but afterwards it starts decreasing.

So, it is not correct to fix the number of iterations and take whatever the solution, we get at the end of the iterations. It is necessary that we have a look at the convergence plot and decide whether the algorithm has converged or not.

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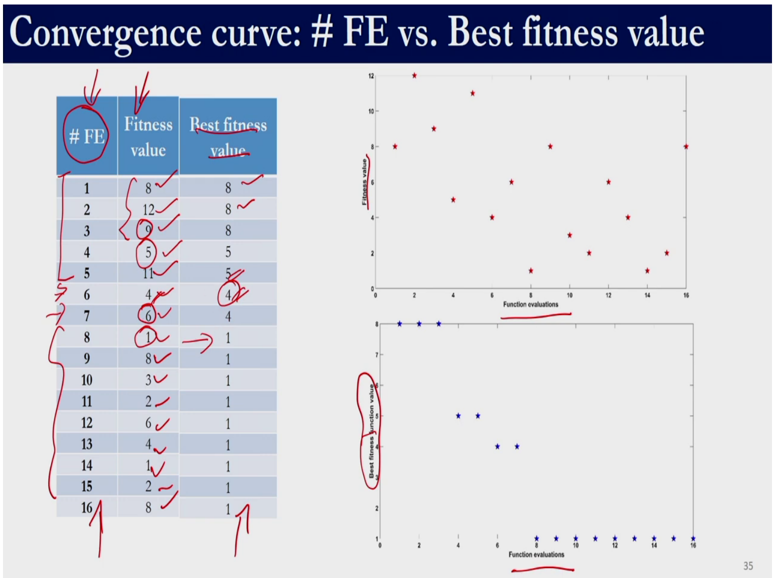

# FE	Fitness value	Best fitness value
1	8	8
2	12	8
3	9	8
4	5	5
5	11	5
6	4	4
7	6	4
8	1	1
9	8	1
10	3	1
11	2	1
12	6	1
13	4	1
14	1	1
15	2	1
16	8	1

So, there is another type of convergence curve which is plotted between the number of fitness function evaluation and the best fitness value right. Let us say in this case the x axis since not going to be iteration, then x axis is going to be the number of functional evaluation.

So, what we are trying to plot is every time I evaluate the function what is its objective function value; that is what we are going to plot. Let us say my initial population was 5 right. So, initially I would have evaluated the objective function 5 times right, remember this is not iteration it is number of functional evaluation. So, the first time when I evaluated the fitness function value let us say it is 8, the second for the second number it was 12, for the third number it was 9, for the forth number it was 5, for the next number it was 11 and this was the initial population right.

So, now let us say solution one underwent teachers phase and we got a new solution which had a fitness function value of 4 right. So, when I evaluate that fitness function value, I make note of this entry that when I evaluated the fitness function for the 6 time, we had obtained the value of 4 right. Similarly when it underwent student phase and it generated a new solution, the fitness function was evaluated for this new solution; let say the fitness function is 6.

So, when I evaluated the fitness function for the seventh time right, the first 5 are the initial population size right. So, you evaluate that even before you begin the algorithm right. The sixth one was because the first member underwent teacher phase so, got a new solution. So, we have to evaluate the fitness function. Similarly therefore, solution underwent learner phase, we generated a new solution; we had to evaluate the fitness function of the new solution.

Similarly every time you evaluate the fitness function right, let us say we make a note of these values right. Now, if we plot this, it will look something like this right. It is scattered, we cannot make much out of it. If we keep doing iterations; obviously, there it is going to exhibit down word trend in a whole. But the inference is not that clear because you are plotting every functional evaluation.

So, for example, let us say you have generated a solution which is actually inferior right. So, since you evaluate the fitness function of that particular solution, you also will end up plotting it if you are going to plot between functional evaluation and fitness value right.

So, what we do is we plot the best fitness values. So, want we are saying is when I evaluated the fitness function for the first time right, I got 8. What is the best value so far? So, that would be 8. When I evaluated the fitness function for the second time, but what is the fitness function value that I have so far best fitness function value? So, this remains 8, right. So, let us say for the third time when we evaluate the fitness function let say, it turns out to be 9 right, but what is the fitness function value best fitness function value that I have? So, it is between these 3, right. So, the between these 3 is still it is 8 right.

So, let us say forth time, I get a solution which has a fitness function value of 5 right. In this case, I need to compare 8 and 5 which is better 8 and 5 is better. So, this 8 become 5, then lecture fifth time you get a objective function value of 11. So, 5 is retained right. So, sixth time let us say, we get a solution 4. So, the best we have so far is 5, but now we have received 4 right. So, we will the best fitness function value so far obtained is 4 right. So, we keep doing this right. So, since this since we encounter a 1 over here, this becomes 1 and bellow this there is no value less than 1. So, it remains at 1 right.

So, instead of plotting this column versus this column what we are going to do is we are going to plot this column versus this column right. So, that plot would look something like this. So, this is functional evaluations right ah. When I evaluated the fitness function for the first time, what was the best fitness function value right? This best fitness is important; this is not fitness value, but best fitness function value right. So, now, if you see this plot will be monotonically converging right.

So, even if I so, for example, let say here the best solution was 4, but I encountered the solution with an objective value of 6. So, I do not consider that because it is poorer than the best that I have so far right. So, the conversions curve can either be between iteration and the best fitness function value of the population right or the convergence curve can be between the functional evaluation and best fitness function. So, if you observe, you would have that in multiple places we had employed random numbers right our initial step itself was to generate a random population.

So, if two people are working on the same problem with the same algorithm, they might start with a different set of random numbers right and it is perfectly possible that both of them end up with a different result right.

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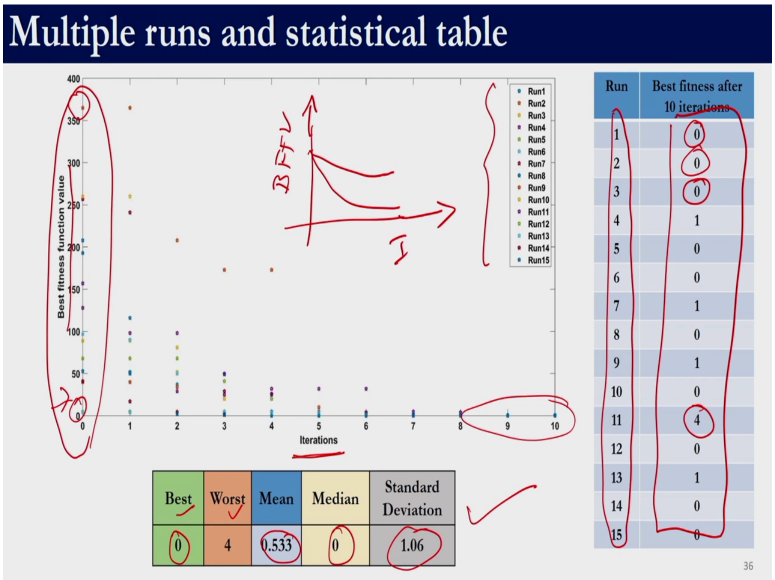

So, since this has stochastic techniques, what is to be done is the algorithm on the same problem has to be run multiple times. If you run it multiple times, you are expecting that the stochasticity which involved will be taken care of and you will be probably towards you will be reaching the optimal solution right.

So, what is done is this curve is between iteration and best fitness function values right which we have first seen right. So, for every fit for every iteration what is the fitness function value, best fitness function value in the population. So, that is what is being plotted iteration versus best fitness function value of the population in that particular iteration right. So, there would be 15 convergence curve right. So, if you remember the convergence curve, this plot is very straight forward for one run, I have one convergence curve right. So, for 15 runs, we will have 15 such curves right.

So, these shows 15 curves right. So, each color indicates one run. So, if you observe that in this 15 runs in one of the runs, the initial population itself had a very good solution right because you chose it randomly. It so, happened that one of the random which you generated is actually closer to the optima right whereas, in one of the run the solution the best solution of the initial random population itself was you can say very bad because it was above 350 right.

So, for this particular run, the algorithm did not have to do much right for the one variant you had the initial population itself which was closer to the optimum. The algorithm did not have too much right where as for this particular run the algorithm had to bring it down right from some 350 plus all the way up to 0 right. So, this shows a the consolidated plot of all the runs right.

So, in this case, it happens that all the runs have probably converge to the same or almost equal values right, but it is not necessary not necessary. We may end up with curves like this where in let us say the first run is like this, where as the second run is like this right. I mean you complete the number of iterations. So, this is iterations this again your best fitness function value right. So, it can so it is perfectly possible that this can happen right.

So, how do we statistically analyze this figure is that we generate stat statistical table right. So, let us say I had this 15 runs. So, that is what is in this column all the 15 runs and in each of the run we find out what is the best solution that was obtained right after the completion of the required number of iterations right.

So, let say run 1, I had to do 10 iteration I completed 10 iterations and at the end of 10 iterations, I have a population; the best value in the population right. That is what we use to use for plotting the convergence. So, the that best value is taken over here right; same this for run 2 run 3 and so on. For example, here if you see for run 111, the algorithm had converge to a value of 4 where as the optimal solution is 0 right. So, this can happen right.

So, now we have this vector. So, we can generate the statistical table right. So, this is the best value. So, in best value what we do is we report the minimum value among this. Since we are talking about a minimum minimization problem so, the best value in this is or the minimum value in this is 0 right that is what is done right.

What is the worst value right? So, worst value if I see the maximum value right. So, maximum value was 4 right and mean value is the mean of this all this numbers all this numbers in the second column, the mean of that is 0.533 median is 0 and the standard deviation is 1.06. So, now, this table tells us how robust is the algorithm.

So, if I do multiple runs, this statistical table shows me the performance of the algorithm across the run. If the standard deviation is less; that means, the algorithm is consistently converging it to the same solution right ah. So, this is how a statistical table is generated for an algorithm [FL]. So, far we have seen how to plot the results that we get. Let us see what would happen if I want to compare two algorithms right.

So, if I wanted to compare two algorithm both algorithms; obviously, I will have to run 15-15 times. So, this graph is going to have 30 curves; 30 conversion curve and it is going to be little bit difficult to interpret things out of that. So, what will next mean convergence curve right?

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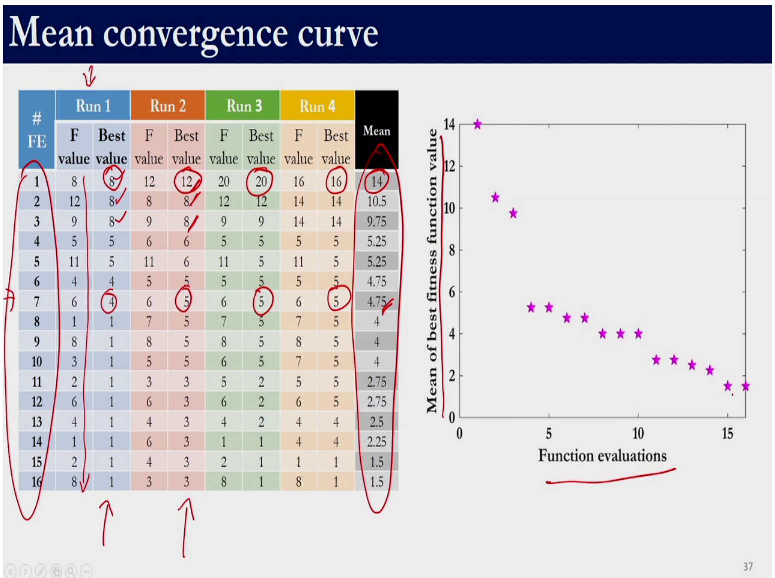

# FE	Run 1		Run 2		Run 3		Run 4		Mean
	F value	Best value	F value	Best value	F value	Best value	F value	Best value	
1	8	8	12	12	20	20	16	16	14
2	12	8	8	8	12	12	14	14	10.5
3	9	8	9	8	9	9	14	14	9.75
4	5	5	6	6	5	5	5	5	5.25
5	11	5	11	6	11	5	11	5	5.25
6	4	4	5	5	5	5	5	5	4.75
7	6	4	6	5	6	5	6	5	4.75
8	1	1	7	5	7	5	7	5	4
9	8	1	8	5	8	5	8	5	4
10	3	1	5	5	6	5	7	5	4
11	2	1	3	3	5	2	5	5	2.75
12	6	1	6	3	6	2	6	5	2.75
13	4	1	4	3	4	2	4	4	2.5
14	1	1	6	3	1	1	4	4	2.25
15	2	1	4	3	2	1	1	1	1.5
16	8	1	3	3	8	1	8	1	1.5

So, for mean convergence curve is plotted between the functional evaluation right; the number of functional evaluation and the mean of best fitness function value. Let us see how it is to be calculated right.

So, let us assume that I run the algorithm first time right; when I executed it first time, these are the objective function values that I obtained right. I evaluated the objective function for 16 times. So, the value obtained against each evaluation is reported here right and we have seen how to find out the best fitness function value till the current functional evaluation right. So, this is the second column indicated that right.

So, first time when I evaluated 8 have nothing to compare. So, I will take it as 8 second time I had obtained 12 right so, but 8 is better than 12. So, the best fitness function value obtained till this functional evaluation is still 8 right. Third time I obtained the function evaluate to be 9, but still the one data I obtained previously is better. So, I retain 8. So, we have basically seen how to do this; how to get this best value column right. So, this I can do for the second run right

For first run I have this column, for the second run I can generate this column right. For the second run when I did first functional evaluation, I obtained a value of 12, right. So, since again I do not have anything to compare with I take 12 as the best value, for the second functional evaluation I get a value of 8. So, since 8 is better than 12, we get 8 third functional evaluation. I get a functional value of 5 9 still since 8 is better I will retain 8 and so on and so forth right. So, I can do this for run 2 as well as run 3, run 4 right.

So, now, we have this thing. So, what we will do is we will find out the mean of the best value will give us 14 right. So, this is the average of the best fitness function value at functional evaluation 1. So, let us see for this it will be this 4 plus 5 plus this 5 this 5 right, the average of these 4 numbers would turn out to be 4.75.

So, what we are saying is when I perform the seventh functional evaluation across all the runs, the mean of the best value is 4.75. So, what we are going to do is we are going to plot this functional value and this mean of the best functional value. So, we will get a curve like as shown in the figure. So, this completes mean convergence curve.

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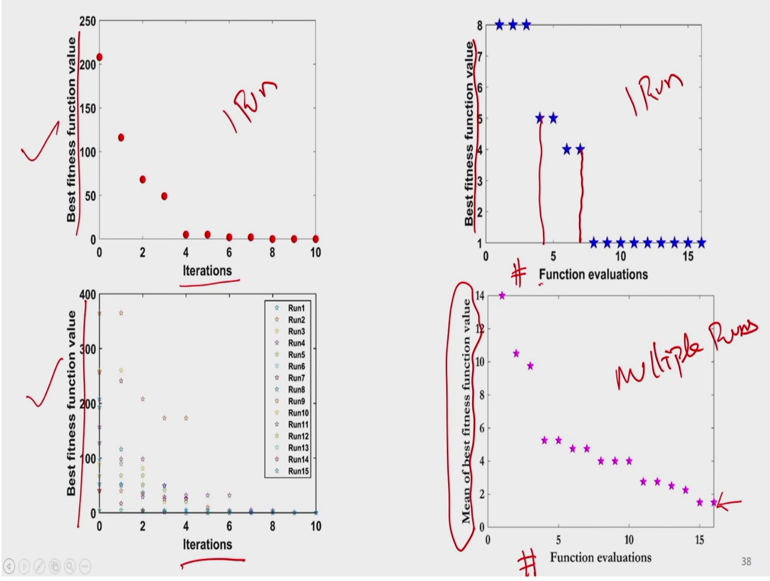

So, since we have discussed fewer convergence curve, let us just try to consolidated. The first one that we stated was plotting iteration versus best fitness function value. So, at the end of every iteration the best value population is plotted right. So, this is convergence curve. When I do for multiple runs this plot, the first plot will have multiple curves that is what is shown over here iteration and best fitness function value since we took 15 runs; there are 15 such curves right.

So, these two are done right in both of this the x axis was iteration and the y axis was best fitness function value right. The third plot which we saw was number functional evaluation. So, here what we did was that if we are evaluating the objective function for the seventh time right, at the end of seventh time your you are plotting what is the best you have obtained so far. That is what we are doing in this best fitness function value.

So, till this particular evaluation. So, if till this particular functional evaluation what is the best solution you have got so far; so, that is what we are plotting over here right. So, in this case so, for till the fourth functional evaluation the best solution that was obtained was 5 that is what this the third plot shows. And this final plot is to show the mean, this plot is for one run right and this plot is also one run right. So, if I have multiple runs I will have multiple curves such curves right. So, the average of that curve is nothing, but this curve. So, this takes care of multiple runs.

So, now if I have multiple algorithms right, I can plot multiple such curves and try to make some inferences from that plot because this particular curve consolidates the information of all the runs for us right. So, I do not need to worry about runs because that information has been captured because of this mean of best fitness function value right. So, these are the four types of convergence curves right. We will come back to it in the next lecture also right; so, over the next 2 or 3 lectures, you will become comfortable with the plotting of this curve and the use of this curve right So, let us now discuss how will we compare algorithms right.

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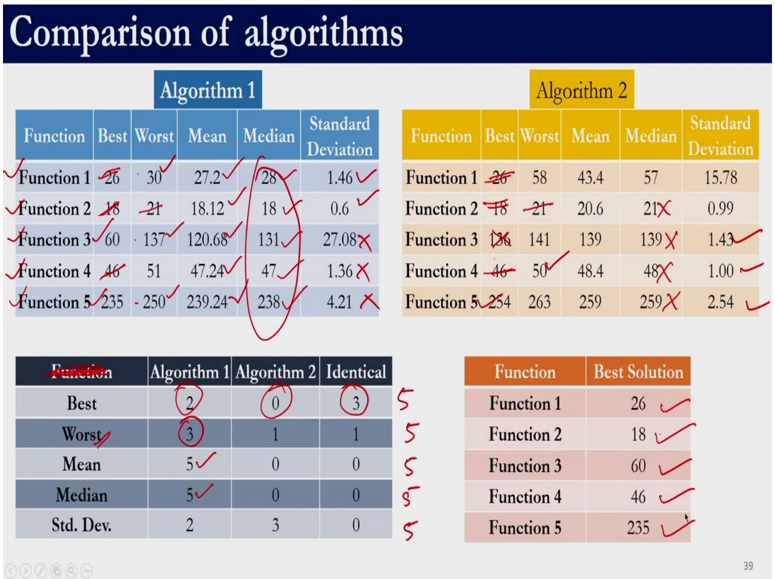

Algorithm 1

Function	Best	Worst	Mean	Median	Standard Deviation
Function 1	~~26~~	30	27.2	28	1.46
Function 2	~~18~~	~~21~~	18.12	18	0.6
Function 3	60	137	120.68	131	27.08
Function 4	~~46~~	51	47.24	47	1.36
Function 5	235	250	239.24	238	4.21

Algorithm 2

Function	Best	Worst	Mean	Median	Standard Deviation
Function 1	~~26~~	58	43.4	57	15.78
Function 2	~~18~~	~~21~~	20.6	21	0.99
Function 3	136	141	139	139	1.43
Function 4	~~46~~	50	48.4	48	1.00
Function 5	254	263	259	259	2.54

~~Function~~	Algorithm 1	Algorithm 2	Identical	
Best	2	0	3	5
Worst	3	1	1	5
Mean	5	0	0	5
Median	5	0	0	5
Std. Dev.	2	3	0	5

Function	Best Solution
Function 1	26
Function 2	18
Function 3	60
Function 4	46
Function 5	235

Assume this is the statistical table of a particular algorithm. So, we have seen few minutes back as to how to generate the statistical algorithms; this statistical table right. So, that time we have generated for only one problem. So, the same concept if I am trying to solve 5 problems let us say I have 5 problems; function 1, function 2, function 3, function 4, function 5; I will get one row for each problem. So, that is what we have discussed previously right. So, we know how to get this table.

Given five problems, the algorithm has to be run multiple times on each of the problem and then you consolidate your results to get this statistical table right. So, let us say I perform the same activity with another algorithm and I get a table like this. So, now, we have two algorithms which were executed on 5 functions same functions for a specified number of runs right.

So, now we have two statistical table. So, with this statistical table we would want to say which algorithm is better. In view of the nature of this course, what we will be doing is a very preliminary comparison right. There are advanced techniques to compare these two tables. To begin with, let us just take the best value right. So, for function 1, algorithm 1 as well as algorithm 2 gave the same best value right.

So, for the second function, same value; for the fourth function, it is the same value right for the third function algorithm one was able to determine a better optimal solution than algorithm 2 because here I have sixty from algorithm one for function 3. Here for algorithm 2 for function 3, the best value it was able to find out is 136 right. So, this is an inferior value whereas, this is a better value right; same thing for function 5 if we see right so, 235 is better than 254 right.

So, I can make this conclusion right. So, that is just the statistic right; so, algorithm one algorithm 2 ah. So, what we are saying is how many times algorithm one was better than algorithm 2 with respect to the best statistics right. So, here in two cases, in two of the 5 problems, algorithm 1 was clearly better than algorithm 2 right and algorithm 2 is not better than algorithm 1 in any of the cases. So, we have this value of 0 right. So, and in 3 cases both of them gave give us identical results. So, that is what we have to strike of 26, 18, 46. So, in three cases it gave us identical results.

So, similarly we can do for worse right. So, worst means between 30 and 58, 30 is actually a good solution right. So, this is doing good over here, 21 is same. So, let me just cross it 137 is better than 141 right. So, this is we let me put a tick mark over here; 51 is actually bad right 50 is better.

So, let me put a tick mark over here and 250 is better 263 is bad right. So, with respect to the worst in three cases; case 1, case 2 and case 3. In three cases algorithm 1 was better than algorithm 2 with respect to the worst and in one case they had identical value because here also we have for function 2, we have a value of 21 as well as in function 2 for algorithm 2 also, we had a value of 21.

So, now we can also do the same thing for with respect to mean right. So, for all the five problems the mean is better for algorithm 1. So, 27.2, a 43.4, 18.1 to 20.68, 120.68, 139, 47.24, 48.42, 39.24 is better than 259 right. So, in all the five cases, algorithm 1 is better than algorithm 2 with respect to mean. Similarly you can do it for median right

So, median also if you see 28 is better than 57, 18 is better than 21, 131 is better than 139, 47 is better than 488 and 238 is better than 259. So, in all these five cases, algorithm 1 out performed algorithm 2 right. So, this table tells how algorithm 1 is better than algorithm 2 from the first column right.

The second column says that how in how many of the cases was algorithm 2 better than algorithm 1 right and the third columns is in how many cases they gave identical results right. So, similarly we can do it with respect to the standard deviation right. So, standard deviation if you see for these two first problems, algorithm 1 has a lower standard deviation whereas, for these three problems algorithm 2 has a better standard deviation right. So, the sum of all this rows would turn out to be 5 because there are 5 problems right.

So, this table gives a comparison of both the algorithms right. If you are not really interested in comparing the algorithms, but want to know the best fitness function value; then we can just for all the function irrespective of from which algorithm is it coming, we can find out the best solution. In this case, it happens that the best solution can be determined just by algorithm 1 with respect to the best solution right.

So, 26 for the first function, 18 for the second, 60 for the third, 46 for the fourth and 235 for the fifth. So, if you are not necessarily interested in comparing algorithms, but are only interested in the optimal solution; then whichever algorithm gives the better solution for a particular problem that solution is to be taken. With that, we will end the session.

Thank you.