Mass Transfer Operations II
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Lecture 03 - Humidification and Air Conditioning

Hi, welcome back to the course Mass Transfer Operations 2, we were discussing on humidification and air conditioning. In the previous class, we discussed the step by step procedure of humidification tower design and today we will be discussing on the humidification problem.

HUMIDIFICATION PROBLEM

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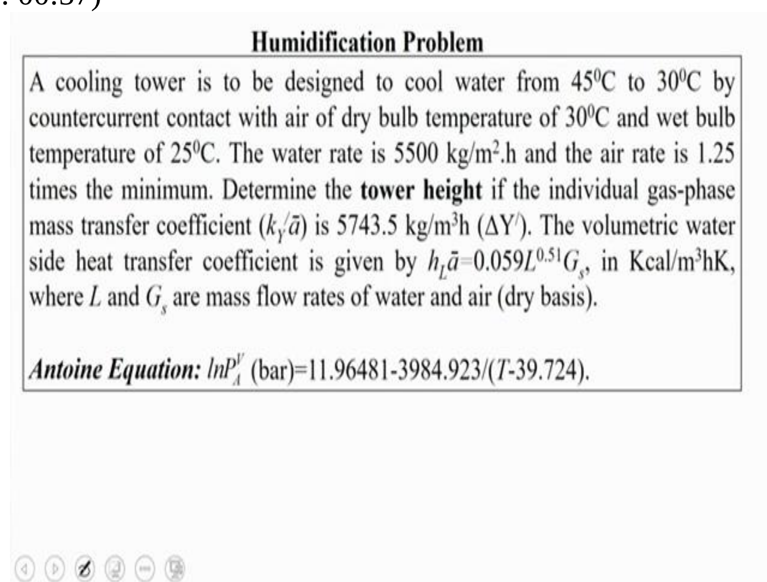

The problem is like that a cooling tower is to be designed to cool water from 45 degree Celsius to 30 degree Celsius by counter current contact with air of dry bulb temperature of 30 degree Celsius and wet bulb temperature of 25 degree Celsius, the water rate is 5500 kg per meter square hour and the air rate is 1.25 times the minimum. Determining the tower height if the individual gas phase mass transfer coefficient k Y a prime is 5743.5 kg per meter cube hour, the volumetric water side heat transfer coefficient is given by the relation h L a prime is equal to 0.059 into L to the power 0.51 into G s in kilo calories per meter cube hour Kelvin where L and G s are mass transfer rates of water and air on dry basis. The Antoine Equation is ln P A V is equal to 11.96481 minus 3984.923 divided by T minus 39.724 where this temperature is in Kelvin and the pressure is in bar.

Before the start of the solution of this tower design, we will recapitulate the step by step procedure of the cooling tower design. The steps are like this:

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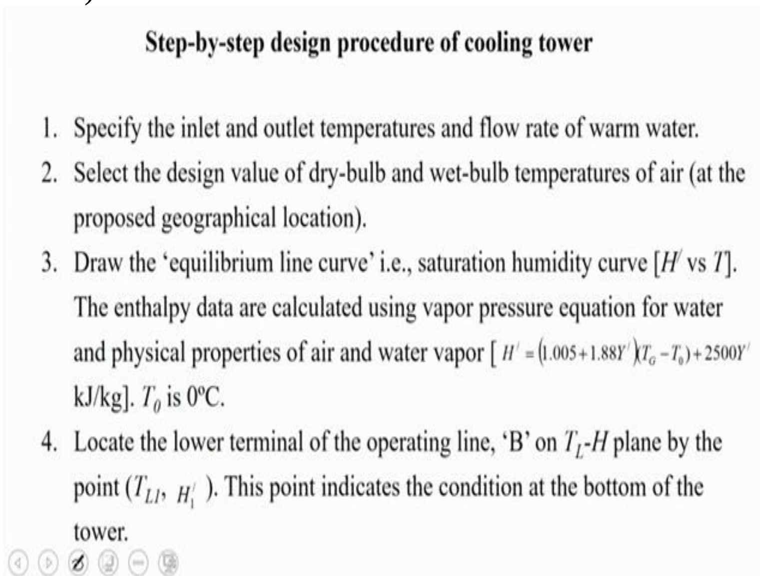

Firstly we have to specify the inlet and the outlet temperature and flow rate of warm water. Say for our practical case the inlet and outlet temperature and flow rate of warm water is given. Then we have to select the design value of dry bulb and wet bulb temperature of air at the proposed geographical location. Here also in this problem the dry bulb and wet bulb temperature of air is applied.

Then third step is draw the equilibrium line curve that is saturation humidity curve which is nothing but the H prime versus temperature, the enthalpy data are calculated using this vapor pressure equation and the physical properties of air and water vapor just by H prime is equal to 1.005 plus 1.88 into Y prime into T G minus T 0 plus 2500 into Y prime that will be in kilojoule per kg, where the T 0 or we can say the reference temperature is taken as 0 degree Celsius. Then the fourth step will be we have to locate the lower terminal of operating line that is B on this TL H plane by the point TL1 and H1 prime, this point indicates the condition of the bottom of the tower that is whenever this cold water will be leaving the cooling tower.

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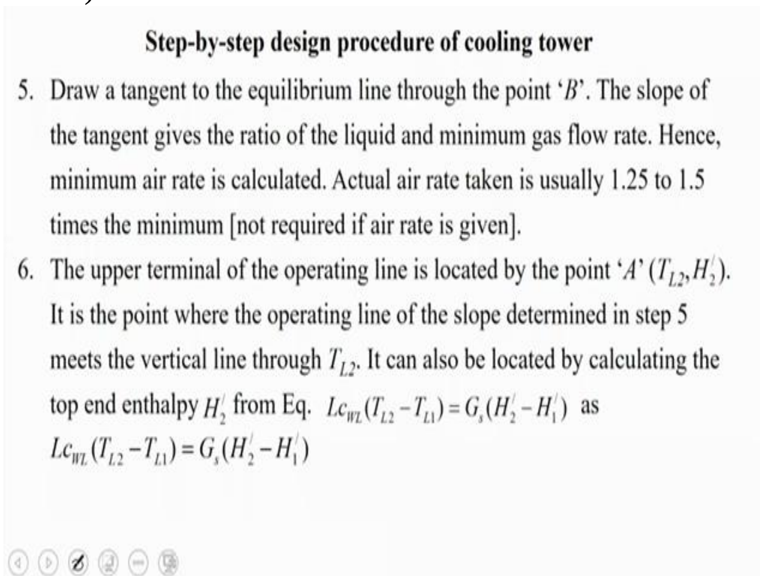

Then this fifth step will be, this we have to draw the tangent to the equilibrium line that we have already drawn from this HT diagram and we have to draw a tangent through this point "B" or we can say this the whatever the bottom point of the cooling tower, the slope of the tangent gives the ratio of the liquid to minimum gas flow rate or that is L by G s minimum, hence the minimum air rate is calculated and in this particular problem the actual air rate is taken as 1.25 times the minimum. And then upper terminal of the operating line is located by point A that is whenever we will be getting this G s minimum and if we multiply by 1.25 we will be getting the actual operating line for this cooling tower.

Then we will be getting just A by say the moving through this line TL 2 or we can say this one in which temperature hot water is entering at the top and whatever the slope we will actually getting by multiplying 1.25 of this gas flow rate. Okay this can also be calculated by locating the top end enthalpy just H2 prime of this we can say hot water which is entering by equation L c WL into TL 2 minus TL 1 is equal to G s into H2 prime minus H1 prime which we will be getting from the envelope two or we can say this one whenever we will be getting the overall enthalpy balance this one across this envelope two or we can say this one across the entire column.

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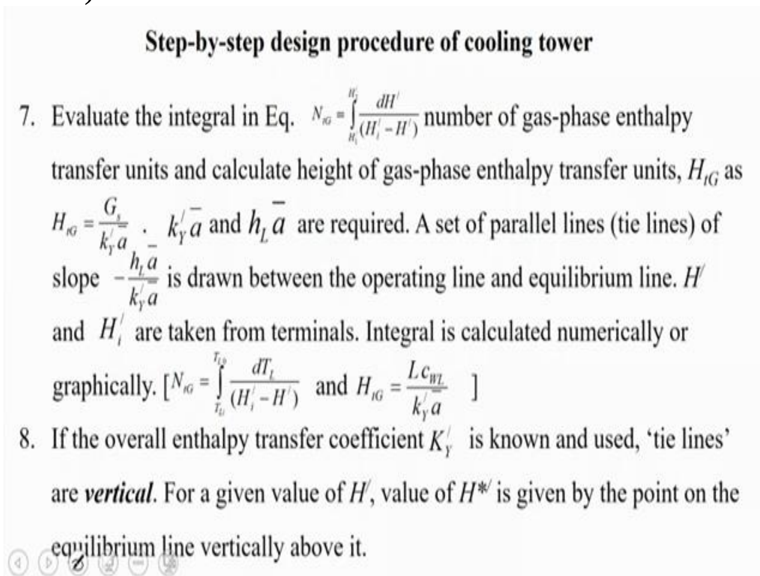

Then seventh step will be to evaluate the integral in equation this N tG we can say the number of gas phase transfer and enthalpy transfer units by integrating dH prime by Hi prime minus H prime from H1 prime to H2 prime. So this one starting from the bottom to the top whatever the change in the enthalpy we can say we will be getting by integrating the integral part and this one we can calculate the height of the gas phase enthalpy transfer unit that H tG like using this equation G s by k Y prime into a bar and k Y prime H L a prime these are required, for this particular problem this are given. So a set of parallel lines that we can say tie lines with the slope of this minus H l into a prime by k Y prime into a bar that is drawn between the operating line and the equilibrium line.

Equilibrium line we have drawn from the saturation enthalpy line with temperature and the operating line we have obtained from the between these two stations of this operation, that is bottom line of the cooling tower and the top line of the cooling tower, so in between this we can see this operating line we have, so in between this whatever the area under the curve that we can get from graphically or numerically just by integrating this d TL by i prime minus H prime from TL1 to TL 0 or TL 2 and H tG will be L c WL by k Y prime into a bar.

So for this case we do not require any we can say this one as this all the gas page enthalpy transfer unit is expressed in term of individual gas phase enthalpy transfer units.

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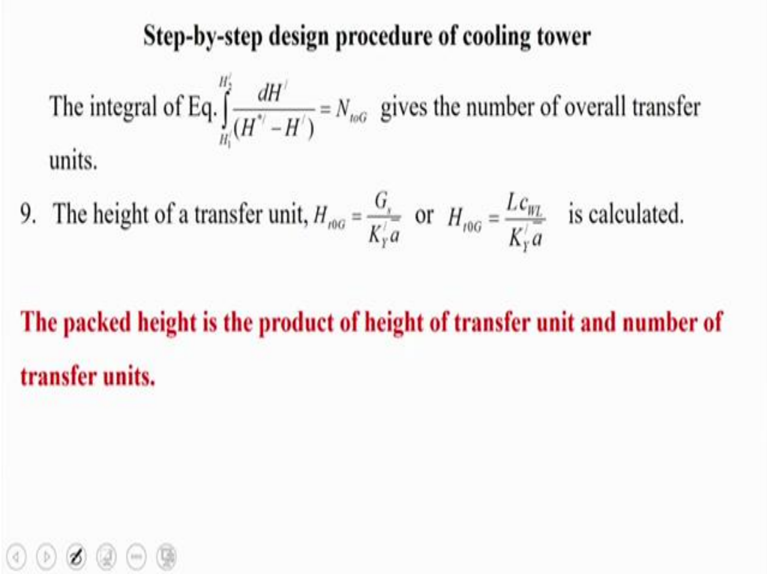

So the step eight and step nine will not be required. So now we can start the solution of this given problem.

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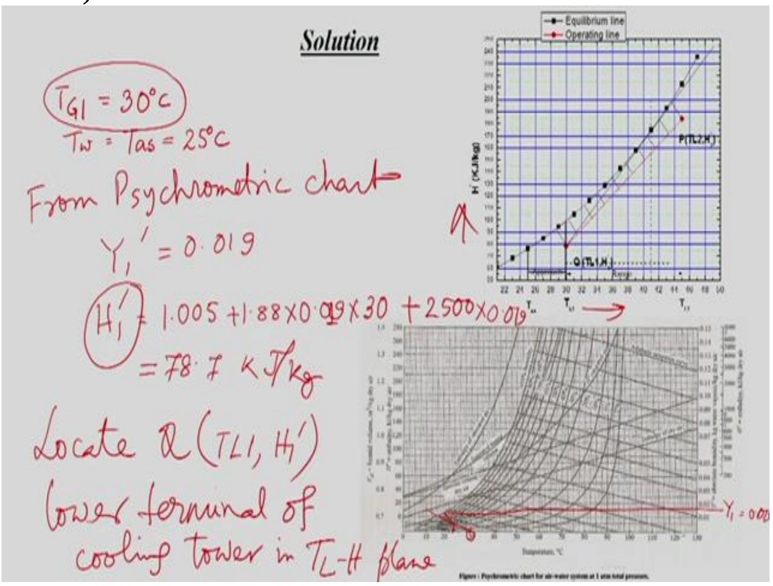

Now we shall start solving the problem for that whatever the data we have we shall first write down those like say T G1 that is given as 30 degree Celsius and T w is equal to T a s for this air water system is given as 25 degree Celsius. So we will be getting this whatever the humidity we have, so we will be getting from the psychrometric chart.

Let us start with this say T G is equal to 30 degree Celsius like this point and then we will be following this we say adiabatic saturation line so we will be following this line like this and we have 25 degree Celsius somewhere it is here. And say we will be following this and from there we will be moving towards this 100 percent saturation line or 100 percent humidity line, from there we will be getting whatever the humidity. So that we can say Y1 will be getting as say this is we are getting as so from psychrometric chart so we shall get this Y1 prime that is equal to 0.019. Okay?

Now we can get this humidity in that 0.019 humidity what is the we can say the enthalpy value or we can say this one bottom condition of the cooling tower we will be getting calculating this enthalpy value which is say of the liquid water or cold water leaving the we can say the cooling tower. So that H1 prime so that actually we will be getting as from this we can say equation we have this 1.005 plus 1.88 into this Y1 prime that is 0.019 into temperature that is T G1 minus T 0 that is we can say 30 minus 0 that is 30 plus 2500 into say Y1 prime so 0.019, so the enthalpy value is coming out as 78.7 kilojoule per kg.

So now we have this H1 prime and say T G1, so we know now the condition of the bottom of this cooling tower, so that we can say this Q point we need to locate now. Locate this Q point that is we can say suppose TL1 and H1 prime. Okay? So that we can say lower terminal of this cooling tower in say TL H plane. Like whatever we have this TL H plane here we will be getting this.

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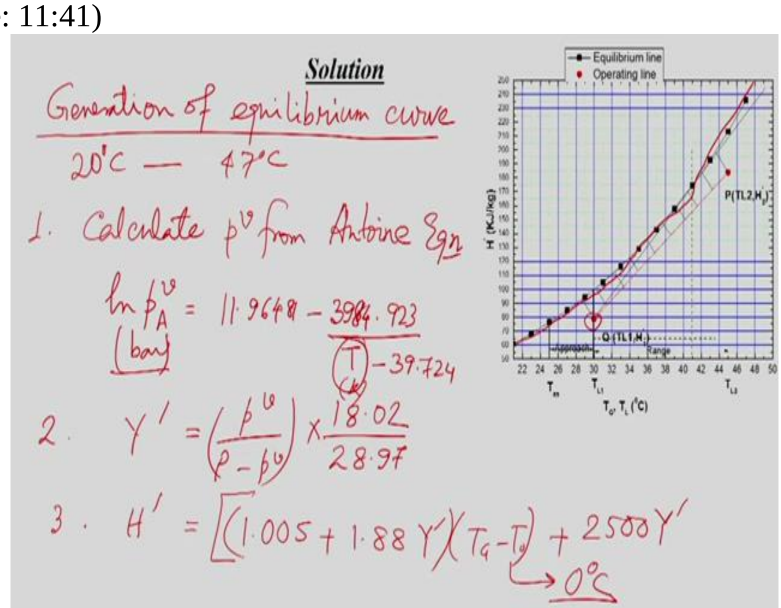

So that say TL1 we can say this TL 1 is equal to say 30 degree Celsius that is given and enthalpy value is 78.7 kilojoule per kg. So we have this lower point here. Okay? So that is actually we have identified.

Now we will be doing this say equilibrium curve generation, say whatever the equilibrium curve we have we shall generate this one, so generation of equilibrium curve. So that we will be doing this one we will be getting this equilibrium curve we can generate very easily for this we can say temperature range say from where actually we have started. Say let us take for our case we can start from 20 degree Celsius to say 47 degree Celsius. In this range the entire cooling tower will be operating. Okay? So for that case we will be doing the step by step calculation like say calculate this vapor pressure from this Antoine equation that is nothing but ln P V A is equal to that is in bar actually, that is equal to so 11.96481 minus 3984.923 divided by T minus 39.724. Okay?

So that temperature in Kelvin and pressure actually is in bar, so from here we will be getting that P A value or we can say vapor pressure value we will be getting from there and then we will be getting, we can say the second step will be like: This say Y prime that is we can say humidity value that will be like this PV by total pressure minus PV that is into molecular weight of water 18.02 by molecular air 28.97. Okay? So we will be getting this Y prime then we will be getting this H prime actually, enthalpy we need. So for a particular temperature whatever the enthalpy value will be, so we will be taking again that equation 1.005 plus 1.88 into Y prime that is into T G into T G minus T 0 plus 2500 into Y prime. Okay?

The T 0 we actually take this one as this we can say this as reference temperature that is 0 degree Celsius. Okay?

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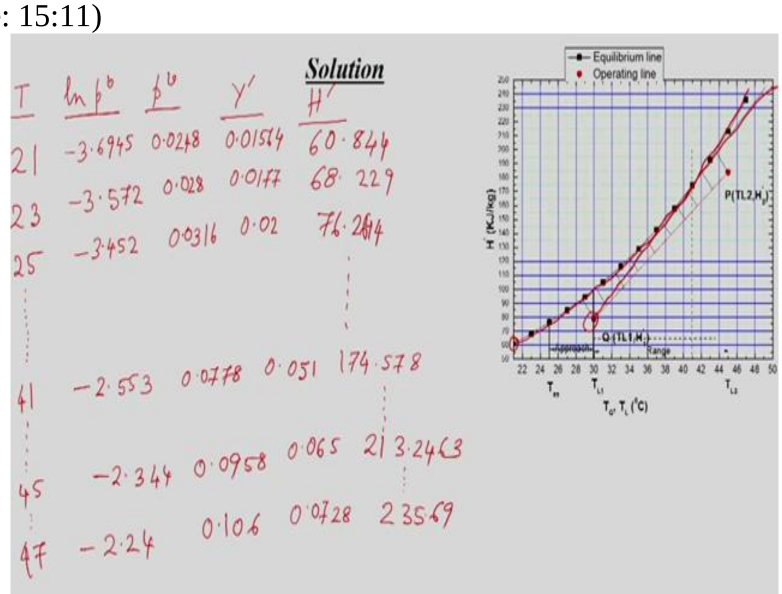

Now we will be generating this we can say equilibrium curve values like we can say we will be getting for a particular temperature what is this enthalpy value. Okay? So now we can start with this suppose we can take one temperature, so then for that we will be getting this from Antoine equation we will be getting ln P b value. From there we will be getting just exponential of this ln P b will be P V and then we will be getting Y prime from there we will be getting H prime. Okay?

So for every temperature we will be getting this enthalpy values, then we will be putting this H prime t curve or the plane then we will be getting this equilibrium line. Okay? So let us start from say 21 degree Celsius, then from this Antoine equation we will be getting this Ln P b value is equal to minus 3.6945 okay? Then P V value will be this very small value 0.0248, okay? And from there we will be getting this humidity value that is also very less, 0.01564 and the enthalpy value we will be getting from that equation 1.005 plus 1.88 into Y prime into T G minus T0 plus 2500 into Y prime. So from there we will be getting 60.844.

So for 21 degree Celsius suppose this is the point, now for 23 degree Celsius again we will be getting that value actually 3.572, then PV value we will be getting 0.028, then Y prime we will be getting to some extent higher 0.01773, then enthalpy value will be say greater than the enthalpy value obtained at 21 degree Celsius that is at 68.229. So then we will be getting suppose for 25 degree Celsius we will be getting minus 3.452, then PV value we will be getting 0.0316 and Y prime we will be getting 0.02, then enthalpy value we will be getting 76.264.

So that way we will be getting, let us take say we can get this one for 41 degree Celsius, the Ln P b value will be say minus 2.553 and P V value will be say 0.0778 and then Y prime value will be 0.051 and this enthalpy value will be 174.578. Then again say 45 degree Celsius we will be getting say minus 2.344 and the P V value will be 0.0958 and Y prime value will be obtained as 0.065 and the enthalpy value will be obtained as 213.2463. And for 47 degrees Celsius this ln P V value will be minus 2.24 and P b value will be 0.106 and Y prime value will be 0.0728 and the enthalpy value will be 235.69. Okay? So all these values actually now we need to put in this enthalpy temperature plot and then we will add these lines to get this saturation humidity.

We need to remember that say the Antoine equation is like this ln P b is equal to 11.9648 minus 3984.923 divided by T minus 39.97 and temperature should be converted into Kelvin from Celsius temperature and Y prime will be PV by P minus P b into molecular weight of water by molecular weight of air. And then enthalpy will be say 1.005 plus 1.88 into Y prime into T G minus T0 plus 2500 into Y prime. Okay?

Now we have identified this point that is we can say the bottom point of the cooling tower and we have this equilibrium saturation line or we can say this one equilibrium curve. Now we need to draw a tangent from this bottom point to the saturation curve, so we have to draw this tangent and it will be like this:

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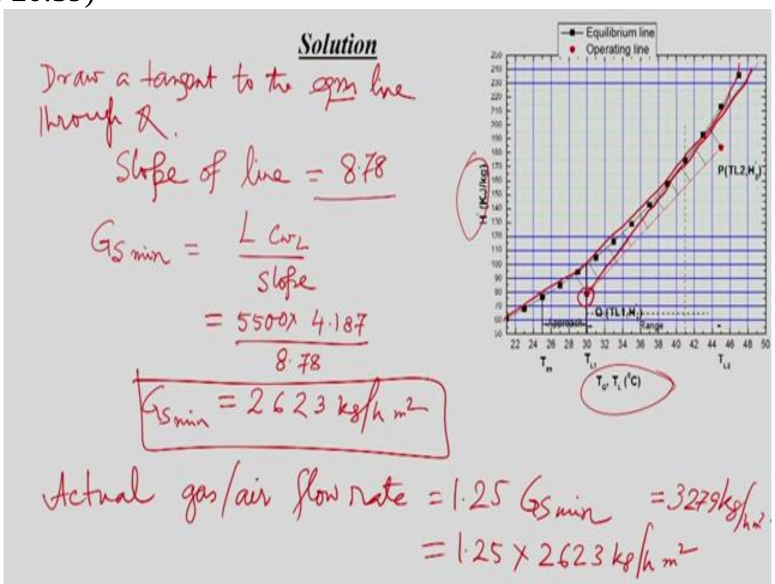

So whenever will be drawing, draw a tangent to the equilibrium line say suppose through Q suppose this one this black line actually we can say we need to draw this, so first we have identified this bottom point then we have drawn this equilibrium line and then we have drawn a tangent to the we can say equilibrium line through Q.

So then we need to find this slope, slope of this we can say this line is obtained as we can say 8.78, so from this for any point actually we need to take for any two points if we take that what is the value of enthalpy and what is the value of the temperature. From the enthalpy difference and the temperature difference we will be getting the slope. Okay? So that slope indicates that the minimum air flow rate required to run this cooling tower. So that we can say this one G s gas flow rate minimum we will be getting like just by using that equation whatever we have derived for this design of this cooling tower L c wL divided by slope.

So now we have this L is equal to say 5500 into C wL we know that this 4.187 divided by slope we have obtained at 8.78, okay? So from here actually we will be getting this G s minimum or minimum air flow rate required to run this cooling tower to cool this one from 45 to 30 degree

Celsius, so we are getting as say 2623 kg per hour meter square. So this G s minimum is this one, so 2623 kg per hour meter square. But in the problem it is given as the actual flow rate gas or we can say this air flow rate is equal to 1.25 times of G s minimum.

So it is coming out at 1.25 into 2623 kg per hour meter square. So it is coming as 3279 kg per hour meter square.

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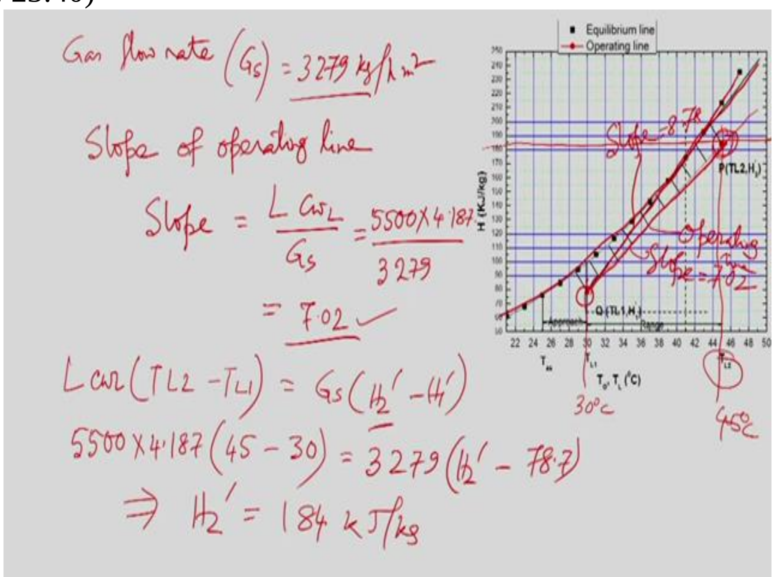

So now we have got this gas flow rate or air flow rate we can say G s is equal to 3279 kg per hour meter square. Now we have this one suppose we this equilibrium line and then we have this bottom point and we have this tangent, from there we got this minimum gas flow rate but now we have this 3279 kg per hour meter square actual gas flow rate. So using that we will be getting the what is actual slope of this cooling tower, so for that we can say this one slope of this operating line, if we say this equilibrium condition or the minimum requirement, but what is the slope of the actual operating line of the say cooling tower? Slope of say operating line.

So that we will be getting as, so we can say the slope using the same formula like we can say L c wL by gas flow rate. Okay? So L c wL by this gas flow rate actual G s, so that is coming out as 5500 into 4.187 divided by 3279. So now we are getting the slope as 7.02, so this you see lower than this one but initially minimum flow rate it was 8.78, now the slope is 7028, just using this slope of the 7028 suppose this slope was the slope of the line which touches the equilibrium line that was 8.78, now we will be drawing one line whose slope is equal to 7.02, so now we have this, this is called we can say operating line.

This is operating line. So now we will be doing this mass balance. So whenever we will be doing this L c wL into TL 2 for the entire cooling tower Lc wL into TL2 minus say TL1, that will be like this G s into H2 prime minus H1 prime. So now everything is on except this H2 prime. So we will be putting this 5500 into 4.187 into TL 2 is given that 45 degree Celsius that TL 1 is the 30 degree Celsius is equal to G s is obtained as 3279 into say H2 prime minus H1 prime is obtained as 78.7, so from here actually we will be getting H2 prime that is the enthalpy of this water entering from the top of this cooling tower means from the station two. So that is actually obtained as 184 kilojoule per kg.

So now we have this suppose 184 kilojoule per kg like this one, we will be coming to this, so this is we can say whenever the operating line touches the 184 then we can say and also you see there is 45 degree Celsius we see this is 45 degree Celsius say TL 2 that is we can say 45 degree Celsius and TL 1 that is say 30 Celsius. So we have this point, so we can say top of this cooling tower is now identified.

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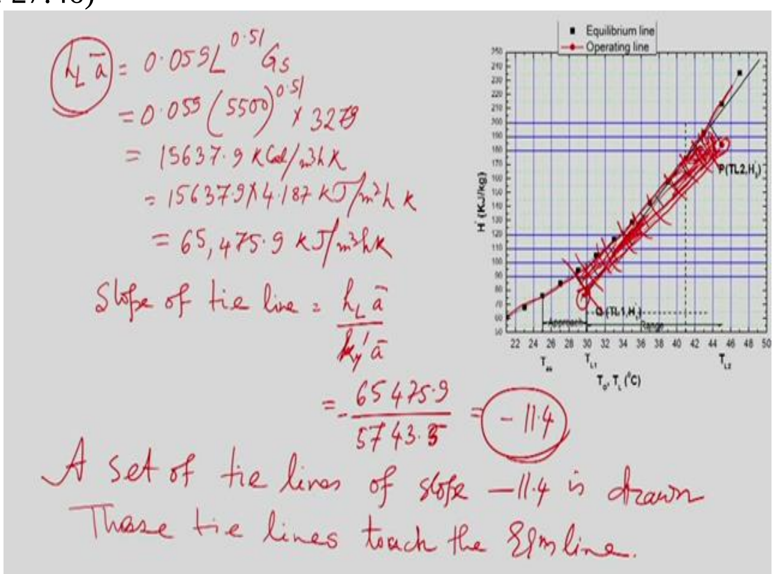

So now we know this one the relation is given say h L a bar is equal to is given as 0.059 into L to the power 0.51 into G s. Okay? So that is nothing but 0.59 into liquid flow rate as 5500 to the power 0.51 into G s is equal to that 3279, okay? That is coming out as a 15637.9 kilocalorie per meter cube hour Kelvin, that is we can say is equal to say 15637.9 into say 4.187, so that is kilojoule per meter cube hour Kelvin. So that is coming out as 65475.9 kilojoule per meter cube hour Kelvin. So we will be getting the slope of the tie line because for that we need this h L a, so slope of the tie line because you see the slope of the tie line we do not know the in which direction, suppose we know now this equilibrium line then we know this bottom point, we know this top point, we have this operating line and then this operating line, now we will be drawing the slope.

If we know the slope of the tie line, then we will be drawing from this point to equilibrium line and in between this one whatever the area under the curve from there we will be getting h toG.

So the slope of the tie line actually we can say this tie line that is we can say h L a by k Y prime a. So that is we have got this h L a is 65475.9 divided by k Y a prime is equal to, that is given that 5743.5, okay? So that is minus actually so that is coming out as minus 11.4. So now we have the slope of this tie line as minus 11.4, so that will be like this, we can draw so many tie lines from bottom point to the top point of the cooling tower, means in between this one we are trying to get the area under the curve. So for that actually we are drawing these tie lines, in between we can say P1 this is 1.2 to 2 point or we can say this one in between the two stations of the cooling tower or bottom point to the top point.

Now say a set of tie lines of slope minus 11.4 is drawn, so that depends on the individual, so how many slopes actually we will be drawing if we draw more slopes, tie lines then we can say the accuracy will be more. If we draw very less number of tie lines so we can say that accuracy the area under the curve will be less. So all the tie lines touch the equilibrium line and it started from the operating line.

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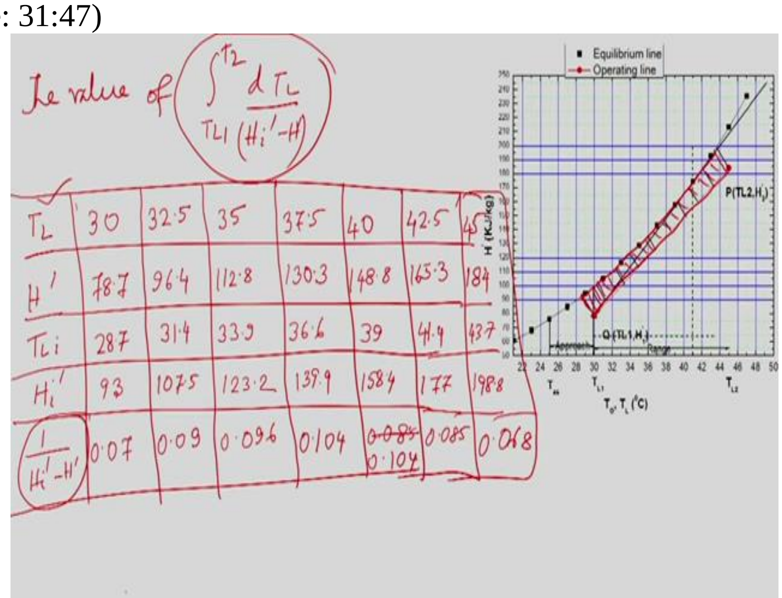

Now we need to get the value of this integration, we can say TL 1 to TL 2 dTL by so Hi prime H prime, so we need to get this one. So for that we need to do the graphical evaluation means what we have to do we have this area under curve so in between this operating line and this equilibrium line. Okay? So we will be doing this one graphically this part actually we will be doing graphically, so for that what we can do? We can take any TL for that we can say H prime and then we can get say TL i then we can say get Hi prime ultimately we will be getting Hi prime minus H prime.

So we will be computing this, very simply we can calculate those and let us take this TL1 is 30 degree Celsius so we will be continuing from 30 degree Celsius to 45 degree Celsius. So for 30 degree Celsius so H1 prime will be like this, 78.7 that is already calculated and for that actually TL i that is on the equilibrium line that will be obtained as 28.7 and the corresponding enthalpy value will be say obtained as from here we will be actually getting like 93. And then 1 by Hi prime minus H prime that will be obtained as 0.07.

So now for any temperature what is the value of 1 by Hi prime minus H prime that we have obtained, like that we can say we can write 32.5 degree Celsius. Similarly, we will be getting this H prime is equal to 96.4 and then TL i that is on equilibrium line will be getting at H 31.4 and enthalpy value at equilibrium point will be 107.5. We can read from this enthalpy temperature curve or we can get this from Antoine equation we can also get the same for this equilibrium enthalpy value if we know this equilibrium temperature. Okay? And then we will be getting 1 by Hi prime minus H prime as 0.09.

Now we will be getting this for 45 degree Celsius, we will be calculating this we know this H prime will be like 112.8 and this TL i will be like this 33.9 degree Celsius and then enthalpy value will be 123.2 and this 1 by Hi prime minus H prime that will be 0.096, okay? Then 37.5 degree Celsius we will be getting this enthalpy value as 130.3 and the corresponding TL i will be 36.6. And then enthalpy value will be 139.9 and then 1 by Hi prime minus H prime will be 0.104. So for 40 degree Celsius we will be getting this enthalpy value as 148.8 and then temperature at the equilibrium line that will be 39 and enthalpy value will be corresponding and the enthalpy value will be 158.4 and the difference will be 0.085. That is also that is 0.104, okay? And then for 42.5 degree Celsius the enthalpy value will be 165.3, temperature will be at the equilibrium 41.4 and the enthalpy value will be 177 and then 1 by Hi prime minus H prime will be 0.085 and foe 45 degree Celsius the top of this cooling tower that enthalpy value will be 184 that is we have already obtained this 184 and whenever it will be reaching this equilibrium line there temperature will be say 43.7 and at that temperature the enthalpy value will be 198.8. So then 1 by Hi prime minus H prime will be 0.068. So now we have this all this TL value with 1 by Hi prime minus H prime.

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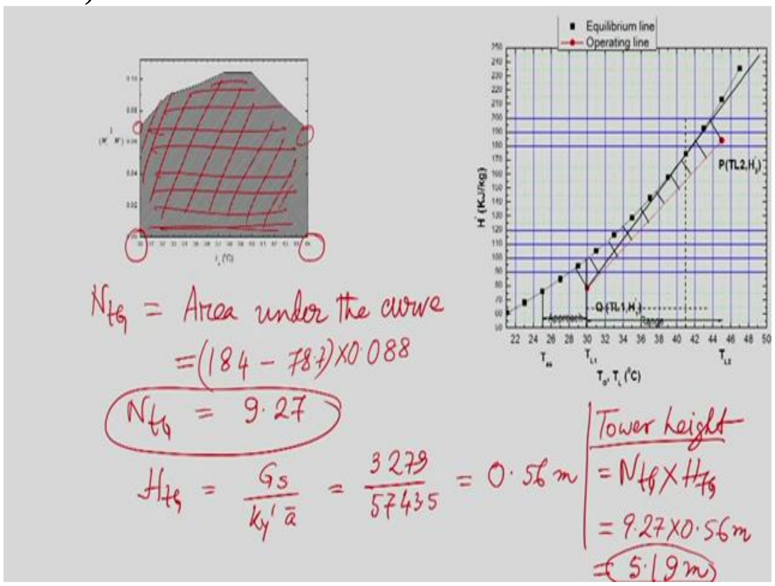

The integration we will be doing now, we will be now putting this one in this graph paper like this we will start from this 30 degree Celsius to 45 degree Celsius and then these all the values whatever we have this starting from this for 30 degree 0.070 to 45 degree Celsius that is 0.068, so this area under the curve actually we will be getting this one. Okay? So the entirely we will be getting this area under the curve. So now we will be getting that whatever the area under the curve from there we will be getting this N tG, okay? That is we can say area under the curve.

So that will be we can start from this whatever this enthalpy 184 minus 78.7 and what is the average value of this 0.088, okay? So from here we will be getting this 9.27, so we can say this N tG is 9.27 and say H tG that is we can say this G s by k Y prime into a bar, okay? So G s that is we have 3279 divided k Y a prime that is obtained as 5743.5. So now it is obtained as this height of every transfer unit is obtained as 0.56 meter. So now we have we can say total tower height will be say N tG into h tG, so that is we can say 9.27 into 0.56 meter, now it is obtained as 5.19 meter. So the total tower height is obtained as 5.19 meter. So that is the end of this calculation.

Tower height= 9.27×0.56 m=5.19 m (Ans.)

So thank you very much, so in the next class we will be discussing on the design calculation of the cooling tower based on the overall enthalpy transfer unit.