Computer Aided Applied Single Objective Optimization
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Lecture – 31
Simplex Method for LP

Welcome in this session, we look into how to solve linear programming problems using Simplex Method.

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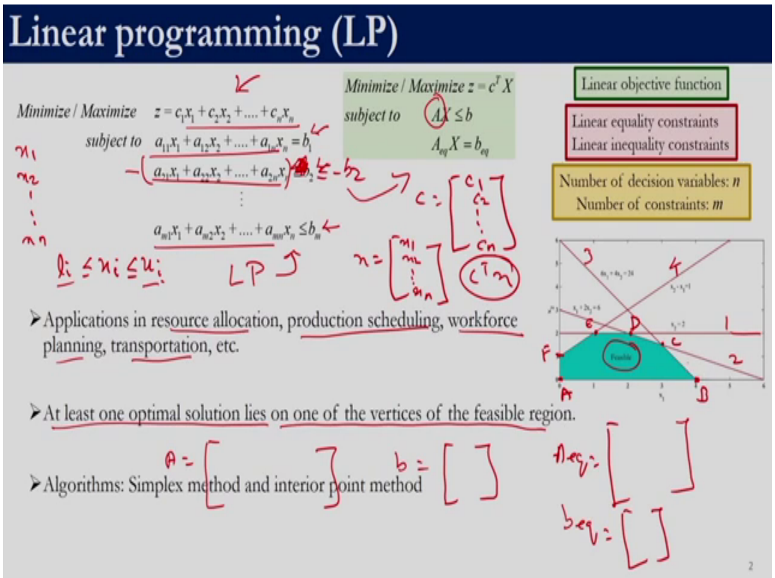

So, in a linear programming problem the objective function is linear and the constraints are also linear. So, in this case the decision variables are x 1 x 2 all the way up to x n. So, here we can have equality constraint as shown over here, here we can have greater than or equal to constraints and also less than or equal to constraints right.

So, this qualifies to be a linear programming problem the only requirements are the variables need to be continuous and the constraints need to be linear. So, this can also be compactly represented as this one right. So, all the greater than or equal to constraint can be converted into less than equal to by multiplying by a minus sign. So, this will become less than equal to minus b 2.

So, in this case c will be c 1, c 2 all the way up to cn x would be x 1, x 2 all the way up to x n right so, that when we do c transpose x we get this equation. So, A in this case would be a matrix. So, it will have the coefficients involved in the inequalities right where as b is the right hand side value of the inequality constraints; A equality would be the coefficient matrix of the equality constraints and b equality would be the right hand side value of the equality constraints. Over here the variables x can have their own lower bounds and upper bounds.

The so, large number of optimization problems can be formulated as linear programming problem; some of the applications are in resource allocation, production scheduling workforce planning transportation among others. A mathematical property of linear programming formulation is that at least one optimal solution will lie on one of the vertices of the feasible region. So, for example, we have these constraints constraint 1, 2 3 and 4 and this shaded region is the feasible region .

So, for this problem it is guaranteed that the optimal solution will lie either at this vertex A or at this vertex B or at this vertex C this one or this one or this one. So, the optimal solution is guaranteed to lie at one of these vertices. So, the method which we are going to discuss is simplex method. So, the simplex method is designed in such a way that it explores only the vertices right and when it moves to one vertice to the other vertice it is ensured that the vertice which is being currently visited is better than any of the previously visited vertices.

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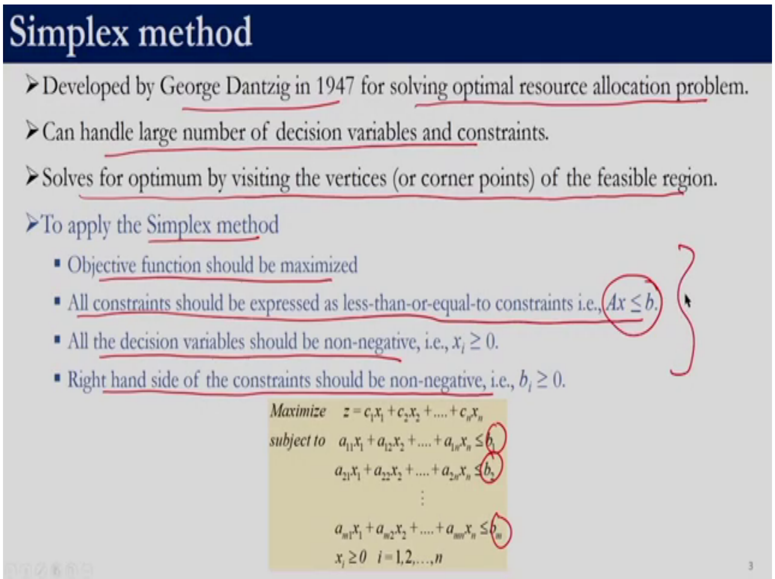

So, simplex method was developed in 1947 by George Dantzig for solving optimal resource allocation problems right. So, the method is designed to handle large number of decision variables and constraints. So, it solves for the optimum solution by visiting the vertices of the feasible region. So, to apply the simplex method as discussed in this course right, we should have an objective function which needs to be maximized. So, if we have a minimization objective function it has to be converted into a maximization form right and all the constraints should be expressed in less than equal to form right.

So, A x less than equal to b right. So, if there are constraints which are greater than or equal to; we can multiply by a minus sign and bring it in a less than equal to form and all the decision variables should be nonnegative right. So, remember linear programming does not require all the decision variables to be nonnegative, but the way we are discussing simplex method requires the decision variables to be nonnegative and the right hand side of the constraints should be nonnegative. So, the right hand side of the constraints are these 1 b 1 b 2 till bm those should be nonnegative. So, we need to ensure these 4 things right.

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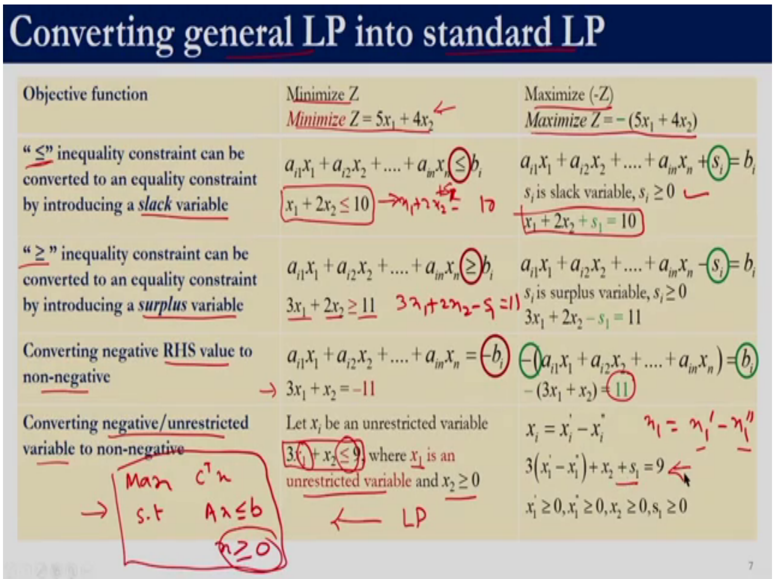

So, as discussed we require the objective function to be in the maximization form right. I you have an optimization problem which is to minimize Z, then instead of minimizing Z if we maximize minus Z we will get the same solution right. So, for example, over here if the objective function is minimized 5 x 1 plus 4 x 2 then instead of minimizing this we can convert it into an equivalent maximization problem wherein we maximize minus of 5 x 1 plus 4 x 2 right.

So, this is how we can convert the objective function into the standard form that is required. If there are less than equal to constraints right we add slack variable on the left hand side and convert it into an equality constraint right. So, over here if we see we have this equation x 1 plus 2 x 2 is less than equal to 10. So, we want to convert this equation into an equality constraint right. So, this 10 as states x 1 plus x 2 x 2 states right.

So, since this is a less than equal to we add a positive variable let us say, s 1 right so, that it becomes an equality constraint; that is what is given over here and the slack variable need to be positive right. If we are given a greater than or equal to constraint, we can subtract a positive variable known as surplus variable right. So, for example, here we have 3 x 1 plus 2 x 2 greater than equal to 11 right.

So, this equation can be written as; So, s 1 has to be positive right equal to 11 this is how we can convert a greater than equal to constraint right. So, one other requirement which we had was that the right hand side value of the equation should be nonnegative right. So, for example, if we have this 3 x 1 plus x 2 is equal to minus 11, then what we can do is multiply by a minus sign on both the sides right so, that the right hand side becomes a positive value. If there are negative variables or unrestricted variables so, remember the standard form that we want is maximize let us say C transpose x right subject to all the constraints we would require in this form less than equal to form right and the variables need to be greater than or equal to 0.

So, the way we are going to discuss simplex method is applicable only for this problem right. So, simplex method requires the variable to be positive, but linear programming does not require the variable to be positive. So, the linear programming can have variables which are unrestricted or can take both positive and negative values right. So, in that case what we do is that to convert that linear programming problem into a form that is compatible to simplex; we will replace each unrestricted variable by the difference of two other variables right.

So, for example, let us say 3 x 1 plus x 2 is less than equal to 9 and let us say over here x 1 is a variable which is unrestricted right and x 2 is a great than 0. So, what we will do is we will replace x 1 by the difference of 2 numbers, x 1 prime and x 1 double prime right. So, these two numbers are positive their difference can be negative right. So, wherever we have x 1, we replace it by x 1 prime minus x 1 double prime. Since this is a less than equal to constraint we add a slack variable to make it as an equality constraint.

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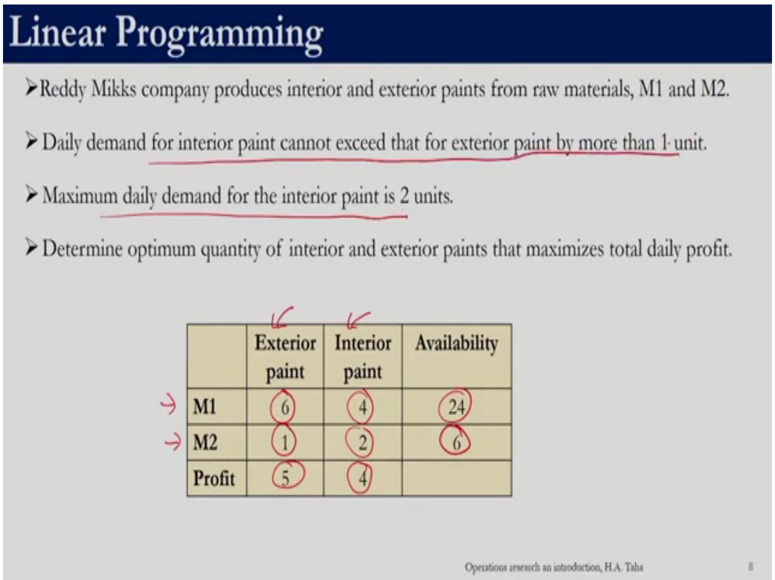

So, this problem is something that we have discussed previously. So, there is a paint company which can manufacture exterior paint as well as interior paint. So, the profit per unit of exterior paint is 5 the profit obtained per unit of interior paint is 4r. So, both the paints require raw material M 1 and M 2 right. So, exterior paint requires 6 units of M 1 and interior paint requires 4 units of M 1, the total amount of raw material M 1 that is available is 24 units similarly for exterior paint and interior paint if we require raw material M 2 and the quantity required is 1 unit and 2 unit respectively.

The total amount of M 2 that is available is 6 units right. So, we have additional 2 constraints right. So, one is that daily demand for the interior paint is 2 units and demand for interior paint cannot exceed that for exterior paint by more than 1 unit.

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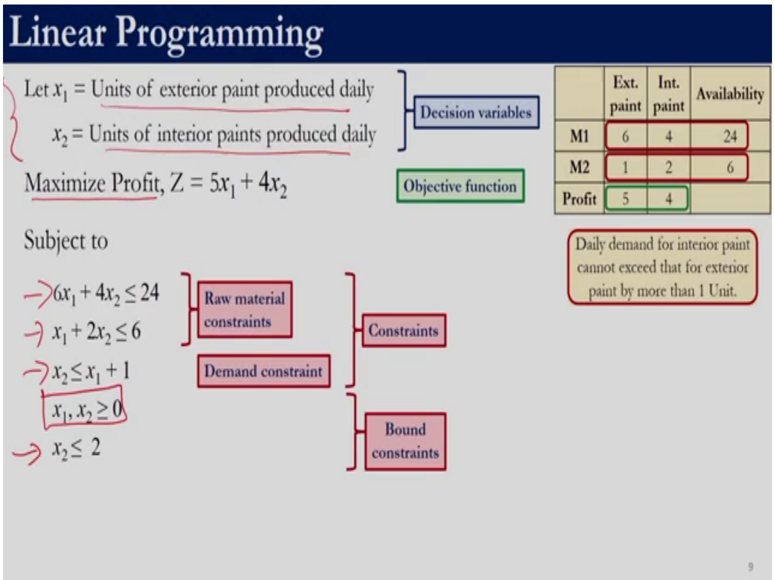

We will not discuss into this formulation we have discussed this previously. So, these are our 4 equations 1, 2, 3 and the fourth 1 over here these are bounds on the decision variable x 1 and x 2. So, the optimization problem is to maximize profit, x 1 and x 2 are our decision variables; x 1 denotes units of exterior paint produced daily whereas, x 2 indicates the units of interior paints produce daily.

This problem is now, to be transformed into the standard form required for solving using simplex method.

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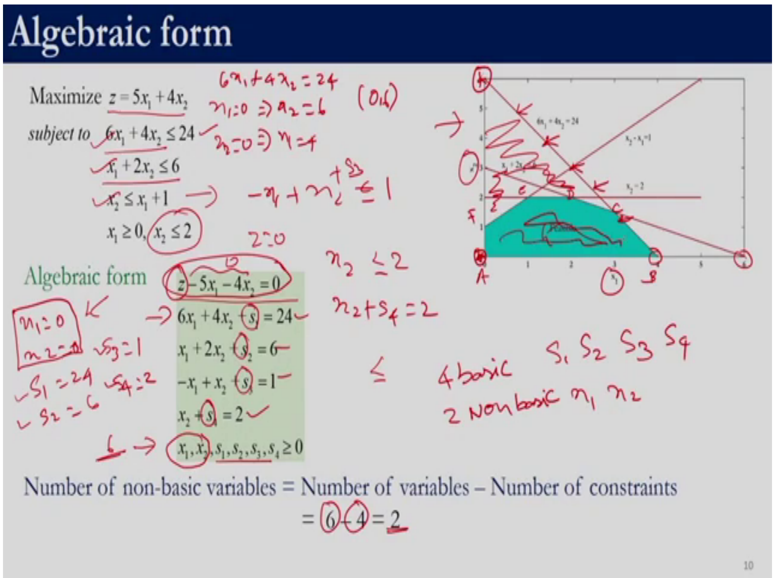

So, in simplex method the objective function as well as the constraints are required in algebraic form. This equation can be written as z minus 5 x 1 minus 4 x 2 equal to 0 over here as we discussed earlier for less than equal to constraints, we can add slack variables this equation is not in the required form.

So, we convert it into first into minus x 1 plus x 2 is less than equal to 1 and then add a slack variable s 3 to make it as a equality constraint. Similarly this constraint x 2 is less than equal to 2 we can add a slack variable s 2 plus s 4 is equal to 2. So, now, we have 4 slack variables right and 2 decision variables right. So, our decision variables are 2 since we have 4 constraints of the form less than equal to, we added one slack variable for each of the constraint. So, we have now 6 variables. So, z is not to be considered as a variable. So, we have 6 variables over here.

So, the number of variables is 6 and the number of linear equality is which we have is four. So, 1 2 3 and 4; so, 6 minus 4 is equal to 2. So, we have 4 basic variables and 2 non-basic variables right. So, 4 basic variables are s 1 s 2 s 3 and s 4 the two non-basic variables are x 1 and x 2. So, an initial solution for this problem is if we take x 1 equal to 0 and x 2 is equal to 0 right. So, this equation will give us the value of s 1 to be 24 the second equation will give us the value of s 2 to be 6 the third equation will give us a value of s 3 to be 1 and the fourth equation would give us the value of s 4 to be 2 right.

So, these two are non-basic variables right whereas, these four are basic variables right this solution satisfies all the constraints right. So, for this solution the objective function is 0. Now we have this 0 comma 0 as the initial starting point. So, this figure over here shows a graphical representation of this problem right. So, here we have two variables. So, this is x 1 and this is x 2. So, if you remember we can plot all of these lines. So, if we take the first equation 6 x 1 plus 4 x 2 right.

So, let me write it as equal to 24 right. So, in this equation if we substitute x 1 is equal to 0 we will get the value of x 2 to be 6. So, the first point is 0 comma 6 which is over here to get the second point we can put x 2 is equal to 0. So, this will give x 1 is equal to 4. So, we will get this point right. So, we can draw this. So, this is the line which is 6 x 1 plus 4 x 2 is equal to 24. So, the feasible region is below it. So, let us take the point 0 0.

So, 0 comma 0 will satisfy this constraint. So, this point is a feasible solution right. So, the feasible region for this particular constraint is all of this right. So, similarly we can plot the other two constraints. So, the common feasible area corresponding to this constraint first second constraint, third constraint and fourth constraint is the shaded region. So, simplex method is designed in such a way that it will only visit the vertices right.

So, though we have such a big feasible region over here, simplex method is only going to visit the vertices. So, this is vertice A B right. So, we have another one over C D over here E and F. The optimal solution is located at one of this vertices and the simplex method itself is designed in such a way that it will only visit this vertices. It is not going to explore anywhere inside this feasible region.

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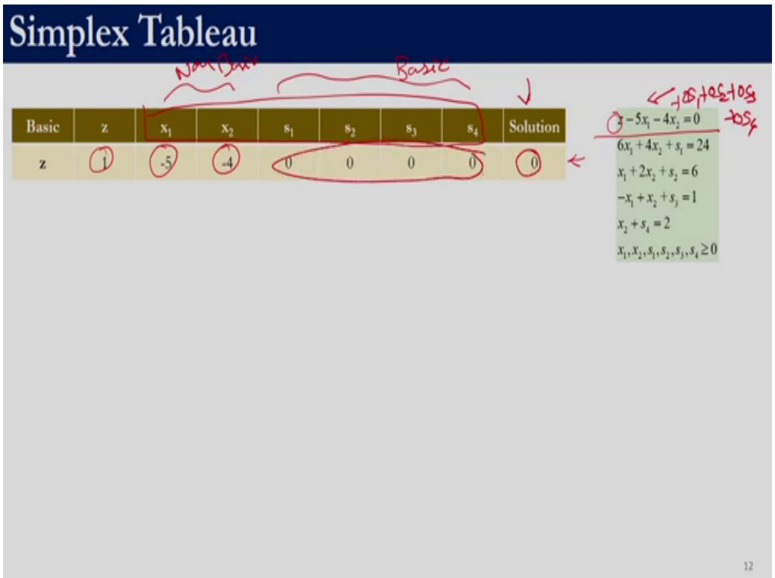

So, now we have this algebraic form over here. Let us construct what is known as the simplex table right. So, these are the 6 variables which we have now, x 1 and x 2 are non-basic variables and s 1 s 2 s 3 s 4 are our basic variables right.

So, this is basic variable, this is non-basic variable and this column is the right hand side which we have for all of this equation right. For each of this equation there will be a row in the simplex table right. So, for the first equation if we see the coefficient of z is 1 right, the coefficient of x 1 is minus 5 the coefficient of x 2 is minus 4 and this equation does not involve s 1 s 2 s 3 s 4 or it is equivalent to 0 s 1 plus 0 s 2 plus 0 s 3 plus 0 s 4 right.

So, we write 0s over here and the right hand side is 0 right. So, in the next row the basic variable is s 1.

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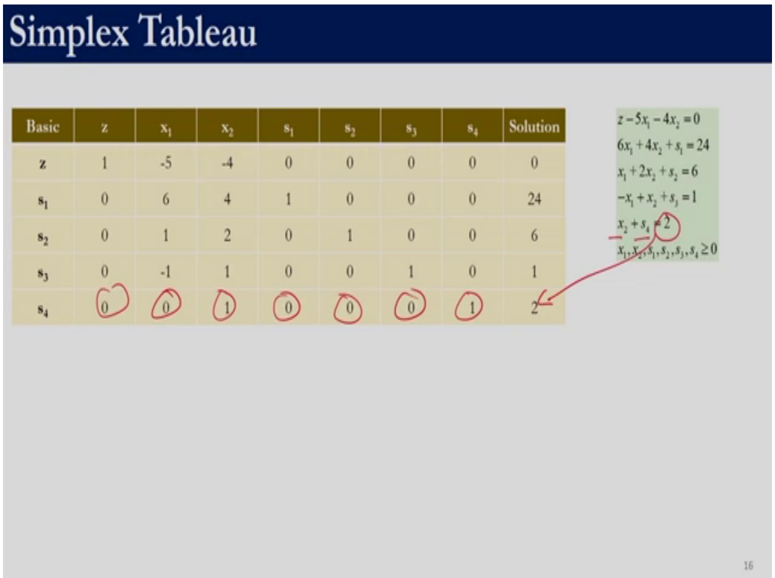

So, there is no z which is involved over here in this equation. So, the coefficient of z is 0, the coefficient of x 1 is 6 and x 2 is 4 right and it involves only the slack variable s 1. So, the slack variable s 2 s 3 s 4 are not involved in the first equation. So, we have 0s over there and this right hand side is 24.

So, the second row is for this equation. So, here the basic variable is s 2. So, we write s 2 over here the coefficient of s 1 is 0 the coefficient of s 2 is 1 s 3 and s 4 are not involved the right hand side is 6 and the coefficient of x 1 and x 2 are 1 and 2 respectively. It does not involve z. So, we have a coefficient of 0 similarly we write for the next equation. So, here z is not involved. So, the coefficient of z is 0, s 3 is the slack variable over here or that is the basic variable and the coefficient of x 1 and x 2 are minus 1 and 1. So, coefficient of x 1 x 2 is minus 1 and 1 and the equation does not involve s 1 s 2 or s 4.

So, their coefficients are 0 it involves s 3. So, its coefficient is 1 and the right hand side value is 1 over here. So, similarly we can also write the next equation right. So, the next equation involves only x 2 and s 4. So, we have the coefficient 1 1 over here, rest of all the coefficients are 0 and the right hand side over here is 2. So, we write the 2 over here. This completes the construction of the initial simplex table, but now we look into certain terminologies and rules to be followed for simplex table.

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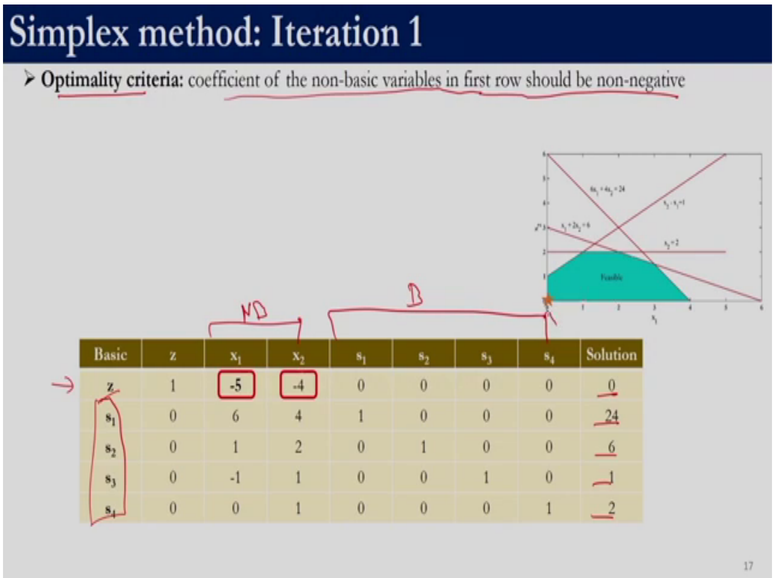

Simplex method is an iterative method. So, the question is when do we stop the method right.

So, the method is stopped when we attain the optimality criteria. So, the optimality criteria is attained when the coefficient of the non-basic variable in the first row are nonnegative right. So, here if we see these two are our non-basic variables right whereas, these four are our basic variables. The first row is the equation that we had from the objective function right. So, the coefficient should be nonnegative. Right now we have minus 5 and minus 4 right. So, that means, that we have not reached the optimality criteria.

So, since x 1 and x 2 are not over here though they are non-basic variables and their value is now 0 comma 0 right. So, the value of s 1, s 2, s 3, s 4 is 24 6 1 2 the value of the objective function is 0 and the value of x 1 and x 2 as they do not appear in the basic variable and since they are non-basic variable their value is 0. So, if we see this plot we are currently at this point.

So, the starting point of the simplex method is this point which would be a feasible solution.

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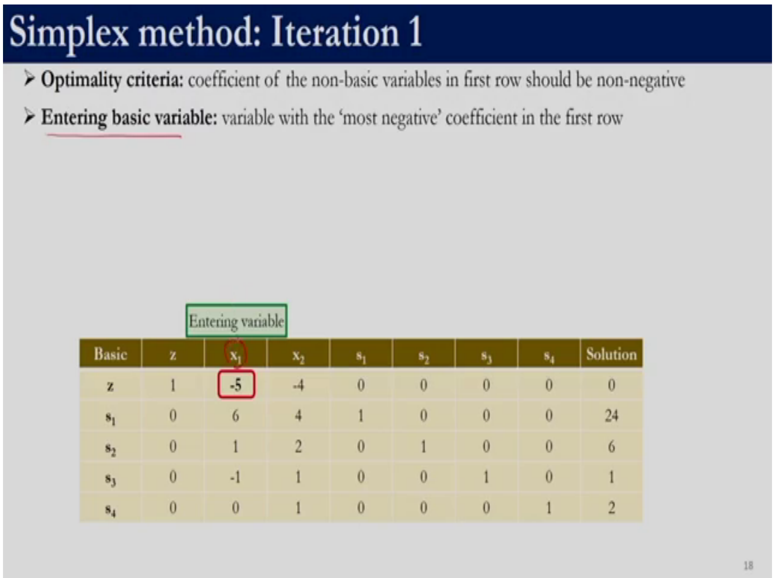

Next we have something called as entering basic variable. So, entering basic variable is the variable which has the most negative coefficient in the first row right. So, for example, now in this first row we have minus 5 and minus 4 over here. So, minus 5 is the most negative. So, this is called as entering variable right. So, this variable which is a non-basic variable will now become a basic variable; pivot column is the column of the entering basic variable right.

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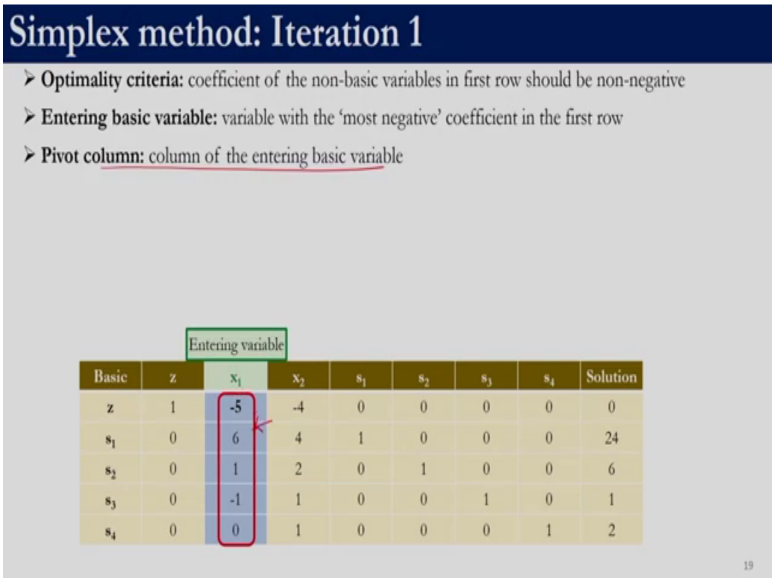

Basic	z	x_1	x_2	s_1	s_2	s_3	s_4	Solution
z	1	-5	-4	0	0	0	0	0
s_1	0	6	4	1	0	0	0	24
s_2	0	1	2	0	1	0	0	6
s_3	0	-1	1	0	0	1	0	1
s_4	0	0	1	0	0	0	1	2

So, here we had identified that x 1 is the entering variable right. So, since x 1 is the entering variable the column corresponding to x 1 is the pivot column.

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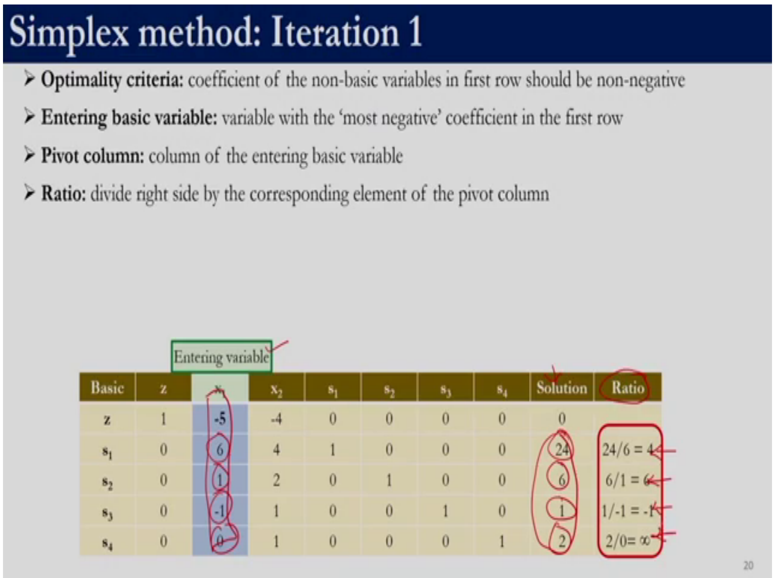

So, once we have identified entering variable the pivot column, now we need to identify something called as pivot row. So, to determine the pivot row we need to first find out the term ratio. So, ratio is the right hand side value whatever is written solution over here divided by their corresponding coefficient in the pivot column. So, over here 24 is the value and its corresponding coefficient in the pivot column is 6 so, 24 by 6. So, here it is 6 by 1 here it is 1 by minus 1 and here it is 2 by 0 right.

So, if we calculate it comes out to 4, 6 minus 1 infinity right. So, that is how we determine ratio.

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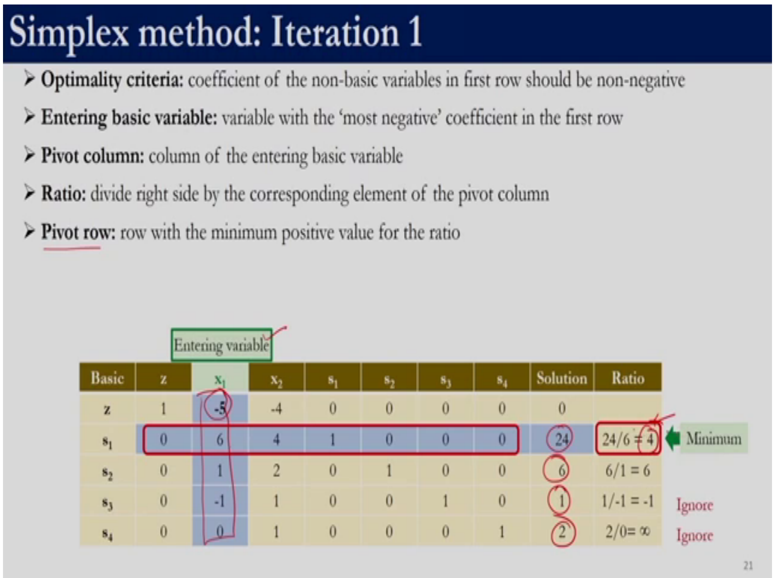

Basic	z	x_1	x_2	s_1	s_2	s_3	s_4	Solution	Ratio
z	1	-5	-4	0	0	0	0	0	
s_1	0	6	4	1	0	0	0	24	24/6 = 4
s_2	0	1	2	0	1	0	0	6	6/1 = 6
s_3	0	-1	1	0	0	1	0	1	1/-1 = -1
s_4	0	0	1	0	0	0	1	2	2/0= ∞

Once we have determined the ratios, we need to determine the pivot row. Pivot row is the row which has the minimum positive value. So, in this case if we see 4 is the minimum positive value in this column right. So, that row corresponds to the pivot row. So, first we identified the entering variable.

So, entering variable was the one which had the most negative coefficient right. Its corresponding column is the pivot column, then we calculated the ratios ratio is the right hand side values divided by the corresponding element in the pivot column. So, that will give us the ratio and the minimum positive value in the ratio helps us to identify the pivot row.

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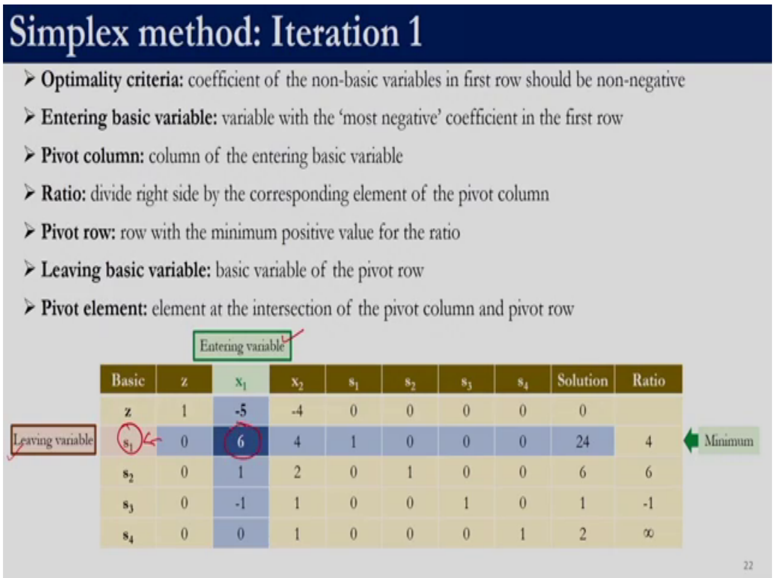

So, now once we have identified the pivot row, the basic variable corresponding to the pivot row. So, in this case which is s 1. So, that is the leaving variable. So, first we identified entering variable, now we have identified the leaving variable and the intersection of the pivot row and pivot column. So, this element is the pivot element.

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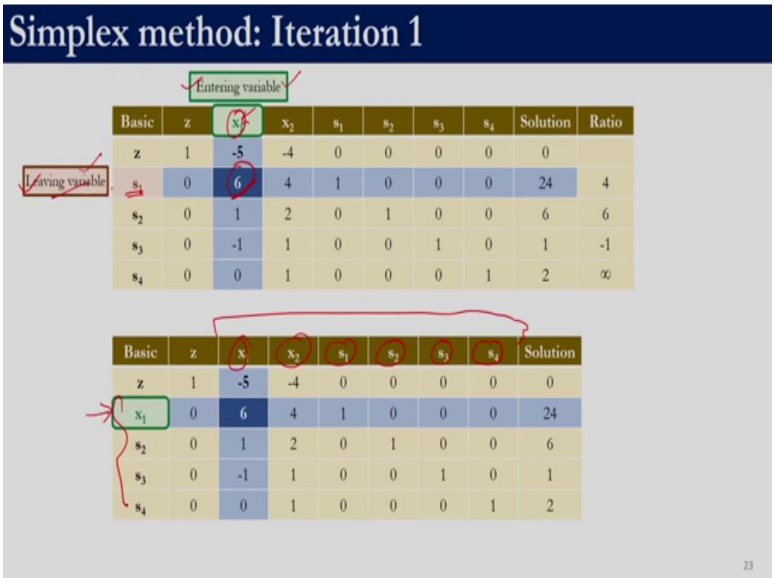

Once we have identified the entering variable, leaving variable and the pivot element, we need to include the entering variable into the basic variables and remove the leaving variables. So, for example, if we see x 1 is the entering variable over here right. So, x 1 enters as a basic variable whereas, s 1 is removed from the basic variable. So, s 1 which was here is no longer in the set of basic variables.

So, this remains. So, we do not make any changes in this x 1 x 2 s 1 s 2 s 3 s 4 it is just that one of the non-basic variables will become a entering variable and one of the basic variables will become the leaving variable right. So, first was to identify entering variable leaving variable once we identify entering variable and leaving variable we will know the pivot element, once we have identified the pivot element we bring in the entering variable into basic variable and remove the leaving variable from the basis vector. So, the next step is to determine the next feasible solution.

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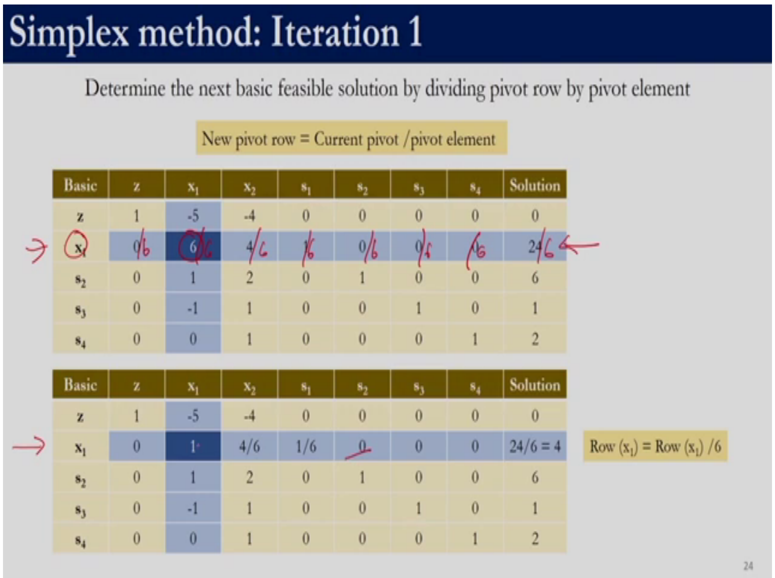

So, to determine the next feasible solution we need to divide the pivot row by the pivot element right. So, over here this is the pivot row, this is the pivot element. So, we need to divide this entire row by the pivot element right. So, 0 by 6, 6 by 6, 4 by 6, 1 by 6, 0 by 6 0 by 6, 0 by 6 and 24 by 6 right. So, that is what we get the new row.

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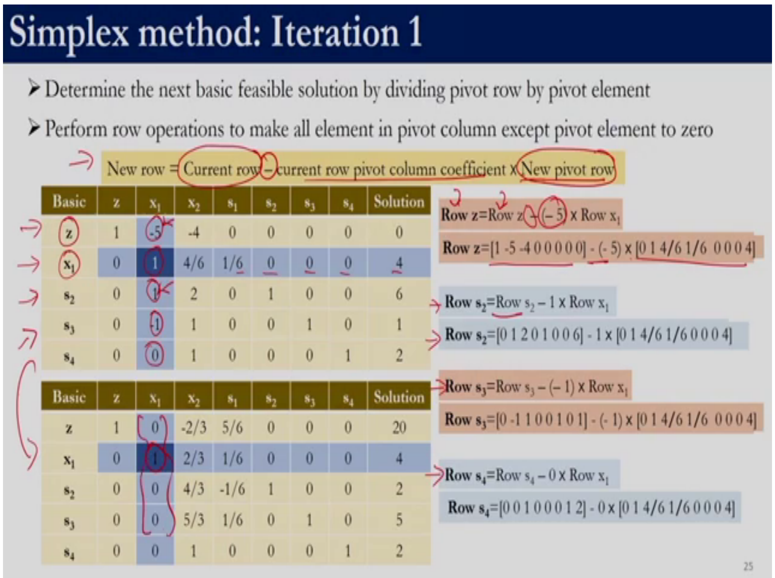

So, now that we have made the pivot element as 1, the next step is to make other elements in the pivot column as 0 by appropriate row operations. So, this can be implemented using this right. A new row is given by its corresponding current row minus the corresponding pivot column coefficient multiplied by the new pivot row right.

So, our new pivot row is 0 1 4 by 6 1 by 6 0 0 0 and 4 right. So, let us see how to do it for the first row. So, the new row corresponding to z is the old row corresponding to z right. So, which is 1 minus 5 minus 4 0 0 0 0 0. So, that is what is written over here minus the pivot column coefficient right. So, we are operating on this row right. So, for this row the pivot column coefficient is minus 5 right. So, we write that minus 5 over here multiplied by the new pivot row.

So, in our case the new pivot row is the pivot row corresponding to x 1 right. So, the elements are 0 1 4 by 6 1 by 6 0 0 0 and 4. So, that is what we have written over here similarly we can calculate the new row s 2 right. So, the new row s 2 is the old row s 2 minus the corresponding pivot column coefficient. So, that is one in this case multiplied by row of x 1. So, that is how we get row s 2.

So, similarly you can find out the row corresponding to the basic variable s 3 and the row corresponding to basic variable s 4 right. So, this table which we had before the row operation will get transformed into this table. So, in this case if we see except for this pivot element 1 all the rest of the elements are 0. So, this completes the first iteration of simplex method.

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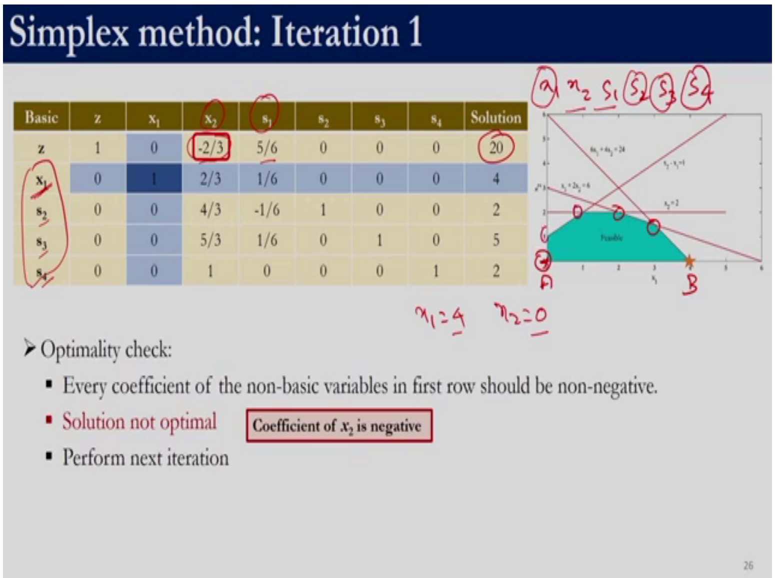

So, now that we have completed the first iteration of simplex method, we need to check for the optimality condition right. So, optimality condition is that the coefficient of non-basic variables should be nonnegative.

So, out of the 6 variables which we have x 1 x 2 s 1 s 2 s 3 and s 4 right x 1 s 2 s 3 and s 4 are basic variables. So, the 2 non-basic variables are x 2 and s 1 right. So, this x 2 and s 1 these are non-basic variables. So, we need to look into their coefficients in the first row. So, their coefficients in the first row if they are nonnegative, then we have reached the optimal solution. So, in this case we have not reached non negative values. So, this is a negative value right.

So, still we have not achieved the optimality right. So, we need to go from the second iteration of simpler method. So, now, if we see initially we were at this position right. So, at the end of first iteration what we have is the value of x 1 is 4 right and x 2 does not appear in the basic variable right. So, the value of x 2 is 0 right. So, 4 comma 0 is the new solution that we have at the end of iteration 1, the objective function value corresponding to this 4 comma 0 is 20 right. So, 4 comma 0 is shown over here. So, initially we were at this 0 comma 0 right at the beginning of the first iteration right. So, from vertex A we have moved to vertex B which is 4 comma 0. So, the next iteration will take us to one of these other vertices.

So, for second iteration of simplex method we need to repeat the same procedure right.

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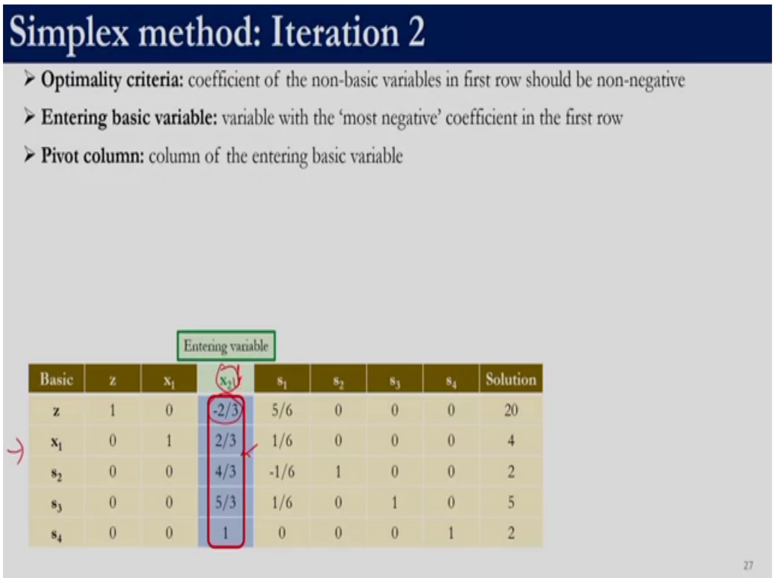

Basic	z	x_1	x_2	s_1	s_2	s_3	s_4	Solution
z	1	0	-2/3	5/6	0	0	0	20
x_1	0	1	2/3	1/6	0	0	0	4
s_2	0	0	4/3	-1/6	1	0	0	2
s_3	0	0	5/3	1/6	0	1	0	5
s_4	0	0	1	0	0	0	1	2

So, over here we have the same table which we got at the end of first iteration right. So, in this case the most non negative is minus 2 by 3 that corresponds to x 2. So, that column becomes the pivot column and x 2 is the entering variable. So, we have identified the pivot column now we need to identify the pivot row. So, to identify the pivot row we need to divide this column by the pivot column right.

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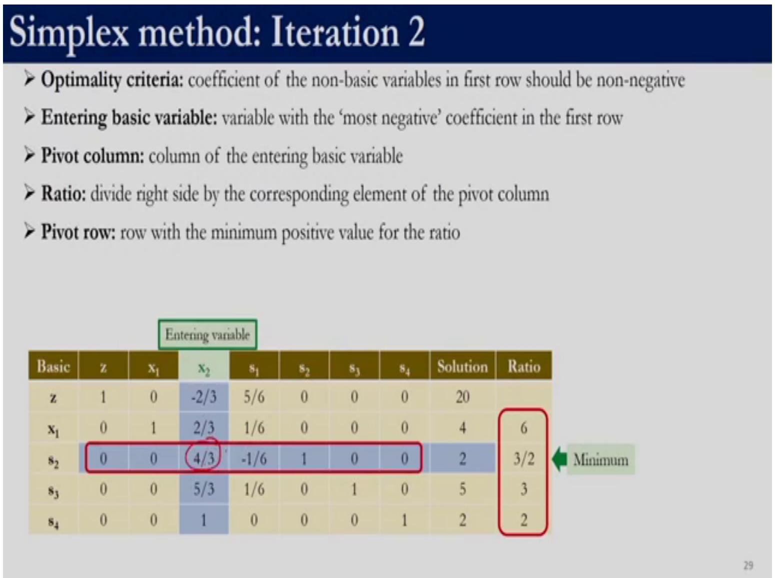

So, these are the ratios which we have to identify the pivot row, we need to look for the minimum positive value right. So, the minimum positive value over here is this 3 by 2 which is 1.5 right.

So, that is the minimum. So, the row corresponding to it is the pivot row right. So, now, we have identified the pivot column, we have identified the pivot row. So, this intersection of pivot row and pivot column is our pivot element. So, this pivot row corresponds to the basic variable s 2.

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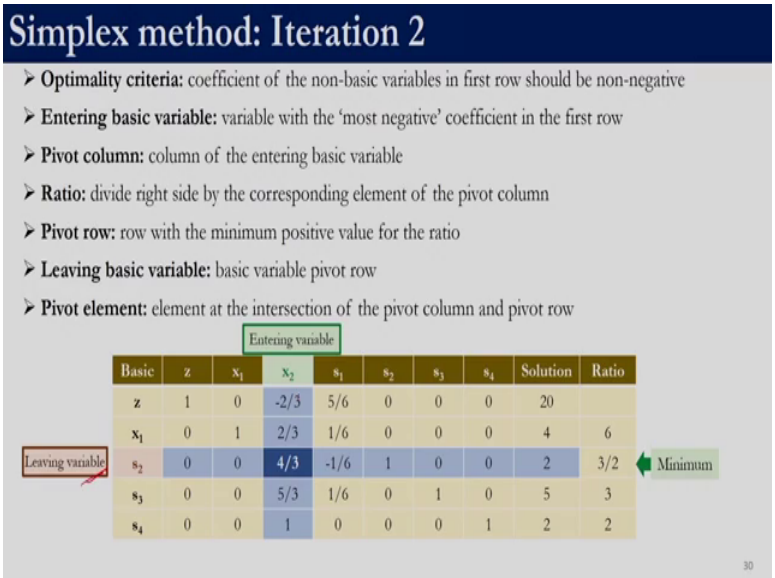

So, now s 2 will become the leaving variable right. So, now, we have identified the entering variable and we have identified the leaving variable. Now we need to make this pivot element as 1 right and all the other coefficients in the pivot column should be 0 right.

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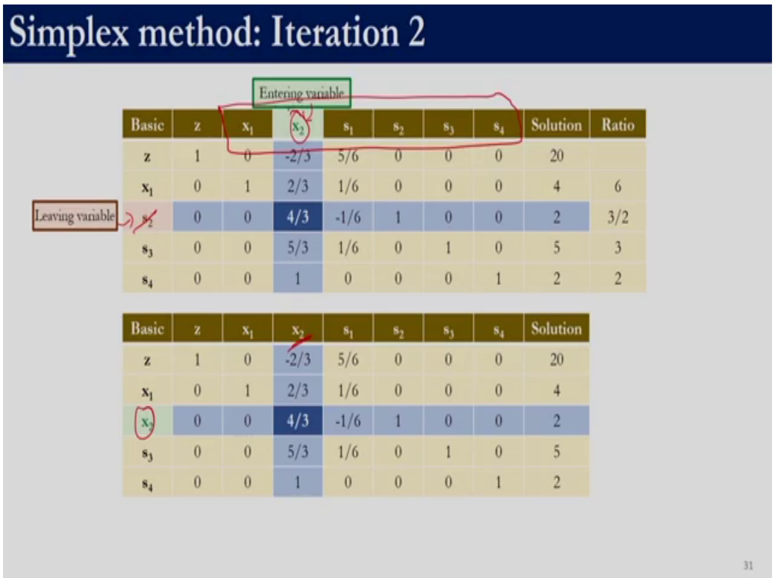

So, the entire row operation has to be repeated x 2 is the entering variable and s 2 is the leaving variable right. So, over here we replace s 2 by x 2 and this x 2 remains the same right. So, remember the leaving variable is not to be written over here, this set of variables over here remain as such just that the leaving variable is no longer a basic variable whereas, the entering variable becomes a basic variable.

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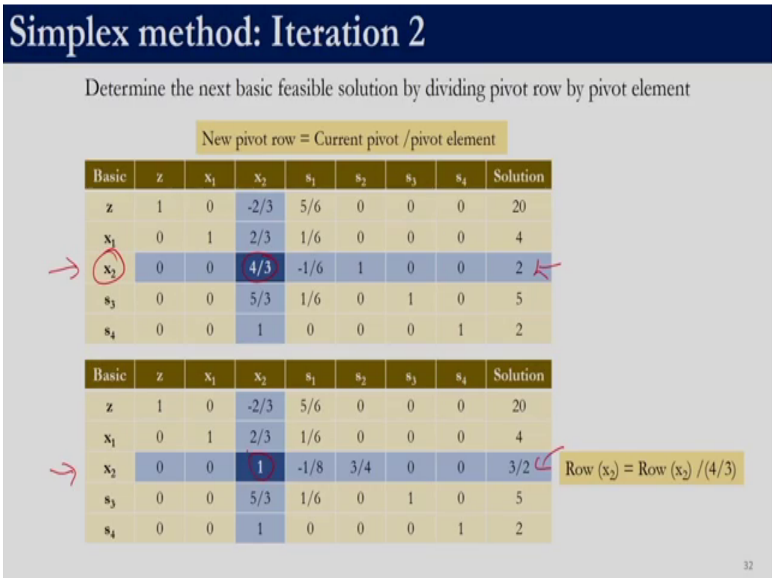

So, now that we have made this entering and leaving variable, we need to now make this pivot element as 1 right. So, to make that pivot element as 1 we need to divide this entire row the third row or the row corresponding to the basic variable s 2 by 4 by 3 right. So, if we divide it by 4 by 3, we would get it as 0 0 1 minus 1 by 8 3 by 4 0 0 3 by 2. Once we have made the pivot element as 1, we need to make the rest of the entries in the pivot column as 0.

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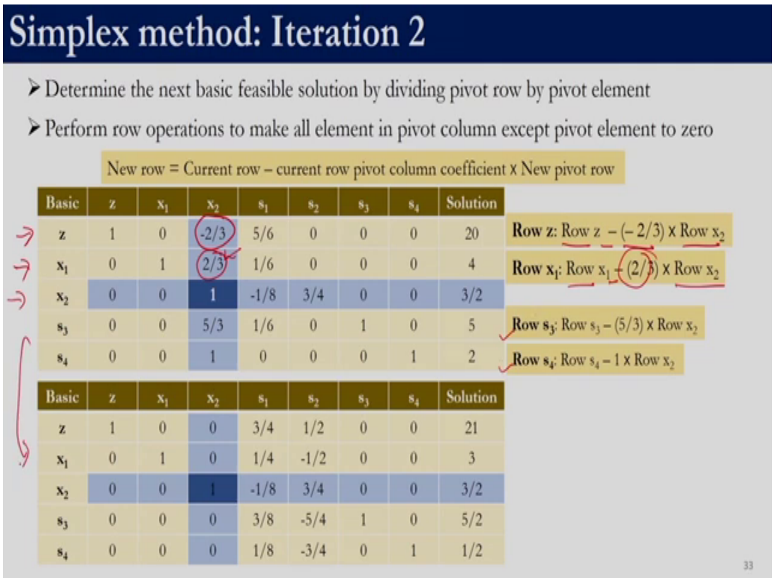

So, again we employ the same operation the new first row is the old first row minus the corresponding element in the pivot column which is minus 2 by 3 over here multiplied by the new pivot row right. Similarly the new row corresponding to x 1 is the old row corresponding to x 1 minus the corresponding element in the pivot column.

So, in this case the corresponding element in the pivot column is 2 by 3. So, this 2 by 3 multiplied by the new pivot row right. So, the new pivot row is 0 0 1 minus 1 by 8, 3 by 4 0 0 and 3 by 2. So, similarly we can get row 3 and row 4. So, if we perform this operation this table will get transformed into this one right.

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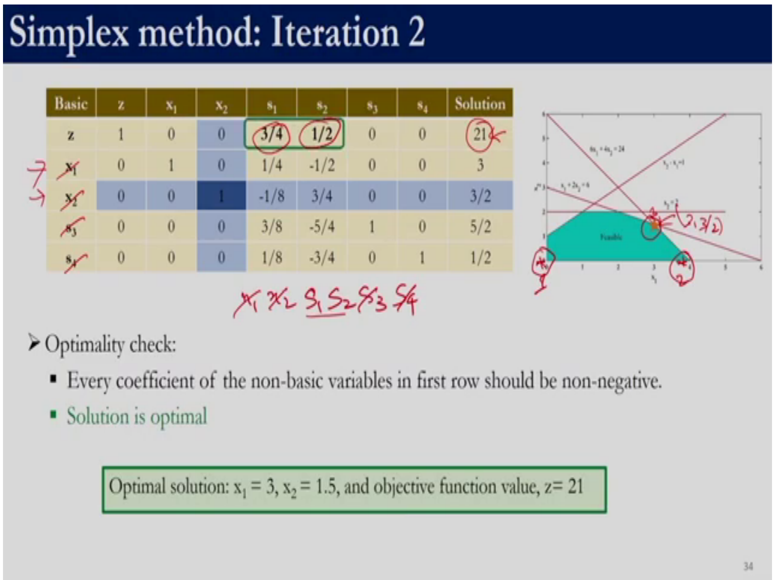

So, this completes the second iteration, if you remember when we started we started with this first point then we moved to this point right and then now, we have achieved this point. So, this point because the value of x 1 is 3 and the value of x 2 is 3 by 2 right. So, this point corresponds to 3 comma 3 by 2 right. So, that is the point which we are currently at right the objective function at this point 3 comma 3 by 2 is 21. So, we have completed the second iteration right. So, we have moved from this point to this point now we have come to.

So, in this case it happened that we are moving from 0.1 to 0.2 to 0.3 right. So, if you see we have only visited the vertices. So, our initial simplex table corresponds to this 0 comma 0, the second solution which we had corresponded to 4 comma 0, the third solution which we have at the end of second iteration corresponds to 3 comma 3 by 2 right. So, before proceeding for the third iteration we need to check whether we have reached the optimal solution or not right. So, for that we need to check the coefficients of the non-basic variable.

So, our variables are x 1 x 2 s 1 s 2 s 3 and s 4 right among this x 1 x 2 s 3 s 4 x 1 s 2 s 3 s 4 are basic variables. So, now, our non-basic variables are s 1 and s 2 and their coefficients are positive right. So, since their coefficients are positive we can confirm that 21 is the optimal solution corresponding to x 1 is equal to 3 and x 2 is equal to 3 by 2 now that we have solved it right.

The simplex table not only gives us the value of the decision variables right, but also gives certain additional information.

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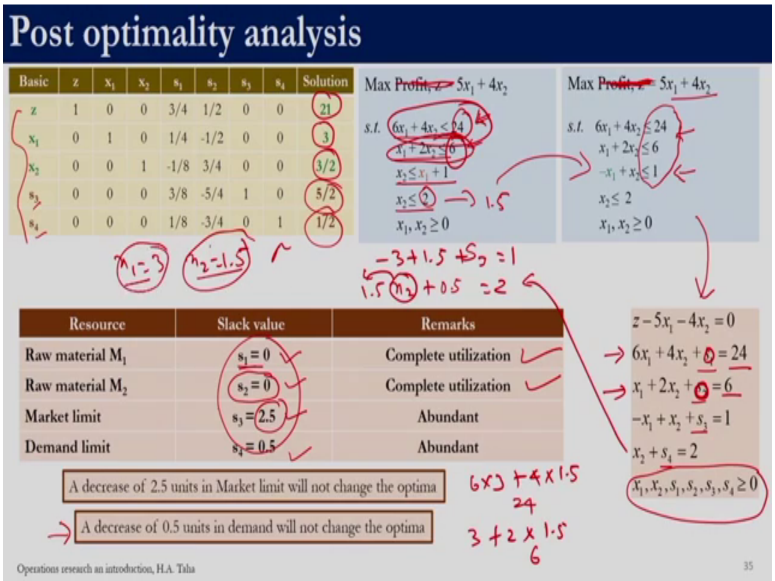

So, this is the optimization problem which we started with right to maximize 5 x 1 plus 4 x 2 right and the constraints were 6 x 1 plus 4 x 2 is less than equal to 24, x 1 plus 2 x 2 is less than equal to 6. So, if you remember the context of this problem these two indicated the amount of raw material 1 and 2 which were available and then we had this requirement about the quantity of interior paint and exterior paint that is produced and we had this demand constraint wherein we were not allowed to produce more than 2 units of interior paint right.

So, this problem we had transformed into this form wherein we got all the constraint to be in the less than equal to form and then from this optimization formulation we went to this algebraic formulation which satisfies the requirement of simplex method as discussed in this course right. So, the objective function was to be maximized.

The right hand side of the equations should be positive and we added slack variables which are also positive. So, we added this 4 slack variables since we had 4 constraint and all of the slack variables are positive. So, this is what we solved and after two iterations we ended up with this table. So, in this table we know that x 1 is equal to 3 and x 2 is equal to 1.5. So, x 1 is equal to 3 and x 2 is equal to 1.5 and at this point x 1 is equal to 3 and x 2 is equal to 1.5 the profit that we would be making is 21 units.

So, this is the information which is what we sort out for the optimal values of the decision variable and its corresponding objective function right. But we can also see that the value of s 3 and s 4 is 5 by 2 and 1 by 2 right. So, the value of s 3 is 2.5 5 by 2 and the value of s 4 is 1 by 2 and since s 1 and s 2 do not appear over here as basic variable their values are 0.

Now, let us look at what do these values signify right. So, s 1 was related to the first constraint and s 2 was related to the second constraint right. So, in this equation at the optimal solution this is 0 and this is 0 right. So; that means, the entire quantity of raw material 1 and 2 which are available are utilized right. So, if you want to check that what we can do is 6 into 3 for this constraint plus 4 into 1.5 this is equal to 24 which is what we have on the right hand side right.

So, the slack variable is 0. So, that is what it indicates that for the amount of paint 1 and point 2 that we are producing we are utilizing entire 24 units similarly for the second slack variable s 2 is also 0 right. So, let us look into this constraint x 1 plus 2 x 2 it is less than equal to 6. So, x 1 is 3 plus 2 into 1.5. So, this happens to be 6 and the right hand side is also 6.

So, since s 2 is equal to 0 we can directly say that the entire quantity of 6 is being used. So, that is why we have these two remarks over here complete utilization so, similarly for s 3 and s 4 right. So, these equations would be satisfied so, minus x 1. So, minus 3 plus 1.5 plus s 3 is equal to 1. So, this equation would be satisfied only when s 3 takes a value of 2.5 similarly for the final constraint. So, x 2 plus s 4 is equal to 2 this constraint over here. So, x 2 is currently 1.5. So, we can even add 0.5 to meet this constraint right.

So, what it basically means that, even if we decrease this 2 to 1.5 right the optimal solution would not change right because x 2 is anyway 1.5 right. So, even if market demand where to reduce for paint to right there will not be any change in the optimal amount of x 1 and x 2 that has to be produced.

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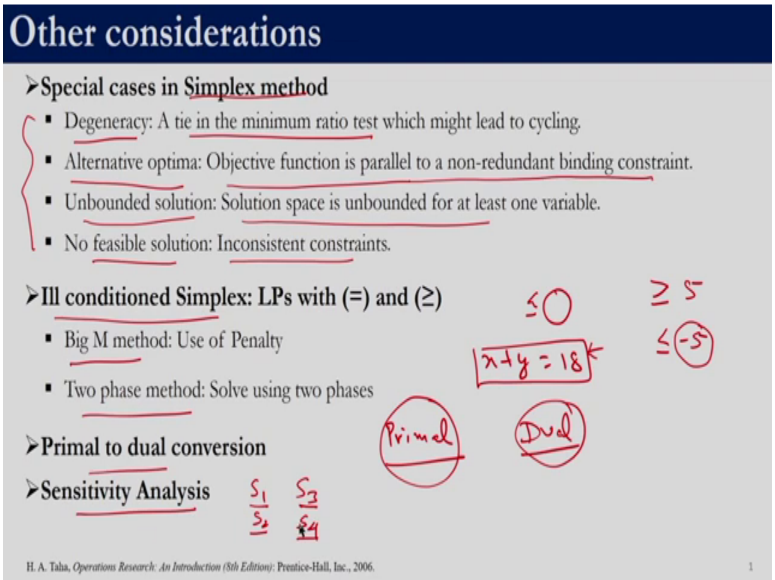

So, the simplex method which we have discussed is the basic simplex method right there are several other considerations that we need to take care of.

So, in simplex method we might encounter these 4 different cases right 1 is degeneracy. So, this happens when there is a tie in the minimum ratio test. So, remember once we have identified the pivot column to identify the pivot row we had to do the minimum ratio test. So, it might happen that two values in that particular column can be identical in that case we can take any one of them and continue right, but at times it can lead to iterations which do not necessarily improve in the objective function values. Another issue that can occur in simplex method is when we have alternative optima right.

So, over here what happens is that the objective function is parallel to a non-redundant binding constraint. So, the solution lies on the constraint and this constraint is parallel to the objective function. So, just like we drew the constraints we can also draw the objective function and if it happens that the objective function is parallel to a non-redundant binding constraint in that case we have alternative optima. The other circumstances when we have unbounded solution. So, under this circumstance the solution space is unbounded for at least one decision variable and if the constraints are inconsistent right if they are conflicting one another then there would be no feasible solution. So, in such cases we need to employ special strategies for simplex method.

So, we can also encounter ill conditioned simplex. So, for the discussion whatever we had done so, far we had assumed that we will be able to convert all the constraint into less than equal to form as well as the right hand side would be a positive value right. Additionally we did not look into the constraints let us say which is great than or equal to 5; if we convert this greater than or equal to constraint into a less than or equal to constraint we end up with a minus 5 right. So, now, the right hand side is no longer a positive value in that case we cannot directly employ the simplex method which we have discussed additionally if we have equal to constraints. So, let us say x plus y is equal to 18 right.

So, we did not consider constraints which are of this nature right. In these 2 cases when we have a greater than or equal to constraint and the equality constraint we need to introduce additional artificial variables right just like we introduced slack variable right and then we will have to employ either a big M method or a two phase method to solve such problems. So, that is something that we have not discussed in this course.

The problem which we solved is known as the primal problem given a primal problem we can also construct a dual problem. The solution of this dual problem and the primal problem would be identical in many circumstances it is better to solve the dual problem rather than the primal problem. So, conversion of primal to dual is something that we have not done as part of this course and we can also do a sensitivity analysis over here we did only preliminary sensitivity analysis right. As in like when we had values of the slack variables at the end of the simplex procedure we interpreted what do they indicate.

So, the sensitivity of the solution can also be studied with respect to various other factors that is something that we have not done as part of this course right.

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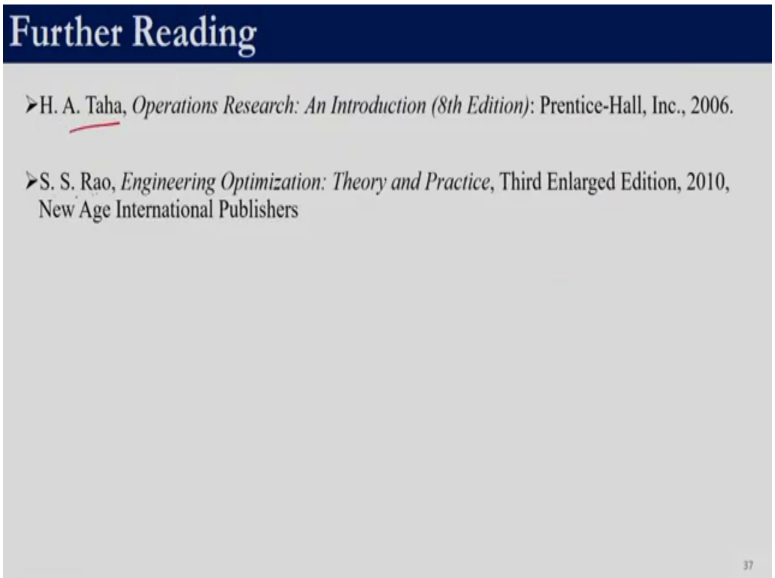

For all this you can look into any standard book of optimization one of them is by Taha and the other one is by S.S Rao with that we will be concluding this session on linear programming using simplex method. In the next session we will be looking at branch and bound method to solve mixed integer linear programming problems.

Thank you.