Membrane Technology
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Lecture-25
Problems and solutions based on RO & MF

Good morning students today is lecture 25 under module 9 and under this module basically, we will be discussing in today's lecture as well as in subsequent lecture, I will solve some of the numerical. So today's lecture will solve some numerical based on reverse osmosis, and microfiltration. So, let us see the problem one.

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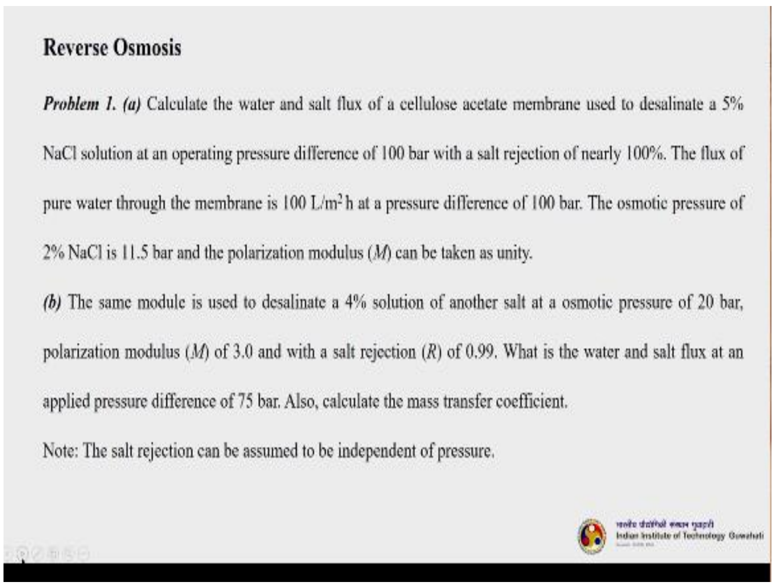

Is being given something like this, like calculate the water and salt flux of a cellulose acetate membrane used to do desalinate a 5% sodium chloride solution at an operating pressure difference of 100 bars, with the salt rejection of nearly 100 %. The flux of pure water through the membrane is 100 liters per meter square hour at a pressure difference of 100 bar. The osmotic pressure of 2% sodium chloride is given is 11.5 bar, and the polarization module M, can be taken as unity.

Now, in the second one in the same problem it has been asked in the same module is used to desalinate a 4% solution of another salt at an osmotic pressure of 20 bar and polarization module M is given as 3.0 with a salt rejection of 0.99. So what is the water and salt flux at an applied pressure difference of 75 bar also calculate the mass transfer coefficient. Note that the salt rejection can be assumed to be independent of pressure.

Now, let us try to solve this problem. So it is a good practice that whenever a problem has been given. So, please note down what is the initially when you try to solve it. Please note down whatever the data is given, and the equations that you require. So, it has been given that pure water flux.

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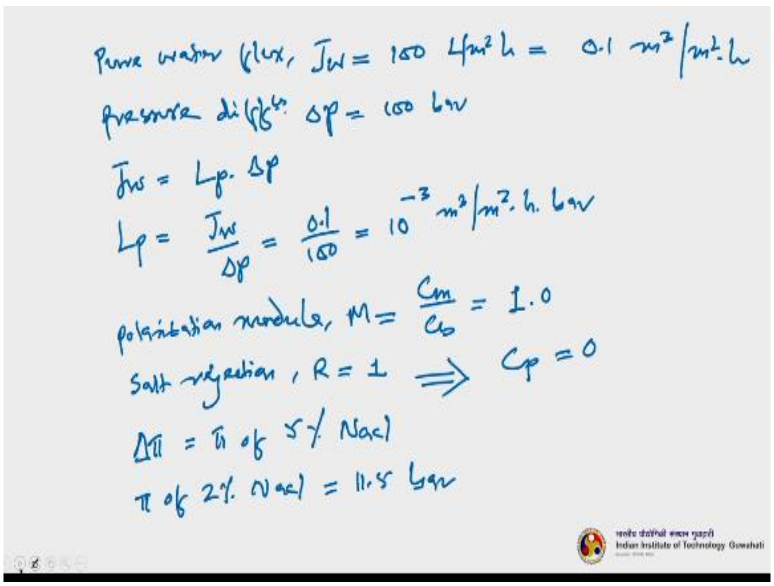

That is, we can write a Jw, it has given as 100 liter per meter square hour. So we can write that 0.1 meter cube per meter square hour. Then pressure difference is given that is delta pi. So delta pi is given 100 bar. So your pure water flux or I will write just Jw equals to Lp into delta pi, Lp is the permeability you know that from our earlier discussions and equation, so we can find Lp from this equation.

Lp equals to Jw by delta p. So Jw is given 0.1, and delta p is 100. So it is 10 to the power of -3 meter cube per meter square hour and bar. So, this is your permeability Lp. Now, we have to calculate the water flux for a 5% NaCl solution as well as salt flux. So, it is given that the polarization module, we can write that is M, capital M is nothing but Cm by Cb. It has given as 1 in this particular case, and it is also given the salt rejection coefficient R equals to 1.

So, this indicates when the salt rejection is unity. So, this indicates Cp equals to 0. So that was the salt concentration in the permeate side is equals to 0. So now we can calculate the osmotic pressure difference the delta pi between the feed and permeate. So due to calculate the delta pi between the feed and the permeate. So, delta pi equals to pi of 5% NaCl. So it is given that the osmotic pressure of 2% sodium chloride solution is 11.5 bar. So it is given pi of 2% NaCl is 11.5 bar.

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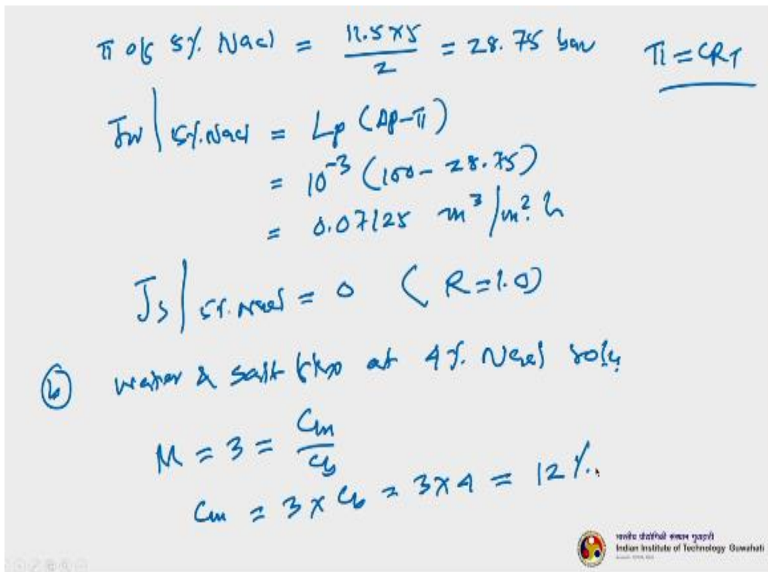

Now we can calculate, Pi of 5% NaCl from that delta of 2% NaCl, so basically you see that it is 11.5 into 5 divided by 2 which is 28.75 bar, you know this is taken from this equation pi equals to CRT. Now water flux Jw for 5% NaCl equals to Lp into delta p - pi, that is 10 to power of -3 just we have calculated Lp to the power of -3 into delta p is equal to 100 - pi is 28.75, so you get by 0.07125 the unit will be meter cube per meter square hour.

So this is the water flux for 5% of sodium chloride solution. Now you know that salt flux Js of 5% of NaCl is equal to 0, because your salt rejection is 1, R is given 1. Now we will try to solve second one, which is called the same module. The same module is used to desalinate 4% NaCl solution at a osmotic pressure of 20 bar and polarization module is given as 3 and salt rejection R is given as 0.99.

Now you have to calculate the water and salt flux at an applied pressure difference of 75 bar and you have to calculate the mass transfers. Let us see water and salt flux at 5% NaCl solution. So your concentration polarization M is given is Cm by Cb is 3. You can calculate Cm here Cm equals 3 into Cb, so 3 into Cb so 3 into Cb is given as 4% sodium chloride so estm is 12%.

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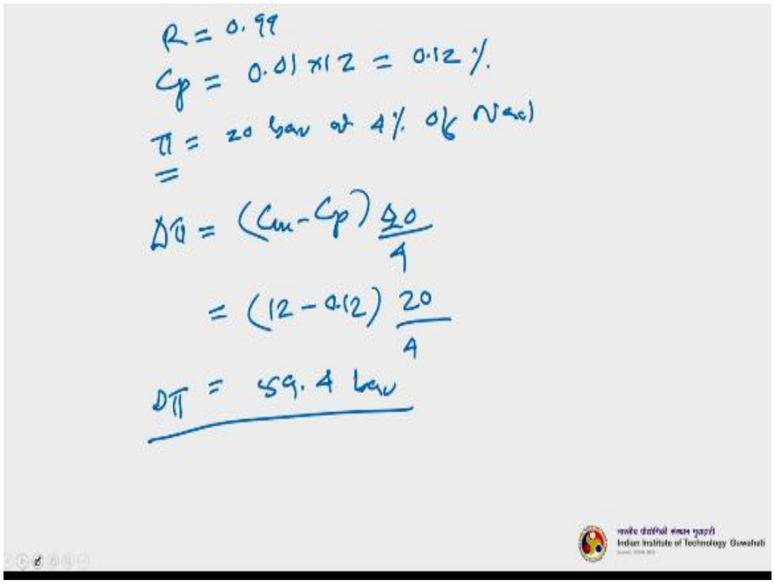

Now, salt rejection R is given as 0.99. So we have to calculate Cp so Cp is the concentration of the salt in the permeate side. So it is 0.01 into 12 is 0.12% do you get it 99% of salt rejection means only 0.01% salt is going inside the permeate side. So Cp equals to 0.01 into 12, 12 is the concentration of salt. Now pi is equals to 20 bar at 4% of NaCl, it is given in the problem statement so what is this?

This is the osmotic pressure difference between the feed and permeate. So now we have to calculate delta pi. So, delta pi equals to Cm - Cp into 20 divided by 4. So which is 12 - 0.12 into 20 divided by 4, delta pi becomes 59.4 bar. So then we have to calculate the water flux. So this your osmotic pressure difference between the feed and the permeate for this particular case of 4% sodium chloride solution, so now we have to calculate the water flux.

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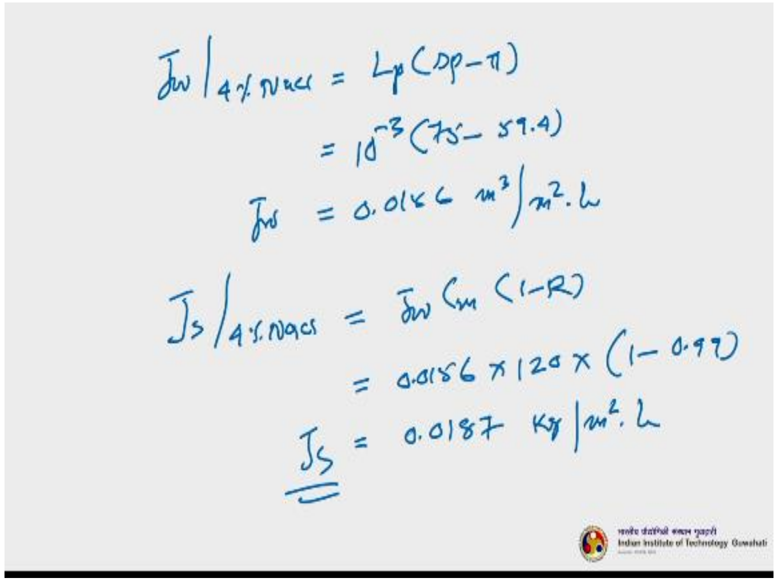

So your water flux Jw is at 4% NaCl. Is Lp into delta p – pi. So LP is that, 10 power of -3 into delta p is 75 and this is we have calculated is 59.4. So Jw here, it comes to be 0.0156 meter cube per meter square hour. So this is your water flux that we have calculated for 4% sodium chloride solution with the same module that is been used. So then we have to calculate the salt flux at 4% NaCl.

So, this is nothing but Jw into Cm into 1-R. So Jw you know 0.0156 we have just calculated into Cm equals to 120 into 1 – R is 0.99. So we will get 0.0187 kg per meter square hour. So this is your salt flux for 4%, Sodium chloride solution. So now next we have been asked to calculate the mass transfer coefficient. So to calculate the feed side mass transfer coefficient you can use the equation.

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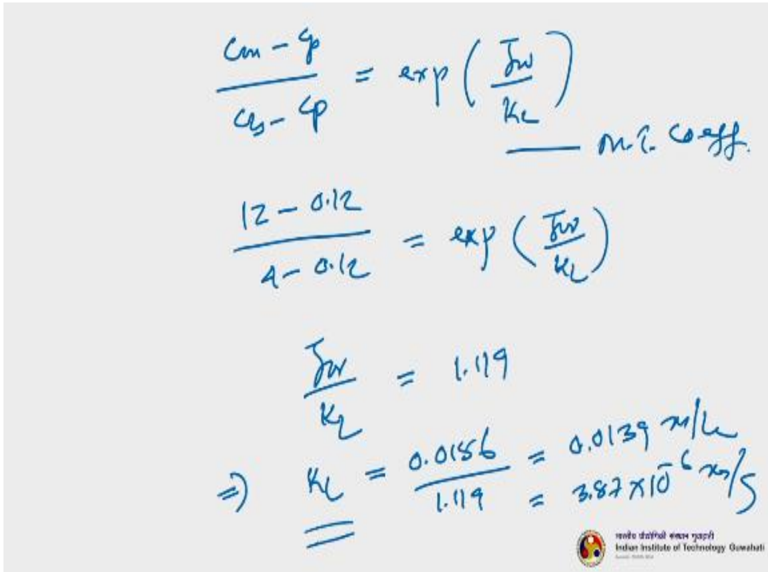

Cm - Cp divided by Cb - Cp equals to exponential to the power of Jw by KL. So this KL is the mass transfer coefficient. So now you substitute all the values, and then you will calculate what is this is so basically it is Cm is 12 - 0.12 divided by Cb is 4 - again 0.12. So equals to exponential to the power of Jw is by KL. So we will find out, Jw by KL equals to 1.119. So we know what is Jw, so from here, we can calculate what is KL.

So KL the mass transfer coefficient is 0.0156, divided by 1.119, which is 0.0139 meter per hour, or we can write it 3.87 into 10 power of -6 meters per second. Usually mass transfer coefficient is being expressed as meter per second, so you get your mass transfer coefficient on the feed side to be 3.87 into 10 power of -6 meters per second. Now, this is how actually you use to calculate the salt flux, as well as water flux. So, you can see that, the first one, we have to calculate salt and salt flux and water flux.

The rejection is 100%, so the salt flux is equal to 0. So everything is getting written on the surface of the membrane so the permeate does not contain any salt so it is R equals to 100 and the next case R is given as a 0.98, so that means the only 0.01% of the salt is going to the permeate side. So with that we have calculated the salt for this one, water flux as well as the salt flux and again when we have calculated mass transfer coefficient using this particular equation.

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$$\frac{C_m - C_p}{C_b - C_p} = \exp\left(\frac{J_w}{k_L}\right)$$

k_L — m.t. coeff.

$$\frac{12 - 0.12}{4 - 0.12} = \exp\left(\frac{J_w}{k_L}\right)$$

$$\frac{J_w}{k_L} = 1.119$$

$$\Rightarrow \underline{\underline{k_L}} = \frac{0.0156}{1.119} = 0.0139 \text{ m/h} = 3.87\times10^{-6} \text{ m/s}$$

So this is one classic example of that how you can calculate water flux and salt flux provided you know the concentration polarization modules and some other data like pressure difference, of course you can calculate osmotic pressure, if osmotic pressure data of salt solution is given a particular value then you can extrapolate, interpolate and calculate it.

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Reverse Osmosis

Problem 2. In a cellulose acetate RO membrane of 12 μm thickness, the water flux is 5.2 μg/cm^2.s and the salt flux is 0.015 μg/cm^2.s at 25 °C. On the feed side of the membrane, a 4% NaCl solution is used and the permeate side salt concentration is kept at zero by continuous flushing with distilled water. The transmembrane pressure difference is 90 atm and the osmotic pressure of 4% NaCl solution is 38 atm. The water sorption can be taken as 0.15 g/cm^3. Calculate the water diffusivity in the membrane.

If the equilibrium sorption of NaCl follow the Henry's law, $C_s = H_s C_f$, where C_s is g of NaCl per cm^3 membrane volume and C_f is g of NaCl per cm^3 feed. H_s = 0.037. Calculate the salt diffusivity in the membrane.

So next, another problem let us see another reverse osmosis problem. So here in another problem says that in a cellulose acetate RO membrane of 12 micron thickness, the water flux is 5.2 microgram per centimeter square second, and the salt flux is 0.015 microgram per centimeter square second at 25 degrees centigrade. So on the feed side of the membrane a 4% sodium chloride solution is used, and the permeate side salt concentration is kept at 0 by continuous flushing with distilled water.

The transmembrane pressure difference is 90 atmosphere and the osmotic pressure of 4% sodium chloride solution is at 38 atmosphere. The water sorption can be taken as 0.15 grams per centimeter cube, calculate the water diffusivity in the membrane. So it is given that, if you have to calculate the water diffusivity. This is what is being asked. So, some data is given that if the equilibrium solution of sodium chloride follow the Henry's law.

Then the Cs is given by this equation Hs into Cf where Cs is gram of sodium chloride per centimeter cube membrane volume and Cf is gram of sodium chloride per centimeter cube of feed. So, just Henry's constant is given as 0.037. So we calculate the salt diffusivity? So you have to calculate the water diffusivity as well as salt diffusivity, so let us see how we will do it. So, this is a classic example from the from the solution diffusion model, we have use this solution diffusion model to solve this problem.

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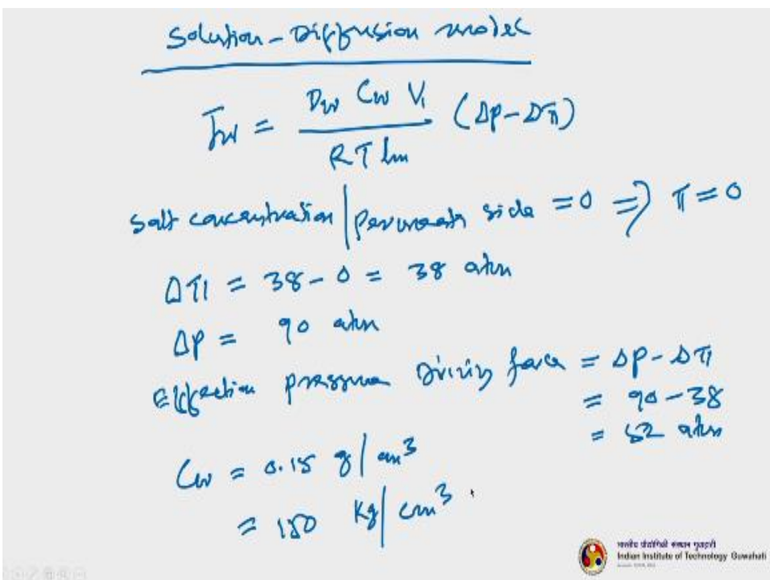

So the solution diffusion model, so, you know, this model is very important in membrane this one calculations, especially when we are talking about. So, I just rewrite this equation Dw Cw V1 divided by RT small lm into delta p - delta pi. So you have to calculate the diffusivity, Dw is the water diffusivity. The Jw is the water flux, V is the volume of the molar volume of water, Rt is all mini temperature T is the temperature, lm is the membrane thickness, delta p is the pressure that is applied delta pi is the osmotic pressure.

Now, it has given them the salt concentration on the permeate side that is given as 0. So which indicates that the osmotic pressure. Osmotic pressure pi equals to 0 in the permeate side. Now you can calculate the osmotic pressure difference. So delta pi equals to 38 - 0. So there is nothing but 38 atmosphere, because the osmotic pressure at the permeate side equals to 0.

So the overall osmotic pressure difference is 38 atmosphere. So the delta p is the transmembrane pressure it is given 90 atmosphere so effective pressure difference or we can say driving force is nothing but delta p - delta pi. So it is 90 - 38 equals to 52 atmosphere. So 52 is the effective pressure driving force so, other data that is given, it is given Cw equals to 0.15 grams per centimeter cube, you can write in terms of kgs 150 kilogram per centimeter cube.

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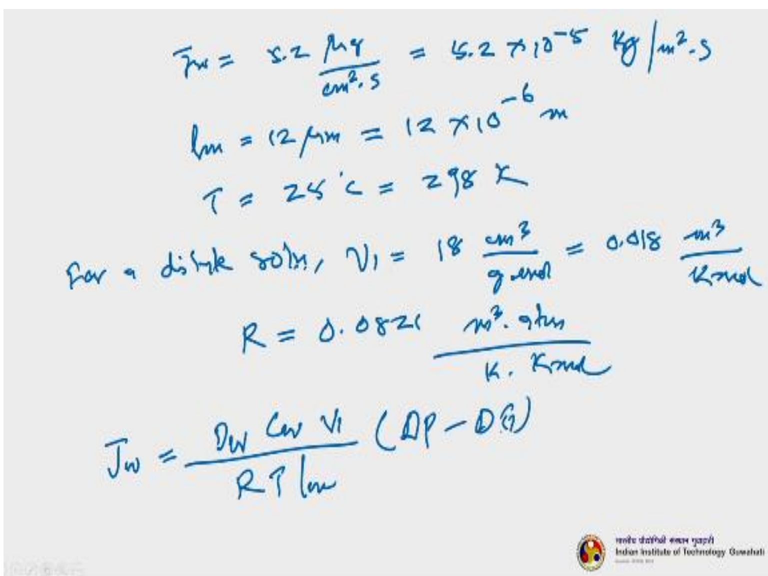

So, Jw which is here, water flux is 5.2 this is micrograms per centimeters square second, so we can write is 5.2 into 10 power of - 5 kg per meter square second and membrane thickness lm is given as 12 microns. So 12 into 10 power of - 6 meters. Just converting everything in terms of meters and kilograms, so that there will be a unit consistency, temperature T it is given as 25 degree centigrade equals to 298 kelvin.

So, for a dilute solution they have to see the V molar volume of water, V1 equals to 18 centimeter cube per gram mole. So, we can write 0.018 meter cube per kilo moles and your gas constant R equals 0.08 to 1 meter cube atmosphere divided by Kelvin kilo mole. Then we have to calculate now, substitute everything in this equation, the solution diffusion equation.

So Jw equals to again I am just writing Dw Cw V1 divided by RT lm is the membrane thickness, then delta p - delta pi.

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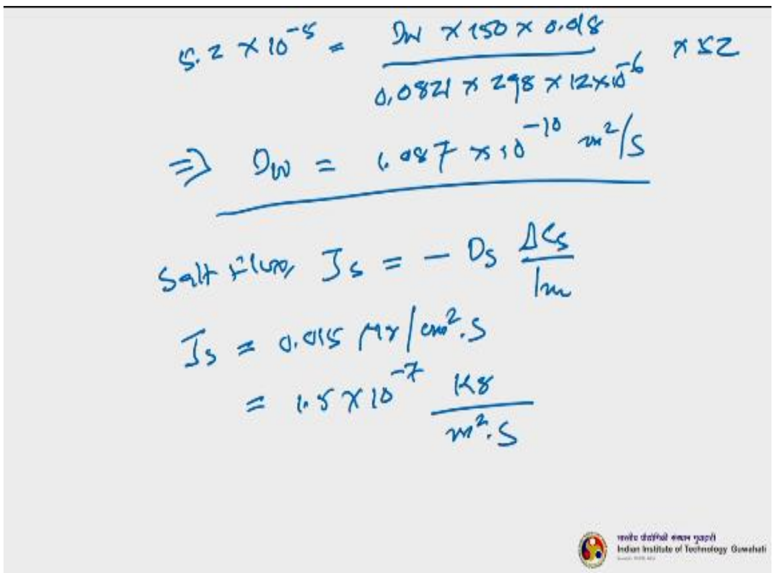

So, just substitute 5.2 into 10 power of - 5 equals to the Dw that is what we do not know we are calculating into 150 into 0.018, then divided by 0.0821 into 298 into lm is 12 into10 power of - 6, we have converted everything into meters into 52 - delta pi through diving force, so from here we will calculate the Dw the water diffusivity equals 0.087 into 10 power of - 10 meters per square second. So this is your water diffusivity.

Now you have to calculate the salt diffusivity which is DS, you know, salt flux is given as, so flux that is Js. Js equals to - Ds the salt diffusivity into delta Cs divided by lm, lm is the membrane thickness. So, Js is given is 0.015 micrograms per centimeter square second or we can write, 1.5 into 10 power of - 7 kg per meter square second. If you neglect the concentration polarization then the feed side salt concentration can be given as.

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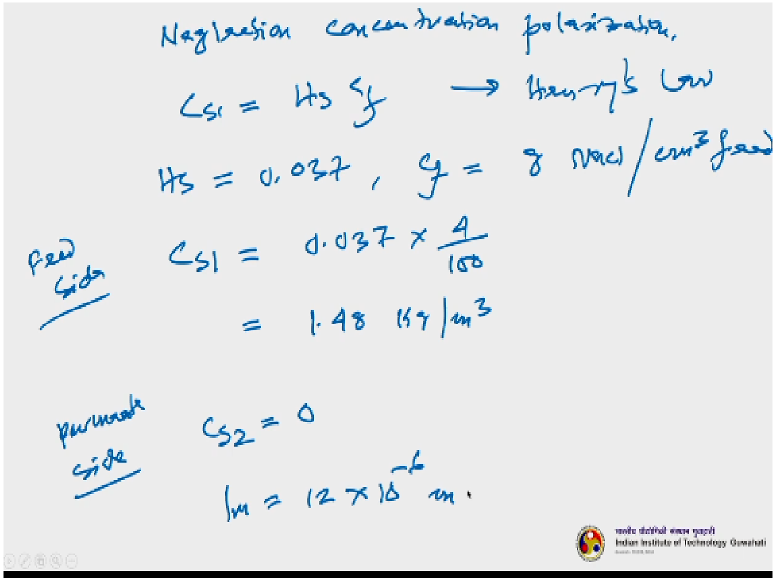

So you can write that neglecting concentration polarization. So, you can calculate the feed side salt concentration then Cs1, is equals to Hs into Cf, so this is from Henry's law, this is also given problems statement is given. So Hs is given. So, Hs is given 0.037 and Cf equals to gram NaCl per centimeter cube, feed. That is Cf is also given. So your Cs1 is equals to 0.037 that is Hs into 4 by 100. So its 1.48 kg per meter cube.

So, then this is feed side similarly, you have to calculate the permeate side that is Cs2. Cs2 equals to 0, because there is no diffusivity then lm membrane thickness is given into 12 into 10 to the power of - 6 meter. So, you can calculate Js by substituting it that equation.

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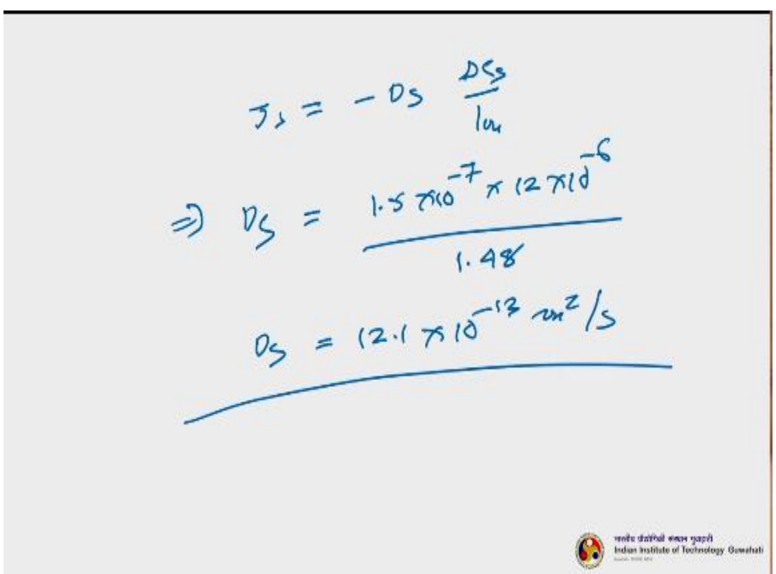

So, Js equals to - Ds delta Cs by lm. So here we can calculate Ds from here Ds salt diffusivity equals to 1.5 into 10 power of -7 into 12 into 10 power of -6 divided by delta Cs is 1.48. So Ds equals to 12.1 into 10 to the power of -13 meter square per second. So this is your salt diffusivity, so you know, I hope you understand how we have proceeded and calculated the salt diffusivity.

As well as water diffusivity inside the membrane by using the classic solution diffusion model. Please solve similar types of problem which are there in many of the books.

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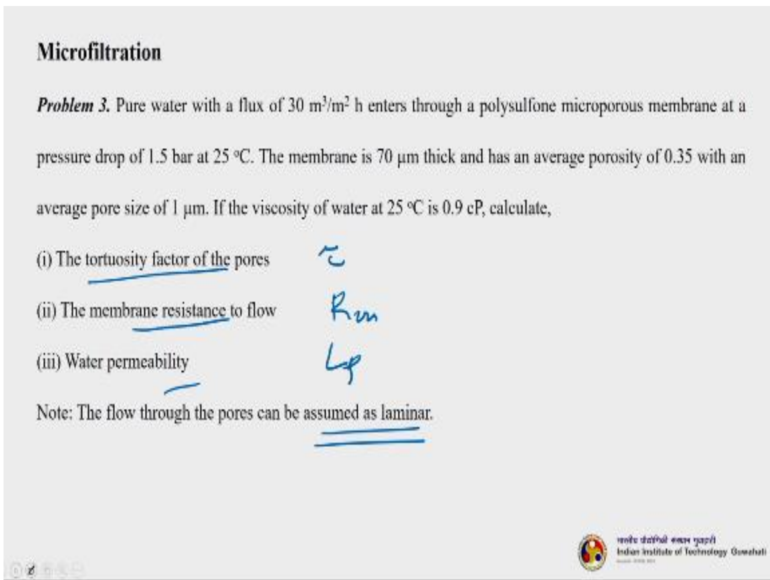

So we will see another problem which is actually based on a microfiltration data. So, the problem goes something like this, pure water with a flux of 30 meter cube per meter square hour, and enters through a polysulfone microporous membrane at the pressure drop of 1.5 bar at 25 degree centigrade. The membrane is 70 micron thick and has an average porosity of 0.35, with an average pore size of 1 micron.

Now if the viscosity of water at 25 degree centigrade is 0.9 cP, calculate the tortuosity factor of the pores that is tau. The membrane resistance to flow. What is that? Rm, then the water permeability that is Lp. Now it is given that flow through the pores can be assumed to be laminar it is a small problem. Let us see how we will do this, see you have to use the classic classical hidden possible equation to solve this particular problem. So, the data provided is.

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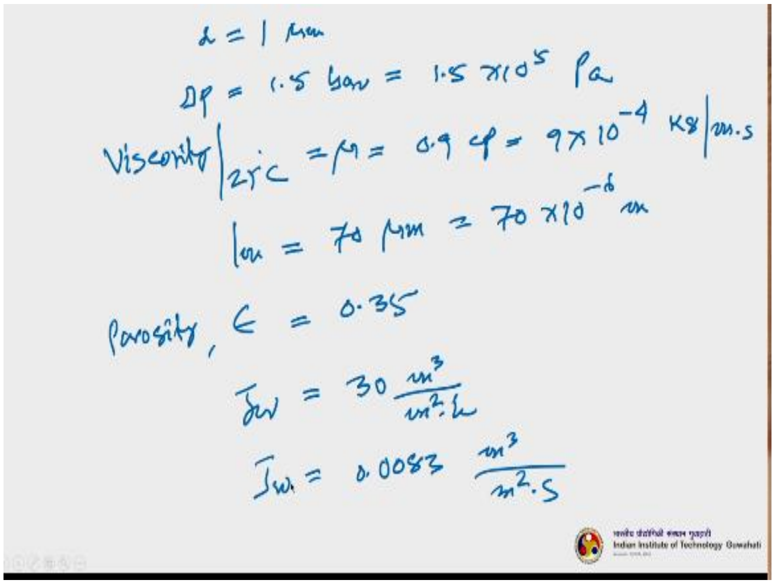

So, d diameter is 1 micron, 4 sides 4 diameter as pressure drip delta p is given is 1.5 bar. So 1.5 into 10 power of 5 Pascal. So viscosity at 25 degrees centigrade. That is mu equals to 0.9 cP equals 9 into 10 to the power of -4 kg per meter second. So then membrane thickness lm is membrane thickness is given as 70 microns. So 70 into 10 power of -6 meters, again, converting everything into meters and kilograms in that so that consistence will be there.

Then porosity epsilon is also given 0.35 and water flux Jw is given 30 meter cube per meter square hour. So you convert we get 0.0083 so meter cube per meter square second. So this is your water flux. So this is all data is been given to us. Now, you use the Hagen poiseuille equation.

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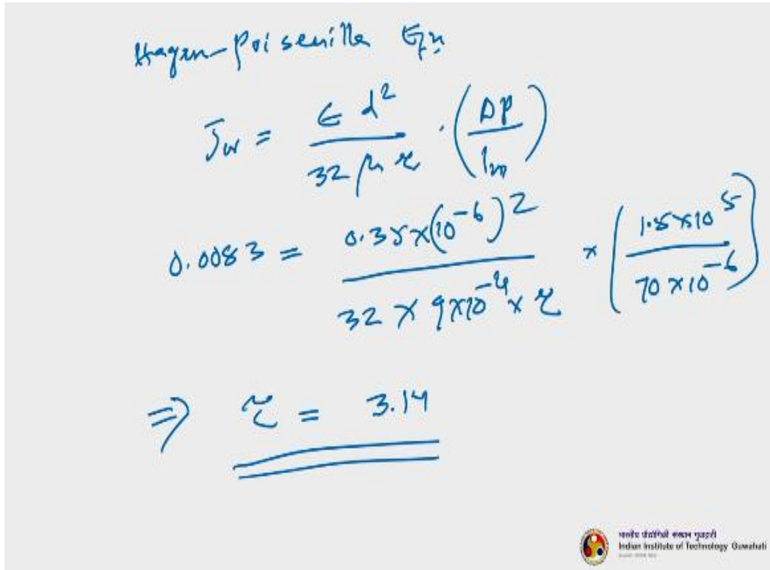

Use the Hagen poiseuille equation, so what is that Jw equals to epsillon d square divided by 32 mu into tau into delta p by lm, the thickness of the membrane. So you substitute everything 0.0083 equals to 0.35 into 10 to the power of -6, so this square divided by 32 into mu is your 9 into the 10 to the power of -4 into tau this is what we are going to calculate 1.05 into 10 to the power of -5, not -5 that is the pressure drop divided by thickness.

That is 70 into 10 to the power of -6, the membrane thickness. So from here, you can calculate tau. So, that is tortuosity equals to be calculated to be 3.14. So the tortuosity factor for this particular problem is 3.14. Now, you have been asked to calculate the resistance to the flow, so the resistance to the flow can be calculated. from this equation.

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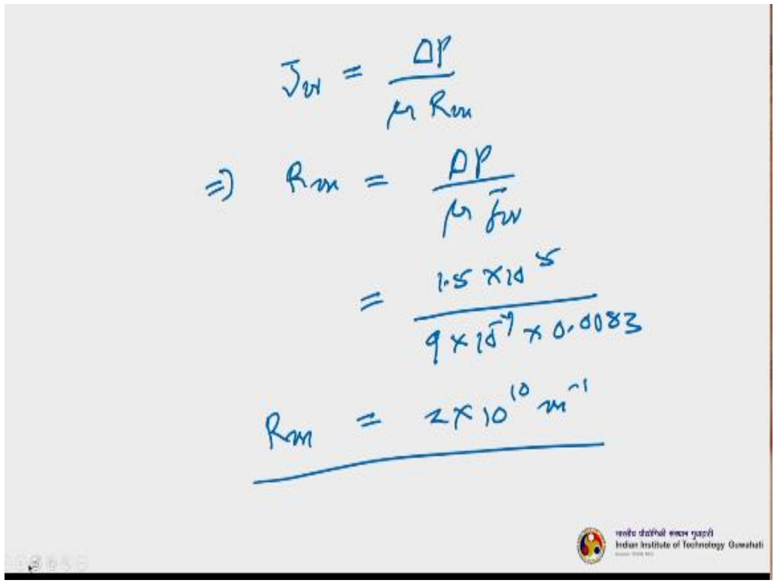

So Jw equals to delta p divided by mu Rm resistance to flow offer by the membrane Rm. So from here, we can write that Rm equals to delta p by mu into Jw. So substitute, 1.5 into 10 power of 5 delta p divided by mu equals to 9 into 10 power of -4 into Jw equals to 0.0083. So here Rm the resistance to flow that is offered by a membrane. It is been calculated to be 2 into 10 power of 10 meter inverse, so this is the resistance. So now we have to calculate the water permeability.

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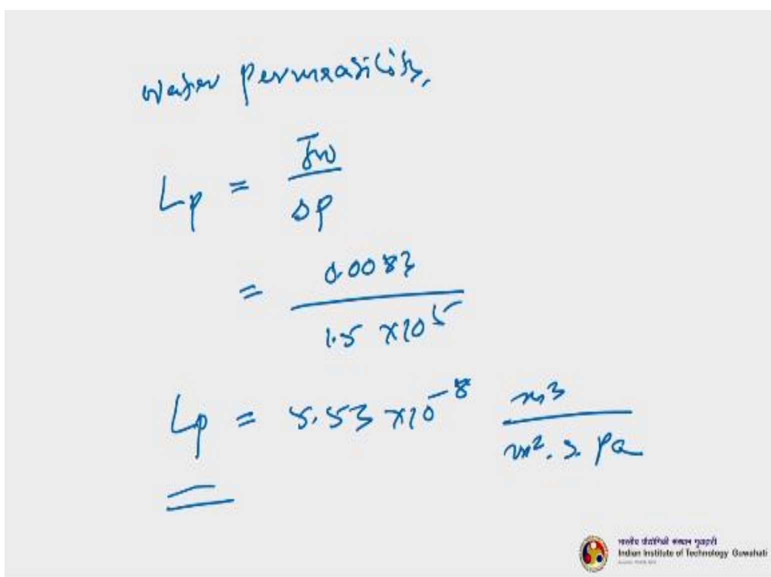

So, water permeability, which is known as Lp, so Lp that is also calculated equals Jw by delta p, so here Jw equals to Lp into delta p. So 0.0083 divided by 1.5 into 10 to power of 5. So your Lp we get 5.53 into 10 power of -8. So the units is meter cube per meter square second Pascal. This is 8. So this is the water permeability so you see this is a very small problem.

So this is how we can calculate the water permeability tortuosity factor, as well as the resistance to the flow that is being offered by membrane using the classical Hagen poiseuille equation. So I am not solving any complicated problems because that takes so much of time and you need so much of data from other handbooks and other things also. So, we are showing it here it is been difficult.

And to show you that how from how to take from handbook and all is bit difficult actually, so when you actually face exam and all, so my intention is just to show you what type of problem, the simple problems that can be solved using the classical equations we have seen the RO equations using the concentration polarization modulus equation. Then we have calculated mass transfer coefficient we have calculated salt flux water flux.

Then in the RO again, using the classical solution diffusion model, we have calculated the salt diffusivity, as well as the diffusivity of water inside the membrane. Now, for the microfiltration problem we have used the Hagen Poiseuille equation to calculate various parameters, whether it is tau or tortuosity. Then we have calculated the membrane resistance and as well as the liquid permeability, water permeability.

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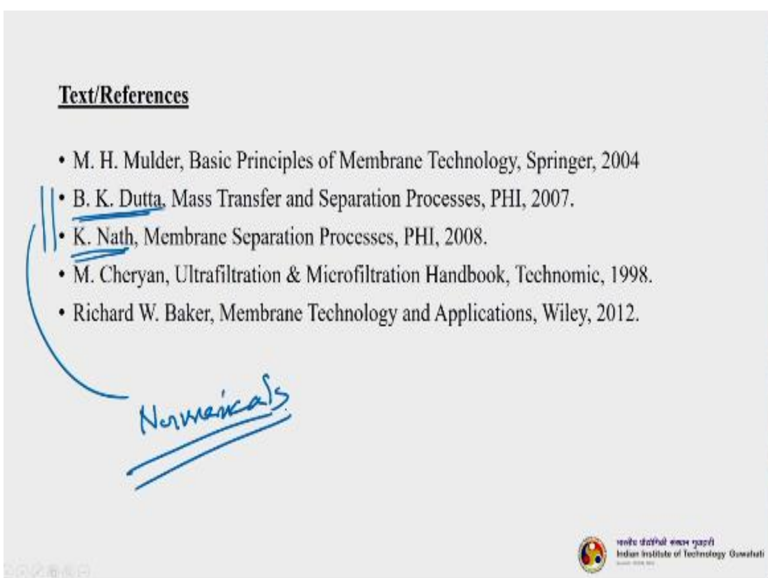

So with this today I stop. So, please refer Professor B.K Dutta book and Professor K. Nath book for solving the numerical, there are so many numerical which are solved as well as far are given there, kindly refer to that and.

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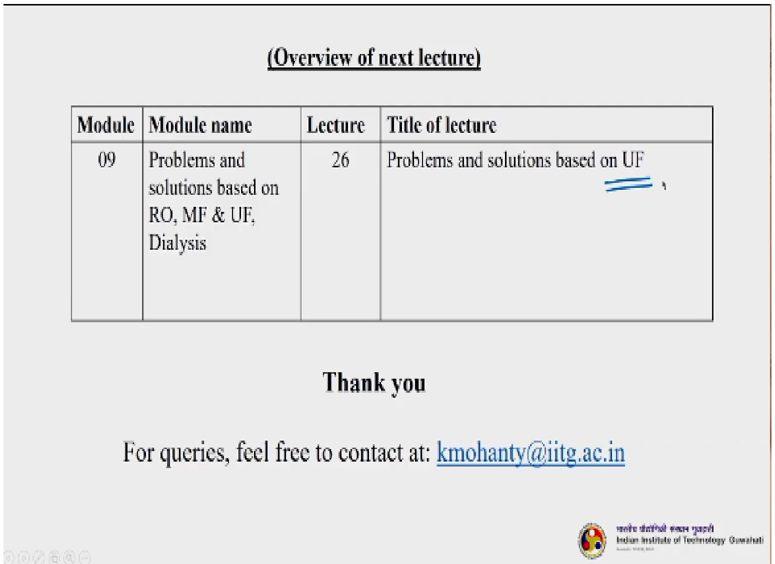

In case you have any query, you can write to me, I will definitely answer your queries. So in the next lecture, we will be solving some of the problems based on the ultrafiltration. So thank you very much, in case you have any query please feel free to write to me at kmohanty@iitg.ac.in Thank you.