Computer Aided Applied Single Objective Optimization
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Lecture - 23
Case Study: Production Planning

Welcome. In this session, we will be looking at a Case Study involving Production Planning. So, far we have studied 5 meta heuristic techniques and we have also discussed how to handle constraints right. As part of this case study, we will be looking at the production planning problem.

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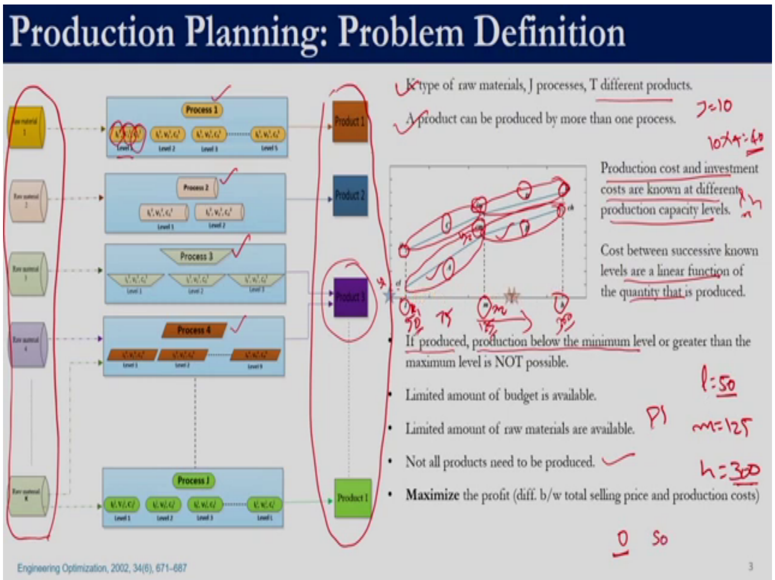

The production planning problem consists of K raw materials and using those raw materials, we can produce different types of products right. So, for the conversion of this raw material into a particular product, we process that raw material right. So, there are various types of processes. So, this is process 1, process 2, process 3 and all the way up to we have J processes. So, K raw materials when processed can give us product right. So, some of these processes can produce identical product right.

So, for example, process 3 and process 4 if you see, it produces product 3; so that is possible right. For each of this process, we know the production cost right and the investment cost for a specified level right. So, for example, if we produce l1 1 right, the investment cost is v 11 and the production cost is c 1 1. So, similarly we have different levels at which we know the production and investment cost. So, for example, let us say for a particular process, we know that the production cost is known at so let us say level l, level m, level h right.

So, it is like if we produce 50 kg per day, we know that this is the production cost. If we produce let us say 125 kgs per day, the cm denotes it is production cost. If we produce let us say 300 kgs per day, the ch denotes the production cost. So, the production costs are known at these 3 levels; level 1, level 2, level 3 right. So, that is what this level corresponds over here level, 1 level 2, level 3 and all the way up to it can have any number of levels. Similarly, at this production level we also know the investment cost right. So, il is the investment cost to produce 50 kgs per day, im is the investment cost to produce 125 kgs per day and ih is the investment cost to produce 300 kgs per day.

So, this is known. So, now, if we have to find out the production cost, let us say at 75 right. So, we know the production cost at l at 50 and we know the production cost at 125. So, the production cost at 75 is a linear function right. So, that is part of the problem definition. We are not assuming it right. So, as part of the problem definition that has been given. So, in that case we know this point. So, if we call this point as let us say x 1, x 2 right. So, cl is nothing but y 1 and cm is nothing but y 2 right. So, we know x 1 y 1, we know x 2 y 2 and if we are told that it is a linear function between x 1 and x 2, we can calculate the production cost for any value between 50 and 125 right.

So, similarly between m and h, the production cost is linear just like production cost the investment cost is also linear. But the slope and intercept are different as shown in the figure. So, these 2 put together right is a piecewise linear function, it is linear in this range and it is linear in this range. So, similarly for investment cost, it is linear in this range and it is linear in this range. So, this is a piecewise linear function.

So, here if we see right. So, if we have 3 levels right l, m and h right and if we have J process; here, we have shown that each process can have different number of levels right. To simplify let us say the number of processes that we have is let us say 10 and for each process, we know the production and investment cost let us say at 3 levels l, m, h right. So, here for every process if you see we will have 4 lines right line A, line B, line C and line D right. So, if we have a 10 processes, we are going to have 10 into 4, 40 such lines.

So, this we have seen that there are K type of raw materials, J processes and which can produce T different products right. So, a product can be produced by more than one process that is what we showed you for this product 3 right. The production cost and investment cost are known at different production capacity levels right. So, in this figure if they were known at l, m and h right. Cost between successive known levels are a linear function of the quantity that is produced right.

So, this also we have discussed that between l and m, it is a linear function; between m and h, it is a linear function; should there be some other level between h and that particular level again the function is linear right. So, here our job is to find out right which of the products need to be produce right. So, for each product we know that there are different types of processes which we can use to produce it and depending upon how much we are producing right, the production cost and the investment cost is going to be different right. So, here there is no compulsion that we need to produce all the product right. So, not all the products need to be produced right. We are going to produce only those products which are profitable.

So, here the constraint is that if a product is produced right, the production below the minimum level. So, the minimum level in this figure is l right. So, if we say let us say l is 50, m is let us say 125 and h is let us say 300. Let us say this is for process 1. So, if we decide to produce from P 1 right, we can choose not to produce anything from P 1 0 is allowed right.

But if it is greater than 0, then it has to be definitely greater than 50 and less than 300. Here if you see this figure right.

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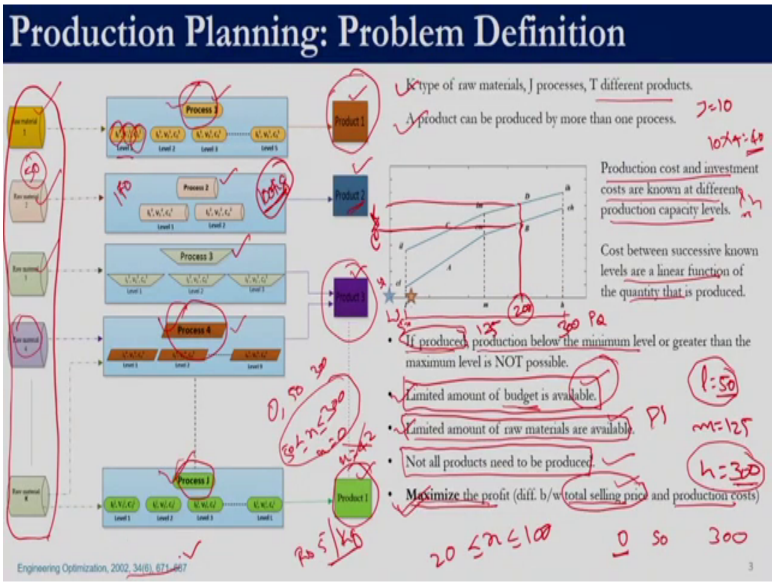

So, we do not have a procedure to calculate the production which is greater than 0, but less than l right, we have a procedure to calculate the cost between l and h right, but we do not have a procedure which can help us to determine the cost which is greater than 0, but less than l right. So, if produced this is important, if produced it is not necessary that we produce. If we decide to produce, then we need to make sure that the production is greater than or equal to 50 and less than equal to 300 right. So, that is a unique feature about this problem. Most of the problems that you would find in conventional optimization textbooks, mostly they would have problems which will give you domain as let us say between 0 to 100 or let us say between 20 to 100 right.

Here, we have a domain where in 0 is permissible right. So, 0 is permissible and any value greater than 50 and less than 300 is permissible. So, x is between 50 and 300 right and x can be also 0 right; but x cannot be let us say 42 because 42 is greater than 0, but it is less than 50. So, the objective here is to maximize the profit. Let us say the selling price of product 1 is 5 rupees per kg right. So, if you decide to produce 10 kgs, then the revenue that we will earn is 5 into 10 all right. So, that selling price is known. What we need to decide is how much to produce. Once we decide how much to produce, we can calculate what would be the revenue right.

So, that is the total selling price right and as we have discussed depending upon how much I choose to produce. So, let us say it is 50 here, this is 125 and this is 300; let us say if we decide to produce 200 right. So, then this is the production cost from this particular process right. So, the sum total of all the production cost. So, we may even choose to produce from process 1, process 4 and let us say process J right. So, the total production cost is production cost incurred here, production cost incurred here and the production cost incurred here. Similarly, the total revenue which is earned is revenue earned by selling product 1, revenue earned by selling product 3 right because process 4 produces product 3 and the revenue incurred by selling product I right.

So, the profit is given by the difference between the total selling price and the production cost right. So, here if you think about the problem, if there are no constrains right, then one may choose to employ all the processes right; all the processes and produce the maximum quantity that will fetch the maximum amount of profit right. So, but here we have 2 constraint; the budget to establish this plant right. So, right now these are our options right; it is not like this plant is already existing right. So, we need to choose what to produce. Depending upon we decide to produce, we will employed appropriate process and establishing a process itself is going to incur us a cost that cost is nothing but investment right.

So, here if we decide to produce 200, then the production cost is what we have shown here right. If we decide to produce 200, then the investment cost is to found from this line. This is going to be our investment cost. So, here the y axis is cost right and the x axis is the production quantity. Depending upon which are the processes that we select, we can also determine what is the total investment that is required right. So, the total investment that is required should be less than or equal to the budget available. So, it is more like let us say 500 rupees are available and you are going to go and shop something. So, whatever you decide to shop should be less than or equal to 500 right. So, that is what this constraint is there.

Similarly, not all of this raw materials are available in abundance right. So, limited amount of raw materials are only available. Let us say we decide to produce 100 kgs from process 2. So, the amount of raw material that would be required to produce this quantity, it should be less than or equal to what is available. So, let us say if only 50 kgs are available of raw material 2 and producing 100 kgs of the product to requires, let us say 170 kgs of raw material 2 right. Then, we cannot even produce 100 kgs of product 2 from process 2. So, whatever the total amount of raw material is required should be less than equal to what we have over here right. So, that is another constraint.

So, these 2 are the explicit constraints that we have right; Budget constraint and Raw material constraint. Again, raw material constraint is a type of constraint; we can have one constraint on raw material 1, one constraint on raw material 2, one constraint of raw material 3. If raw material 4 is available in unlimited quantity, then we will not have a constraint on raw material 4 right. So, it depending upon how many raw materials we have right and their availability, we may have that many constraint right and here you need to remember that the problem does not require that all products we produce right. We are given the choice to produce the products that are most profitable.

So, this is the definition of production planning problem right. So, for additional details you can look into this reference right. So, in this reference they use mathematical programming approach to solve this problem. If you remember for this course, we had classified optimization techniques into 2 categories right; one is meta heuristic techniques and the other one is mathematical programming techniques right.

So, since we have only looked into meta heuristic techniques as of now right, we will see how to solve this problem right with the help of meta heuristic techniques. The solution procedure that we are going to discuss in this lecture is not given in that paper, but the problem definition is exactly the same that was handled in this research work. So, this case study is reported in that research work right.

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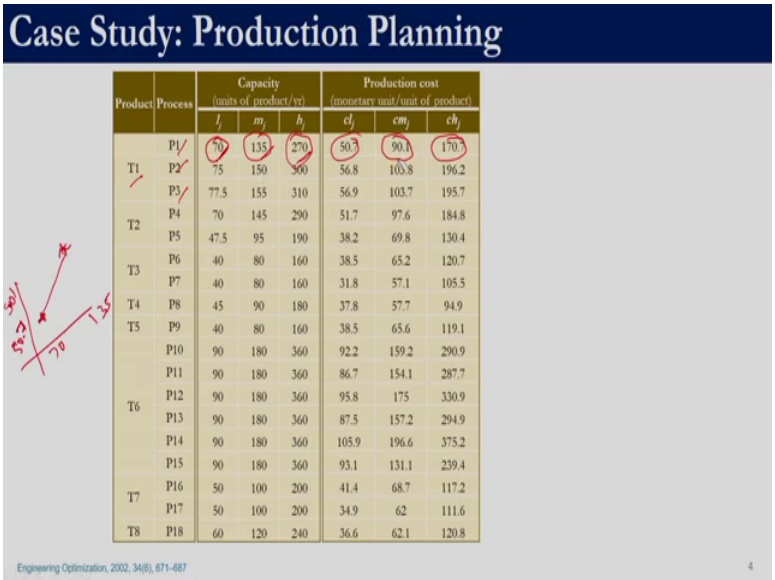

Product	Process	Capacity (units of product/yr)			Production cost (monetary unit/unit of product)		
		l_j	m_j	h_j	cl_j	cm_j	ch_j
T1	P1	70	135	270	50.7	90.1	170.7
	P2	75	150	300	56.8	105.8	196.2
	P3	77.5	155	310	56.9	103.7	195.7
T2	P4	70	145	290	51.7	97.6	184.8
	P5	47.5	95	190	38.2	69.8	130.4
T3	P6	40	80	160	38.5	65.2	120.7
	P7	40	80	160	31.8	57.1	105.5
T4	P8	45	90	180	37.8	57.7	94.9
T5	P9	40	80	160	38.5	65.6	119.1
T6	P10	90	180	360	92.2	159.2	290.9
	P11	90	180	360	86.7	154.1	287.7
	P12	90	180	360	95.8	175	330.9
	P13	90	180	360	87.5	157.2	294.9
	P14	90	180	360	105.9	196.6	375.2
	P15	90	180	360	93.1	131.1	239.4
T7	P16	50	100	200	41.4	68.7	117.2
	P17	50	100	200	34.9	62	111.6
T8	P18	60	120	240	36.6	62.1	120.8

So, here they have 24 products right. So, as of now in this slide, we have shown you only 8 products. So, T 1 to T 8 are the products right. So, T 1 can be produced by P 1, P 2 and P 3; all the 3 process are capable of producing T 1 right. So, we may choose to produce the product T 1 from P 1, as well as P 2, as well as P 3 right. So, as of now there is no requirement that a product has to be produced from only 1 process right. So, all these 3 process are capable of producing it, if required we can use all the 3 processes.

So, these are the capacity levels right at which the production cost and investment cost are known right. So, for units of product, we can take it as tons right. So, if we decide to produce

70 tons per year, we know the production cost and the investment cost. If we decide to produce 135 tons per year, the production cost and the investment cost is known right. And if we decide to produce 270 tons per year, we know the production cost and the investment cost right. So, here if we produce 70 tons per year right, the production cost is 50.7 monetary units per unit of product right. So, if we decide to produce let us say 135, the production cost is 90.1. If we produce 270, the production cost is 170.7.

Remember a common misconception is that we need to decide between 70, 135 and 270. So, that is not correct right. We can still choose to produce anything that is 70 or greater than 70, but less than or equal to 270 right. So, then in that case how do we determine the production cost is using that piecewise linear function which we have discussed right. So, if it is 70. So, 70 is this 50.7 on the y axis right.

So, this is one data point and other data point is 135 right and 90.1. If we decide to produce something which is greater than or equal to 70 and less than or equal to 135, we can use this linear line right. So, this point is known; for this point the production cost is 90.1 right. So, this this is also known. So, in between 70 and 135 we can still estimate the production cost right. Similarly, between 135 and 270 we know the production cost is 90 and 170.7.

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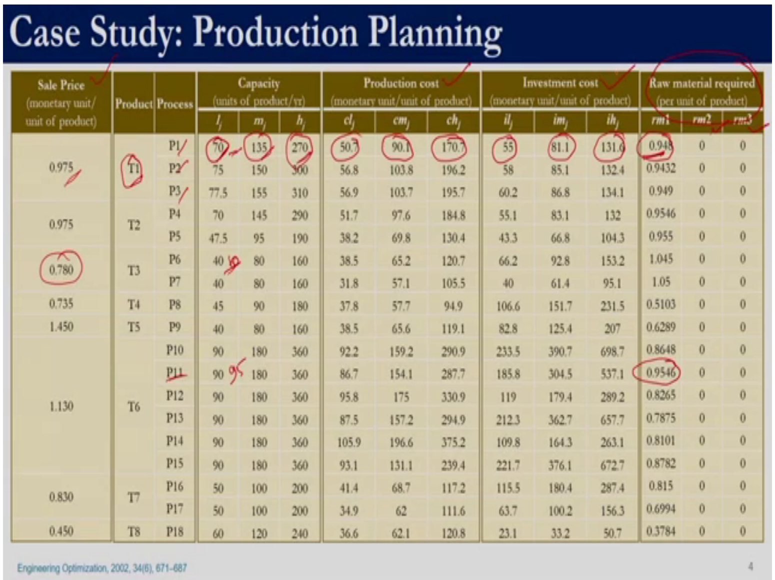

Case Study: Production Planning

Sale Price (monetary unit/ unit of product)	Product	Process	Capacity (units of product/yr)			Production cost (monetary unit/unit of product)			Investment cost (monetary unit/unit of product)			Raw material required (per unit of product)		
			l_j	m_j	h_j	cl_j	cm_j	ch_j	il_j	im_j	ih_j	rm1	rm2	rm3
0.975	T1	P1	70	135	270	50.7	90.1	170.7	55	81.1	131.6	0.948	0	0
		P2	75	150	300	56.8	103.8	196.2	58	85.1	132.4	0.9432	0	0
		P3	77.5	155	310	56.9	103.7	195.7	60.2	86.8	134.1	0.949	0	0
0.975	T2	P4	70	145	290	51.7	97.6	184.8	55.1	83.1	132	0.9546	0	0
		P5	47.5	95	190	38.2	69.8	130.4	43.3	66.8	104.3	0.955	0	0
0.780	T3	P6	40	80	160	38.5	65.2	120.7	66.2	92.8	153.2	1.045	0	0
		P7	40	80	160	31.8	57.1	105.5	40	61.4	95.1	1.05	0	0
0.735	T4	P8	45	90	180	37.8	57.7	94.9	106.6	151.7	231.5	0.5103	0	0
1.450	T5	P9	40	80	160	38.5	65.6	119.1	82.8	125.4	207	0.6289	0	0
1.130	T6	P10	90	180	360	92.2	159.2	290.9	233.5	390.7	698.7	0.8648	0	0
		P11	90	180	360	86.7	154.1	287.7	185.8	304.5	537.1	0.9546	0	0
		P12	90	180	360	95.8	175	330.9	119	179.4	289.2	0.8265	0	0
		P13	90	180	360	87.5	157.2	294.9	212.3	362.7	657.7	0.7875	0	0
		P14	90	180	360	105.9	196.6	375.2	109.8	164.3	263.1	0.8101	0	0
		P15	90	180	360	93.1	131.1	239.4	221.7	376.1	672.7	0.8782	0	0
0.830	T7	P16	50	100	200	41.4	68.7	117.2	115.5	180.4	287.4	0.815	0	0
		P17	50	100	200	34.9	62	111.6	63.7	100.2	156.3	0.6994	0	0
0.450	T8	P18	60	120	240	36.6	62.1	120.8	23.1	33.2	50.7	0.3784	0	0

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So, similarly the investment cost is given right. So, if we decide to produce 70, the investment cost is 55 right. So, 55 monetary units per unit of product right. If we decide to produce 135, the investment cost is 81.1. If we decide to produce 270, the investment cost is 131. So, if we decide to produce anything between 135 to 270 the investment cost can be calculated by the points connecting 135 comma 81.1 and 270 comma 131.6 right.

So, there are 3 different type of raw material which are required right. So, process 1 to process 18, do not require raw material 2 and raw material 3 right, only raw material 1 is required. So, the amount of raw material required is 0.948. So, if we decide to produce 1 unit of product 1 from process 1, we require 0.948 units of raw material 1 right.

So, if we decide to produce let us say 95 over here right, the amount required for 1 unit is 09546. So, the amount required for 95 would be 95 into 0.9546 right. So, raw material 2 and raw material 3 are not required for any of the first 18 processes and we also know the selling price of each of the product right. So, product T 1 for 1 unit of product T 1 can be sold for 0.975 monetary units right.

So, if we decide to produce let us say 60 over here right, the 1 unit can be sold for 0.780. So, for 60 units, it will be 60 into 0.780 right. So, the selling price and the raw materials can be directly calculated, only for the production cost and the investment cost, we need to use that piecewise linear function to determine the actual production and investment cost.

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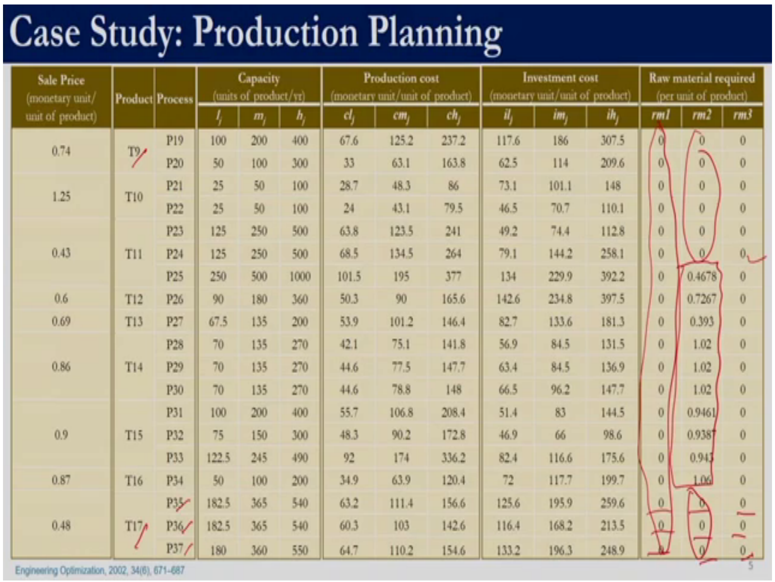

Case Study: Production Planning

Sale Price (monetary unit/ unit of product)	Product	Process	Capacity (units of product/yr)			Production cost (monetary unit/unit of product)			Investment cost (monetary unit/unit of product)			Raw material required (per unit of product)		
			l_j	m_j	h_j	cl_j	cm_j	ch_j	il_j	im_j	ih_j	rm1	rm2	rm3
0.74	T9	P19	100	200	400	67.6	125.2	237.2	117.6	186	307.5	0	0	0
		P20	50	100	300	33	63.1	163.8	62.5	114	209.6	0	0	0
1.25	T10	P21	25	50	100	28.7	48.3	86	73.1	101.1	148	0	0	0
		P22	25	50	100	24	43.1	79.5	46.5	70.7	110.1	0	0	0
0.43	T11	P23	125	250	500	63.8	123.5	241	49.2	74.4	112.8	0	0	0
		P24	125	250	500	68.5	134.5	264	79.1	144.2	258.1	0	0	0
		P25	250	500	1000	101.5	195	377	134	229.9	392.2	0	0.4678	0
0.6	T12	P26	90	180	360	50.3	90	165.6	142.6	234.8	397.5	0	0.7267	0
0.69	T13	P27	67.5	135	200	53.9	101.2	146.4	82.7	133.6	181.3	0	0.393	0
0.86	T14	P28	70	135	270	42.1	75.1	141.8	56.9	84.5	131.5	0	1.02	0
		P29	70	135	270	44.6	77.5	147.7	63.4	84.5	136.9	0	1.02	0
		P30	70	135	270	44.6	78.8	148	66.5	96.2	147.7	0	1.02	0
0.9	T15	P31	100	200	400	55.7	106.8	208.4	51.4	83	144.5	0	0.9461	0
		P32	75	150	300	48.3	90.2	172.8	46.9	66	98.6	0	0.9387	0
		P33	122.5	245	490	92	174	336.2	82.4	116.6	175.6	0	0.943	0
0.87	T16	P34	50	100	200	34.9	63.9	120.4	72	117.7	199.7	0	1.06	0
0.48	T17	P35	182.5	365	540	63.2	111.4	156.6	125.6	195.9	259.6	0	0	0
		P36	182.5	365	540	60.3	103	142.6	116.4	168.2	213.5	0	0	0
		P37	180	360	550	64.7	110.2	154.6	133.2	196.3	248.9	0	0	0

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So, this is from product 9 to 17 right. So, here if you see raw material 1 is not required right and raw material 2 is not required for these processes. Raw material 2 is required for these particular processes and none of these processes use raw material 3 right. So, for example, for this processes if you see P 35, P 36, P 37, it does not use any of this 3 raw materials right. So that means, it will uses some other raw material on which we do not have any constrains right. So, here we are only considering those raw materials on which we have availability constrains right.

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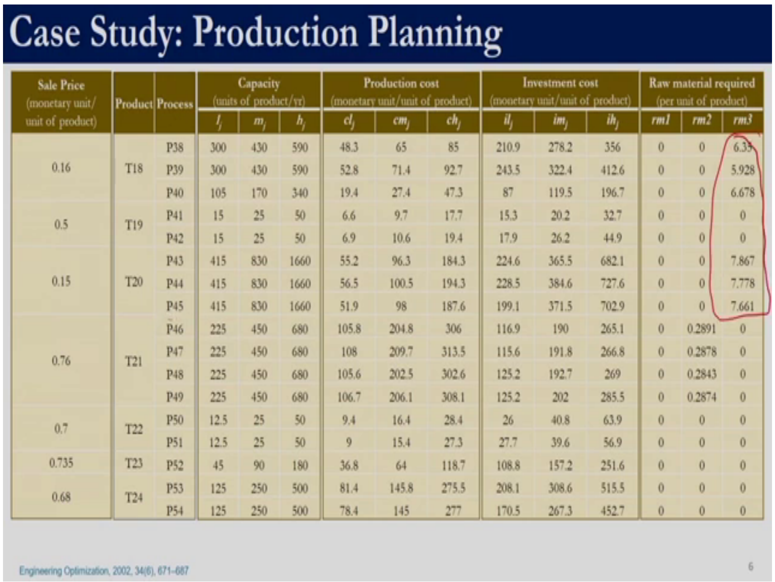

Sale Price (monetary unit/ unit of product)	Product	Process	Capacity (units of product/yr)			Production cost (monetary unit/unit of product)			Investment cost (monetary unit/unit of product)			Raw material required (per unit of product)		
			l_j	m_j	h_j	cl_j	cm_j	ch_j	il_j	im_j	ih_j	rm1	rm2	rm3
0.16	T18	P38	300	430	590	48.3	65	85	210.9	278.2	356	0	0	6.35
		P39	300	430	590	52.8	71.4	92.7	243.5	322.4	412.6	0	0	5.928
		P40	105	170	340	19.4	27.4	47.3	87	119.5	196.7	0	0	6.678
0.5	T19	P41	15	25	50	6.6	9.7	17.7	15.3	20.2	32.7	0	0	0
		P42	15	25	50	6.9	10.6	19.4	17.9	26.2	44.9	0	0	0
0.15	T20	P43	415	830	1660	55.2	96.3	184.3	224.6	365.5	682.1	0	0	7.867
		P44	415	830	1660	56.5	100.5	194.3	228.5	384.6	727.6	0	0	7.778
		P45	415	830	1660	51.9	98	187.6	199.1	371.5	702.9	0	0	7.661
0.76	T21	P46	225	450	680	105.8	204.8	306	116.9	190	265.1	0	0.2891	0
		P47	225	450	680	108	209.7	313.5	115.6	191.8	266.8	0	0.2878	0
		P48	225	450	680	105.6	202.5	302.6	125.2	192.7	269	0	0.2843	0
		P49	225	450	680	106.7	206.1	308.1	125.2	202	285.5	0	0.2874	0
0.7	T22	P50	12.5	25	50	9.4	16.4	28.4	26	40.8	63.9	0	0	0
		P51	12.5	25	50	9	15.4	27.3	27.7	39.6	56.9	0	0	0
0.735	T23	P52	45	90	180	36.8	64	118.7	108.8	157.2	251.6	0	0	0
0.68	T24	P53	125	250	500	81.4	145.8	275.5	208.1	308.6	515.5	0	0	0
		P54	125	250	500	78.4	145	277	170.5	267.3	452.7	0	0	0

So, this is the last section of the data right. So, here raw material 3 is required for these processes.

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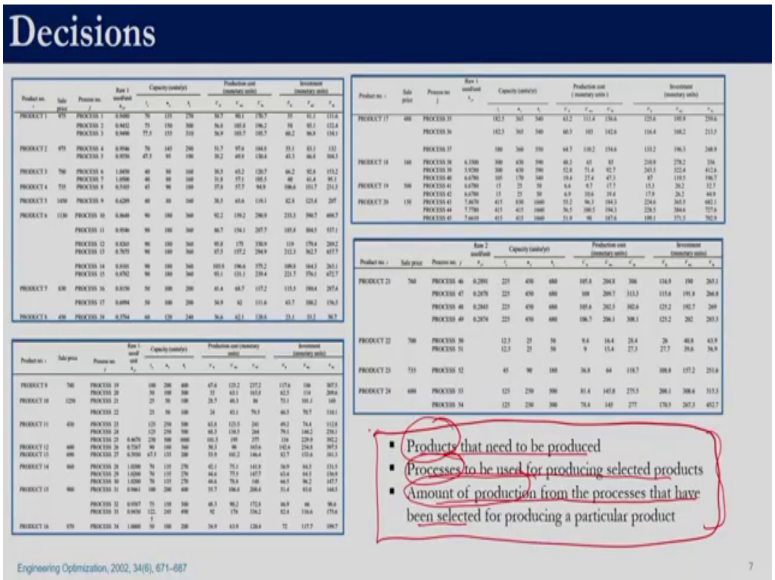

So, this is the same data which we have shown you right. So, over here we now need to decide what are the products that need to be produced right. So, we have the choice to produce the product right. So, that is why it is an optimization problem. We need to decide what are the products that we are going to produce right and for each product that we decide to produce, we need to select the process also. So, as you have seen, so some of the products have more than one process which can produce it right. So, we need to select the process and we also need to decide on how much quantity we are going to produce right.

So, the amount of production from the processes that have been selected. Products have to be selected, processes have to be selected and then we need to also select amount of production right. So, these are the decisions that we need to make right. Such that the profit that we obtain from whatever we decide to produce should be maximum and whatever production plan that we come up with should require an investment which is less than the available budget and the raw material required for the production plan should also be less than or equal to the raw material that is that is available.

So, now, we have discussed this right. So, this is the problem right. So, recollect the metaheuristic techniques that we had discussed right. So, for that first thing we need to give is lower bound, upper bound and the fitness function right. So, now the question is what are our decision variables right. Only if we know our decision variables, we will be able to give the lower bound and upper bound right. So, now, we need to resort that what would be our decision variables right, whatever we have discussed here this is what is required. So, we will be discussing 2 approaches.

So, for the first approach is the one that usually strikes us particularly when we see these 3 things. So, we will discuss what is the disadvantage of that approach right and then we will be using the approach with which we will be able to solve this problem. So, for metaheuristic techniques to work well right, it is necessary that we choose the decision variables carefully. We need to select the decision variables and the constraint such that it captures the complete definition of the problem right.

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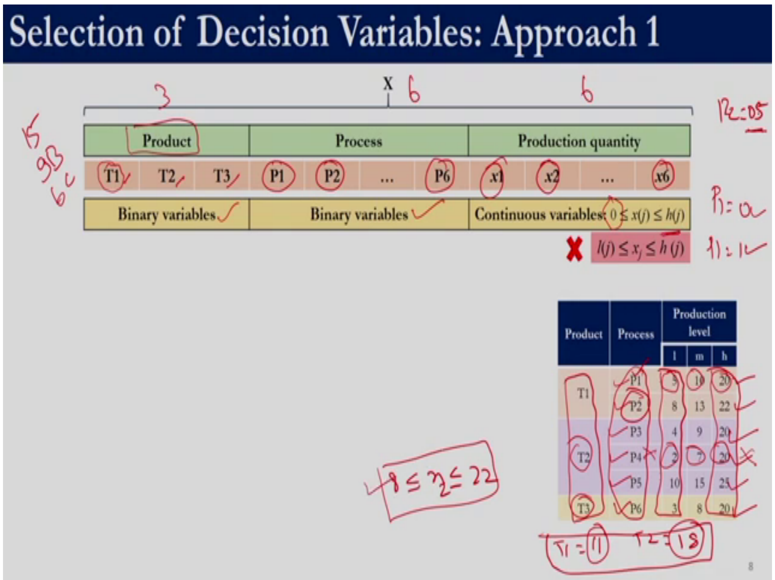

And we should try to avoid excessive decision variables. So, in this case just for easier understanding, we are working with only 3 products right; product T 1, T 2, T 3. So, product T 1 can be produced from P 1, P 2. Product T 2 can be produced from process 3, process 4, process 5. Product T 3 can be produced from processes 6. And these are the levels at which their production cost and investment cost are known.

Production cost and investment cost for process 1 are known at 5, 10 and 20 right. Similarly, the production and investment cost for process 4 are known at 2, 7 and 20. So, anything above 20 is not permissible; anything below 2 is not permissible right, but 0 is permissible. If he decides not to produce a product that is fine right. So, there is no compulsion that we need to produce a product.

So, the production quantity can be 0, but it cannot be greater than 0, but less than 2. One way to approach this problem is to define 3 binary variables right, indicating whether a product is produced or not right. So, whether T 1 is produced or not, this is our decision variable right. So, any of the metaheuristic techniques that we discussed will be required to give us the values right. So, it has to give either 0 or 1 right. So, this is the first section. The second section is we have another 6 binary variables right.

So, for example, for this process if I just say that T 1 is required in let us say 11 quantity, T 2 is required in let us say quantity 18 right. So, if we say this is the production plan, then it is not a complete production plan. We are only saying that we want to produce 11 units and 18 units. We are not saying which of the process, we will use and depending upon the process that we use the production cost and the investment cost should be varying right.

So, we also need to make a call on the processes right. So, here again we defined 6 binary variables right P 1, P 2, P 3, P 4, P 5 and P 6. So, we have define variable which are binary variables, those variables can take only 0 or 1 right. It cannot take any value in between 0 and 1; it cannot take a value of 0.5 right; it has to be either 0 or 1. Because if I say P 2 is equal to 0.5, it does not help me to infer whether the process is selected or not right. So, P 1 is equal to 0, we will say that the process is not selected; P 1 is equal to 1, the process is selected right. So, if it is 0 or 1, we can decide whether the process is being used or not.

So, it is should not contain continuous values right, it has to be either 0 or it has to be 1. So, again this is not sufficient right, we also need to know what is the production quantity right. So, since there are 6 processes, we will ask the algorithm to give 6 values which will indicate the amount that is being produced right. So, that would be 6 variables. So, right. So, here it is 3 binary variables, again 6 binary variables and 6 continuous variable right. So, we have 15 variables right. Out of 15 variables 9 are binary right, the first 3 and the next 6 are binary and the rest of the 6 are continuous variable right.

So, this binary variables, the lower and upper bound is clear because it is a discrete decision that we need to make whether the process is used or not and the product is produced or not right. So, those are binary variables. Whereas, for this continuous variables, the question is what should be the domain right. The upper bound is clear right. So, the upper cannot exceed these values 20, 22, 20, 20, 25 and 20 right. So, the upper bound is clear that is h j right. So, it is j indicates the jth process. So, there are 6 variables; the upper bound of the first variable is 20, the upper bound of the second variable is 22, the upper bound of the third and fourth variable are 20, the upper bound of fifth variable is 25. And the upper bound of the sixth variable is 20 right. So, the upper bound is clear.

Coming to the lower bound, so the question is should the lower bound be 0 as we have shown here or the lower bound should be 5, 8, 4, 2, 10, 3. So, if we choose this as lower bound right, then we are implicitly enforcing a constraint that every product has to be produced. So, if we consider process 2 right. So, if we declare this as the bounds right. So, then x 2 cannot take a value of 0 right. If x 2 cannot take a value of 0, we are implicitly forcing that process 2 has to be used right which is not part of the problem definition right.

So, since the problem definition allows a process not to be used, we need to give lower bound as 0 right. So, this bound is not allowed right. So, this is wrong. If we are enforcing this right, then we are implicitly enforcing the constraint that every process has to produce a product which has to be between its lower and upper bound right. So, here we are for process 2, we are enforcing though the problem definition does not have the requirement that process 2 has to be used. If we take this as bound, then we are enforcing that constraint that process 2 should produce at least 8 and at most 22.

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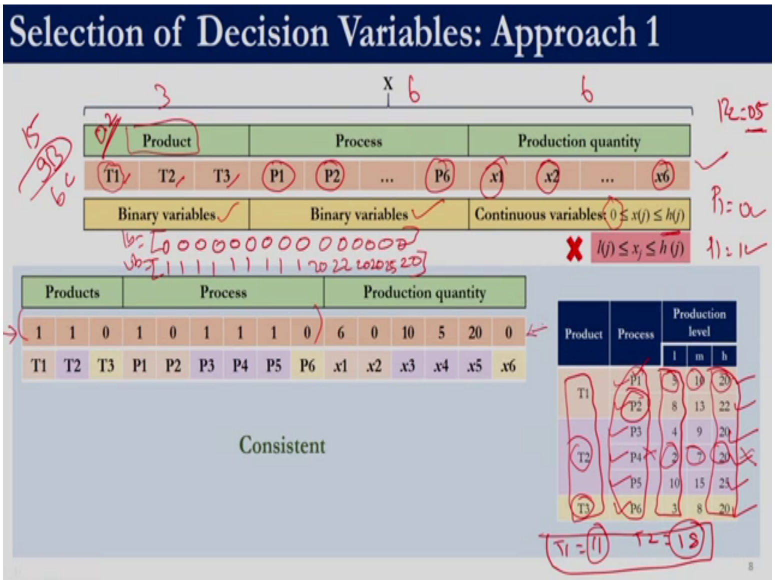

Now, if we are let us say if we have to use TLBO. So, the lower bound is we will give 0 0 0; 9 times right and then we will give again 6 another 0's right and the upper bound is 9 times 1 right and then hj right. So, the upper bound would be 20, 22, 20, 20, 25 and 20. So, this is the lower bound and this is the upper bound right. So, all the algorithms which we have discussed are guaranteed to give values between the lower and upper bound right.

So, for the first 9 variables, if you think about it. It will definitely give a value between 0 and 1, but there is nothing in the algorithm which will make sure that the value that it returns is an 0 or 1. So, for example, consider the teaching phase right. So, in teaching phase when you employee that equation, you may end up with the value of 0.2 right. So, that is what is going to be fed to the fitness function right, but a value of 0.2 for the first decision variable over here right. So, let us say value of 0.2 does not make any sense, it has to be 0 or 1. So, that is the first problem with this encoding. So, this is how we are coding the solution right. So, this is called as encoding right.

The first problem is the algorithms that we discussed are for continuous variable and here, we are having a binary variables right. So, that is the first issue. So, one way to overcome that issue is we will say whatever values are written by any of the 5 metaheuristic techniques that we discussed right, we will round it off right. So, if it is 0.2, we will round it to 0; if it is 0.7, we will round it to 1. So, that way we can handle that that problem. So, let us look at some of the other issues right. So, for example, consider this solution, let us say we get this from TLBO, either we get this or whatever we get for the first 9 variables, we round it off right. So, then let us say this is our solution; 1 1 0 1 0 1 1 1 0 and then 6 0 10 5 20 0 right. So, this is what we will get from the algorithms. So, what does this mean right?

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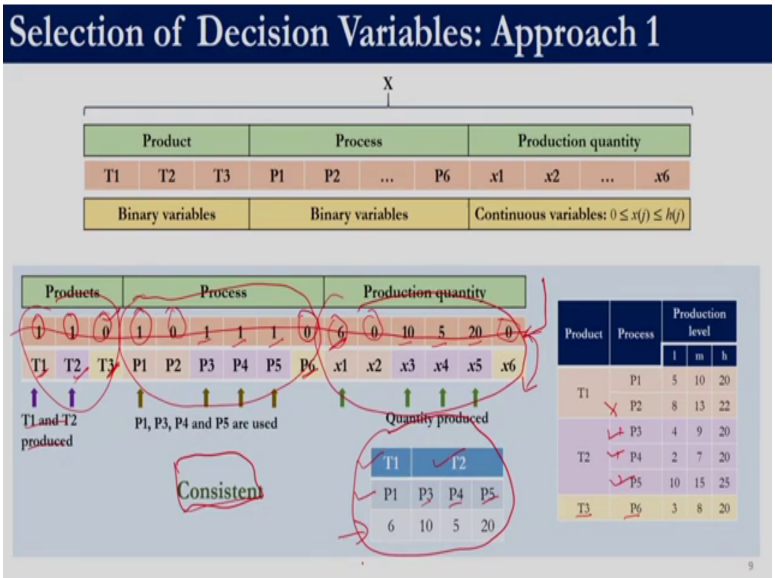

So, this solution means that product T 1 and T 2 are produced right because here we have 1 right. Product T 3 is not produced right. So, since product T 3 is not produced, P 6 has to be 0, here if you see there is only 1 process. So, if this is 0, this one has to be 0 and in this case, it happens to be 0. So, if T 3 is 0, P 6 is 0 means that process 6 not selected. So, if process 6 is not selected, the quantity that we produced from process 6 should be 0 right.

So, over here also we have a 0 right. So, for product 3, it is consistent. Let us check for product 1 right. So, for product 1, we are using the process 1 right; process 2, we are saying that we will not select process 2 right. So, since we are selecting process 1, we need also know how much we are producing. So, this is let us say 6 right.

So, process 2 we are not using right. So, 0. So, x 2 is also 0 which is consistent and if we talk about product 2 right P 3, P 4, P 5 right. So, all of the processes are selected. So, product 2 is being produced from process 3, from process 4, from process 5; all the tree process and all of these are non-zero values 10, 5, 20 right.

So, in this case what the encoding that we chose is consistent right. So, what information that we derived from here consistent with this and these 2 information are consistent with this right. So, that is why we have termed this solution as it is consistent right. So, what this solution basically tells is that product T 1 and product T 2 are produced.

We are using process 1 for product 1 right. We are not using process 2 and for product 2, we are using all the 3 processes right P 3, P 4, P 5 and this is the production quantity right 6, 10, 5, 20 right. So, if this happens if we get a solution like this right, we can decode that solution right. So, now, this is the decoded solution. So, whatever metaheuristic techniques, we have discussed, they will give a solution like this what we have mentioned over here. So, this is the solution that it will return right and that solution we can decode right.

So, now, we know the decoded solution. So, now we can go and calculate what is the production cost, what is the investment cost, what is the profit, whether we are satisfying the raw material constraint and all those things right.

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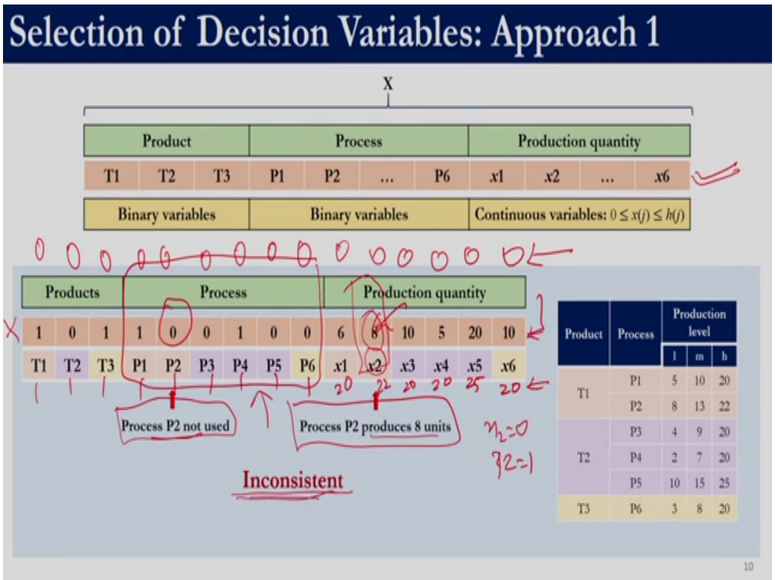

So, but assume had we got something like this right. So, remember our lower bound was 0; 9 times, 1 2 3 4 5 6 7 8 9. Our upper bound was 1. For all of this is upper bound was 1 and the lower bound for this 6 were also 0. The upper bound over here was. So, 20, 22, two times 20, 25 and 20 right. So, this is the lower bound; this is the upper bound. So, it is perfectly possible that we get a solution like this as given over here right. So, this is the decision variable which we obtain from any of the metaheuristic techniques right.

So, now, let us see what does this solution indicate right? So, this solution indicates that process P 2 is not used because it is 0 right. So, that was our encoding scheme. If it is 0, it means we are not using that right. Whereas, look at x 2 right. x 2 what does it denote? How much we are producing from process 2 which is 8 right. So, here this part of the information is saying that we are not using process 2; whereas, this part of the information is saying that we are using process 2 and producing 8 units.

So, that is inconsistent, what we are getting from here and what we are getting from here are inconsistent right. Now, it becomes a problem has to which one is correct. Should we make this x 2 as 0 right because P 2 is 0 or we should make P 2 as 1 because x 2 is 8? Only then, this information would become consistent and we will be able to calculate the production cost right. If we select this encoding scheme, so we may end up with this inconsistency right.

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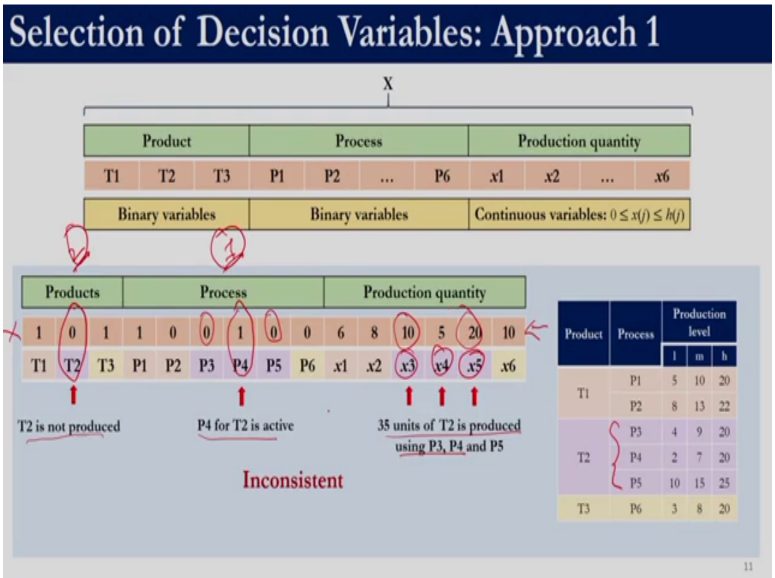

So, couple of more example for inconsistent solution. Again, this is a solution that we can get from metaheuristic technique. Let us say with this is the solution which we get from metaheuristic technique and the first 9 values were not binary, we rounded it off so that we get binary right. So, over here if we see this part says that T 2 is not produced right; whereas, P 4 is 1 right.

So, the algorithm does not know that we are solving a production planning problem and that if T 2 is 0, then P 4 has to be 0. The algorithm does not know; for the algorithm, these are decision variables right. So, nothing in the algorithm says that the second variable and the seventh variable should be consistent right. So, the algorithm will give values between the lower and upper bounds, that is what we have discussed right.

It if it is to give values only between the lower and upper bounds, we may end up in situations like this, wherein the first part of the information is not consistent with the second part right and over here if you see x 3, x 4, x 5 as we discussed indicate the production from process 3, process 4 and process 5 right. So, here we are saying T 2 is not produced right, but the process 4 is active, process 3 and process 5 are not active. Again, we have a complication here right that we are saying process 3 is not active, but we are saying we are producing 10 from process 3; process 5 we are saying it is not active, but we are producing 20 from process 5 right. So, again it leads to an inconsistent solution.

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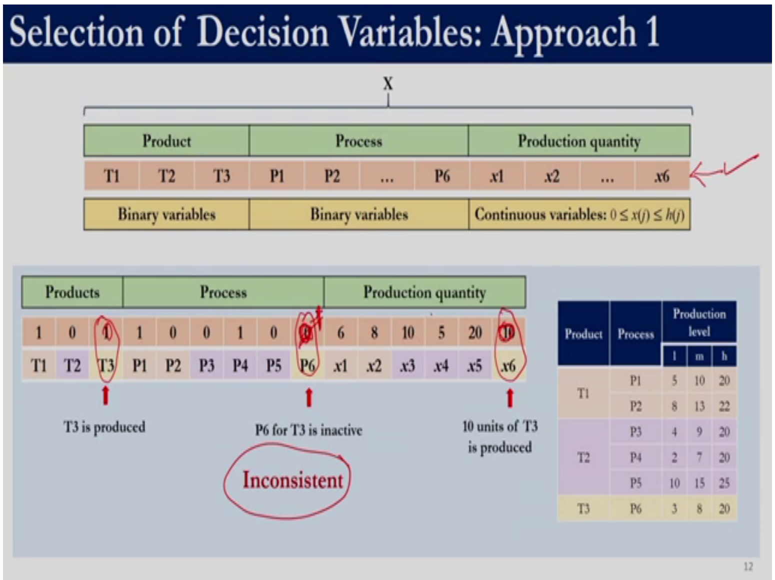

So, similarly this is another example of an inconsistent solution right. Here the first part of the information tells that we can produce T 3, but P 6 is 0 and x 6 is 10. So, now, the question is should we make this as 1 or we need to trust that this 0 as correct and make this as 0 and make this is 0 right, that call needs to be taken. So, that is the problem with this approach. Remember the problem does not tell us what are the decision variables. It is we who are to identify what are the decision variables right.

So, if we decide to choose the decision variables in this pattern right, then we may end up in situations where in the solution that we are getting from the algorithm itself is not consistent right. So, now, we can either go ahead with this scheme right and as soon as we get the solution with make modifications to the solution and then, do something and calculate the cost or the question is there a better approach to select the decision variables right. So, so far whatever we discussed was Approach 1. So, now let us look at Approach 2.

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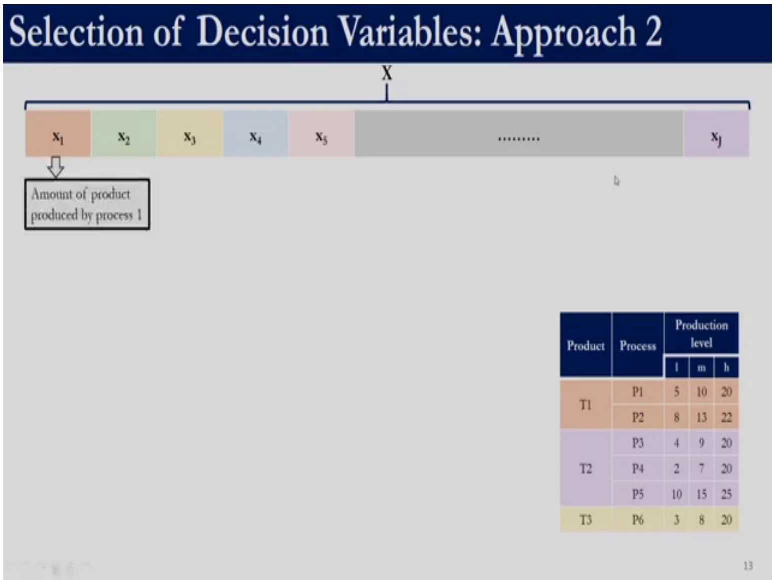

So, in approach 2, what we are saying is that we will have only j decision variables.

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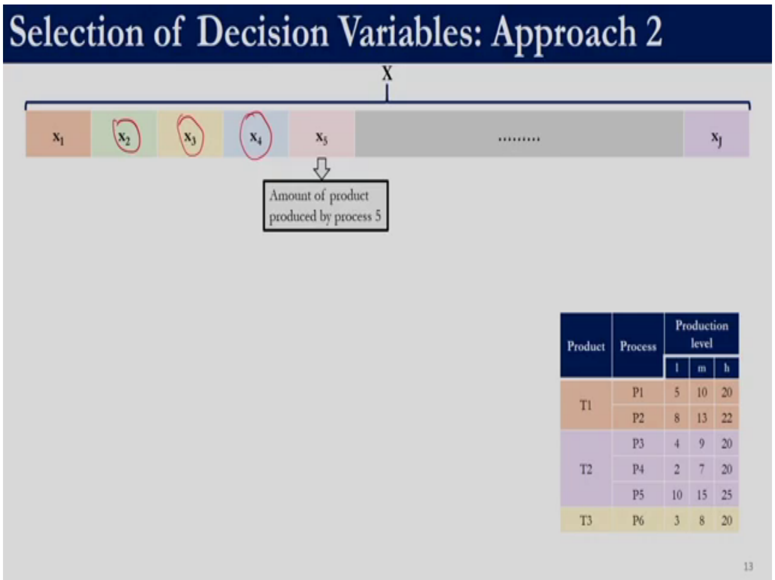

The first variable will denote how much we are producing from process 1; the second variable will tell us how much we are producing from process 2; third variable will tell us how much we are producing from process 3; fourth variable will tell us how much we are producing from process 4 and so on right.

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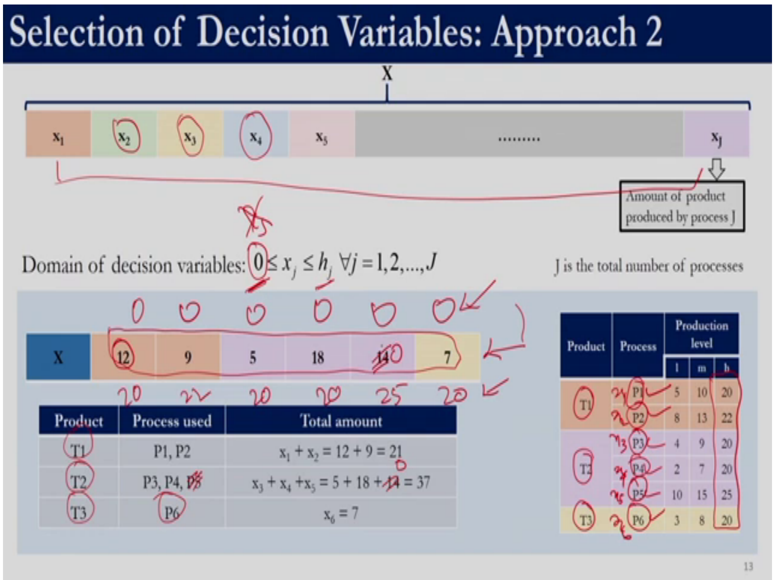

So, we are saying that now, we will define only j variables right. So, for this example they are saying that we will only define x 1, x 2, x 3, x 4, x 5, x 6. We will define only 6 variables right. We are not making an explicit call whether we are producing product 1 or not whether we are producing product 2 or not whether we are producing product 3 or not right and similarly, we are not making an explicit call whether we are using process 1, whether we are using process 2, process 3, process 4, process 5 and process 6. So, the first nine variables which you saw right that we have removed right. We are only sticking with the last 6 decision variables right.

So, the question is if I know these values, can I construct additional information which is required. So, for this 6 variable, the lower bound will be 0. We have discussed couple of times why the lower bound has to be 0 and not lj right. It should not be lj. The lower bound has to be 0 right because the problem definition does not require us to use all the processes right. So, to capture the feature that we can choose not to produce from a process is possible with this encoding only if we take the lower bound as 0.

And hj is the upper bound right. So, this this is the upper bound of the 6 decision variable right. So, if we give to the algorithm 0 0 0 0 0 0 as the lower bound and if we give this as the upper bound right 20, 22, 20, 20, 25 and 20. Let us say we give this lower bound; we this give upper bound. So, between this lower and upper bound, it will generate let us say end a random solutions right. So, let us say one of the solution looks like this 12, 9, 5, 18, 14 and 7. Now, the question is this sufficient amount of information for the problem. Remember when we define the problem decide, we need to decide whether product is produced or not; which is the process that we are going to use and how much we are going to produce.

So, in the previous approach the answer to this questions, we put it explicitly; the first part will tell whether a product is produced or not the second part will tell whether a particular process is used or not and the third part will tell us how much we are producing from that particular process right. So, with that encoding, we saw that we may end up in inconsistent solutions right.

So, rather than getting an inconsistent solution from an algorithm and then working it out, right now we are seeing whether if we choose only the last 6 that how much is being produced from each process, with that information can we answer all the 3 questions right. So, the answer to that is yes right. So, here if we see x 1 is 12 right. So, x 1 is 12 means we are using process 1 right. x 2 is 9 right which means we are using process 2 right.

All these are non-zero values which means we are using all the processes right. So, process 1, process 2 is used, process 3 process 4, process 5 is used, process 6 is used. So, how much we are producing it is explicitly obtained from the algorithm itself right, whether we are using a particular process or not can be derived from this information. So, let us say instead of 14, this had been 0 let us say right. So, this this is not over there; this is 0. So, which means we are using P 1, P 2 to produce product 1. We are using P 3, P 4 to produce product 2. We are using P 6 to produce product 3 and P 5 is not used; had it been 0 right.

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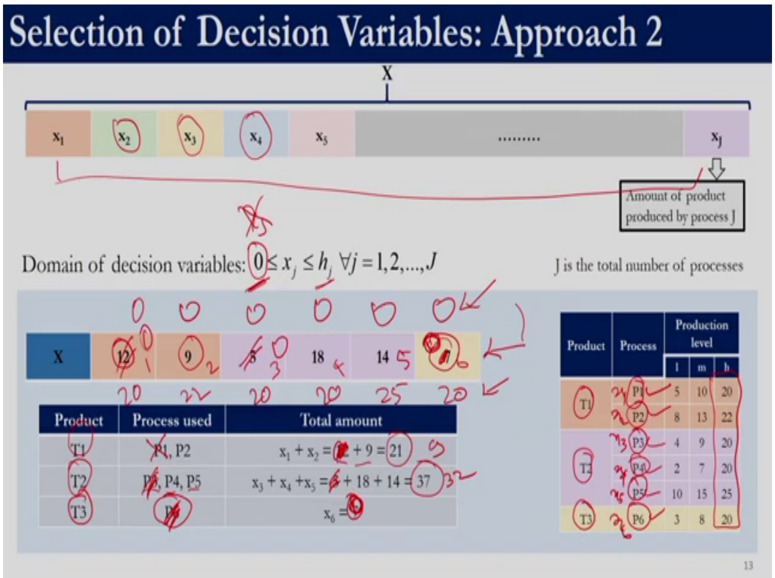

So, now let us see that for this how much product we have produced right. So, for example, T 1. So, T 1 is produced by P 1 and P 2. So, we can add this 12 and 9 right. So, that means, that we produced 21 units of T 1. For T 2, we need to add all that we obtained from process 3, process 4, process 5 that works out to be 37 and for product 3, we use process 6 right because this is non zero, 7 is non zero and the total amount produced is 7 right. So, let us say if this was 0, if this was 0 right; so that means, P 1 would get away right and this 12 would be 0 right. So, we will have only 9. So, this 5 would go away. So, process 3 would go away right.

So, this is process 1, process 2 process 3, process 4, process 5 and process 6. So, this 5 would go away. So, this will be 18 plus 14; this will be 32 and this stays right, had this been also been 0 right. So, then this will be 0 right; process 6 is not used right and the amount of T 3 produced is 0 right. So, here in the second approach, we are saying that it is sufficient if the algorithm tells us how much is to be produced from each process.

The algorithm does not need to explicitly decide whether we are producing a particular product or not and it does not explicitly need to decide whether we are using a particular process or not right. So, that information can be derived by knowing how much we are producing from each process right.

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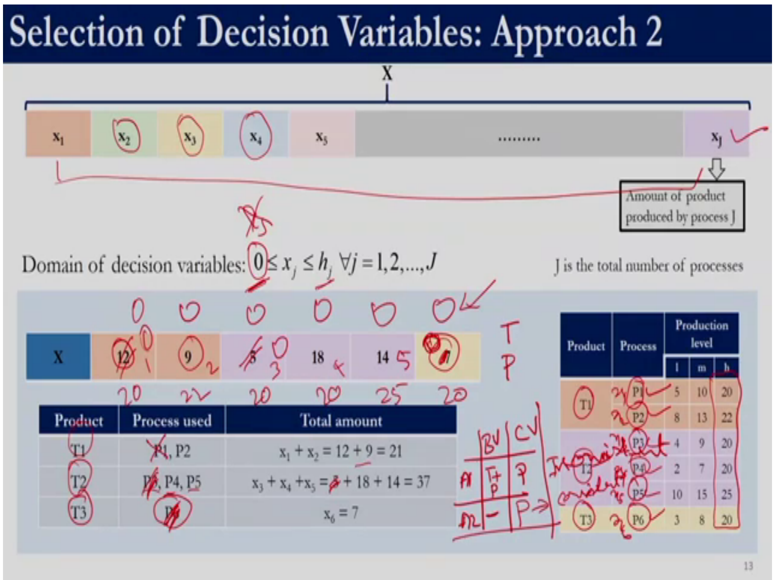

So, let us consider approach 1 and approach 2 right. So, let us say these are binary variables and these are continuous variable right. Let us say we had T products right and we have P processes right. In that case the number of binary variable that we had was T plus P. First T will tell whether a product is produced or not the second P will tell whether the process is used or not and again, we had P continuous variables right which tells us how much we are producing from each process right. Whereas, in the second approach if you see and despite using these many variables, this was giving us inconsistent.

So, this can give us an inconsistent solution which we need to again sort it out before we can evaluate the fitness of that solution right. Whereas, in the approach 2 if you see we have not used any binary variable right. So, the binary variable is 0 right and we need to use only P continuous variable right and this information what we get from here is actually consistent right. By the second approach, we will be getting consistent solution. Remember we are using the word consistent and not feasible right. Feasible is one which satisfies all the constrains right. Here, we are using the terminology consistent to understand that there is no conflicting information within the solution given by the metaheuristic techniques right.

So, in approach 1, we were using large number of variables, but still the information that we may get from the algorithm was inconsistent; whereas, in the second approach we are not using any binary variables which is in a way good right because the algorithm itself is not designed for binary variables right. So, we are not using binary variables, we are only using continuous variables and the information which we get from the algorithm will be consistent.

So, now, we will choose this approach right. So, our decision variables are going to be how much we are producing from each process. So that is what the algorithm is going to tell us and that is what the algorithm needs; lower bound and upper bound to come up with a solution right. Once a solution is generated, the next part is to calculate it is fitness function right. So, the rest of the discussion, we will be talking about how to determine the fitness function.

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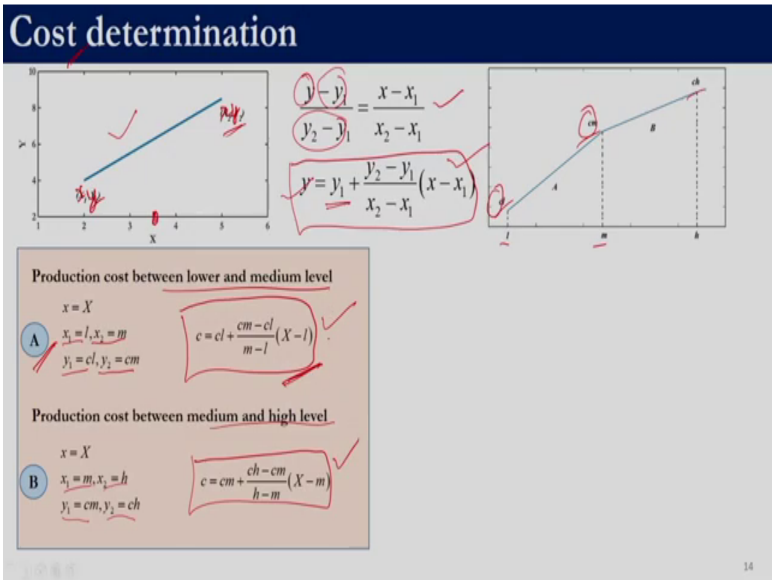

So, to determine the fitness function. So, we need to determine costs just to help you recollect basic mathematics for a line connecting 2 points x 1 y 1, x 2 y 2 right. This is the equation of the line y minus y 1 by y 2 minus y 1 is equal to x minus x 1 by x 2 minus x 1. So, given any particular x right, how do we calculate the value of y right. So, we can rearrange this equation. So, we will keep y on the left hand side, we will take this y 2 minus y 1 to the right hand side right. So, y 2 minus y by x 2 minus 1 into x minus x 1 and this minus y 1, we take it to the right hand side the that becomes y 1. This is the equation that we will be using right.

So, in this case we do not have x 1 y 1, x 2 y 2; we have l, m, c l and cn. So, this has to be lowercase x 1 y 1 right. This is a lower case x 2 y 2 right. So, if we substitute to x 1 is equal to l; x 2 is equal to m; y 1 is equal to cl; y 2 is equal to cm, we will get this equation and similarly, for the second part right. So, this is for part A.

So, if we get any production which is between l and m, we need to use this equation. If we get anything between m and h, we are not supposed to use this equation right because this equation is valid only between lower and medium level, l and m right. So, between medium and high the equation is still same; the interpolation procedure is still same right. So, instead of x 1, we need to substitute m; instead of x 2, we need to substitute h; instead of y 1, cm and instead of y 2, we need to use ch. So, this is the equation to determine costs.

So, now, we know for any production between l and m, how to calculate the cost; for any production between m and h, we know how to calculate the production cost. So, this is just the production costs. These 2 equations are only for the production cost right.

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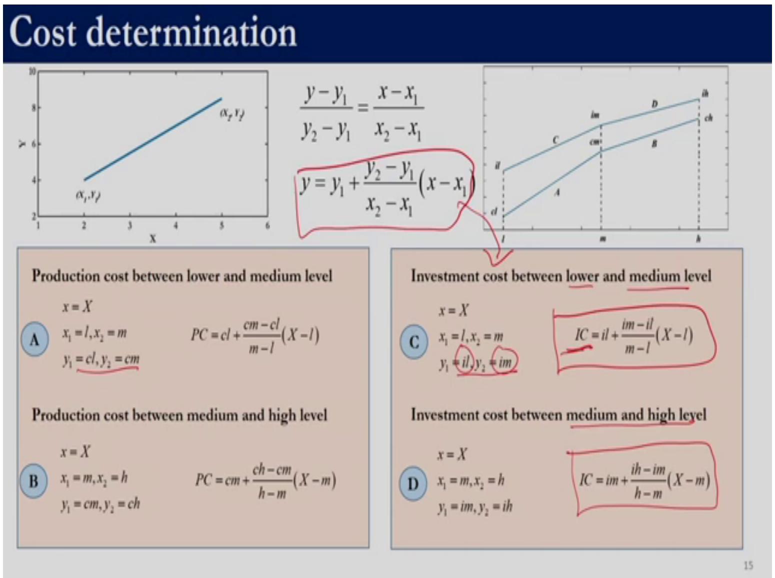

So, similarly we can write a equation for the investment costs also. So, this equation is still the same right. So here we were substituting cl, cm right for finding out the production costs.

Here, we need to substitute il and im right. So, il and im are the investment costs. So, the total investment costs, if the production is between lower and medium level can be calculated by this equation and if the production is between medium and high level, the investment cost has to be calculated using this equation right. So, now we know how to calculate production costs and the investment costs right.

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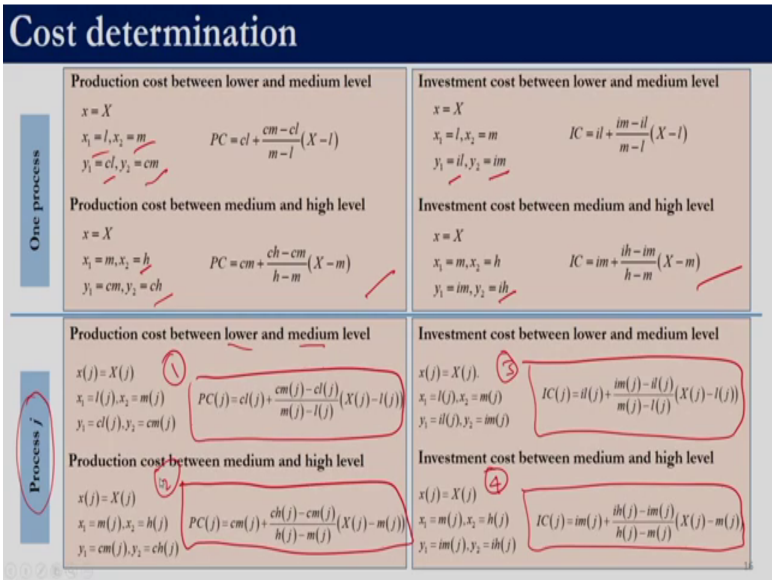

So, this is for a particular process right. So, this is the same set of equation which we showed you in the previous slide right. But there are different processes right. So, remember the table which we showed you at the very beginning, there are 54 processes and for each process, this l, m, h, cl, cm, ch, il, im, ih can be unique right. So, these values can be unique for each process. So, for the jth process, these are the equations which we will be using for calculating the production costs. We will use equation 1, if it is between lower and medium level.

The production is between lower and medium level and you will use this equation 3 to calculate the investment costs. Remember the production can be either between low and medium or between medium and high right. So, if it is between low and medium, we will use equation 1 and 3; if it is between medium and high, we will use equation 2 and four. So, that completes the discussion on the cost determination.

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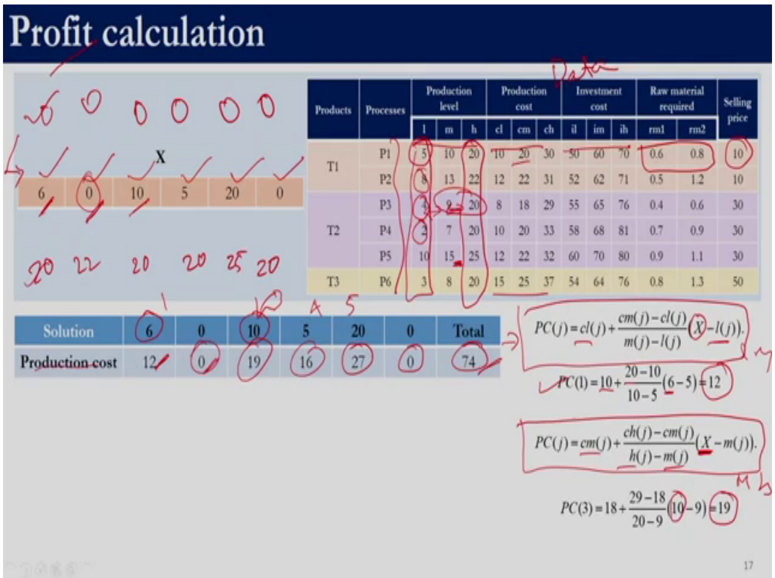

So, now let us discuss how to calculate the profit right. So, let us assume that this is the data which is given, this table is the data which is given right. So, we have been given 6 processes. We have been given production cost at different production levels, we have been given investment costs, we have been given how much raw material it requires for producing 1 unit quantity, we have also been given the selling price right. So, let us say with this data, we gave the lower bound to let us say TLBO 0 0 0 0 0 0 right lower bound is 0; upper bound is 20, 22, 20, 20, 25, 20 right. So, this is the upper bound right. So, remember the lower bound is not this one, the lower bound is 0 right.

So, if we give this to TLBO lower bound and upper bound. So, let us assume TLBO returns back such a solution 6, 0, 10, 5, 20, 0. So, in this case, we have chosen a feasible solution right. So, if it is infeasible, what we will do? We will come to that a little bit later right. So, let us say we come up with this solution right So, 6 if we see, it is greater than 5 right. So, this satisfies the domain or constraint. 0 right is less than 8, but it is 0 right. So, 0 is permissible.

So, this is permissible and this is permissible right; 10 right the lower bound is 4 and we are say the upper bound is 20 right. So, and for 10, we can use this portion to calculate the production costs. So, 10 is fine.5 right; so, the l is 2 right. So, anything above l and below h, we can calculate cost. So, for this, we will be able to calculate cost and similarly, for this we will be able to calculate cost because it lies over here. For each of this, we need to calculate the production cost. If we decide to produce 6 units from process 1. So, production cost. These are the 2 equations. This equation is valid between l and m right and this equation is valid between m and h.

So, the amount that is produced is 6, in this equation we need to substitute 6 over here right and the low level is 5, the medium level is 10. So, wherever l is there, we need to put 5; wherever m is there, we need to put 10 right. For cl, we need to substitute 10 and for cm, we need to substitute 20 right. So, everything is known. So, we will be able to calculate the costs.

So, in this case the cost turns out to be 12. So, this is the production cost for producing 6 units from process 1. For process 2 the production is 0. So, the production cost is 0. So, process 3, so process 3 we are producing 10 units right. So, process 3, 10 units; it is not between low and medium, it is between medium and high. We need to use the third equation and instead of x, we need to substitute whatever the algorithm has given. So, we need to substitute 10 right and cm, ch, m and h, you can look at the table and substitute. So, the cost would turn out to be 19 right.

So, similarly the cost for these two can also be calculated right. So, we have shown you for process 1 and process 3. You can calculate for process 4 and process 5. For process 2 and process 6, the production cost is 0 right. So, the summation of all this right comes out to be 74. So, that is the total production costs. So, if we decide to produce 6 units from process 1, 10 units from process 3, 5 units from process 4 and 20 unit from process 5, the total production cost is 74 right.

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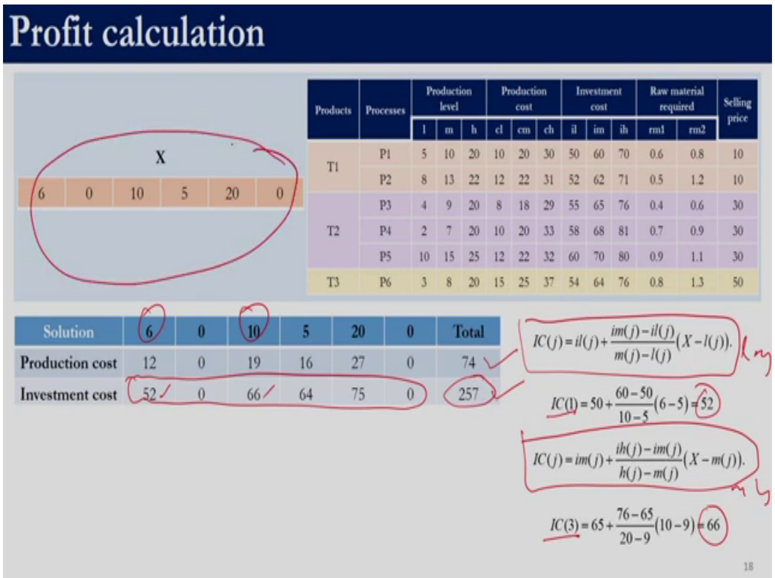

So, similarly we can calculate the investment cost right. So, this equation is for between l and m right and this equation, it is between m and h right. So, for the first and the third one. So, first and the third one, we have shown you the determination of cost. So, this is 52 and 66. This is 52 and this is 66. So, for the rest of the processes, you can calculate and if you sum this values right, if you sum this values it will turn out to be 257. So, the production cost is 74, the investment cost is 257 if we choose to produce as per this plan.

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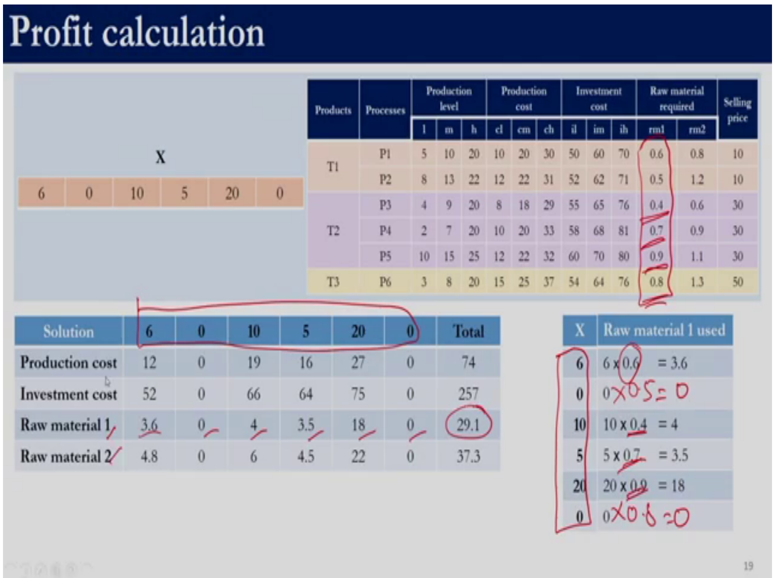

So, for raw material 1, this vector we have directly written over here right and this is the raw material 1 that is required for each of the process right. So, here it requires 0.6. Here it requires 0 into 0.5. So, this is 0. So, 10 into 0.4, 5 into 0.7, 20 into 0.9 and 0.8 into 0. So, this is the raw material required for each process. So, 3.6, 0, 4, 3.5, 18, 0 right. So, the total raw material required is 29.1. Similarly, you can calculate for raw material 2, it should work out to be 37.3 right.

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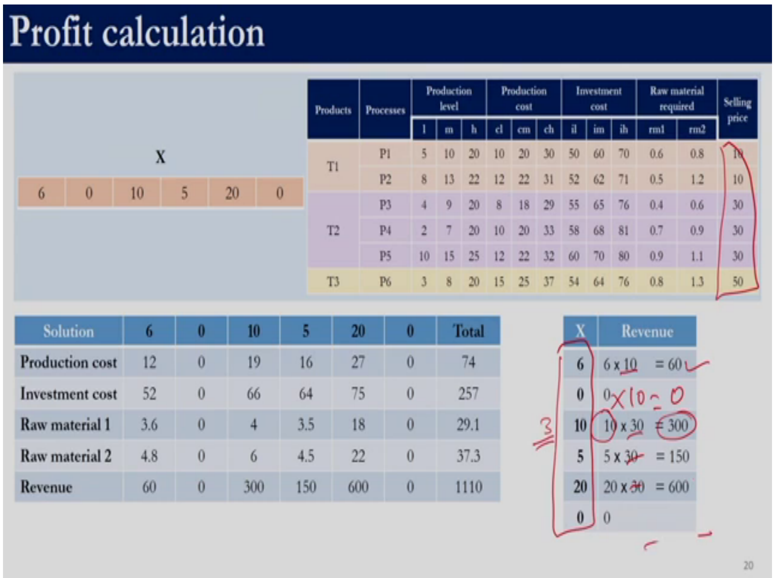

So, then we can calculate the revenue. So, again we have written this directly 6, 0, 10, 5, 20, 0 and this is the selling price per unit right. So, 10; 0 into 10, 30 three times 30 30 30. So, this is the revenue earned by selling 6 units of whatever comes from process 1 right, for process 3 we are producing 10 units. So, each unit the selling price is 30. So, we get a revenue of 300 right and so on right. So, if we sum this up right, this will come as 1110, that is what is given over here.

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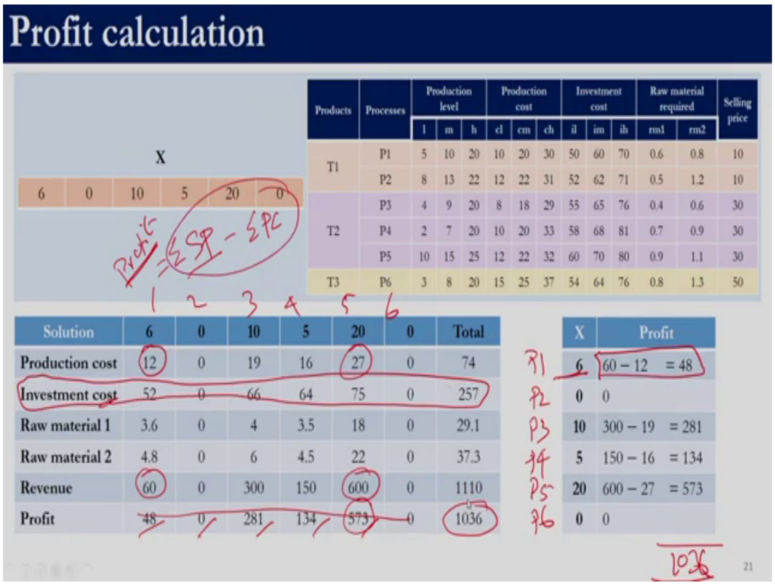

So, then profit for each process, we can calculate right. So, here the production cost is 12, here revenue earned is 60. So, 60 minus 12. So, that is the profit from process 1 if we produce 6 units right. So, similarly from process 2, process 3, process 4 5 and 6, it can be calculated right. So, here it will be 600 minus 27 right. So, that will be 573. So, this is the profit that is obtained from individual process right; from process 1, process 2, process 3, process 4, process 5 and process 6 right.

So, the total profit is the summation of all this right. So, the summation of all this is 1036. So, remember investment costs does not play a direct role in the determination of profit right. So, profit is selling price minus the production cost of all the process. So, this is what is going to give us profit right. So, it is going to be a scalar value and this investment cost is not used to calculate the profit.

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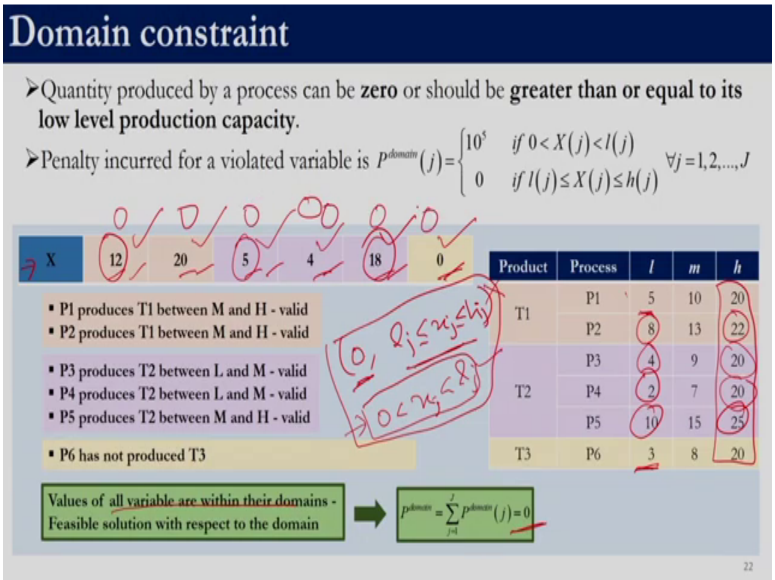

So, now let us consider what happens if we get a solution like this. This is x right. So, TLBO gives is 12, 20, 5, 4, 18, 0. So, the lower bound of all the processes are 0; the upper bound for all the processes are this one 20, 22, 20, 20, 25 and 20. So, that is the upper bound.

So, now, 12, if you see, it is greater than 5 and less than 20 right, so it is in the valid region. 20, if we see it is greater than 8 and less than 22, so this is also in the valid region. 5, if we see it is greater than 4 right and it is less than 20, so this is also in the valid region. 4 is greater than 2 right and it is less than 20, so this is also valid region. So, this 18 is greater than 10, but less than 25, again in the valid region right. 0 is less than 3, so that means, this product is not produced. Only if it is produced, it has to be l or greater than l right; if it is not produced then it is 0.

So, in this case none of the 6 variables are violating their domain constraint. So, all variables are within their domain. So, the domain is 0 comma lj xj and the hj right. So, it has to be either 0 or it has to be in this range right. If it is greater than 0, so this is problematic right; xj is greater than 0, but less than lj. So, if a solution falls in this range right, then it is an in feasible solution. So, this is the domain constraint right. So, solution if it is in this category and not in this category right, then it is violating the domain constraint. So, in this case all the 6 variables as in their domains right. So, there is no penalty. The penalty associated with each of them is 0.

So, in domain constraint the quantity that is produced by a process can be 0. So, 0 is permissible, but I has to be greater than or equal to it is low level production capacity right and if you have thoroughly understood metaheuristic techniques, you would also know that we will never get above h. Because any solution that we get from the metaheuristic techniques is within the domain, it is going to be between 0 and h right.

But here we additionally wanted to be either 0 or it has to be greater than l. So, whenever a variable satisfies this right that it is greater than 0, but less than l right, then we will assign a penalty of 10 power 5. So, remember when we discuss constraint handling in the previous session, we said one of the approach was hard penalty. So, here we are using a hard penalty approach.

No matter how much it violates, we are going to assign a penalty value right, if it is not satisfying something that is required. So, here what we require is that xj has to be either 0 or it has to be in this range right; but if it happens to be greater than 0 and less than l, then we will assign a penalty of 10 power 5. So, this is for each variable. So, every variable that violates the domain constraint, we are going to assign a penalty; penalty of 10 power 5 and it is a hard penalty right. So, in this case if you see, so none of these 6 variables fall under this category right. So, wherever it is non zero 18, 4, 20, 12 fall under this category that it is greater than l and less than equal to h right.

So, that is why this solution does not incur any penalty. The penalty incurred by this variable is 0; the penalty incurred by this variable is 0; the penalty incurred by this variable is 0 and the penalty incurred by the rest of the 3 variable is also 0. So, the total penalty that is incurred is 0 because this solution does not violate the domain constraint.

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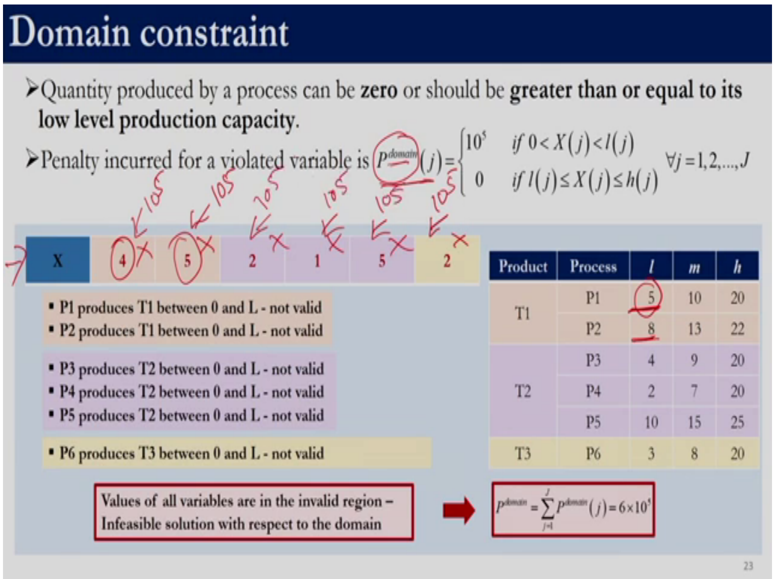

So, this is an example, where in some of the variables violate the domain constraint right. So, for example, let us say we get such a solution from 1 of the metaheuristic techniques right. So, 4; so 4 is not allowed because it has to be either 0 or 5 or greater than 5 right. So, 4 is a problem. Similarly, 5 right; so, the low level is 8 right. So, it is less than 8 right. So, this is not permissible.

So, similarly if you see the rest of the 4 variables each of the 4 variables violate the domain constraint. So, each variable incurs a penalty of 10 power 5. So, 10 power 5; 10 power 5 because here we have P domain. So, this is the name of a variable right. So, P for penalty and domain for to indicate that it is a domain constraint. So, all these 6 variables incur a penalty of 10 power 5 right.

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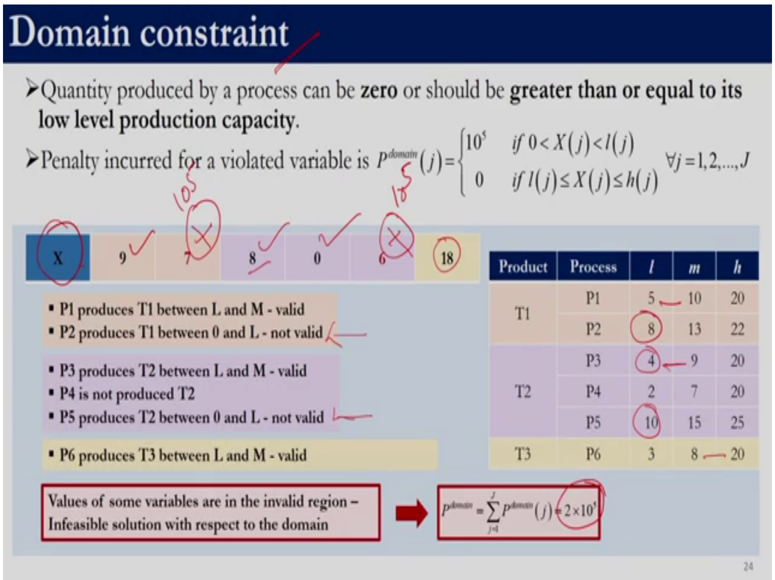

So, the total penalty is 6 into 10 power 5. So, in this case if we see right. So, 9 is permissible right because 9 is here. 7 is not permissible because 8 is the minimum right. So, 8 this is permissible right, minimum is 4; 0 is any way permissible. So, 6 is again not permissible right because it is less than 10 right and 18 is permissible because it lies in this range right. So, the fifth variable and the second variable actually violate right. So, the second variable and the fifth variable actually violate the domain constraint right.

So, this variable incurs a penalty of 10 power 5 and this variable incurs a penalty of 10 power 5. The total penalty for this solution right for violating the domain constraint is 2 into 10 power 5. Remember there are multiple constraints. Right now, we are only discussing about that domain constraint. So, if a solution satisfy the domain constraint, no penalty is assigned; but if it violates, then for each variable that is violating the domain constraint, we assign a hard penalty of 10 power 5 right and the total penalty for that particular solution is the summation of all the individual penalties. So, that is how we are handling domain constraint right.

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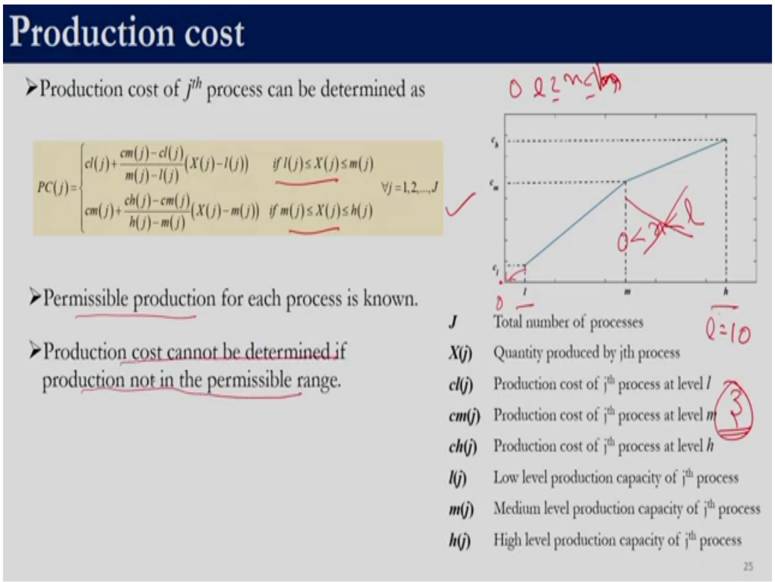

So, now let us see how to calculate the production costs right. So, this we have discussed. These other two equations which we have discussed right. So, if a production is permissible. So, what do we mean by permissible? 0 right and greater than or equal to l and less than or equal to h right. So, it can be here or it can be this 0; x cannot be greater than 0 and less than l right. So, this is not permissible. The algorithm does not ensure this right.

So, the algorithm knows this is lower bound and this is the upper bound, so it can give any value between the lower and upper bound right. So, now, if we get a solution over here right, let us say lower bound is 10 and the algorithm is giving a value of 3 right. So, then we do not know how to calculate the production cost right. So, production cost cannot be determined, if production is not in the permissible range.

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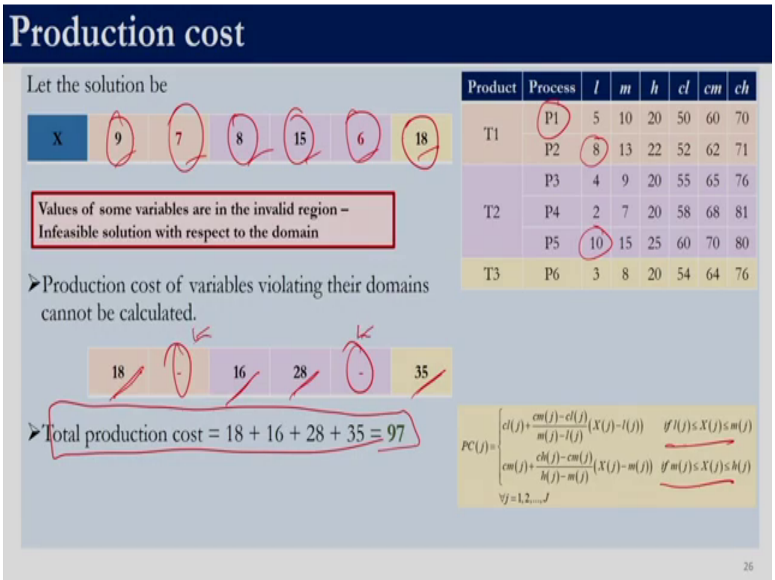

Let us assume that this is the solution that we get from one of the metaheuristic techniques 9, 7, 8, 15, 6, 18 right. So, some variables are not in the valid region right. So, 7 and 6 are not in the valid region; for process 2 minimum is 8, but what we have is 7; process 5 minimum is 10, what we have is 6 right. So, these two variables are violating the domain constraint. So, the production cost for these two variables cannot be calculated right. So, that is what is indicated and for the rest of the four variables, we have already discussed how to calculate the production costs right. It is based on where it lies, one of this equation has to be used with the respective lm, h and cl, cm, ch to calculate the production costs right.

So, the production cost for producing 9 units from process 1 turns out to be 18 right; for process 3 to produce 8 unit, it turns out to be 16; for process 4, it is 28 and for process 6, it is 35 right. So, the total production cost is 97, but the solution will have a penalty for violating the domain constraint, for a solution which does not satisfy the domain constraint. This is how we calculate the total production cost.

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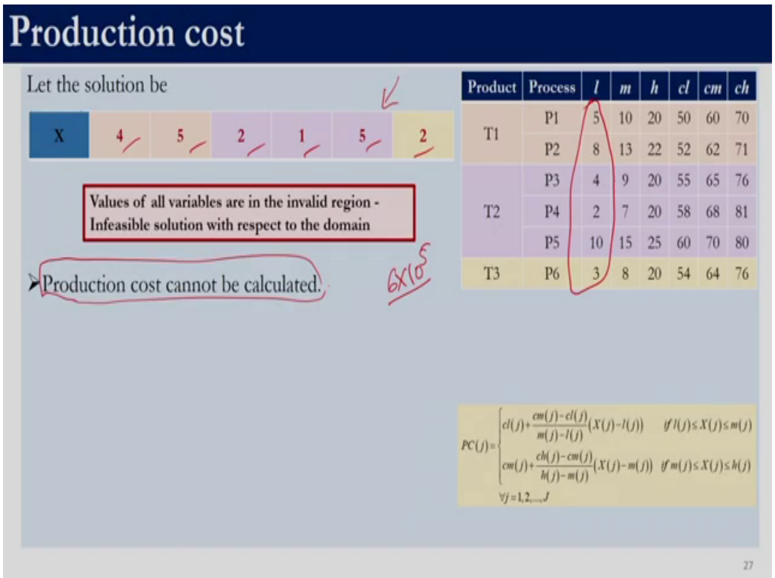

If the solution is like this let us say if the solution is 4, 5, 2, 1, 5, 2. This you can compare and you will be able to infer that all of these variables are actually violating the domain constraint right. So, if the if a variable is violating the domain constraint as we just saw we cannot calculate the production costs right. Here the production costs cannot be calculated. So, it will be assumed to be 0, but there will be a penalty of 6 into 10 power 5 because for each variable that is violating, we put penalty of 10 power 5 and here are all the 6 variables are violating the domain constraint. So, that is the discussion about production cost.

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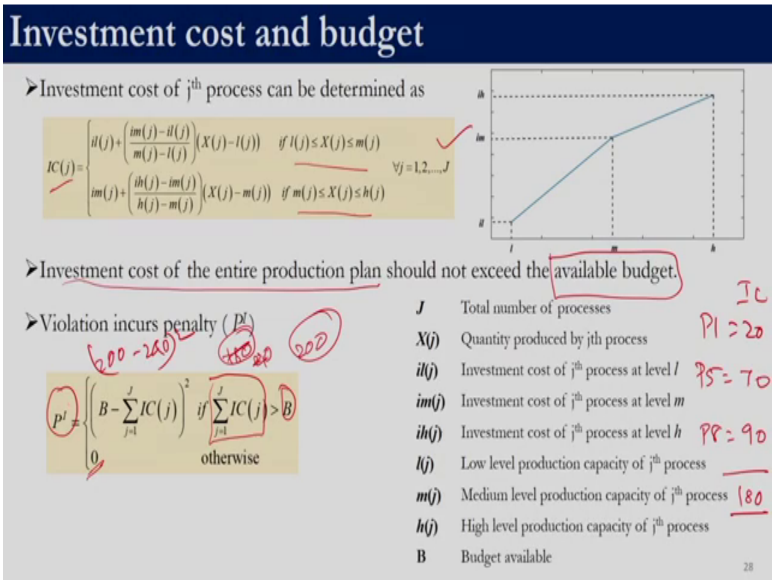

Now, let us look at investment cost and budget. This equation we have discussed previously that if a production is between l and m, we use this equation and if the production is between m and h, we use this equation to calculate the cost. This will tell us investment costs with respect to each process right. So, that has to be summed for all the process right and whatever investment cost, we get for the entire production plan.

Let us say for process 1, we require investment costs to be 20. For process 5, we require investment cost of 70 and for process 8, let us say we require investment cost of 90, then the total production cost for this is 180 that should be less than the available budget. This is an information which is available as to how much money is available to set up the plant.

So, the cost of whatever we decide to set up a should be less than or equal to the amount of money that we have right. So, the amount of money B is given right. So, this is the total production cost right. Let us say in this case 200 was available right and this cost comes out to be 180. So, 180 is not greater than 200 right. So, we have 200, we require only 180. So, the penalty incurred is 0 right.

Otherwise, let us say if we had required let us say 240. So, the penalty would be 200 minus 240, the whole square. So, that would be the penalty for investment cost. So, this P stands for penalty and the superscript I is to denote investment cost. So, we have penalty. So, if we violate domain constraint, we are going to incur penalty. If you violate the investment cost, we are again going to incur penalty.

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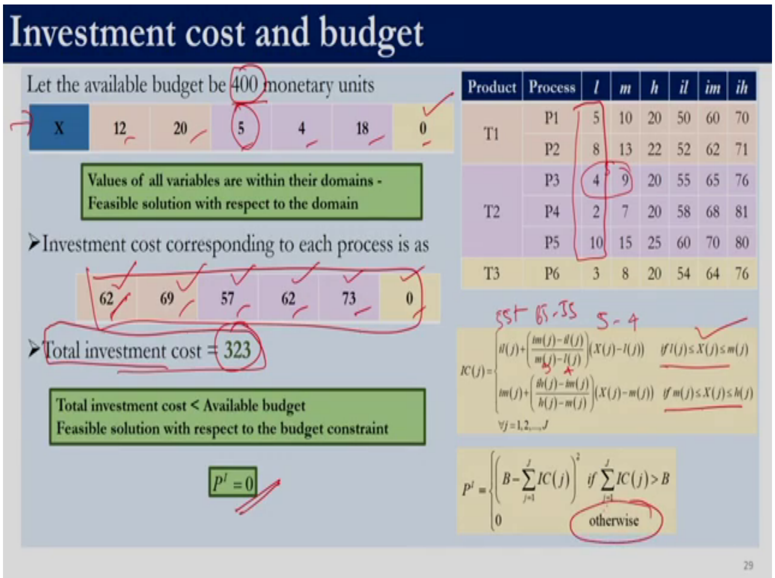

So, let us see an example right. So, let us assume that the total amount of money that we have is 400 monetary units and this is the production plan which is suggested by one of the meta heuristic techniques 12, 20, 5, 14, 18, 0 right. So, all of this satisfy the domain constraint right 0 is anyway permissible. The rest of the 5 processes have a production which is greater than their lower level.

We can calculate the investment cost for each of the process. So, again we have shown you previously how to calculate investment cost. So, for example, for process 3, we are producing 5 units right. So, process 3, 5 is over here between 4 and 9. So, we need to take the equation which is valid for between l and m right.

So, we need to use this equation instead of x, we need to substitute 5; instead of l, we need to substitute 4; im and il are 65 and 55 right; m is 9 and l is 4 right il is again 55 right. So, if you compute this, you can calculate the investment cost. Similarly, depending upon which region the production lies, you can use one of this equation to calculate the production cost of all the other variables. Also, if we sum this up right, so this is the investment cost for process 1, process 2, process 3, process 4, process 5 and process 6.

So, the total investment cost which you require is the summation of this 323 right. So, we require 323, but what we have is 400 right. So, what we require is less than what is available right. So, there is no penalty to be assigned. So, it falls in this category. So, no penalty to be assigned.

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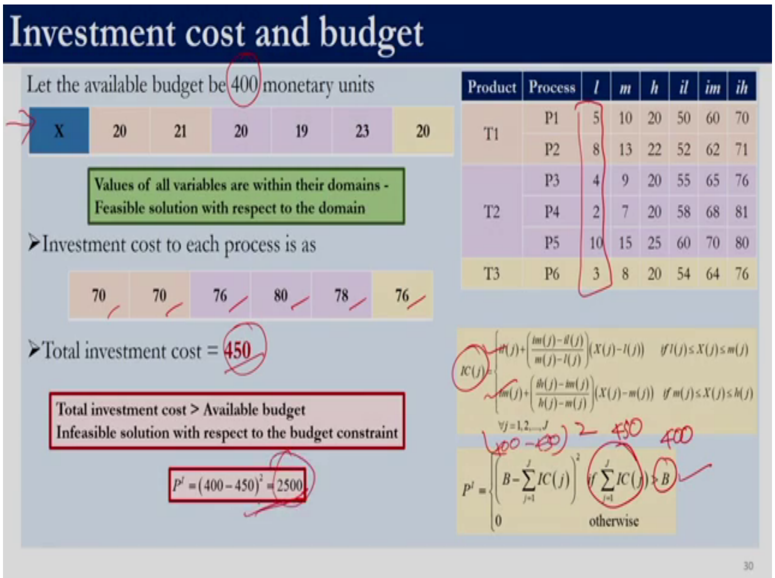

So, let us consider a case, wherein let us say this is the decision variable 20, 21, 20, 19, 23 and 20. Again in this case all of these have their production greater than l depending upon what is the production we have to use one of this equation to calculate the investment cost right. So, the investment cost turns out to be 70, 70, 76, 80, 78 and 76 and the total investment cost is 450, but what we have is 400. So, we require is more than what we have right.

So, in this case this term turns out to be 450 right and this is 400 right. So, this solution falls under this equation right. So, it will be 400 minus 450 the whole square that is the penalty right. So, 50 square, 2500. So, this is the penalty due to the violation in investment cost right.

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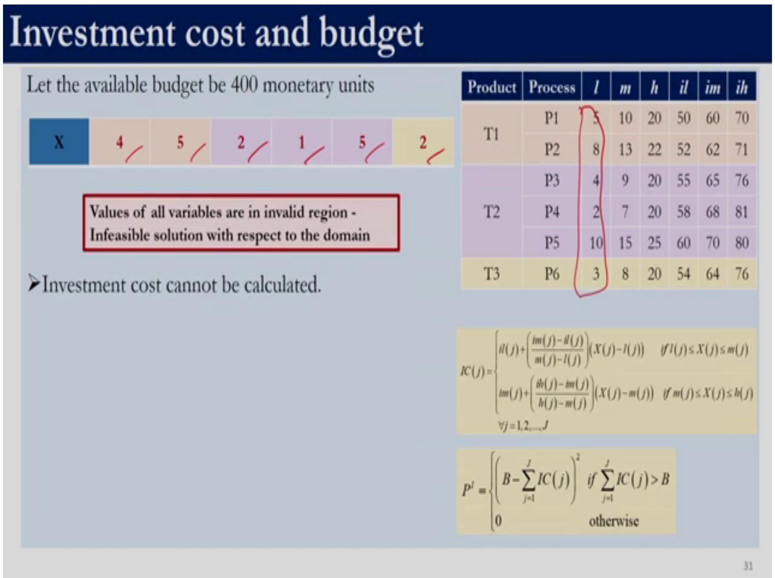

This is another example right. In this example all the 6 variables actually violate the domain constraints right. So, if you see 4, 5, 2, 1, 5, 2; it is actually less than these values right. So, in this case the investment cost itself cannot be calculated.

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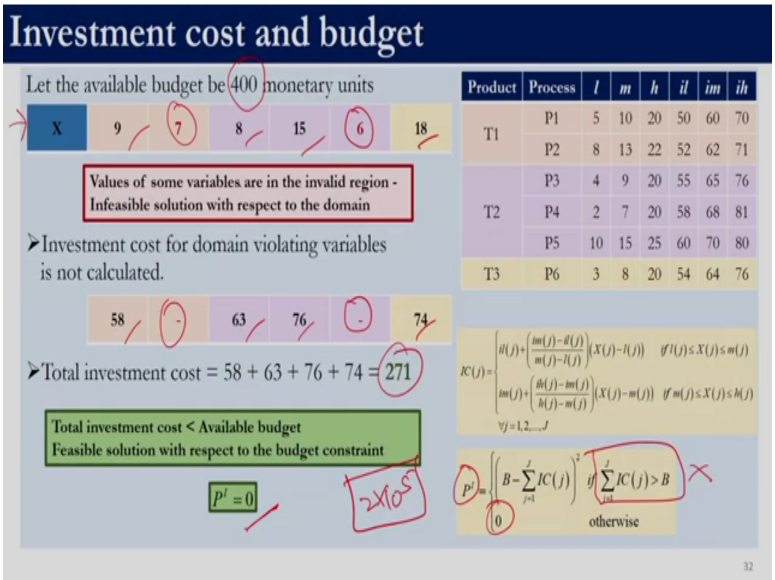

So, over here we have two variable which are not satisfying the domain constraint right. So, for those two variables, we cannot calculate the investment cost right. For the rest of the 4 processes right, you know how to calculate the investment cost right. So, the total investment cost over here is 271 right. The amount that is available is 400 right. So, this condition is not satisfied for this solution. So, the penalty that is incurred for violating the investment cost is 0 because it is not violating the investment cost constraint; but this solution will nevertheless have a penalty of 2 into 10 power 5 due to violation in the domain hole constraint.

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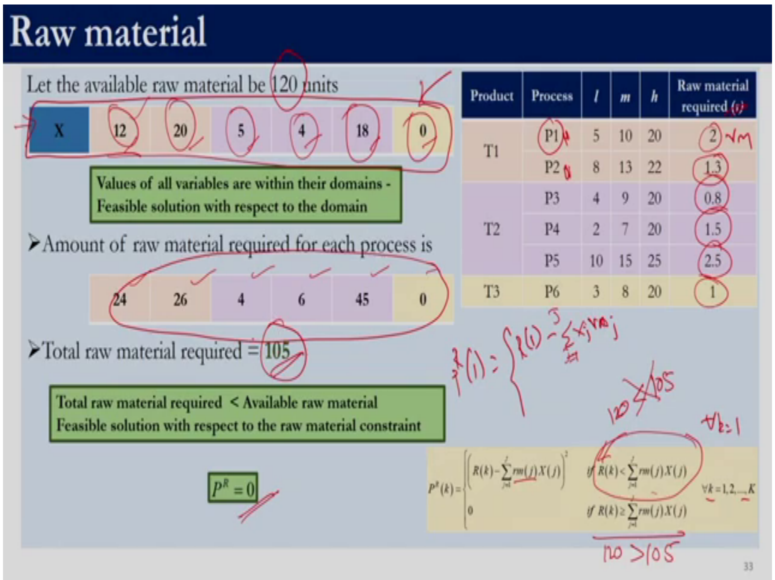

Now, let us look at the raw material constraint. So, let us assume that this is the solution that we get from the meta heuristic techniques right. So, from process 1, we are producing 12; process 2, 20; process 3, 5; process 4, 4; process 5, 18 and process 6, 0 right. And we know how much raw material is required for producing 1 unit from process 1.

So, if we produce 1 unit from process 1, we required 2 units of raw material right; if we produce 1 unit from process 2, we require 1.3 units. Here we are producing 12 units right. So, it will be units 12 into 2 right, 20 into 1.3, 5 into 0.8, 4 into 1.5, 18 into 2.5 and 0 into 1. So, that is the raw material required for each of the process right.

So, the total amount of raw material that will be required is the summation of all of this right. So, in this case, it turns out to be 105. So, the amount of raw material that is available is 120 units. What we require if we decide to produce according to this solution right, then we will require only 105 right. So, that lies over here. So, here k indicates the type of raw material right. So, in this case for this example, we have only one raw material. So, this is for k equal to 1. So, is equal to just 1. So, wherever you have k you just need to put 1. So, P R of 1 is equal to R of 1 minus summation of what is over j right. So, summation of X j into rm j.

So, this has to be rm right. So, rm is given right. X j is what we get from the algorithm right. So, this sum across sum across all the process right, if we have j process; R is what is available right. So, here we have 120. So, 120 is less than 105 right. So, this is not satisfied; 120 is not less than 105. So, it will fall in this category. So, 120 is greater than 105. So, the penalty incurred with respect to the raw material constraint is 0. So, no penalty is incurred, we required only 105 units whereas, what we have is 120 units.

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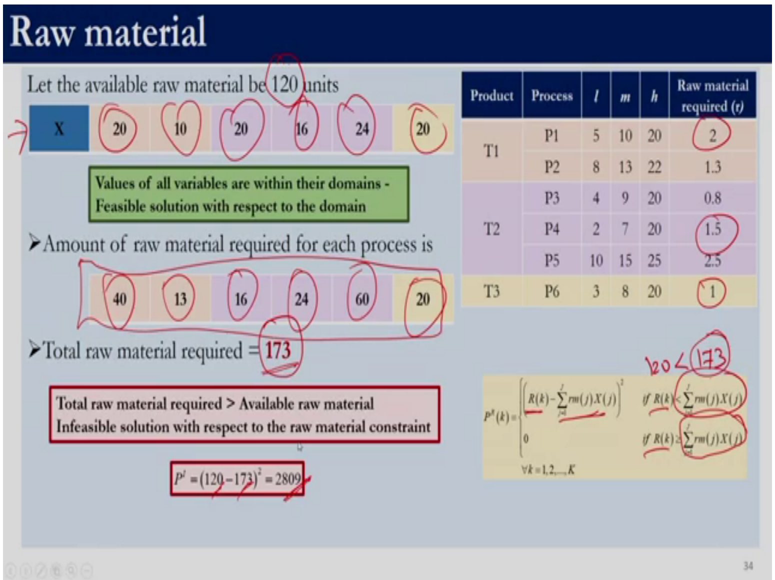

So, this is a case wherein, we are violating the raw material constraint right. So, if this the solution again 20 into 2 is 40 right; 16 into 1.5 will give us 24 right and this 20 into 1 will be 20; for remaining 3 processes also, you can calculate right and the total raw material that will be required is the summation of this which it turns out to be 173 in this case.

So, what we have is 120 and what we require is 173 right; what we have is 120 right and this is less than what we require right 173. This is what is we required, this is the required amount and this is the available amount right. So, in this case we need to calculate the penalty by what is available which is 120 minus what is required which is 173. So, 120 minus 173 the whole square. So, that turns out to be 2809. So, this is how we handle raw material constraint.

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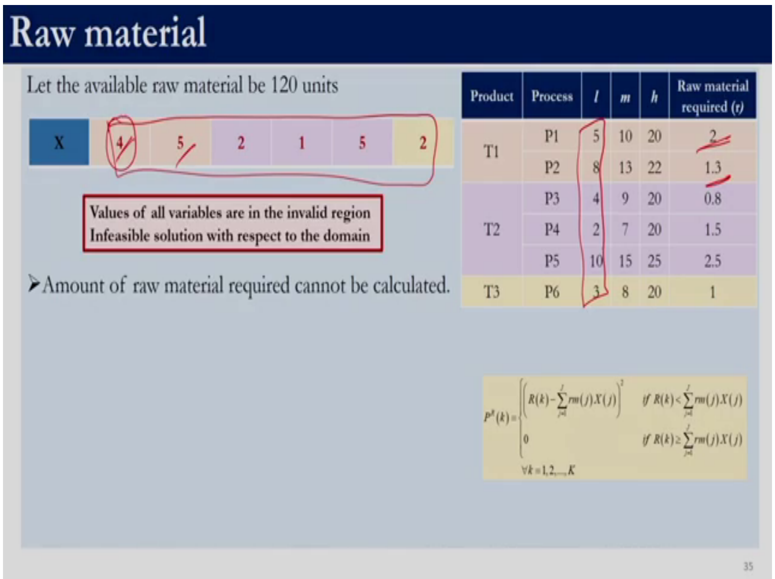

So, similar to our previous discussion, if we get a solution which is actually violating the domain constraint itself. So, these 6 values are less than these 6 values and they are not 0 right. So, in this case we cannot calculate how much raw material is required. So, the raw material here, we cannot say that it is 4 into 2 or 5 into 1.3 right. So, that is because this production itself is not permissible. So, if the product itself is not permissible, there is no point in calculating the raw material that is required. If a variable violates the domain constraint, so the raw material required is not to be computed.

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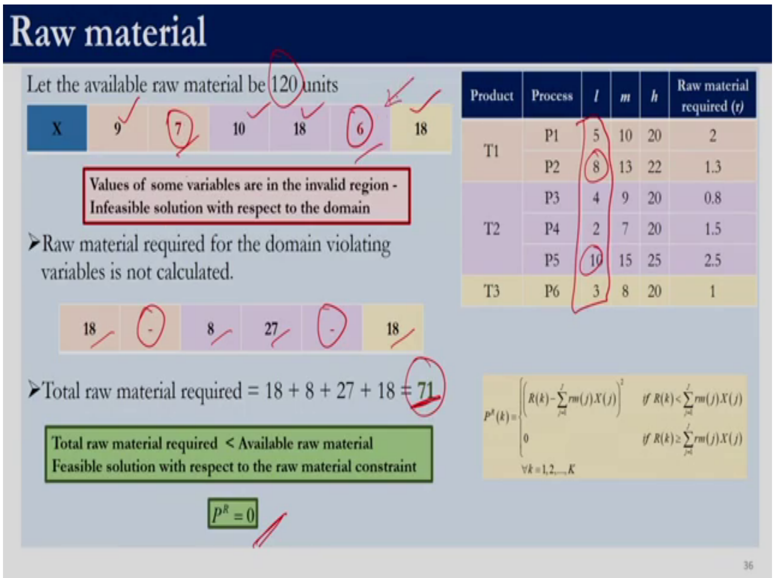

So, this is a case where in two variables violet right. So, we can calculate for 9, 10, 18, 19 right because those are greater than this l values and they will be less than h also right. But for the 7 and 6. So, this is the fourth and the fifth one right. So, 8 and 10 is the lower level right, so we can produce 8 and above 8 or we should not produce 0 is permissible right. So, for here the raw material consumption is for these 2 processes, it is not calculated; for the rest of the 4 processes we calculate the raw material consumption right and the total raw material required is 71. What we have is 120, what is required is 71 right. So obviously, there won't be any penalty with respect to raw material.

So, this we are repeating multiple times just because it does not incur raw material penalty, does not mean this solution will not have penalty; it will have penalty with respect to the domain constraint right, but with respect to raw material it does not have a penalty. Now, we have seen how to handle raw material constraint, we have seen how to handle the investment cost constraint and we have also seen how to handle the raw material constraint right. So, now, let us look at how to evaluate the profit right.

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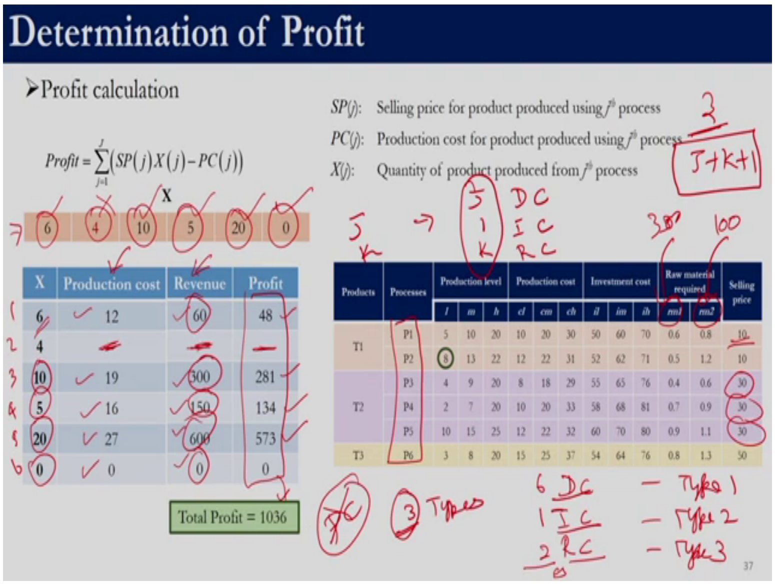

So, profit in this case, let us assume that this is the solution that we get from let us say one of the meta heuristic technique. So, 6, 4, 10, 5, 20 and 0; 0 is anyway permissible for any process. Let us compare the other ones. 6 which is right, 4 is not permissible because the lower level is 8. So, 4 is not permissible, 10 is permissible, 5 is permissible, 20 is permissible right. So, for whichever process satisfies the domain constraint, we can calculate the production cost right. Remember those two equations, you need to see whether it is between l and m or m and h and use the appropriate equation and calculate the production cost right and we can also calculate the revenue right.

So, revenue in this case is 6 into 10 right. So, that is 60 right. So, the second variable itself is violating the domain constraint right. So, we will not calculate production cost, we will not calculate revenue and the profit from that is also not calculated right. So, from process 3, the production is 10 right. So, and the selling price is 30 right. So, the revenue earned is 300 right.

So, similarly 5 into 30 is 150; 20 into 30 is 600; 0 into whatever is the selling price will still give us 0; this is the revenue. So, the profit is revenue minus production cost. Remember it is not the investment cost; investment cost is one time cost which is required to establish the plant production cost is the recurring cost right.

So, profit can be calculated for each process 300 minus 19, 281; 150 minus 16, 134; 600 minus 27, 573 and this will be 0 right. So, the sum total of this is the total profit right. So, right now what we have done is we have seen how to calculate profit for a feasible solution as well as an infeasible solution. We have seen how to calculate the penalty for a feasible solution as well as infeasible solution, for all the 3 type of constraints right; the domain constraint, the investment cost constraint and the raw material constraint right.

So, right here when we are saying, we are saying types of constraints right. So, there are 3 types of constraints that does not mean that we have only 3 constraint. For example, take this data right. So, we have 6 processes right. So, we will have 6 domain constraints, for each process we will have domain constraint. So, this is one type. So, let us say this is type 1 right.

We will always have one investment cost as per the problem definition irrespective of any number of process we will have only one investment cost constraint right. So, this is type 2 and this case we have 2 raw materials; raw material 1 and raw material 2 and if both are available in limited quantities, let us say this is available in 300 and this is available is only 100 right. So, if this is available in 300 and if this is available in 100 right, then we have 2 raw material constraints. So, this is type 3 constraint right.

So, we will have a total of 9 constraints over here right. So, if we have J processes, we are going to have J domain constraint, we are going to have 1 investment cost constraint and if we have K raw materials which are available in limited quantities, we are going to have K raw material constraint. So, the total number of constraint is J plus K plus 1. This is the total number of constraint; the type of constraints are 3. Three types of constraint; domain constraint, investment cost constraint and raw material constraint. Remember as per the problem definition, we do not have any constraint on production cost. So, production cost is required to calculate the profit.

So, now we know how to calculate objective function, how to calculate the violation with respect to each of the constraint right. Remember for production planning problem what is to be returned is the fitness function right and not the objective function right. So, objective function plus penalty is the fitness function for a minimization problem. Right now, we have a maximization problem. So, we need to convert the maximization problem to a minimization problem and then, add penalty to the objective function. So, that we get fitness function right.

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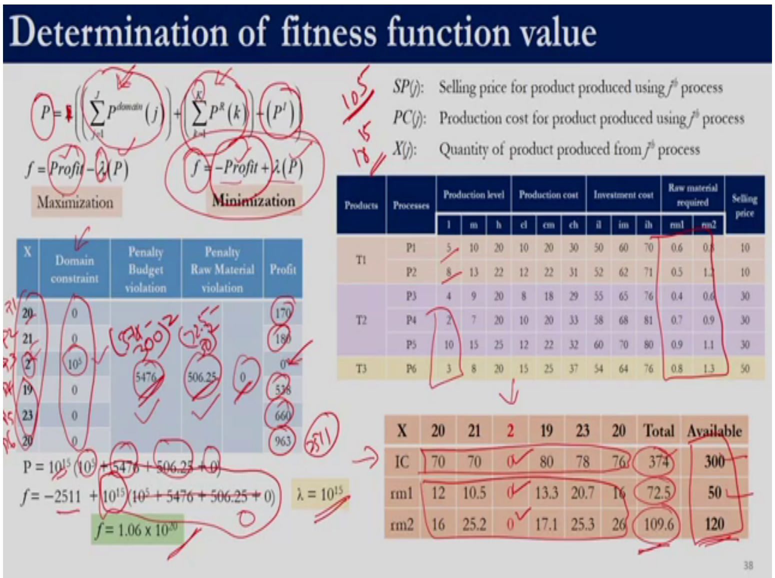

So, we know how to calculate profit, since there is only one investment constraint cost we have the penalty incurred due to violation in investment cost constraint. This is the sum total of all the penalties that is the total penalty incurred for violating the domain constraints right for all the variables and then, we have k raw materials right. We know how to calculate the penalty for every raw material right. So, there will be k such terms. So, we need to sum this this is one category of penalty; this is a second category of penalty and investment cost we will have only one penalty if the solution violets the investment cost constraint.

So, we will have only one value for violating the investment cost constraint right. So, if you violated the investment cost constraint, there will be a penalty and it will be just a scalar value right. There is no investment cost constraint on each process. The investment cost constraint is on the entire production plan. So, the total penalty is addition of all this penalties right. So, we add all this penalties so that is our total penalty right. So, we know how to calculate profit, we know how to calculate penalties right. So, this lambda is the penalty factor that we have discussed previously right. So, this is for a maximization problem.

So, if we were to solve a maximization problem right, then our fitness function would be profit minus lambda into penalty right. But since all the meta heuristic techniques which we discussed and which we coded were for minimization problems right. So, the fitness function we need to multiply by minus right. So, this profit becomes minus profit and this minus lambda P becomes plus lambda P right. So, this is the fitness function value.

So, we know how to calculate profit, we know how to calculate the individual penalties and hence, the total penalties and we can combine this. So, this is a consolidated picture of whatever we have discussed. So, let us assume this is the production plan given by one of the meta heuristic techniques.

So, for process 1, we need to produce 20; process 2, we need to produce 21; similarly 3, 4 5 and 6. Now, if we see 20 and compare with this data, this is not violating the domain whole constraint; similarly this is not violating the domain whole constraint. This violets the domain whole constraint. So, we have a penalty of 10 power 5 right.

Similarly, these three values do not violet their low level production right. So, no penalty. So, this is the domain constraint, penalty incurred for domain constraint right. For this production plan you know how to calculate the investment cost. So, depending upon in which region it lies, you need to use the appropriate equation right and you will be able to calculate the investment cost. So, we have given calculated the investment cost and given over right.

So, this data plus whatever the equation which we have discussed in for this production plan will lead to this investment cost right. So, for here if you see the third process is violating the domain constraint right. So, the investment cost what we were saying is not calculated is considered to be 0 right. So, the total investment cost is 374 and the budget which is available. So, this is part of given data right. So, what we require is 374 and what we have is 300 right. So, we require more than what we will incur a penalty. So, that penalty will be 374 minus 300 the whole square right. So, that will work out to be 5476 right. Similarly, for this production plan the amount of raw material required for each process can be calculated as shown over here.

So, the total amount of raw material 1 that is required is 72.5; the total amount of raw material 2 that is required is 109.6 right. So, what we have for raw material 1 is only 50 and what we require is 72.5. So, again we will incur a penalty which 72.5 minus 50, the whole square right. So, that will work out to be 506.25. Here, we require 109.6 right, what we have is 120. So, what we require is less than what we have. So, the penalty is 0. Similarly, we can also calculate the profit for all the processes right. We do not calculate the profit for the third process because the value of the decision variable is actually violating the domain constraint.

So, whenever variable violated the domain constraint, we do not calculate the investment cost, the raw material required and we do not calculate the profit, we take it to be 0. So, the total profit if you sum this up should come out to 2511 right. So, 2511 is the profit for this plan, but this plan is not feasible right, it is violating the budget constraint, the raw material 1 constraint and the domain constraint for one of the variable. So, 10 power 5 is the penalty that we have over here right. So, 10 power 5 plus this 5476 plus 506.25 plus 0 right. So, for this problem we have taken the lambda value to be 10 power 15.

So, the fitness function will be minus profit it is given over here. So, minus profit which is minus 2511 plus 10 power 15 into all this values right. So, if you work it out it will come to 1.06 into 10 power 20. So, here we I have taken a value of lambda to be 10 power 15 right. So, just to make sure that no feasible solution gets lost to an infeasible solution, if you have understood constrained optimization right for a feasible solution this will work out to be 0 right; 0 penalty. So, this second part will not have any impact on the fitness function right. The fitness function will be minus profit right. So, here we have given a penalty of 10 power 5 for violating the domain whole constraint right and we have given 10 power 15 as a penalty factor right.

So, you can take different values and see what impact it has on the performance of the meta heuristic techniques right.

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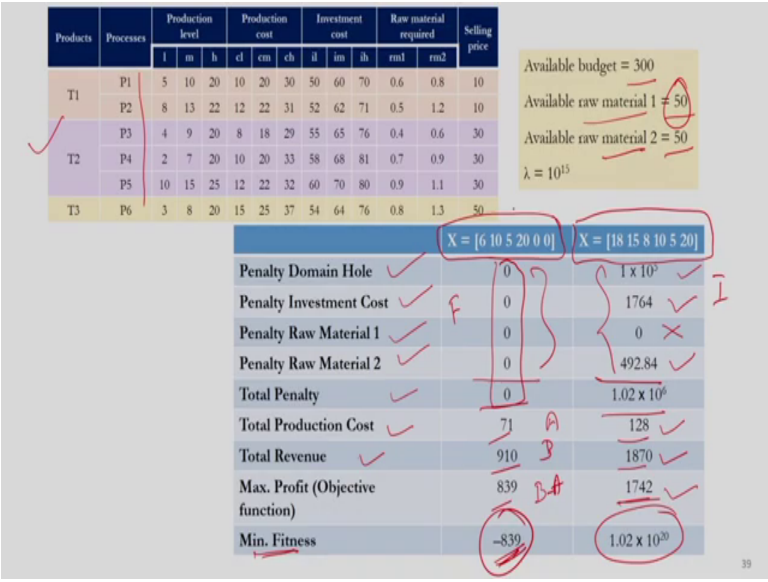

So, we leave this for you to calculate right. So, this is the given data and amount of budget which is available is 300 the amount of raw material 1 and raw material 2 which is available is 50 right, assume the production plan to be 6, 10, 5, 20, 0, 0 right. So, there are 6 processes. So, we have 6 variables and calculate what is the penalty incurred for domain whole; what is the penalty incurred for investment cost; what is the penalty incurred with respect to raw material 1 and raw material 2; what is the total penalty right? So, total penalty will be the summation of this right. You need to calculate what is the total production cost; what is the revenue right. So, if this is A and if this is B. This profit would be B minus A right. So, we leave it for you to calculate right.

So, since it is feasible solution, the penalty is 0 right. So, if you plug that into that fitness function equation, you will still get minus of profit. So, minus 839 is what you should get ah; whereas, this is an infeasible solution right. So, one of this variable violates the domain constraint that is why we are incurring this penalty. Similarly, there is a penalty associated with investment cost and raw material 2. No penalty is incurred for raw material 1 right that is because whatever we require is less than or equal to 50 right. So, total penalty again should be the summation of this right and the production cost would turn out to be 128, the revenue is 1870 and the profit is 1742.

If you calculate the fitness function, it should turn out to be this. So, this is an infeasible solution this is a feasible solution right. All the algorithms which we have are for minimization right and we have converted this problem also as a minimization problem. So, between these 2 solutions, if any of the meta heuristic techniques had to select a solution it would select this one because it has a better fitness function right. We are minimizing it that you need to keep in mind. The problem is the maximization problem, but we converted into a minimization problem solution right. So, this solution would be selected right.

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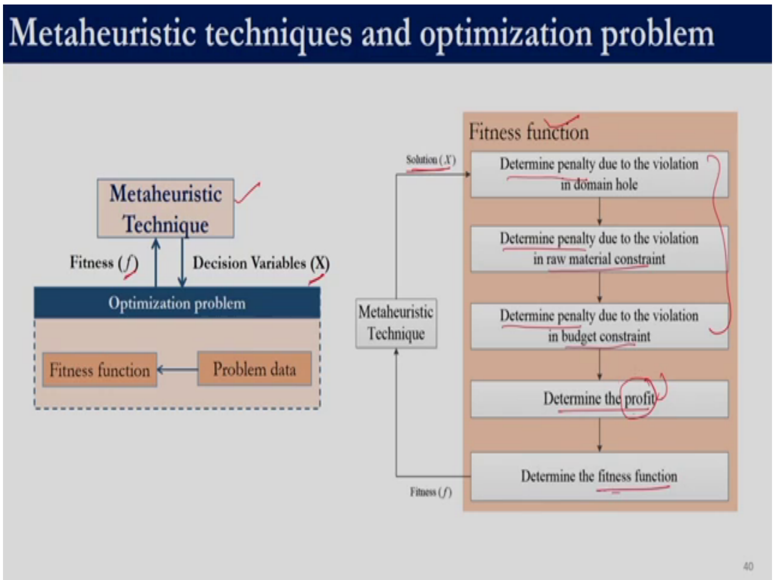

So, this gives a consolidated picture right. So, this meta heuristic technique as we have seen multiple times, it will convey the decision variables to the optimization problem and optimization problem is supposed to send back the fitness function value. For this X, what is the fitness function? For this problem to calculate the fitness function, we need to check for the violation in the domain constraint right. If it is violating the domain constraint, we need to determine the penalty; else the penalty is 0. Then, we need to calculate the amount of raw material that is required.

Amount of raw material required is less than what we have, then no penalty is to be assigned; else we need to determine the amount of penalty. Similarly, we need to calculate the total investment cost which is required right. So, if it is less than whatever we have, then there is no penalty. Else we need to assign a penalty. So, once we have calculated all the penalty, we need to calculate the fitness function and calculation of fitness function requires the objective function right for this problem, the objective function is profit. So, we need to calculate the profit.

So, for calculating the profit we again need the production cost right. So, depending upon how much we are producing from each process, we need to calculate the production cost of each process and then, sum it up. Similarly, we need to calculate the total revenue and the difference between the revenue and the total production cost will give us the profit right. Once we have profit, we can calculate the fitness function as shown earlier.

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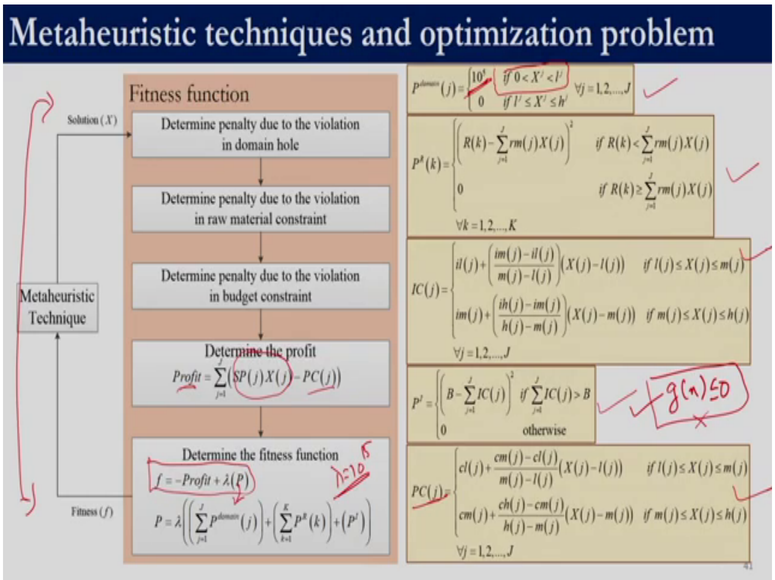

So, over here, we have just compiled all the equations. So, right. So, this figure is the same that we discussed previously right. So, here we have just put all the equations together. So, this is how we assign penalty for violating the domain constraints. This is how we calculate the penalty for violation in raw materials. This is how we calculate the investment cost and then, check for violation in the investment cost this is how we calculate the production cost right. So, once we calculate the production cost, the revenue earned is selling price into amount that is produced.

Once we know the profit, we can use this fitness function equation to calculate the fitness function right. So, the penalty the value of P is nothing but summation of all the penalties right. So, here we have taken lambda value to be 10 power 15 right and here, we took the penalty for violating the domain constraint as 10 power 5. So, the domain constraint, we did not even have in this form g of x is less than equal to 0 right. So, if you remember when we were first talking about how to handle constraint using meta heuristic techniques, we had said that if a condition is not satisfied, you can assign penalty right.

It is not necessary that the condition should be of this form g of x should be less than equal to 0. Here, we had this condition that x is greater than 0, but less than equal to l right. So, that is the power of meta heuristic techniques right. So, if we are not able to pose the problem in the conventional format minimize f of x subject to g of x of is less than equal to 0 or h of x is equal to 0 and x within lower and upper bounds, even if we are not able to pose our problem in that format, even then we can use meta heuristic techniques right.

Why we are able to use meta heuristic techniques even in that case is in meta heuristic techniques all that we require from the problem is the lower bounds the upper bounds and the way to calculate the fitness function value right. So, given a solution what is the fitness function value, that is all that is required for meta heuristic techniques right. So, if we are able to pose the constraint in this form, well and good, we can pose the current in this form and then use mathematical programming techniques as well as meta heuristic techniques right. But if we are not able to pose the constraints in conventional form, we can still use meta heuristic techniques.

With that we will conclude this session. In the next session, we will implement this production planning problem using MATLAB right and then will deploy one of the meta heuristic techniques that we discuss to actually solve this problem.

Thank you.