

**Computer Aided Applied Single Objective Optimization**  
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**Lecture - 20**  
**Comparison of Variation Operators and Survival Strategies**

Welcome, so far we have seen five metaheuristic algorithms; teaching learning based optimization, particle swarm optimization, differential evolution, genetic algorithm and artificial bee colony optimization. So, before moving into constrained optimization we thought we will compare all the survival strategies and the variation operators which we have discussed in this five technique.

So, we thought to put it in a consolidated form, so that it is easier for you to refer back. And also if you are planning to design new algorithms you can make use of this revision. So, first we will be looking into the variation operators. So, variation operators are used to generate new solutions from the existing solutions.

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### Teaching Learning Based Optimization (TLBO)

➤ Teacher phase:

$$X_{new} = X_i + r(X_{best} - T_f X_{mean})$$

➤ Learner phase:

$$X_{new} = \begin{cases} X_i + r(X_i - X_p) & \text{if } f_i < f_p \\ X_i - r(X_i - X_p) & \text{otherwise} \end{cases}$$

X	Current solution
X <sub>new</sub>	New solution
X <sub>best</sub>	Teacher
X <sub>mean</sub>	Mean of the population
T <sub>f</sub>	Teaching factor, either 1 or 2
r	Random numbers between 0 and 1
X <sub>p</sub>	Partner solution
f <sub>i</sub>	Fitness of current solution
f <sub>p</sub>	Fitness of partner solution

So, in TLBO new solutions were generated either using the teacher phase or in the learner phase right. So, in the teacher phase we will get one solution and in the learner phase we will get one solution. In teacher phase we will use the solution X<sub>i</sub> to generate the new solution X<sub>new</sub> right. So, X<sub>i</sub> is the current solution for which we are generating a new solution, r is random number between 0 and 1 right.

So, this has to be 1 cross D where D is the number of decision variable, X<sub>best</sub> is the best solution in the population, X<sub>mean</sub> is the mean of the population. And teaching factor is either 1 or 2 remember it is not between 1 and 2 it is either 1 or 2. And teaching factor is a constant is a scalar value 1 cross 1 irrespective of the dimension of the problem.

So, that is how we will be generating a new solution in the teacher phase. Whereas, in the learner phase we will be generating a new solution by selecting a partner from the population.

So, if we are generating a solution for  $X_i$  the solution  $X_i$  we need to select a partner from the population right, so one of these equations we need to use.

So, if the fitness of the  $i$ th member is less than fitness of the partner then we will have to use the first equation else we will have to use the second equation. Again here  $r$  is a random number it has to be between 0 and 1 and this is also a  $1 \times D$ . This is how we will be generating new solutions in teaching learning based optimization. In particles swarm optimization we had to update the velocity of the particle first then we had to determine the position.

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### Particle Swarm Optimization (PSO)

➤ Particle velocity

$$v_i = w v_i + c_1 r_1 (p_{best,i} - X_i) + c_2 r_2 (g_{best} - X_i)$$

➤ Position of particle

$$X_i = X_i + v_i$$

$v_i$	Velocity of the $i$ th particle
$w$	Inertia of the particles
$c_1$ and $c_2$	Acceleration coefficients
$r_1$ and $r_2$	Random numbers $[0,1]$ of size $(1 \times D)$
$p_{best,i}$	Personal best of $i$ th particle
$g_{best}$	Global best
$X_i$	Position of $i$ th particle

So, for determining the velocity in  $t$  plus 1th generation we will have to use the velocity of the  $t$ 'th generation  $w$   $c_1$  and  $c_2$  are use a defined parameters  $r_1$  denotes random numbers between 0 and 1. And it has to be of the dimension  $1 \times D$ . So, for every decision variable

we will have to generate one random number same thing with  $r_2$  this is also 1 cross  $D \times i$  is the particle position for which we are updating the velocity.

$p_{best}$  is the best solution which was obtained for the  $i$ th particle so far right. So, that is  $p_{best}$  and  $g_{best}$  is the global best of the entire population. So, remember this is not  $g_{best}$  of  $i$ , but only  $g_{best}$  right. So, once we find out the particle velocity we can update the position. So,  $X$  of  $i$  of  $t$  plus 1th iteration is of  $t$ th iteration and  $t$  plus 1 velocity obtained in that  $t$  plus 1 generation. So, here this is how the variation operator is used to generate a new solution.

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### Differential Evolution (DE)

➤ Donor vector using DE/rand/1 mutation strategy

$$V_i = X_{r_1} + F(X_{r_2} - X_{r_3})$$

$\text{Example: } [2 \ 8 \ 9] + 0.8 [ \overset{(6 \ 5 \ 3)}{6 \ 5 \ 3} - \underset{(2 \ 7 \ 9)}{2 \ 7 \ 9} ]$

F      Scaling factor, a constant between 0 and 2

$r_1, r_2, r_3$       Random solutions  $r_1, r_2, r_3 \in \{1, 2, 3, \dots, N_p\}$  and  $r_1 \neq r_2 \neq r_3 \neq i$  where  $i$  is the index of current solution

➤ Trial vector using binomial crossover

$$u^j = \begin{cases} v^j & \text{if } r \leq p_c \text{ OR } j = \delta \\ x^j & \text{if } r > p_c \text{ AND } j \neq \delta \end{cases}$$

$p_c$	crossover probability
$\delta$	randomly selected variable location $\delta \in \{1, 2, 3, \dots, D\}$
$r$	random number between 0 and 1
$u^j$	$j^{\text{th}}$ variable of trial vector
$v^j$	$j^{\text{th}}$ variable of donor vector
$x^j$	$j^{\text{th}}$ variable of target vector

In differential evolution for every particle  $i$  we had to find out the donor vector, while determining the donor vector we had to select three random solutions right. So, this random solution should not be the same member for which we are determining the donor vector.  $F$  is a

scalar value right irrespective of the number of decision variables it is just a scalar value known as scaling factor and it is a constant which is between 0 and 2.

So, once we determined this donor vector then we will have to get the trial vector right. So, trial vector the values of the trial vector will either come from the donor vector or the target vector using these conditions right. So, here  $p_c$  denotes the crossover probability and this  $\delta$  is a randomly generated integer variable between 1 and  $D$  right. This ensures that at least one member is copied from the donor vector to the trial vector in differential evolution this is how we employ variation operators to generate new solutions. Here we have discuss only binomial crossover you can look into the previous videos to see the exponential crossover also.

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### Genetic Algorithm (GA)

➤ Offspring generated using SBX crossover

$$\beta = \begin{cases} (2u)^{1/\eta_c} & \text{if } u \leq 0.5 \\ \left(\frac{1}{2(1-u)}\right)^{1/\eta_c} & \text{otherwise} \end{cases}$$

$r_c$  and  $u$  are random numbers between 0 and 1  
 $p_c$  is the crossover probability  
 $\eta_c$  is distribution index

$$\begin{aligned} O_a &= 0.5[(1+\beta)P'_a + (1-\beta)P'_b] \\ O_b &= 0.5[(1-\beta)P'_a + (1+\beta)P'_b] \end{aligned} \quad \text{if } r_c < p_c$$

$P'_a$  Parent 1     $O_a$  Offspring 1  
 $P'_b$  Parent 2     $O_b$  Offspring 2

➤ Offspring generated (if  $r_m < p_m$ ) using polynomial mutation

$$\delta = \begin{cases} (2r)^{1/\eta_m} - 1 & \text{if } r < 0.5 \\ 1 - [2(1-r)]^{1/\eta_m} & \text{if } r \geq 0.5 \end{cases}$$

$r_m$  and  $r$  are random numbers between 0 and 1  
 $p_m$  is the mutation probability  
 $\eta_m$  is distribution index

$$O = O + (ub - lb)\delta \quad \text{if } r_m < p_m$$

$O$  Offspring solution  
 $ub$  upper bound  
 $lb$  lower bound

In genetic algorithm we had to generate this parameter beta. So, this beta has to be generated for every decision variable right. So, as many decision variables are there that many random numbers we need to generate between 0 and 1; if the random number is less than or equal to 0.5 we update the beta using this equation here  $\eta_c$  is a user defined parameter right. So, once we calculate beta either using this part or using this part we can calculate the two offspring right.

So, over here  $P_a$  and  $P_b$  denote the parents. So, in for generating even one offspring we need two solutions which are the parents solution which we had found from the mating pool right. For this crossover to happen we need to select another random number right which has to be less than the crossover probability.

So, if we have a problem with  $D$  decision variables then we will have to generate  $D + 1$  random numbers between 0 and 1. The first random number will decide whether we will do crossover or not, if we decide to do crossover if this condition is satisfied then we will have to generate the additional  $D$  random numbers to find out the beta values.

Once the beta value is found out we can generate the two offspring using these two equations right. Similarly, for a mutation we need to generate a first one random number between 0 and 1 if that happens to be less than mutation probability then we need to undergo mutation right else mutation is not to be performed. If this condition is satisfied then we need to generate additional  $D$  random numbers between 0 and 1 right.

So, if those random numbers are less than 0.5 we need to use this equation to determine the delta else we need to use this equation to determine the delta.  $\eta_m$  again is a user defined parameter similar to  $sbx$  crossover; in polynomial mutation also we need to generate a  $d$  different random numbers. So, that we get  $d$  different deltas right, once this delta is obtained we can calculate the new offspring using this equation.

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### Artificial Bee Colony (ABC)

➤ Employed bee phase

$$X_{new}^i = X^i + \phi(X^i - X_p^j)$$

➤ Onlooker bee phase

$$prob_i = 0.9 \left( \frac{fit_i}{\max(fit)} \right) + 0.1$$

$$X_{new}^j = X^j + \phi(X^j - X_p^j) \text{ if } r < prob_i$$

➤ Scout bee phase

$$X_k = lb + (ub - lb)r \text{ if } \max(trial_k) > limit$$

$X^j$   $j^{th}$  variable of solution  $X$

$X_p^j$   $j^{th}$  variable of partner solution

$prob_i$  Probability of  $i^{th}$  solution

$fit_i$  Fitness of  $i^{th}$  solution

$\phi$  Random number between -1 and 1

$r$  Random numbers between 0 and 1

$lb$  Lower bound of decision variable

$ub$  Upper bound of decision variable

Handwritten notes:

- [2 3 4]
- [4 9 7]
- [2 3]
- trial
- 5
- [8 5 7 3 (12)]
- 6

In artificial bee colony optimization the equation which is used to generate a new solution is this right. So, the same equation is use in employed bee phase as well as in onlooker bee phase right, so this equation is same. And like the four other techniques right wherein all the variables were change; so for example, if we look into this. So, if we if I have a 3 variable problem 289 let us say  $X_r$  is 289. Let us say scaling factor is 0.8 and the other two solutions are let us say 653 minus 297 right.

So, all the variables would get changed all the variables are modified to get this v i. Since v i is changing most likely the trial vectors will also be having many values from v i right. So, it is not like we will be changing only one value whereas, in artificial bee colony optimization right we will be deciding on which variable to change we will be changing only one variable right.

So, here if we have solutions let us say 2, 3, 4 and the partner solution is 4 9 7, then first we will have to decide which is the variable that we are going to change.

So, that  $j$  variable is to be randomly selected, so between 1 and 3 we will have to randomly select one variable right. So, let us assume that we are selecting the third variable right. So, this 2 3 will remain constant the only this four will get change according to this equation right. So, here again  $X_p$  is a randomly selected partner  $j$  is the randomly selected decision variable and  $\phi$  is a random number between minus 1 and 1.

So, that is unique about artificial bee colony optimization when compared to the other four algorithms that only one variable is changed over here. And then in onlooker bee phase the variation operator is same only thing is that we will have to check this probability. So, if this condition is satisfied in that case that particular onlooker bee explores the  $i$ th food source right. The equation that we use to generate new solution is similar to what we used in employed bee phase right.

So, whereas, in scout bee phase we randomly generate a solution right between the lower bound and upper bound, remember again in scout bee phase we were generating only one new solution right. So, for every solution we had to keep track of the trial vector this trial vector denotes as to how many times the solution got an attempt to generate a new solution, but fail to generate a better solution than itself right.

So, if there are five solutions which exceed this limit like again limit is a user defined parameter if five solutions exceed this parameter limit. So, let us say if the trial for five solutions are 8 9 7 3 2 12 and the limit is let us say is 6 right. So, in that case since this solution has the maximum number of failures we will be updating the solution corresponding to the 12 failures right, so only one new solution is generated ok.



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### Equations used for variation

**TLBO**

$$X_{new} = X_i + r(X_{best} - T_j X_{min})$$

$$X_{new} = \begin{cases} X_i + r(X_i - X_p) & \text{if } f_i < f_p \\ X_i - r(X_i - X_p) & \text{otherwise} \end{cases}$$

**PSO**

$$v_i = w v_i + c_1 r_1 (p_{best,i} - X_i) + c_2 r_2 (g_{best} - X_i)$$

$$X_i = X_i + v_i$$

**DE**

$$V = X_i + F(X_i - X_{r_1})$$

$$u^j = \begin{cases} v^j & \text{if } r \leq p_r \text{ OR } j = \delta \\ x^j & \text{if } r > p_r \text{ AND } j \neq \delta \end{cases}$$

**GA**

$$\beta = \begin{cases} (2u)^{\frac{1}{(u+1)}} & \text{if } u \leq 0.5 \\ \left(\frac{1}{2(1-u)}\right)^{\frac{1}{(u+1)}} & \text{otherwise} \end{cases}$$

$$O_a = 0.5[(1+\beta)P_a + (1-\beta)P_b]$$

$$O_b = 0.5[(1-\beta)P_a + (1+\beta)P_b] \text{ if } r_i < p_i$$

$$\delta = \begin{cases} (2r)^{\frac{1}{(u+1)}} - 1 & \text{if } r < 0.5 \\ 1 - [2(1-r)]^{\frac{1}{(u+1)}} & \text{if } r \geq 0.5 \end{cases}$$

$$O = O + (ub - lb)\delta \text{ if } r_n < p_n$$

**ABC**

$$X_{new}^j = X^j + \phi(X^j - X_p^j)$$

$$probab_i = 0.9 \left( \frac{fit_i}{\max(fit)} \right) + 0.1$$

$$X_{new}^j = X^j + \phi(X^j - X_p^j) \text{ if } r < probab_i$$

$$X_i = lb + (ub - lb)r \text{ if } \max(trial_i) > limit$$

So, this slide shows all the variation operators put in one place for the 5 algorithms which we have discussed. Given this you can now think of developing your own variation operator which can be used to develop a new algorithm. So, now that we have discussed the variation operator let us quickly see how the survival strategies differ from one algorithm to the other algorithm right.

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### Survival strategy

Old Solution		New Solution		Greedy strategy		$(\mu, \lambda)$ strategy		$(\mu + \lambda)$ strategy	
Sol	f	Sol	f	Sol	f	Sol	f	Sol	f
[3 1]	10	[4 3]	25	[3 1]	10	[4 3]	25	[1 0]	1
[1 5]	26	[1 2]	5	[1 2]	5	[1 2]	5	[2 0]	4
[3 3]	18	[4 2]	20	[3 3]	18	[4 2]	20	[1 2]	5
[2 0]	4	[1 0]	1	[1 0]	1	[1 0]	1	[3 1]	10
[4 1]	17	[3 1]	10	[3 1]	10	[3 1]	10	[3 1]	10

Algorithms →

- TLBO
- DE
- ABC (except scout phase)

PSO

Scout phase of ABC

GA

10 ✓  
26 ✓  
18 ✓  
4 ✓  
17 ✓  
25 ✓  
5 ✓  
20 ✓  
1 ✓  
10 ✓

So, consider this case wherein let us say this is the set of solutions that we have and these are the set of solutions which get generated at some point of time in the algorithm right. So, these are old solutions these are new solutions, so this is the decision variable let us say this is a two variable problem.

So, we have  $x_1$  and  $x_2$  and this is the fitness function value corresponding to 3 and 1. Assume that there is some objective function for in which if we substitute  $x_1$  is equal to 3 and  $x_2$  is equal to 1 we get a fitness function value of 10 right. So, here we have these five solutions and they are corresponding fitness function. Similarly, in new solutions let us say these five solutions get generated 4, 3, 1, 2, 4, 2, 1, 0 and 3, 1. And this is their corresponding fitness function value 25, 5, 20, 1 and 10; broadly we employed three different type of survival strategies the first one is the greedy strategy.

So, in greedy strategy what we did is like we compared fitness of this first solution with the fitness of the first new solution, so between 10 and 25 10 is better right. So, we took that, so between 26 and 5; 5 is better, so we took this 1, 2 right, between 18 and 20; this 18 is better right. So, we took that 18 and the corresponding solution is 3 3, so that is taken.

Between 4 and 1; the solution 1 is better and their corresponding solution is 1 0. So, that is what we retain right and between 4 comma 1 and 3 comma 1 the fitness is 17 and 10; so 10 is better right. So, we take a 10 over here right and the corresponding solution, so this greedy strategy we employed in TLBO. Remember in TLBO what we did was the first solution underwent teacher phase and we generated a new solution. If the new solution is better we took that solution into the population same thing we did in the learner phase also right. So, in those cases we were using greedy strategy right.

So, in TLBO we use greedy strategy similarly in ABC also we did the same thing that the first bee undergoes the employed bee phase, if it generates a better solution that better solution is taken into the population right. So, employed bee phase as well as the onlooker bee phase we employed this greedy strategy right. In scout bee phase we did not employ this strategy right we generated a new solution and whether it was good or bad it was taken into the population right.

Similarly, in DE we use this greedy strategy right. So, we had let us say we have 20 solutions we generate 20 more solutions in D compare the first old solution with the first new solution and whichever is better was taken into the new population right. So, this greedy strategy is the one that we widely used in three algorithms right.

The other strategy is mu comma lambda strategy. In this mu comma lambda strategy the new solution is always taken into the population right, we do not care whether it is better or not right it is always taken into the population. So, this if you remember we did it in particle swarm optimization as well as the scout phase of ABC.

So what we basically did was let us say if these were the old solution 10, 26, 18, 4 and 17. And these are the new solutions 25, 5, 21 and 10 right, we did not take the old solutions or we did not do a comparison between 10 and 25, 26 and 5, 18, 24, 1, 17, 10 we did not do that comparison, we directly took the new set of solutions right.

So, here if you see this is identical to the new solutions right. So, this is known as mu comma lambda strategy. So, this we had used in particle swarm optimization as well as scout phase of ABC. And then we had this mu plus lambda strategy wherein we combined all the solutions and we use this only in GA right. So, here what we did was we have this 10, 26, 18, 4, 17 and this new solutions 25, 5, 20, 1 and 10.

So, we had 5 old solutions we generated 5 new solutions, so out of this 10 solutions we had to select 5 best solutions. So, what we did is we selected the 5 best solutions 1, 4, 5, 10, 10, so this is what we did in mu plus lambda strategy. So, these are the three strategies which we have seen in different algorithms. So, here as you can see for example, in ABC one phase we employed greedy strategy and in another phase a in the scout bee phase we employed a mu comma lambda strategy.

So, similarly when you are designing an algorithm you can choose to incorporate any one of these strategies or a combination of these strategies or you can even come up with your own strategies. So, that concludes this session, in this session we basically revised the survival strategies and the variation operators which we came across in this five algorithms.

Thank you.