

Membrane Technology
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Lecture-30
Problems and solutions based on ED & PV

Good morning students and this is lecture 30 under module 10. So, in today's lecture we will be solving some problems so based on electrodialysis and pervaporation.

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Electrodialysis	
□ Summary of Electrodialysis	
Membranes	Cation-exchange, and anion-exchange membrane
Thickness	100-500 μm
Pore size	Non-porous
Driving force	Electrical potential difference
Separation principle	Donnan exclusion mechanism
Membrane materials	Cross linked co-polymers based on di-vinylbenzene with polystyrene or polyvinylpyridine co-polymers of polytetrafluoroethylene or poly(sulfonyl fluoride-vinyl ether)
Application	a. Desalination of water (also food and pharmaceutical industries), b. Separation of amino acids, c. Production of salts



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Now as you know we have discussed this in our few last lectures so I am just again repeating the summary of electrodialysis. So, as you know in electrodialysis we need ion exchange membranes either it is a cation exchange membrane or anion exchange membrane or we have discussed what is bipolar membrane also. So, usually the thickness of this membrane which is doing the separation is not more than 500 micron in the range of 100 to 500 micron there is no pore sizes because mostly these are non porous membranes.

Driving force for electro dialysis is electrical potential difference and the separation principle is not an exclusion mechanism this we have discussed. And membrane materials are usually cross-linked copolymers based on di-vinylbenzene with polystyrene or polyvinyl pyridine copolymers of polytetrafluoroethylene or poly sulfonyl chloride vinyl ether. Application is major application

of electrodialysis is of course desalination of water also it has applications in food and pharmaceutical industries. Separation of amino acids production of salts.

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Electrodialysis

Question 01: Estimate membrane area and electrical energy requirements for an electrodialysis process to reduce the salt (NaCl) content of $32000 \text{ m}^3/\text{day}$ of water from 2500 mg/L to 800 mg/L with a 50% conversion. Assume each membrane has a surface area of 0.5 m^2 and each stack contains 300 cell pairs. A reasonable current density is 12 mA/cm^2 and current efficiency is 0.8 (i.e. 80%).

Solution:

Faraday's constant, $F = 96520 \text{ A/Cu}$.

Volumetric flow rate of diluate, $Q = (32000 \times 0.5) \times (24 \times 3600)$

$= 0.185 \text{ m}^3/\text{s}$

Molecular weight of $\text{NaCl} = 58$.



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So, let us now see one or two or few problems based on electrodialysis right. So, the first one it has been asked to estimate the membrane area and electrical energy requirements for any electrodialysis process to reduce the salt here it is sodium chloride content of 32000 meter cube per day of water from 2500 milligrams per liter to 800 milligrams per liter with a 50% conversion you can assume each membrane has a surface area of 0.5 meter square and each stack contents around 300 cell pairs.

A reasonable current density is given which is 12 milliamperes per centimeter square and current efficiency is 0.8 or we can assume it to be 80%. Now let us see how we can solve this problem. So, as you know that Faraday's constant that is capital F, so that is nothing but 96520 m Spock equivalent and a volumetric flow rate that is volumetric flow rate of diluate, so that is Q so it is given 32000 into 0.5 into 24 into 3600 is just converting some of the units to meter cube per second, so we will get 0.185 meter cube per second.

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current density, $i = 5 \text{ mA/cm}^2$

concentration diffⁿ betⁿ feed & diluate, $\Delta C = \frac{(2500 - 800)}{50.5}$

$\Delta C = 29.6 \text{ mol/m}^3$
 $= 29.6 \text{ equiv/m}^3$

total area of cell pairs,

$$A_m = \frac{1 \times 96520 \times 0.185 \times 29.6}{120 \times 0.8}$$

$$= 5405.22 \text{ m}^2$$

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Then molecular weight of sodium chloride that is 58.5, so it is usually a better practice to write down the data which are given in the problem. So, current density that is the small i is given to be 5 mili amperes per centimeter square, so just note down all the data which are given. Then concentration difference between feed and diluate between so that is on the Delta C actually, so Delta C is directly you can calculate delta C equals 2500 - 800 divided by the molecular weight.

So Delta C is nothing but 29.6 or we can write down it is 29.6 so this is moles per meter cube it is moles per meter cube or we can write also in terms of equivalent what meter cube. So, total area of cell pairs that is the equation A_m actually which was discussed in electrolysis class A_m equals 2, so just substitute in that equation 1 into 96,520 into 0.185 into 29.6, now all divided by 120 into 0.8. So, here area that you will get equals to 5405.22 meter cube. Then you have to calculate and the electrical power and all.

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Each stack contains 300 cell pairs
 Total area = $0.5 \times 300 = 150 \text{ m}^2$
 No. of stacks = $5405.22 / 150 = 45.04 \text{ m}^2$
 Electrical current, $I = \frac{96520 \times 0.185 \times 29.06}{300 \times 0.8}$
 $= 2162.09 \text{ A}$
 Electrical power, $P = 2162.09 \times 300$
 $= 648.626 \text{ kW}$



So, let us do that so each stack contents each stack it is already given each stack content around 300 cell pairs contains around 300 cell pairs. So, total area will be total area of 0.5 into 300 which is 150 meter square. So, number of stacks equals to 5405.22 divided by 150 right so we get 45.04 meter square. So, electrical current through the stack that is I equals to 96520 into 0.185 29.06 divided by 300 into 0.8 I am not writing the original equation it is there I think you know that we have already discussed so that is why I am not writing it.

So it is 2162.09 ampere so electrical power P equals to I into our in that equation .09 into 300 so it is 648.626 kilo watts so this is a very small example so we will see one or two more so this is how you will calculate the basic equations remain same from where you can calculate your electrical current so from that once you calculate that then you can calculate the power P equals to I into R.

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Electrodialysis

Question 02: An electrodialysis experiment is being carried out for desalting a solution containing KCl. The volumetric flow rate of solution is 25 L/min, number of unit cells is 60, feed concentration is 3.4 g/L, diluate concentration is 2.1 g/L. Current is measured to be 20.5 amps and energy consumption is 60670 joules in one minute.

Determine (i) the efficiency; (ii) the resistance per unit cells at c_f

Solution:

$$I = \frac{z F Q (C_f - C_{di})}{E_R} \quad , \quad F = \text{Faraday's const.} = 96520 \text{ amp/g}$$

$$z = \text{Graham's valence}$$

$$E_R = n (\eta_s \eta_w \eta_m)$$



So the next problem is in electrodialysis experiment is being carried out for desalting a solution containing KCl potassium chlorate. So, the volumetric flow rate of solution is 25 liters per minute number of unit cells is 60, feed concentration is given to be 3.5 grams per liter and diluate concentration is 2.41 grams per liter, current is measured to be 20.5 amperes and energy consumption is 6067 joules in 1 minute. Now you have been asked to calculate the efficiency and the resistance per unit cell at C f. Basically the electrical current needed to desalt a solution is directly proportional to the number of ions transport through the membrane and that we can get it from the usual I equation.

So I am just writing one say over here so I equals to Z F Q then C f - C dial divided by Zeta now where F is your Faraday constant, so we just discussed that. So, constant so which is this 96520 equivalent amps per equivalent so Z is electrical balance Q is volumetric flow rate and this Zeta so this is your current utilization. So, the current utilization is directly related to the number of cells n in the stack and the efficiency of the utilization of current so we can write this Zeta equals to so this is eta s eta w eta m.

So where this small n is the number of stack then this 3 eta x eta w and eta m so they are the efficiency due to semi permeability of the membrane water transfer through the membrane and current leads to the membrane. so, if you combine above these two equations.

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$$\text{Electrical Efficiency} = \frac{F Q z (C_f - C_{dil})}{n i}$$

$$F = 96500, Q = 25 \text{ L/min}$$

$$n = 60, i = 20.5 \text{ amp}$$

$$C_f - C_{dil} = (3.4 - 2.1) / 74.5 = 0.017 \text{ equiv/L}$$

$$\text{Electrical Efficiency} = \frac{96500 \times (25/60) \times 0.017}{60 \times 20.5}$$

$$= 0.557$$


So, we will get that electrical efficiency equals to F just steady arranging the equations so C f - C dil divided by n and i, i is the current this one. So, you are given F equals to 96,500, Q is given as 25 liters per minute n is given 60, i is given 20.5 amps then C f - C dil the concentration difference is 3.4 – 2.1 divided by 74.5 so this comes to you know 0.017 equivalent per liter. So, we can write electrical efficiency then so electrical efficiency equals to just substitute these values 96,500 into 25 by 60 into 0.017 so divided by 60 into 20.5, so this comes to be 0.557.

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Assuming the cell resistance to be inversely proportional to concentration,

$$E = \frac{I^2 n t R (C_f)}{\frac{2 C_{dil}}{C_f} - \left(\frac{C_{dil}}{C_f}\right)^2}$$

Rearranging,

$$R(C_f) = \frac{E}{I^2 n t} \left[\frac{2 C_{dil}}{C_f} - \left(\frac{C_{dil}}{C_f}\right)^2 \right]$$

$$R(C_f) = \frac{60670}{20.5 \times 60 \times 60} \left[\frac{2 \times 2.2}{3.4} - \left(\frac{2.1}{3.4}\right)^2 \right]$$

$$= 0.7018 \text{ ohm/volt cell}$$


Now assuming the cell resistance is inversely proportional, so assuming the cell resistance cell resistance to be inversely proportional to concentration proportional to concentration so we can write E equals to I square n t RC R then C f so divided by 2 C dil divided C f - C dil / C f square.

So, on rearranging will get $R C f$ equals to E to the power of I square $n t$ then $2C$ divided by $C f - C$ divided by $C f$ square. So, please substitute these values so here $C f$ will be 60670 divided by 20.5 into 60 into 60 divided by 2 into 2.21 divided by 3.4 – 2.1 divided by 3.4 this is all square.

So you calculate it is 0.7018 so that unit is ohm per unit cell so this is your resistance which is being asked to calculate.

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Electrodialysis

Question 03: A stack of electrodialysis units having 100 cells is to be used to partially demineralised 1,10,000 L/day of water from casein plant. The salt content is 4000 mg/L and the cation (or anion) content is 0.066 g equiv wts per litre. Pilot scale studies using a multicellular stack have been made. It was found that the current efficiency was 0.8, the efficiency of salt removal was 0.4, and the resistance was 4.4 ohms, and the current density normality ratio was 500.

Determine:

- (i) the current required,
- (ii) area of membrane, and
- (iii) power requirement



We will see another small problem the statement goes something like this, so a stack of electrodialysis units having 100 cells is to be used to partially demineralized 110,000 liters per day up water from Casien plant. So, basically it is a dairy unit you can say in that it is being used so the salt content is 4000 milligrams per liter and the cation or anion content is 0.66 gram equivalent weights per liter.

Pilot scale studies using a multicellular stack have been made it was found that the current efficiency was 0.8 the efficiency of salt removal was 0.4 the resistance was 4.4 ohms and the current density normality ratio was 500. Now you are asked to determine the current required area of membrane and the power requirement.

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Solution:

The current I calculated for 110 cells,

$$I = 96500 \times 11000 \times 0.045 \times \frac{1}{86400} \times \frac{0.4}{110 \times 0.8}$$

$$I = 25.13 \text{ amp}$$

Since, the normality equals the no. of gram equivalent weight / L
Normality = 0.045

So let us solve it the current I calculated for 110 cells so you can just calculate using the equation so I am not writing the original equation directly substitution into 11000 into 0.045 into 1 by 86,400 in to 0.4 by 110 into 0.8. So, I comes to be 25.13 ampere. Now since the since the normality equals so normality equals the number of gram equivalent the number of gram equivalent per gram equivalent weight per liter **per liter**. So we can write normality to be 0.045.

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The current density

$$= 400 \times 0.045$$
$$= 18 \text{ mA/cm}^2$$

$$\text{The membrane area} = \frac{25.13}{18} \times 1000$$
$$= 1396.11 \text{ cm}^2$$

$$\text{Power required, } P = RI^2 = 4.4 \times (25.13)^2$$
$$P = 2.77 \text{ kW}$$

Now we can calculate the current density the current density so equals to 400 into 0.045 which is 18 milli ampere per centimeter square. So, the membrane area is 25.13 so divided by 18 into 1000 so this is 1396.11 centimeter square and then you have to calculate the power requirement. So, the power required so that is $R I^2$ so 4.4 ohm and I is 25.13 square so it is 2.77 kilo

watts power. So, this is how you will calculate and you will specifically solve the electro dialysis problems.

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Pervaporation	
□ Summary of pervaporation	
Membranes	Composite membranes with an elastomeric or glassy polymeric top layer
Thickness	~ 0.1 to few μm (for top layer)
Pore size	Non-porous
Driving force	Partial vapour pressure (activity difference)
Separation principle	Solution/ diffusion
Membrane materials	Elastomeric and glassy polymers
Application	a. Dehydration of organic components, b. Removal of organic components (alcohols, aromatics, chlorinated hydrocarbons) from water, c. Polar/ Non-polar (such as, alcohol/ aliphatics or alcohol/aromatics), d. Saturated/ Un-saturated (such as, cyclohexane/ benzene) e. Separation of isomers (such as C8 isomers; o-xylene, m-xylene, p-xylene, p-xylene, ethyl benzene, styrene)

So, let us try to solve one pervaporation problems so though we do not have much further pervaporation things. So, we will try to solve one problem so before we go into pervaporation. So, let us just go through once again the summary of pervaporation. As I told you earlier for pervaporation names comes from the term permeate vaporization or evaporation. So, the permeate is getting vaporized and thereby enhancing the rate of mass transfer.

So the different types of membranes that he used for pervaporation there is really composite membranes with elastomeric or glassy polymeric top layer asymmetric structure basically. Thickness is usually 0.1 to few microns for the top layer I am talking about the top layer thickness as I told you many times that in case of asymmetric membrane or a composite membrane so the top layer thickness that determines the rate of separation.

So, the thickness edgeless as possible so that the resistance to flow by membrane or due to membrane would be less so there is no pore size because these are non porous membranes. Driving force is nothing but your partial vapor pressure or we can call activity difference also. Separation principle is your solution diffusion, so membrane materials are elastomeric and glassy polymers. So, n number of applications are there we have discussed few applications also.

So dehydration of organic components removal organic components like alcohols, aromatics, chlorinated hydrocarbons from water. Polar and non-polar compounds such as alcohol aliphatics, alcohol aromatic saturated and unsaturated like cyclohexane and benzene mixtures then separation of isomers such as C8 isomers, orthoxylene, metaxylene, paraxylene then p-xylene ethyl benzene and styrene right.

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$$T_{avg} = \frac{T_{in} + T_{out}}{2} = \frac{60 + 35}{2} = 47.5^{\circ}\text{C}$$

$$C_{PB} = 0.654 \frac{\text{cal}}{\text{g}^{\circ}\text{C}} = \frac{0.654 \times 74.12}{1000} \left(\frac{\text{cal}}{\text{g}^{\circ}\text{C}} \cdot \frac{\text{kcal}}{\text{cal}} \cdot \frac{\text{g}}{\text{gmol}} \right)$$

$$= 0.048 \text{ kcal/g}^{\circ}\text{C}$$

$$C_{PW} = 1.1 \frac{\text{cal}}{\text{g}^{\circ}\text{C}} = \frac{1.1 \times 74.12}{1000} = 0.081 \frac{\text{kcal}}{\text{g}^{\circ}\text{C}}$$

$$C_{PF} = (0.1 \times 0.081) + (0.9 \times 0.048)$$

$$= 0.051$$

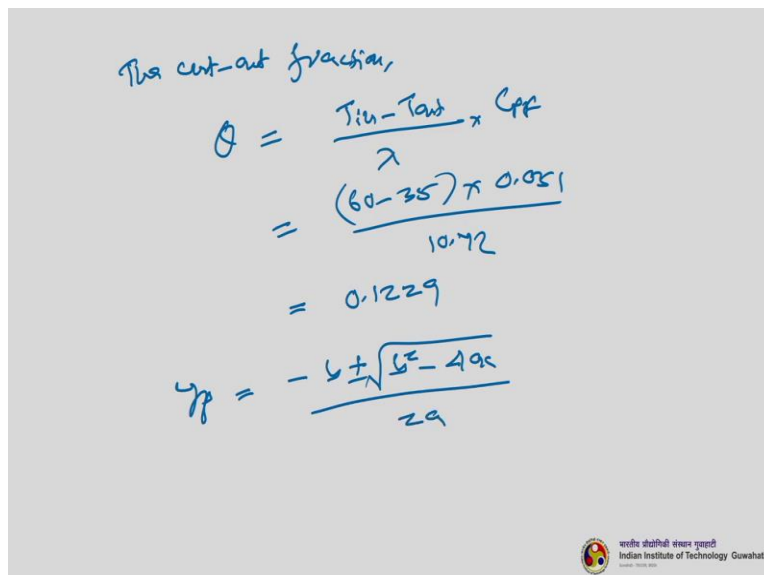
So I have just listed one a small problem so let me just read out the statement then we will solve it. So, it is required to remove water from n-butanol using cellulose 2.5 acetate membrane. The feed is 90 mole percent n-butanol, feed is at 60 degree centigrade, selectivity at 42 flux is 0.15 pound per feet square hour maximum permissible temperature drop is 35 degree centigrade. So, find out the cut fraction theta and the permit and liquid mole fractions. If the flow rate is 100 lb per hour find out the membrane area.

So, given lambda V is 140.6 calorie per gram lambda W is 9.84 kilo calorie per gram mole C p the specific heat of B is 45 degrees centigrade is 0.654 calorie per gram degree centigrade and C pw at 45 degree centigrade again is 1.1 calorie per gram degree centigrade. So, let us now solve this problem so lambda B is 140.6 calorie per gram. So, just converting 140.6 into 74.12 divided by 1000 so this is calorie per gram this is then your kilo calorie for calorie then gram by gram mole. So, you will get 10.42 kilo calorie okay per gram.

So then T average temperature is T in + T out divided by 2 so 60 + 35 by 2 is 47.5 degree centigrade then your C pB so it is given 0.654 calorie per gram degree centigrade. So, that will convert just like we converted the lambda be 0.654 into 74.2 divided by 1000 so that you units are calorie per gram degree centigrade then kilo calorie for this is calorie then your gram per gram mole. So, you get 0.048 kilo calorie per gram degree centigrade.

Just converting into proper units similarly C pw is 1.1 calorie per gram degree centigrade so we will write this 1.1 into 74.12 divided by 1000 it goes to 0.081 kilo calorie per gram degree centigrade C pf is 0.1 into 0.081 + 0.9 into 0.048 so this is 0.051 C pm.

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The cut-out fraction,

$$\theta = \frac{T_{in} - T_{out}}{\lambda} \times C_{pf}$$

$$= \frac{(60 - 35) \times 0.051}{10.42}$$

$$= 0.1229$$

$$y_p = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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So, the cut out fraction which is denoted by theta so is T in - T out divided by lambda into C pf so just substituting the values so we get 60 - 35 into 0.51 divided by 10.42 which is 0.1229 now permeate mole fraction can be found out by solving the following equation so y p equals to -b +/- b square - 4ac under root so divided by 2a right.

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$$\begin{aligned}
 a &= (\lambda_{AB} - 1)\theta = (42 - 1) \times 0.1229 = 5.039 \\
 b &= -[(1 - \theta) + (\lambda_{AB} - 1)Z + \lambda_{AB}\theta] \\
 &= -[(1 - 0.1229) + (42 - 1)0.1 + 42 \times 0.1229] \\
 &= -10.139 \\
 c &= \lambda_{AB}Z = 42 \times 0.1 = 4.2 \\
 y_p &= \frac{10.139 \pm \sqrt{(10.139)^2 - 4 \times 5.039 \times 4.2}}{2 \times 5.039} \\
 &= 0.583 \text{ mol fraction of water} \\
 \text{Molar} &= \frac{Z - \theta y_p}{1 - \theta} = 0.0323 \quad \left| \text{Area} = \frac{\text{Feed rate}}{\text{flux}} = 550 \text{ ft}^2 \right.
 \end{aligned}$$

Where a is $\lambda_{AB} - 1$ into θ so this is nothing but $42 - 1$ into 0.1229 is nothing but 5.039 then b, b is minus of $1 - \theta$ this is $+\alpha_{AB} - 1$ into $Z + \alpha_{AB}$ by B into θ so you substitute everything so $-1 - 0.1229 + 42 - 1$ into $0.1 + 42$ into this 0.1229 so your b comes out to be -10.139 so, we got abc so ab we got now we will calculate c, c is this α_{AB} by B into Z so that is 42 into 0.1 equals to 4.2 so you calculate your y_p mole fraction, so this is 10 point I am not writing the equation again it is being there so $10.139 \pm \sqrt{10.139^2 - 4 \times 5.039 \times 4.2}$ so this is under root divided by 2×5.039 so you get 0.583 there is the mole fraction of water.

So then your x_w out will be $Z - \theta y_p$ divided by $1 - \theta$ so you will get point the substitute I am not substituting here 0.0323 then you have to calculate area. So, area equals to feed rate by flux, so we will get 500 feet square. So, this is how you can solve the pervaporation problems. So, please go through all these things in case we have any doubt please do ask me. So, I will try to clarify either by replying wave mail or during open season.

So, the next module will be module 11 it will be on liquid membranes gas separation and membrane discussion the three most potent and very highly attractive membrane by separation actually liquid membrane and distillation are being the new entrant. Gas separation is being happening since decades of course. So, the first lecture we will discuss about liquid membranes. So, what are liquid membranes how they can be prepared a mechanism and what is the mass transfer in liquid membrane.

Then different types of solvents carriers and applications of liquid membrane so, thank you very much in case you have any doubt please feel free to write to me at kmohanty@iitg.ac.in thank you.