Computer Aided Applied Single Objective Optimization
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Lecture – 33
MILP Formation of Production Planning Problem

Welcome. In the previous session we had looked into the Production Planning problem and we had formulated in a way that can be used with metaheuristic techniques to solve the problem. In this session, we will look into a mathematical formulation for the same production planning problem.

So, whatever formulation we had previously, if you think about it, it cannot be directly used with let us say intlin prog of MATLAB, right. Because, there we require the constraints to be of the form ax is less than equal to b and the model which we had developed, had implication constraints right.

We had equations involving let us say f x is between l and m then we need to use one equation; if it is between m and h then we need to use another equation, right. So that cannot be directly incorporated using intlin prog of MATLAB, right. In this session, we will see the mathematical formulation of the same Production Planning Problem. The constraints will be in the format a x is less than equal to b or a equality x is equal to b equality and the objective function in this case also will turn out to be linear.

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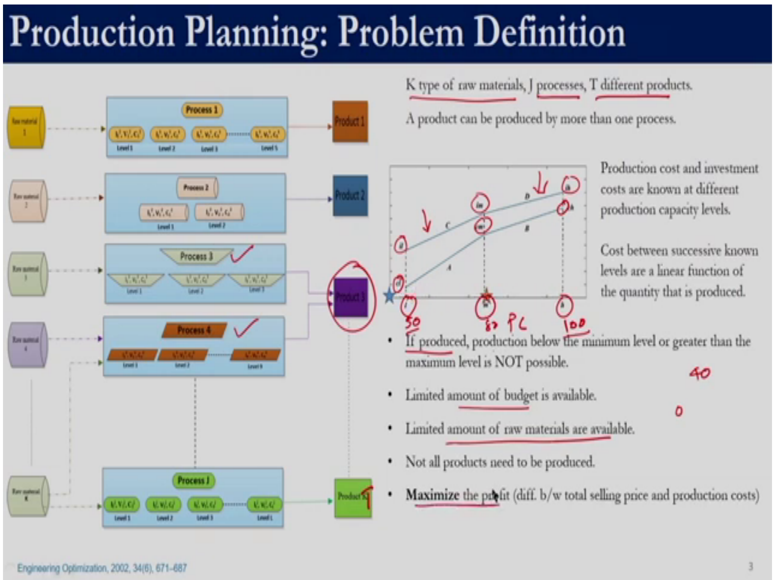

We had previously looked into this definition of the problem, just to refresh our memory, we will briefly go through the problem again. So, we have T products product 1, product 2, product 3, all the way up to T products and there are J different processes which can produce these products. So, some of the processes can produce the same product. So for example, product 3 is produced by process 3 as well as process 4 right, and there are K different type of raw materials which are available, right.

So, the production and investment cost are known at l, m, h. So, this is production capacity let us say this is 50 80 and 100, right. So, if we produce 50 tons per day, the cost is given by this cl, right. Similarly, if you produce 80 tons per day the cost is given by cm, if we produce 100 tons per day the cost is given by ch. So, this information is known for any production between 50 and 80, the production costs can be calculated by using linear interpolation.

So, if you see between l and h, it is a piecewise linear function. Similarly, we also know the investment cost at these 3 production capacities and for any production between 50 and 80, the investment cost can be calculated by this line. Similarly, for any production between 80 and 100, the investment cost can be calculated by using this line. So the decision variables are, which of the products are to be produced; right? And what are the processes which are used to produce the product? And what should be the production amount from each of those selected process, right?

So, here it is not necessary to produce all the products and it is not mandatory to use all the process right. So, if produce, so, if we choose to produce a product from a particular process, the production has to be between l and h, right. It cannot be less than l; it can be 0. So, for example, here it is 50 and 100.

So, the production cannot be 40; but the production can be 0 and the production can be anywhere between 50 and 100. Above 100 there is no mechanism to calculate the cost and below l there is no mechanism to calculate the cost, except for the fact that at x is equal to 0, the production costs and investment costs are 0.

So whatever production plant we decide to implement, we will require an investment cost right. So that investment cost has to be less than the amount of budget that is available. Similarly, for the production plant that we choose, the amount of raw material required should be less than equal to the amount of raw materials that are available. Under this scenario, we need to come up with a production plan which will maximize the profit.

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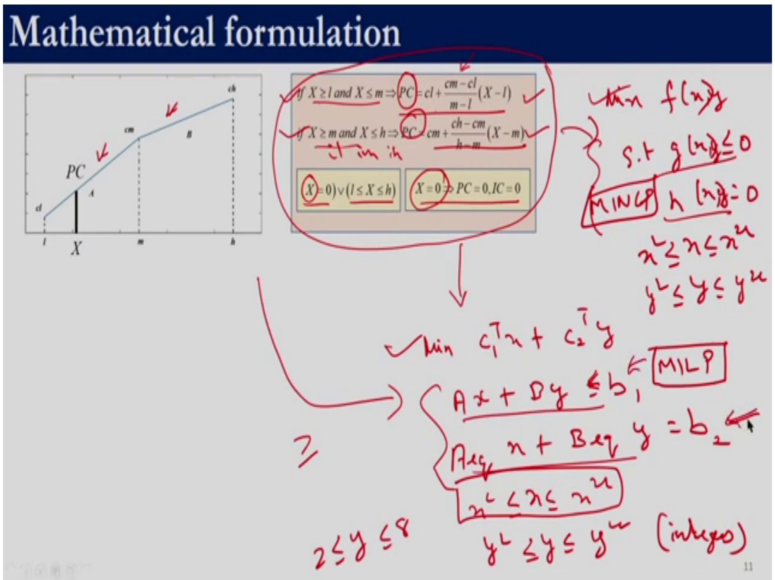

If you remember our previous discussion, we had used these to calculate the production costs if X is between l and m the production cost is given by this expressions. We had previously seen how to derive these two expressions, right. So, the production cost is calculated based on the amount of X right and if X is not produced, then the production cost and investment costs are 0.

So here we have written it only for production costs, but the expression for investment costs will also be similar; instead of cl, which is the cost of production at low level it would be i l, instead of cm it will be i m and instead of ch it will be i h right.

So whatever equations we use for mathematical formulation should capture this production cost if the X value is in this range and if the X value is 0, the production and investment costs should be 0. Additionally, the equation should ensure that whatever the value of X is right; X is amount that is being produced from a particular process that it should either be 0 or it should be between l and h, right.

So, we need to have a set of equations which will capture all of this, right. And this cannot be directly included in a mathematical formulation right, because in mathematical formulation if we are looking at a mixed integer linear programming formulation, we are looking at something minimizing C transpose x plus let us say C one transpose x plus C 2 transpose y. And, we want the constraint A x plus B y is equal to b and let us say A equal to x plus B equal to y, is equal to b 2; let us say this is b 1 right.

And we want x to be between some range l and u, and y also to be between some range. right only thing is that here it is integer, it can take only integer values. So, here if we say y is between 2 and 8 then y can take the values 2, 3, 4, 5, 6, 7, 8. Only the integers are permissible whereas, x are continuous variable it can take any value between x lower and x upper.

So the question is, can we transform this problem? Whatever problem we have, the production planning problem; can it be transformed into this set of equations? If not, this set of equations, at least similar to minimize f of x subject to g of x is less than equal to 0, h of x equal to 0 and x is between lower and upper bounds, right.

So, if we have binary variables then it is x comma y x comma y, x comma y here and again y is between lower and upper bounds, right. So, here it is an MINLP formulation. Here it is a MILP formulation. So right now these equations which we have neither fall under this category, nor do they fall under this category.

So the question is, can we transform this problem into equations of either this type or of this type, so that we have a mathematical formulation, so, that we can use mathematical programming techniques such as branch and bound simplex method and other gradient based methods. What we essentially want to do is capture these two piecewise linear functions into a set of linear or non-linear equations, which are of the conventional form right.

So, this and this we will call it as conventional form or the canonical form for mathematical programming techniques. So over here, since this takes care of the equality constraints, this has to be less than equality. So, this takes care of all the inequality constraints, this one takes care of all the equality constraints.

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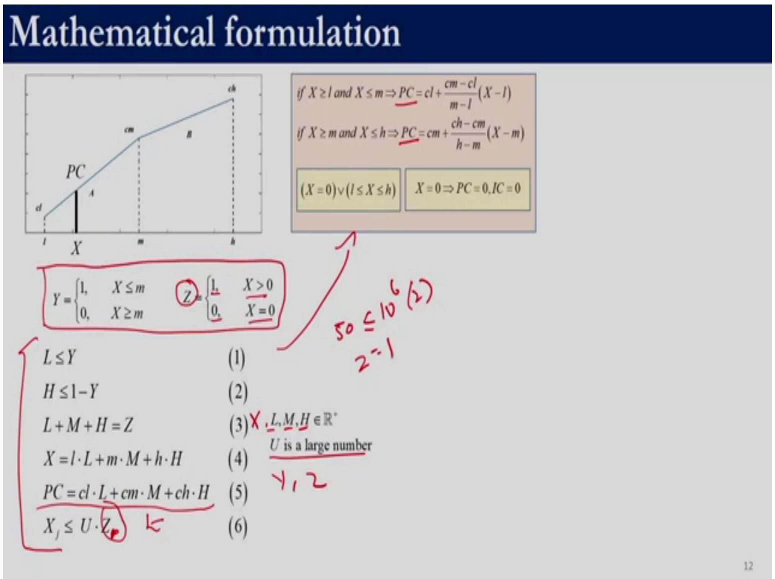

So, let us say we have these equations, right. This equation 1 to 6, right; in equation 1 to 6, L, M, H are real positive numbers right. So they can be anywhere between 0 to infinity, U is a large number and the variables Y and Z are binary variables, ok. We also have X which is a decision variable, right. So, X is again real positive number.

So, the claim is that these equations help us to capture whatever we were doing over here. So, this production costs whatever we calculate over here, that would be the same given by this equation, provided all of these equations are taken together, right. So here the binary variables

Y and Z are supposed to behave in this way, right. So, Z is a binary variable which will indicate whether X is produced or not. So Z will be equal to 0, if it is not produced. If it is produced, it will take a value of 1.

So for example, look at this equation 6 right. So let us say X takes a value of 50 and let us say U is let us say 10 power 6, right. And this Z is a binary variable; there is no J over here and this Z is a binary variable, right. So now, we have this. So, this equation would be satisfied if and only if Z takes a value of 1.

So, if Z takes a value of 0, it will be 50 is less than equal to 0 which is not possible, right. If X takes any non-zero value, this constraint would ensure that Z would be equal to 1 right. So, we can consider this Z as an indicator variable which will have a value of 1 if X is greater than 0; it will be 0 if x is 0.

So coming to this variable Y, so, this binary variable Y would take a value of 1 if X is less than or equal to m. It will take a value of 0, if it is greater than or equal to m.

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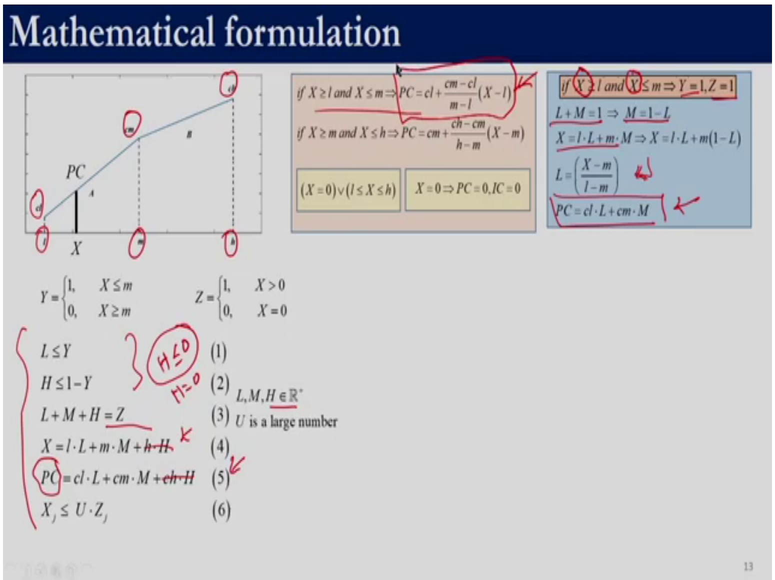

Now, let us look into a case wherein X is greater than l and X is less than or equal to m, right. So X is somewhere in between this. So, if X is somewhere in between this, we want to see what will happen to those equations and if whatever production costs will be calculated by this expression; equation 5, would it be similar to this equation? Because this equation is valid between l and m, right.

So, that is the case that we are discussing as of now. So, if X is between l and m, let us say the variable Y takes value of 1, right and since X is non-zero, Z will be equal to 1, right. So Y is equal to 1, Z is equal to 1. If you look at equation 1 and 2 right, since Y is equal to 1, it means H is less than equal to 0, right. So, since H is a real positive variable, the only value that H can take is 0, right. And if H takes a value of 0, equation 3 turns out to be l plus m is equal to 1, right.

So, if you rewrite it will be m is equal to 1 minus l right. Now let us look at equation 4, right. So equation 4 H is equal to 0 that has been established, right. So, if H is equal to 0, this expression is now only X is equal to l into l plus m into m. So, if we rearrange this equation, we can get this expression for l; l is equal to x minus m divided by l minus m, right. So, if H is 0 right, so again, this part of the equation will go from equation 5. So, PC will be c l into L plus c m into M right. So, the lowercase l, m, h are the production capacities right and cl, cm, ch production costs at l, m and h respectively, right.

So through this equation the production costs that would be calculated is this. PC equal to cl into L plus c m into M. So the question is, now is this cost the same as this cost what we have over here?

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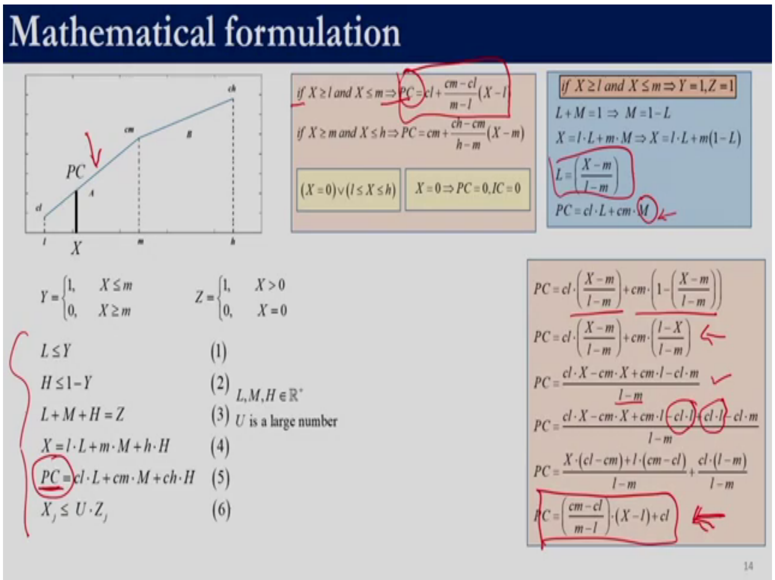

So here in this expression, if we take this expression right and substitute the value of l which we have over here right this and for M; M is nothing but 1 minus L right. So, here we have one minus L, right. If we rearrange this equation, we would get this 1, right. So, if we take l minus m as common term, and expand this equation, we will get this one. So to this equation what we will do is, we will subtract and add cl into L right; and if we rearrange this expression, you would finally be able to get this expression which is the same as what we have over here.

So, what we have established is the cost calculated by this equation if X is between l and m, is the same as what we had previously calculated using the 2 point form of this line, right. And these equations are in the canonical form or the conventional form. So between equation 1 to 6, we do not have any if condition or implication constraints which we had previously, when we solve this problem.

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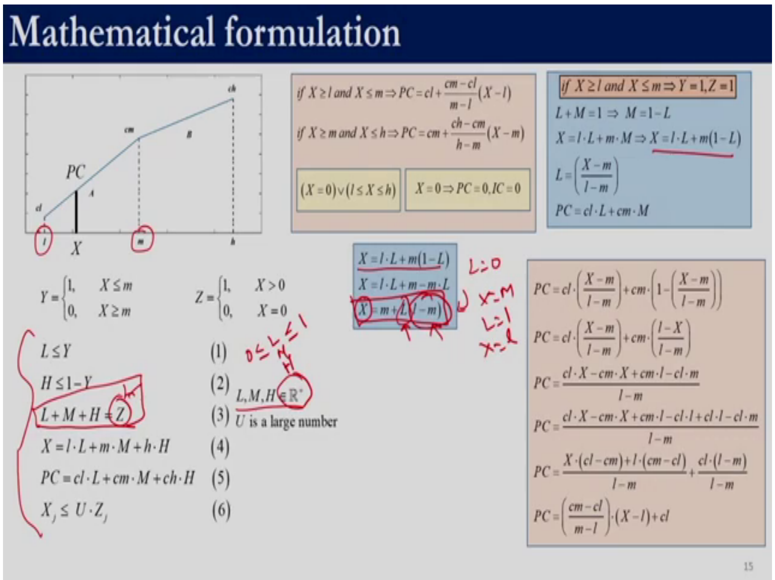

If you look into this expression given over here, and if you expand it you will end up with this one right. So, X is equal to m plus uppercase L into l minus m right. So, l minus m if you see m has to be a value with greater than l right, so this will be a negative, right. So, this term is going to be a negative term and this uppercase L is a decision variable which is going to be positive and this lowercase l which is a decision variable, will be less than or equal to 1 because of this constraint. So, if anything is produced Z will be 1. So, L plus M plus H is equal to 1.

So, though we are defining L, M, H to be in the real positive domain, because of this constraint, it is actually going to be in between 0 and 1. So, all these 3 variables because of this particular constraint, if a product is produced, is going to be between 0 and 1, right. So, this is going to be a negative value right, and this is a number less than 1. So, this this quantity would always be less than or equal to m, right.

So if L is equal to 0 in this expression, right then X is equal to M and if L is equal to 1 then X would be l right. So, this set of equation also makes sure that X can be either 0 or it will be between l and m.

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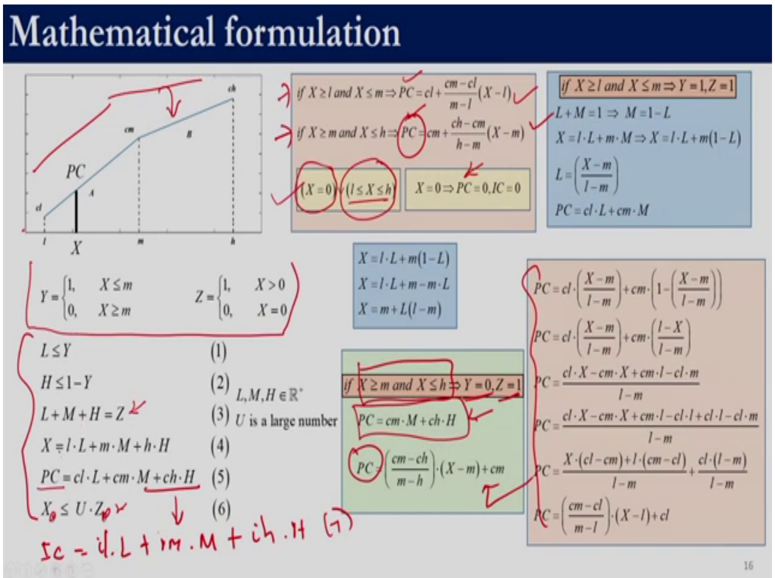

So similar arguments can be made for X between m and h right, so if X is between m and h, Y would take a value of 0 and Z will take a value of 1 because the product is produced, right. So in that case PC would turn out to be cm into M plus ch into H. So, we can employ a procedure similar to this to show that the cost that is calculated by this equation is equivalent to what we have over here; which is what we would get by using this line. So now, we have discussed 2 cases that if X is between l and m and if X is between m and h, right.

So these 2 expressions we have been able to capture and if X is 0, the set of equation will also ensure that PC is 0 which is what we want, right. Similar to this equation, we can add 1 more equation that investment cost is equal to i l dot L plus i m dot M plus i h dot H, right. So, this is similar to equation Y; instead of production cost, now we are writing investment costs because the concept remains the same its just that another line over here is being captured.

So similar to the expression that we would get by using these 2 lines, this investment costs will also correspond to the same cost. Now what we have done is, we have shown that if we use this 7 equations, we can capture all this condition, we will be able to calculate the production costs no matter if the production is between l and m or if it is between m and or if it is 0.

Similarly, we will be able to calculate the investment cost and this constraint also ensure that X can take either value of 0 or it will be between l and m or m and h right. So basically, what we are saying is X will be either 0 or it will be between l and h which is what we wanted, right. So, the domain constraint which we had previously wherein we check like if anything is non-zero, but less than l we were assigning penalty.

With these equations, if we get a solution which satisfies all those equation it will also satisfy the domain constraint.

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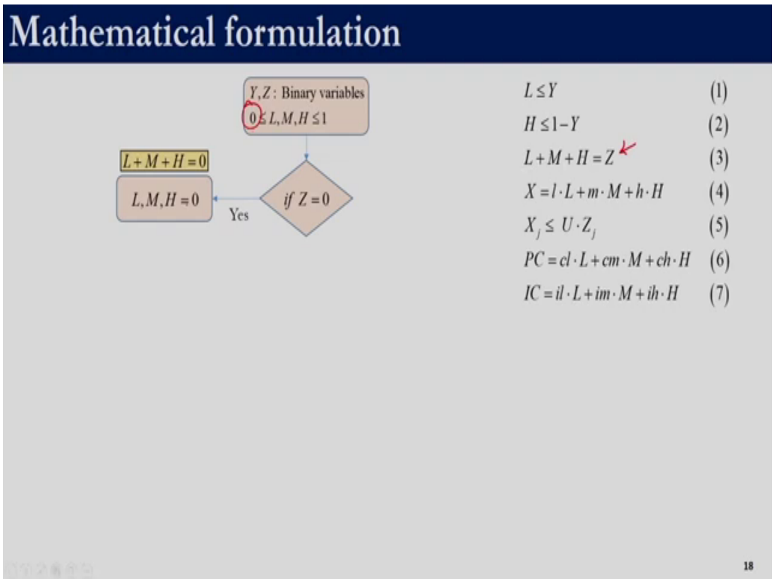

Now, let us consolidate whatever we have seen, right. So Y and Z are binary variables and L, M and H would be between 0 and 1. Though, we define it to be real positive, this constraint would ensure that they are between 0 and 1, right. So let us say if Z is equal to 0. So if Z is equal to 0, equation 3 would ensure that L plus M plus H is equal to 0. Since the lower bound is 0, that condition can be satisfied if and only if L, M and H are 0.

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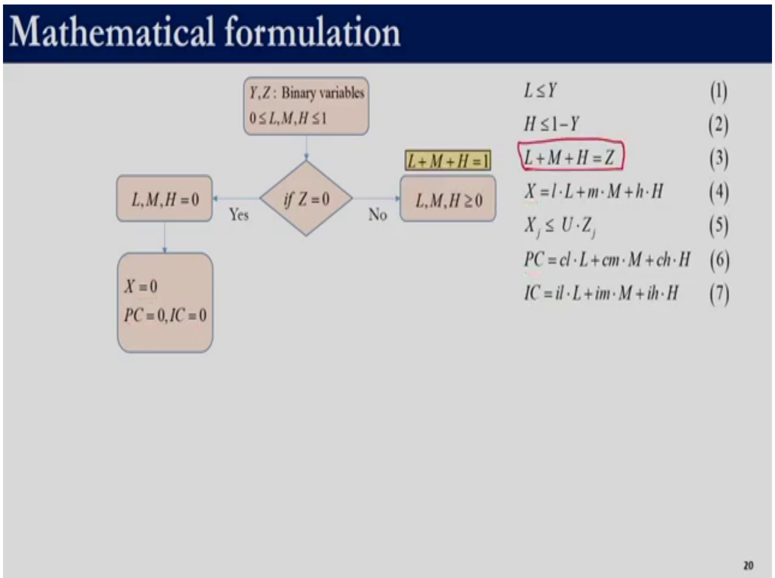

If L, M H is 0, then X will be 0 because of equation 4; and equation 6 and equation 7 would ensure that the production costs and the investment costs are also 0. If this condition is not satisfied Z is not equal to 0, so this constraint will make sure that L plus M plus H is equal to 1, right. So now, all the 3 values cannot take 0, right. So the summation of this has to be equal to 1, right.

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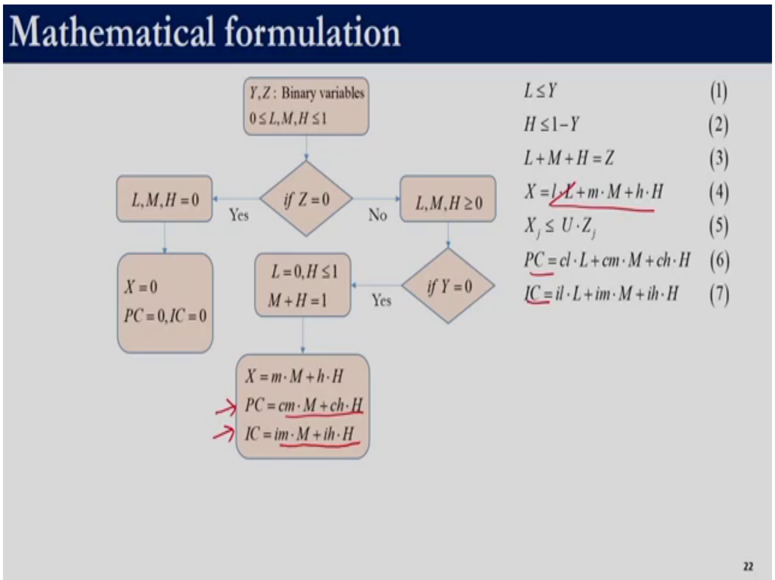

So, now the variable Y it can take either 0 or 1, right. So let us assume that it takes a value of 0. So if Y takes a value of 0, equation 1 would ensure that L will be equal to 0, right, because again L is positive variable. So, L is less than or equal to 0. So, the only value which will satisfy is L is equal to 0, right and over here equation 2 would ensure that H is less than or equal to 1.

Whereas equation 3 would ensure that L plus M plus H is equal to 1, that now becomes 0 plus M plus H is equal to 1; and equation 4 because L is equal to 0, this term would vanish right. So, X will be m into M plus h into H. Similarly, the production costs and the investment cost will be c m M plus c h H and i m M plus i h H, right.

So and we have previously seen that this production cost and this investment cost is nothing, but what we would get by interpolating the line between M and H right.

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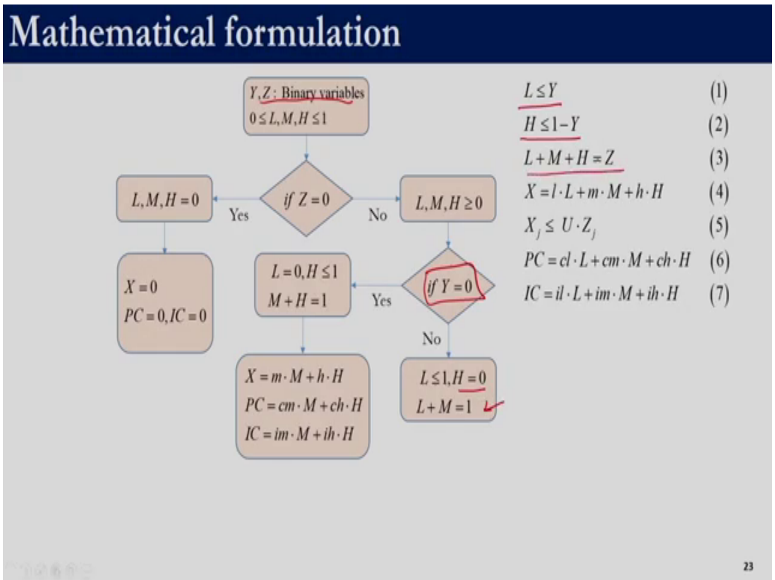

So if this condition failed that Y is not equal to 0, right. So, if Y is not equal to 0 that would mean that Y is equal to 1 because, Y and Z are binary variables, right. So if Y is equal to 1, then l has to be less than or equal to 1 and H would be 1 minus 1 which is 0. So, H has to be less than equal to 0. So, the only value that would satisfy that constraint is H is equal to 0. Similarly, this constraint will become L plus M is equal to 1; because H is equal to 0.

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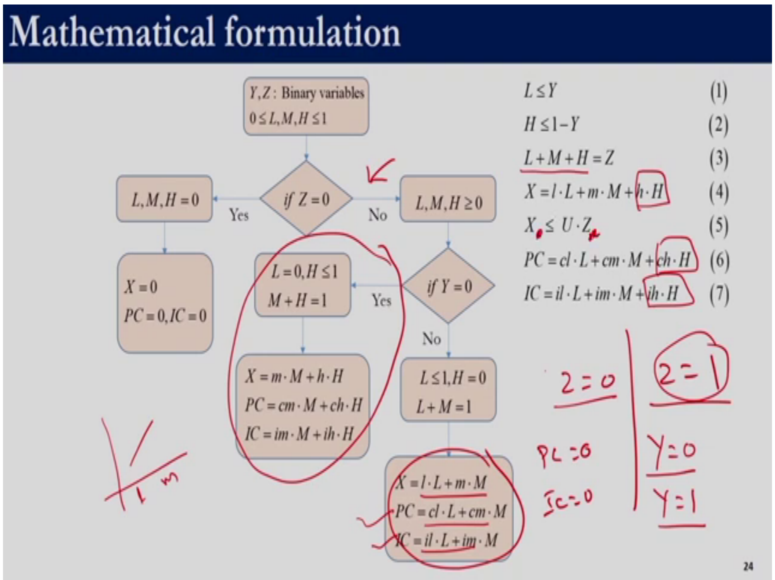

So, equation 4 would now become l L plus m M because H is equal to 0, right. So this term will vanish similarly over here these 2 terms would vanish, right. So this is the cost that would be calculated and we have previously seen that these 2 costs are nothing but what we would get by interpolating between l and m.

So in this flowchart, we have seen the case that what happens if Z is equal to 0 and if Z is equal to 1. So this part is Z is equal to 1 because Z is not equal to 0. So in z is equal to 1, we also saw what happens when Y is equal to 0 and Y is equal to 1, right. So far Z is equal to 0, the production costs and the investment costs turn out to be 0, right; so which is what we wanted, right.

So, if Z is equal to 1 and Y is equal to 0. So then we are talking about this part, right. So in that case, the production is between m and h and if Y is equal to 1 with Z is equal to 1, the production is between l and m, right. Which is how we had defined the variables in the first place. So, since we are discussing for only one process this j is not required, right.

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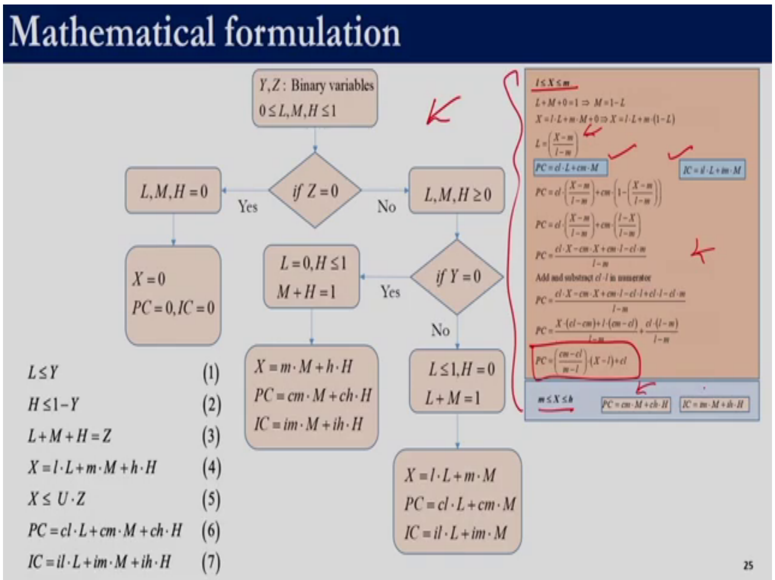

So, this gives a consolidated picture of whatever we have seen right, so this shows all the cases which we discussed and this is the derivation which we had previously discussed that what happens if X is between l and m, right. So what we essentially get is the production cost and the investment cost is this right and this production cost is nothing but, if we use this L in this expression and the fact that l plus m is equal to 1 and do a little bit of rearrangement, we would get this expression which is what we had previously derived.

Similarly, if X is between m and h right, so this would be the production cost and this would be the investment cost; which is the line used for interpolating between m and h, right. So, and these were the equations which we had used along with the fact that Y is a binary variable, Z is a binary variable X, L, M and H belong to real positive.

So with this definition of the variables right, and these equations if we use, we can accommodate the piecewise nature of the production cost and investment cost in terms of the conventional equalities right. So, this discussion is for only 1 variable, for 1 process because we had used just the variable X right.

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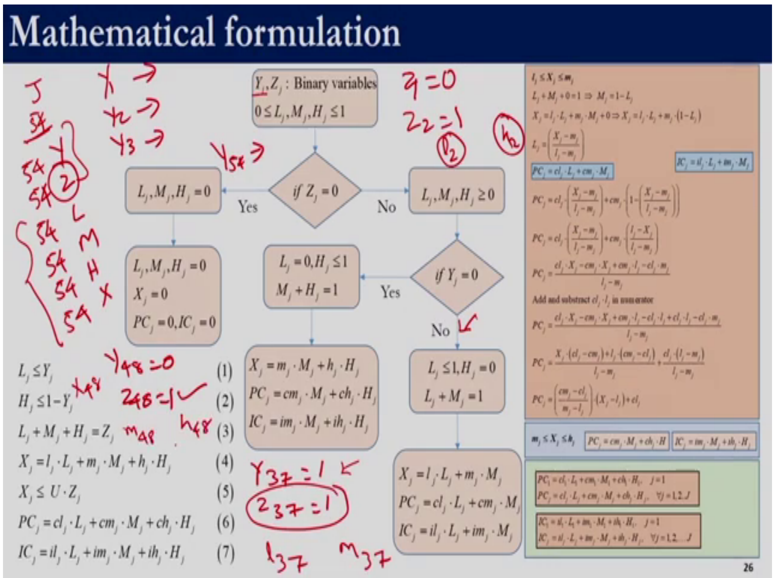

So, if there are J such process then we introduce J variable. So, if there are J processes let us say there are 54 processes right, then we will use 54 Y variables. We will use 54 Z variables we will use 54 L, 54 M, 54 H and similarly 54 X variable, right. So, all of this are real positive variables whereas, these 2 set of variables are binary variables.

So, now we will have 54 such Y variables right. So, Y 1 is for the first process Y 2 is for the second process Y 3 is for the third process. Similarly, we will have all the way up to Y 54, right. Similarly, we will also have Z 1 right. So, Z 1 is equal to 0 will indicate that nothing is produced from process 1, right and as value of Z 2 equal to 1 would indicate that process 2 produces X which is between l 2 because l 2 is the lower capacity level for process 2 and it is between h 2.

Let us consider this process, let us say Y 37 is equal to 1 right and Z 37 is equal to 1. So that means, we are producing something from process 37 that the production is not 0 and Y 37 is equal to 1, right. So that means, it will be this part, right. Is Y 37 equal to 0? So, the answer would be no for this case, right.

So, that would mean that the production is between l 37 and m 37, right. So what we discussed for 1 process that can be extended for as many processes as we have. Similarly, if let us say Y 48 is equal to 0 and Z 48 is equal to 1. So, that would mean that forty-eighth process produces some quantity because Z 48 is equal to 1. So the production would be between m and h. So, X 48 would lie in between M 48 and H 48.

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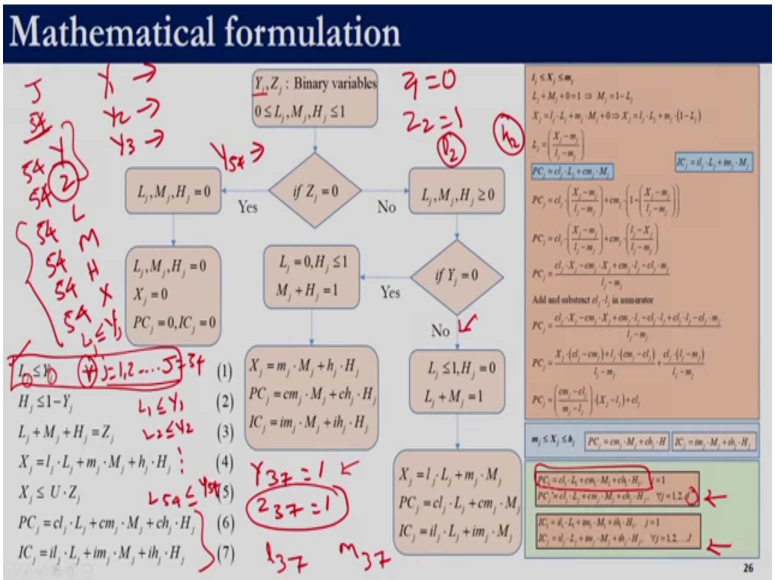

So previously, the equation which we had was L is less than equal to Y, right. So now we add this subscript right and this J is for all the process. So what we do is, for all J is equal to 1, 2 all the way up to capital J where capital J is the number of processes we have, right. So when we write like this, what we essentially mean is that L 1 is less than equal to Y 1, L 2 is less than equal to Y 2 and all the way up to L. Let us say if J is equal to 54 then L 54 is less than or equal to Y 54, right.

So instead of writing these 54 equation, we can compactly write L j is less than equal to Y j and the subscript j runs from 1 to capital J. So, this symbol is for all symbol; this means this equation is to be written for lowercase j is equal to 1, lowercase j is equal to 2, lowercase j is equal to 3 all the way up to 54. Similarly these 2 equations are for equation 6 and 7, right. So if we are talking about one particular process then we can just write this equation. So, if j is equal to 1 then it is this expression. Since we have capital J number of processes instead of this 1 we can just write this subscript j.

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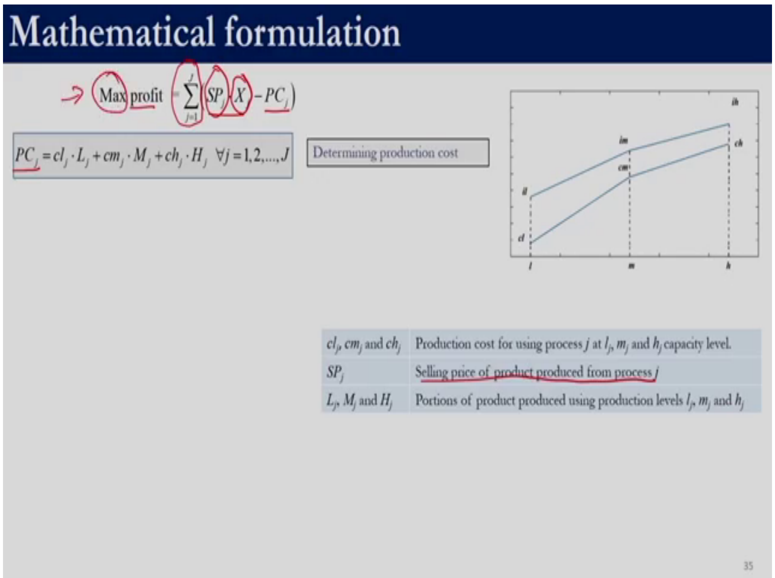

So far what we have done is, to calculate only the production cost and investment cost, right. So, still we need to employ the constraint that investment cost should be less than budget and the total amount of raw material that is consumed for the entire production plant is less than what is available right. So, this is the expression for production cost which we have discussed previously.

So, since X j denotes the amount of product produced from process j and SP of j is the selling price of product produced from process j. So, SP j into X j will give revenue from process j minus PC of j is the production cost from process j, right. So, this is the revenue from process j this is the production cost of process j, right.

So, the difference between them would give profit of process j; if we sum it across all the processes, then we get the total profit right. So, this is what we need to maximize.

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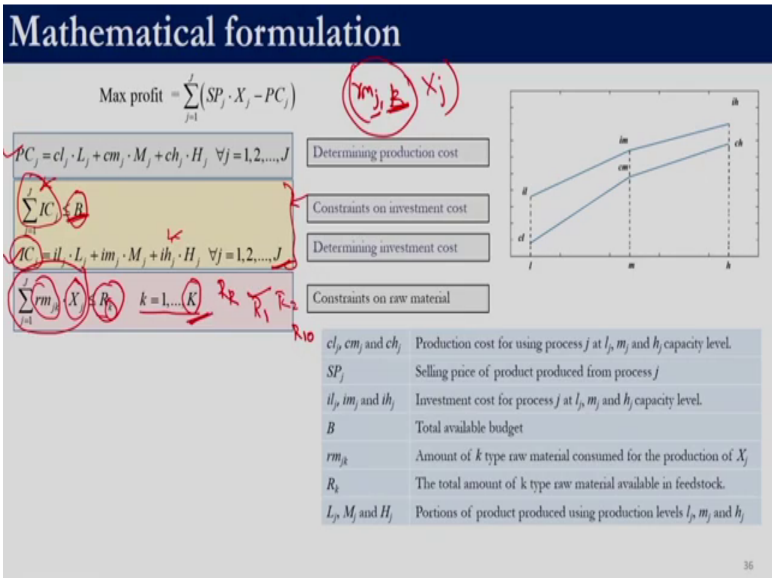

So similarly, we can calculate the investment costs, right its just similar to production costs we can have this one. So, IC of j will indicate what is the total investment cost of process j. If we sum it across all the processes, that will tell us what is the total budget that is required and thus B is the budget that is available. So whatever that is required, which is on the left-hand side should be less than or equal to what we have right. So these two constraints take care of the budget constraint.

So over here, if you see this equation has to be written J times. J is the number of processes, right and this constraint is for raw material constraint, right. Let us say, we have k types of raw material and the amount of raw material of type 1 that is available is R 1. Let us say R 1 is the amount of raw material which is available for type 1, R 2 is the amount of raw material which is available for type 2.

Let us say R 10 is the amount of raw material of type 10 which is available, right. So in general, we can write R of k subscript k, right. So, this k runs from 1 to the total number of raw materials, right. So this is what is available, we know the production from a particular process is X j right, and we also know this rm jk.

So rm jk is the amount of raw material of type k that is required in process j for producing 1 unit quantity. If you remember our previous discussion, this value is given as to how much of raw material is required for producing one unit quantity of product that is given, right. So, here we are determining how much we are producing, this quantity will give how much we require in process j of raw material k.

So, if we sum it up across all the processes, the left-hand side tells us how much raw material is required for whatever production plan is decided and the right-hand side is, how much is available, right. So whatever is required has to be less than or equal to whatever is available, right.

So we will have j such constraints, and over here we will have k such constraints right, because the number of processes and the number of raw materials need not be the same.

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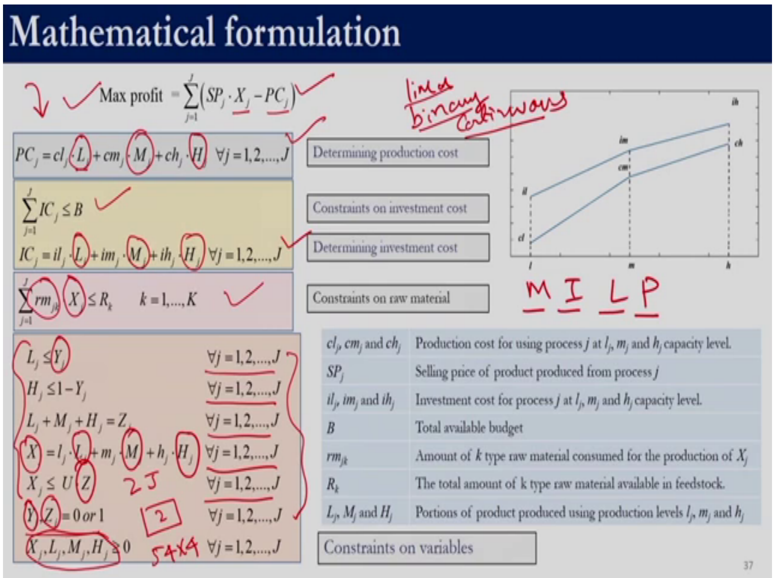

These are rest of the equations which we have discussed previously right, so we will not get into it again. Just that each of the equation is to be written J times where J is the number of processes. So we have 2 binary variables per process, right. So, if there are J processes then we have 2 J number of binary variables, but the type of binary variables are just 2 right, so, Y and Z. Similarly, the decision variable X j, L j, M j, H j is to be used for each process, right.

So, if we have 54 process then we will have 54 into 4; because there are 4 continuous variable per process. So if we have 54 processes then we will have 54 into 4 continuous variables. So, this formulation if you see, we have an objective function, right; the decision variable involved over here is j and PC of j. Again PC of j is given by this expression and the decision variable involved here is L j, M j, H j.

So this is a linear equation, this is also a linear equation. Similarly, this L j, M j, H j are again linear terms, this is also a linear constraint. Over here also X j appears linearly in this equation. That is the only decision variable, this is a known quantity right. So this is also a linear equation and all these equations are also linear, right.

So, all our equations are linear equations right, some of our variables are binary, right. So, Y j and Z j are our binary variables, L j, M j, H j and X j are our continuous variables. So, this formulation which we have, has mixed type of variable. Some variables are binary, some variables are continuous; and all our expressions including the objective function and the constraints are linear. So this formulation is an MILP, Mixed Integer Linear Programming, right.

This formulation can be solved using intlin prog; and if it completes normally, whatever the solution that we get is the best possible solution for this problem, right.

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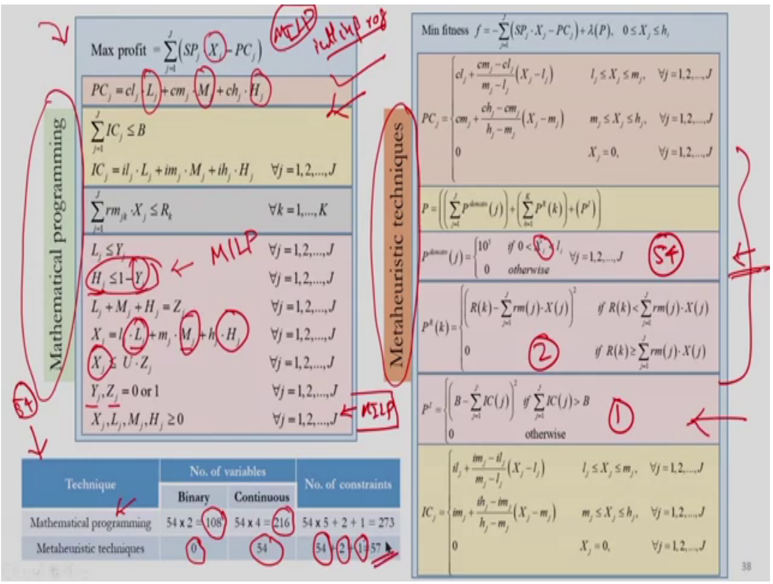

So over here we have given both the formulations which we have, right. So the problem description is same; this is the formulation that we had used when we were trying to solve it with Metaheuristic techniques and this formulation is the mathematical programming formulation, right.

So, this is in the form of Mixed Integer Linear Programming formulation, this can be solved with a MATLAB intlin prog or the other 2 softwares discussed in this course. So, the problem description remains the same, there are 2 different ways to modulate, right. Now the question is, can we use this formulation to solve using Metaheuristic techniques? So the answer to that is, yes. So, the decision variable will be X j, H j, L j, M j, Y j and Z j.

So, 54 into 6 is what the metaheuristic technique is supposed to give and now we need to check for this constraint the L value, M value and the H value given by the metaheuristic technique, whether it is equal to j which is also given by the metaheuristic technique. Now, if we are to solve this using metaheuristic techniques right.

So whatever value of Y we get, right and whatever value of H we will get should satisfy this equation. If they do not satisfy this equation, then we need to assign a penalty. So, this particular formulation can be also solved using metaheuristic technique, we would encourage you to go and solve this formulation right, this MILP formulation using a metaheuristic technique and see if you are able to get a solution from it.

We had shown you that we were getting reasonably good solution using this particular formulation. The question is, can this formulation be used? The answer is yes, but would you be getting equally good results, that is something that we want you to go and try it out. So, here we have a basic comparison of these two formulation, right.

So, in mathematical programming we had these 2 types of binary variables, right. So here this comparison is made for 54 processes which is the case study that we are working with, right. So, the total number of binary variables we would have 108. Whereas, this formulation which we had previously used did not have any binary variables right, it only had continuous variable X j, right. How many such variables would be there?

Capital J which is 54 in this case right. So, here we do not have any binary variables and in metaheuristic techniques we had only 54 continuous variables. Whereas, in this mixed integer linear programming formulation, the number of continuous variables is 4 types L, M, H and X j. So 4 times, so 4 into 54. So, we have 216 continuous variables for this MILP formulation. Whereas, for metaheuristic techniques, we had only 54 continuous variable

Similarly, if we look into the number of constraints, right. So, we had 54 domain constraints, 2 constraints for 2 type of raw material and 1 budget constraint, right. So this was 2, the budget constraint is 1, right and the domain whole constraints would be 54. Because, the domain constraint is for 54 variables, right. So, we had only 57 constraints whereas, if you consider for this MILP formulation, this constraint is also to be written J times, right.

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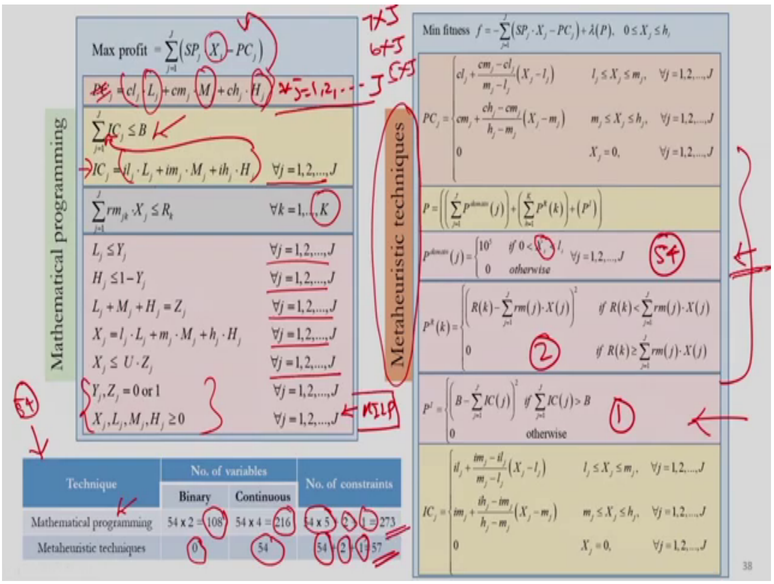

So, we have 1 set of J constraint, second set of J constraint, third set, fourth set, fifth set, sixth set and seventh set, right. So these are the decision variables, right. So we have this 7 set of constraints, right. So, we basically have 7 into J right. So what we can do is, we can totally eliminate this variable right and we can directly have this expression over here. So that way we can reduce J constraint.

So, it will become 6 into J. Similarly, what we can do is we can eliminate this set of J constraint and write this expression over here. So this 6 into J will get further reduced into 5 into J, right. So we have 54 into 5 such constraints, plus 2 constraints for raw material or K constraint and 1 constraint for this budget, right. So here we have 273 constraints.

So for the same problem description, if we formulate it as a mixed integer linear programming problem, we have 54 into 2, 108 binary variables and 54 into 4 continuous variables, plus we have 54 into 5 plus 3 constraints, right.

So, that is for mixed integer linear programming formulation, but if you formulate it in another way, which does not classify it to be a mixed integer linear programming formulation, then we can model that problem with 54 continuous variable and 57 constraints without using any integer or binary variables, right. So, the same problem description can have two different types of formulation.

Now the question is, which one of this is better, right? If you have a mixed integer linear programming formulation, it can be solved to global optimality. Whereas the metaheuristic technique, there is no guarantee that it would get solved to global optimality, right. Additionally, in Metaheuristic techniques, you have the issue of fixing the user-defined parameters. Based on user-defined parameters, we may or may not get the optimal solution.

Whereas for a mixed integer linear programming formulation, there is a guarantee on global optimality; only thing is that there is no assurance that it would get solved in a reasonable amount of time, right. So, if you have a mixed integer linear programming formulation, that can be solved in reasonable amount of time; then that is what has to be preferred. Because, we get a guarantee on the solution that we obtain that the solution is globally optimal solution and there is no better solution than what is discovered.

Again you need to remember, we can use a mixed integer linear programming formulation and solve it with metaheuristic techniques, right. So for example, in this case instead of 54 variables, we can employ 108 binary variables as well as 54 into 4 continuous variables; but the performance of metaheuristic techniques may not be up to the mark. In the next session, we will see how to code this MILP formulation to get the optimal solution.

Thank you.