Mass Transfer Operations II
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Lecture 25 – Adsorption and Ion-Exchange

Welcome back to Mass Transfer Operations II. We were discussing on Adsorption and Ion-exchange. In the last class we discussed on adsorption process that is the types and nature of adsorption, then adsorption isotherm. So now we will be discussing on stage wise and continuous adsorption processes.

Adsorption operations:

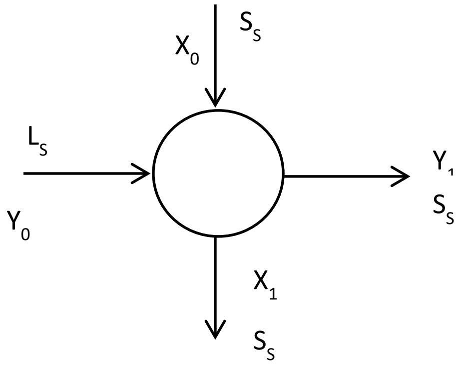
a) Stage-wise operation

- → Single stage operation.
- → Multistage cross current operation.
- → Multistage countercurrent operation

b) Continuous-contacting method

Single stage operation:

- Amount of adsorbed is very small compared to amount of solution
- Solute to be removed is adsorbed more strongly than other constituents present
- Adsorption of other constituents can be ignored
- Adsorbent is insoluble in solution



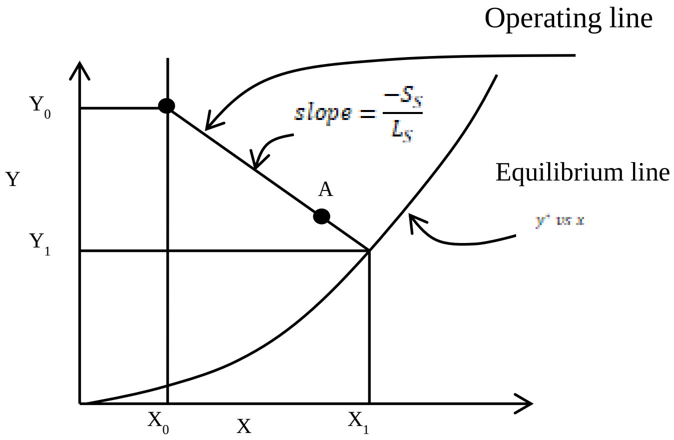
Let, Ls is mass unadsorbed substance, Yo is initial solute conc.

Ss is mass of adsorbent in adsorbate free basis

and Xo is initial solute conc

Xo= 0 if fresh adsorbent is used

$$\therefore L_s(Y_o - Y_1) = S_s(X_1 - X_0)$$



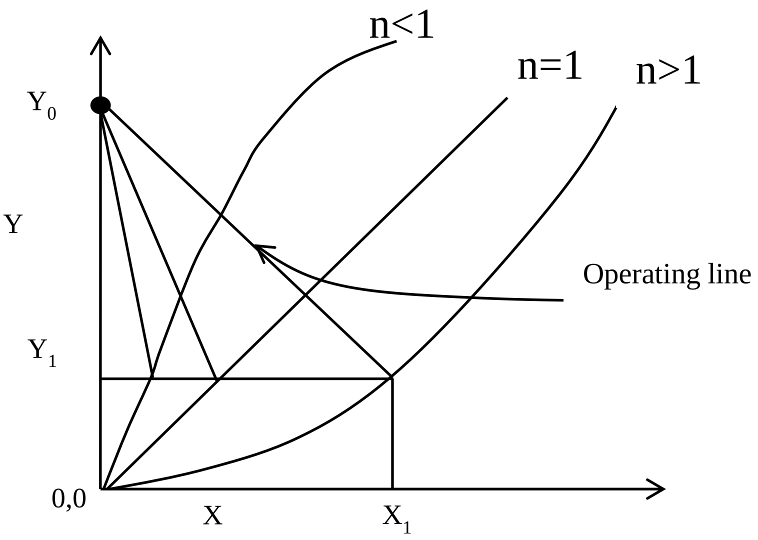
- ❖ If insufficient contact time is allowed, equilibrium is not reached, final liquid and solid concentration corresponds to point A.

Applying Freundlich equation:

$$q = k' p^{1/n}$$

$$Y^* = mX^n \Rightarrow X_1 = \left(\frac{Y1}{m}\right)^{1/n}$$

$$\frac{S_s}{L_s} = \frac{Y_0 - Y_1}{\left(\frac{Y_1}{m}\right)^{1/n} - 0} \qquad [X_0 = 0]$$



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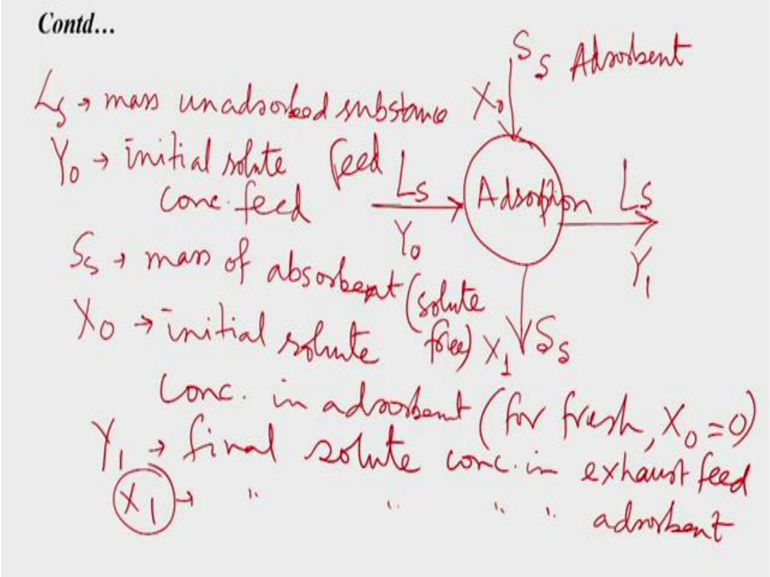
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L_s → mass unadsorbed substance

Y_0 → initial solute conc. feed

S_s → mass of absorbent (solute free)

X_0 → initial solute conc. in adsorbent (for fresh, $X_0 = 0$)

Y_1 → final solute conc. in exhaust feed

X_1 → " " " " adsorbent

So if we discuss this schematic diagram of the adsorption process that will be like this. Suppose this is the single stage adsorption process, then so we can say feed is added here, suppose this feed let us take this LS is the feed flow rate in the solute free basis, then the concentration will be like this Y0 and say this whatever the solute is added that will be say we can say S S with this concentration of X0, means the small amount of the solute also is present or which one is coming from the other adsorption process.

If some amount of solute also is there, so we then we say this one, that is we can say this one adsorbent. Like this, this is we can say feed and this is adsorbent. That is maybe coming from, in general we can say if it is a pure adsorbent then X0 should be 0. Then we can say it will be separated into two streams as the other separation processes also. So whatever the adsorbent is this one, this is also the solute free basis and their concentration will be say X1, this one after adsorption process.

And whatever the exhaust feed will be there, suppose this the flow rate will be LS with we can say this solute concentration in the feed is Y1. So our target will be that Y1 will be reduced to, Y0 will be reduced to Y1. And this whatever if X is equal to 0, X0 equal to 0 and X1 will be the we can say maximum so that the adsorption process will be feasible. Like this so we can say this LS is the mass unadsorbed substance. And then say Y0 will be we can say initial solute concentration, then we can say S S that is the mass of adsorbent. That is we can say solute free basis.

Then we can say X0 that will be the initial solute concentration in adsorbent. That is in feed, so initial solute concentration in adsorbent. For fresh X0 will be 0. And then Y1 say final solute concentration in exhaust feed and say X1 is the final solute concentration in your adsorbent. So our target that this X1 should be maximum, so that will be our target. So we can say if we do the mass balance, then suppose this if we do the mass balances LS plus S S will be is equal to LS plus S S and if we do the component balance, then will be LS into Y0 plus S S into X0 will be equal to LS into Y1 plus S S into X1.

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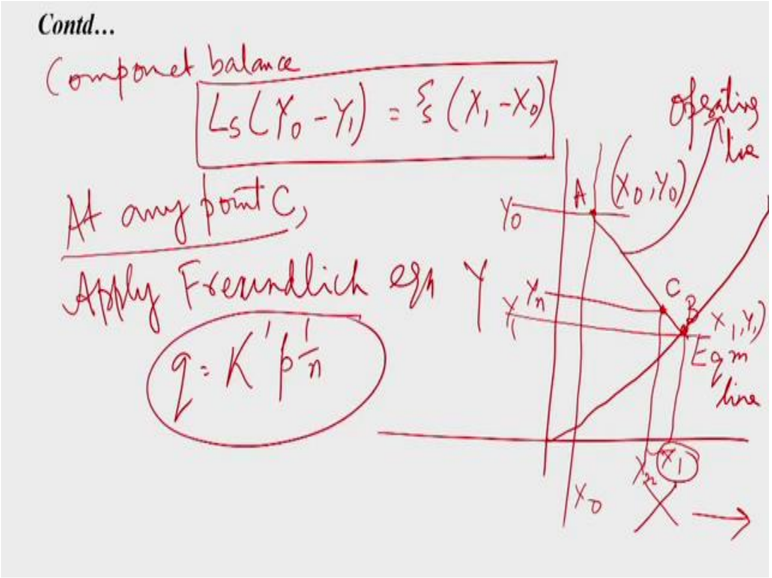

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Componet balance

$$\boxed{L_S(Y_0-Y_1) = S_S(X_1-X_0)}$$

At any point C,

Apply Freundlich eqn

$$q = K' p^{\frac{1}{n}}$$

So from there we will be doing this one in terms of LS and S S like this. We can say this by component balance if we do the component balance, so will be like this: LS into Y0 minus Y1 is equal to S S into X1 minus X0. So that is the we can say this if we do the adsorbent balance or we can say if we do the component balance. So then what we will be doing now?

Now we will be doing this, we will be putting this X versus Y here. So then we have this suppose for a particular system if we know this what is adsorbent and what is adsorbate, then we know this equilibrium relationship that is like this one.

Suppose this is the equilibrium line, let us say this is equilibrium line. And then the beginning condition will be like this, X0 and Y0. Let us take this is the initial condition say X0 and Y0. So we can say this one will be the initial condition. And so when it will be leaving the adsorber like this, we can say let us take this is the X1 and Y1 condition. So whenever it will we can say adsorbent will be leaving the adsorber that time the concentration of the solute in two different streams will be X1 and Y1.

So if we add these two line, so we can say this is Y0 and this is X0. So this is the initial condition and then final condition will be like this, say this is Y1 and this is X1. So this we can say that should be say this line, so we can say if I say this one A and B, so A is the initial condition and B is the final condition. Means before the addition of this adsorbent in the adsorbate, that was the condition at A and whenever it will be leaving the adsorber that is we can say this one B, so with X1 and Y1.

Say equilibrium is reached after certain period of time. So we can say this one when this both adsorption and desorption process becomes same, so then we can say equilibrium is reached. But if we do not allow this system to reach its equilibrium then it will it can stop somewhere. Suppose we are allowing this A to come to this point C and then we are not allowing this one to reach its equilibrium, so that is we can say this operating line.

So in this condition, say suppose this is for any Yn suppose in a particular time, so Yn and then we can say this Xn. So what is happening there, so we are not allowing the system to reach X1 in the adsorbent. Before that actually we are taking out. So what we are doing? So we are losing this amount say suppose this X1 minus Xn part actually we are losing for this system. But if we allow the system for a long period of time, then it will be reaching this point, the final point and then the maximum amount of solute will be separated by adsorbent.

So that is why we can say this any point say at any point C, at any point C we can say this whatever the concentration of the target component in both the we can say this adsorbent and adsorbate, that will be obtained by say suppose Xn and Yn. So there we can say at point C we can apply this Freundlich, suppose this we apply this Freundlich equation, so apply

Freundlich equation like say q is equal to K prime into p to the power 1 by n. So we can apply this equation.

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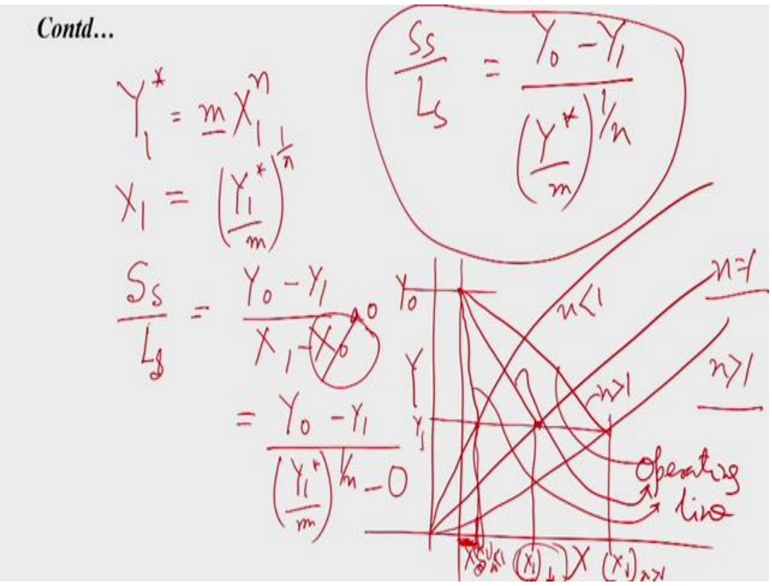
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$$Y_1^* = m X_1^n$$

$$X_1 = \left(\frac{Y_1^*}{m}\right)^{\frac{1}{n}}$$

$$\frac{S_s}{L_s} = \frac{Y_0 - Y_1}{X_1 - \overset{0}{\cancel{X_0}}} = \frac{Y_0 - Y_1}{\left(\frac{Y_1^*}{m}\right)^{1/n} - 0}$$

$$\frac{S_s}{L_s} = \frac{Y_0 - Y_1}{\left(\frac{Y^*}{m}\right)^{1/n}}$$

There then we will be getting like this, we have now this one Y star is equal to say m X to the power n and so then we can say if I take for the single stage that is say we can, Y1 star is equal to m into X1 star. Then we can say this one X1 will be equal to Y1 star by m, m is that slope to the power 1 by n. Then say this one, from the component balance whatever we have got this S S by LS, say S S by so this LS, that will be equal to we can say just by doing this one Y0 by Y1 by X1 minus X0.

That is we can say will be equal to Y0 minus Y1 by X1 is equal to Y1 star by m to the power 1 by n. Minus, so let us take for the time being for the single stage, that will be 0. Because for single stage we are adding this fresh adsorbent there, that is why it is 0. So we can say it is coming as we can say so LS by so that is S S by LS will be equal to we can say Y0 minus Y1 by Y1 star by m to the power 1 by n. So we are getting this that type of relationship. So now what actually we will be doing? For different n values let us take say we have this Y versus X like this.

If I say n is equal to 1, then equilibrium line will be this and say let us take the initial condition that is we can say let us take this is Y0 and X0. From here we can say the operating line will be like this, we can draw any operating line. Suppose for a particular condition we can say, so this is the we can say this one whenever it will be reaching the equilibrium point, it will be like it will be coming here.

So like we can say Y1 will be this. But you see this one, Y1 whatever the n value is suppose n is equal to 1, if I say n is greater than 1 like this, suppose n greater than 1 then Y1 will be like this, we can say Y1 will remain same. That is why this X1 will be here. So when n is equal to 1 this is n is equal to 1, so we can say this one particular X1 for n is equal to 1. X1 is n greater than 1 and whenever this n will be less than 1 like this, so this one will be n less than 1. So this is the case n less than 1, then it will be coming to this point and then we have this X1 that is X less than 1. Okay, this when n less than 1, so in all the cases we can say these are all operating lines.

So suppose this is one operating line for this is n greater than 1, this is the operating line. So when n equal to 1, this is also one operating line. When n less than equal to 1, this is operating line. So we can say for different cases when n will be less than 1, n will be equal to 1 or n will be greater than 1, then the equilibrium lines will be different. Means it indicates that this how much amount of solute will be separated using the different adsorbents like this. So based on that we can say for a single stage operation if the n value is greater than 1 so then we can say this one more solute will be separated.

If n is equal to 1, then the moderate amount of the solute will be separated. When n is less than 1, then small amount of solute will be separated. Just only this part, we can say this amount, only this part will be separated. Suppose initially this is X0 and then after this reaching the equilibrium it will be coming as this X1. So we can say this one with increasing the n values in the adsorption process we can say the adsorption efficiency increases.

Problem 2:

Adsorption equilibrium data of nitrobenzene (in an aqueous solution at low concentration) on a type of activated carbon fit in the Freundlich isotherm, $q_e = 68(C_e)^{0.43}$ [q_e in mg/g; C_e in mg/litre]. One m^3 of an aqueous solution containing 10 ppm of nitrobenzene is to be treated in a batch with activated carbon to reduce the concentration to 0.01ppm. Calculate the minimum quantity of carbon required. If 1.3 times the minimum amount is used, determine the batch time. The adsorption kinetics is fast and the overall process is virtually 'diffusion- controlled'. The effective surface area of the adsorbent for transport of the solute from the bulk is 200 cm^2/g, and the external mass transfer coefficient is $1.8 \times 10^{-5} m/s$.

Solution:

Now we will be solving one problem on single stage operation, very easy we can say problem. Say adsorbent equilibrium data of nitrobenzene in an aqueous solution at low concentration on a type of this activated carbon fit in the Freundlich isotherm, that is equation is given as q e is equal to 68 into Ce to the power 0.43. Like say it is related to the, suppose this q is the so we can say this one mass adsorbed per gram, means milligram adsorbed per gram of the adsorbent and C is the one equilibrium concentration. So that is the time dependent parameter.

That is C is this milligram per liter, the concentration but it is dependent on the time. Means it will be varying with time. So 1 meter cube of an aqueous solution containing 10 ppm or 10 milligram per liter of nitrobenzene is to be treated in a batch this one adsorber with activated carbon to reduce the concentration to 0.01 milligram per liter. So initial concentration is 10 ppm and final concentration in the exhaust feed will be 0.01 ppm. Now we need to calculate the minimum quantity of carbon required. So dose of the carbon or adsorbent amount we need to find out.

And if this is we can say minimum amount of carbon requirement for the separation. If this 1.3 times of the minimum amount is used then determine the batch time. The adsorption kinetics is very fast and the overall process is virtually say diffusion controlled like this. The effective surface area of the adsorbent for transport of the solute from bulk is 200 centimeter square per gram and the external mass transfer coefficient K value is 1.8 into 10 to the power minus 5 meter per second.

So we need to solve this one very, this is very simple problem. We will start with this schematic diagram and whatever the initial conditions, these are already given to us. In this problem everything is supplied, we need to calculate only to get the minimum quantity of the carbon, how much carbon will be required to reduce the nitrogen concentration from 10 ppm to 0.01 ppm.

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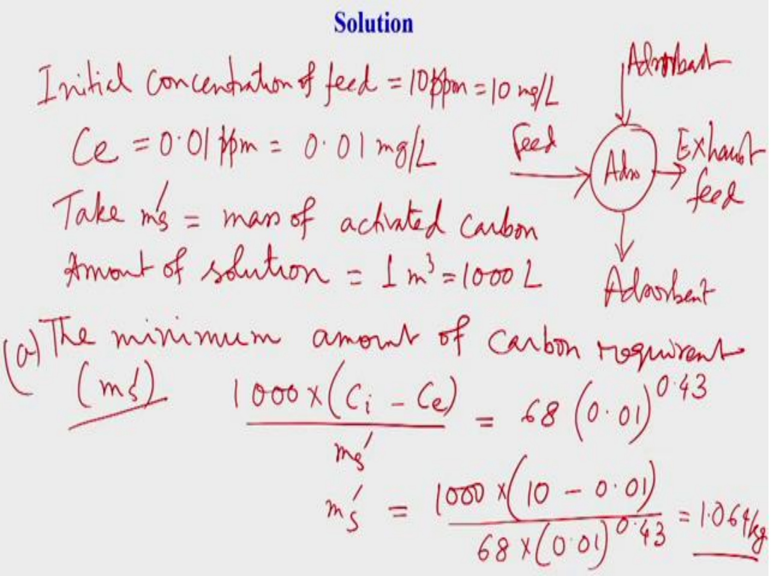


Solution

Initial concentration of feed = 10 ppm = 10 mg/L

$C_e = 0.01$ ppm = 0.01 mg/L

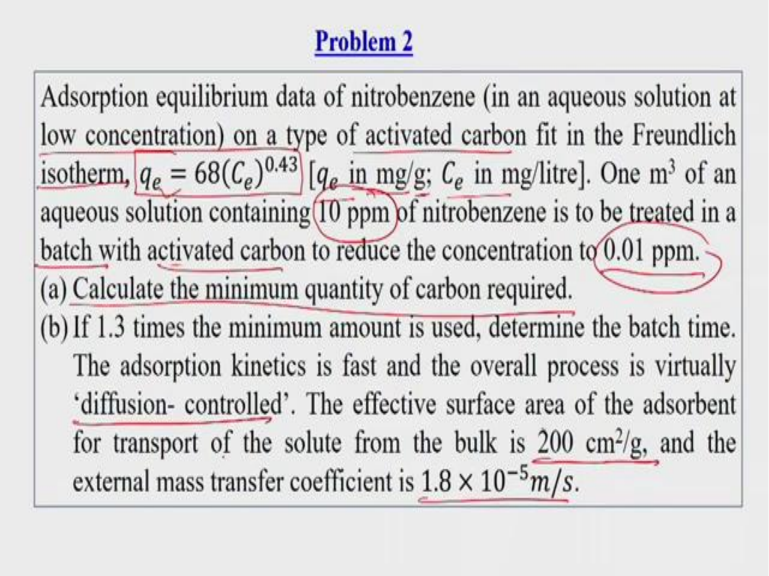
Take m_s' = mass of activated carbon

Amount of solution = 1 m^3 = 1000 L

(a) The minimum amount of carbon requirement (m_s')

$$\frac{1000 \times (C_i - C_e)}{m_s'} = 68\,(0.01)^{0.43}$$

$$m_s' = \frac{1000 \times (10 - 0.01)}{68 \times (0.01)^{0.43}} = \underline{1.064\,kg}$$



Problem 2

Adsorption equilibrium data of nitrobenzene (in an aqueous solution at low concentration) on a type of activated carbon fit in the Freundlich isotherm, $q_e = 68(C_e)^{0.43}$ [q_e in mg/g; C_e in mg/litre]. One m^3 of an aqueous solution containing 10 ppm of nitrobenzene is to be treated in a batch with activated carbon to reduce the concentration to 0.01 ppm.

(a) Calculate the minimum quantity of carbon required.

(b) If 1.3 times the minimum amount is used, determine the batch time. The adsorption kinetics is fast and the overall process is virtually 'diffusion- controlled'. The effective surface area of the adsorbent for transport of the solute from the bulk is 200 cm^2/g, and the external mass transfer coefficient is $1.8 \times 10^{-5} m/s$.

We will start with this schematic diagram. Like this, this is the say adsorption process. So now we will be adding this suppose initial concentration, say we are adding this feed. And we are adding this adsorbent. Then we have this adsorbent and exhaust feed. So in the feed initial concentration, say let us take initial concentration of say nitrobenzene is equal to saY10 ppm milligram per liter. And we can say the Ce whatever will be the equilibrium concentration or add the end of this one, that will be 0.01 ppm. That is 0.01 milligram per liter.

And let us take m S prime that is equal to mass of activated carbon. And amount of solute, amount of solution is equal to saY1 meter cube, that is equal to we can saY1000 liter. Now the minimum is for the first problem, amount of carbon requirement. We can say that is requirement is m S prime. So for that case the Freundlich equation that is given as q e is equal to 68 into Ce to the power 0.43. So we will be taking this one, for this case we will take that so we can saY1000 liter into this C i minus Ce divided by say m S prime.

That is equal to coming out as 68 into let us saY0.01 to the power 0.43. Or just by manipulation we will be getting m S prime is equal to saY1000 into this C i is equal to we can say, 1000 into C i is equal to 10 minus say Ce is equal to 0.01 divided by 68 into 0.01 to the power 0.43. So that is coming out as saY1.064 kg. So that is the minimum amount of active carbon requirement for the separation of nitrobenzene from 100 ppm to 0.01 ppm, is 1.064 kg. So that is the we can say this one for the first case.

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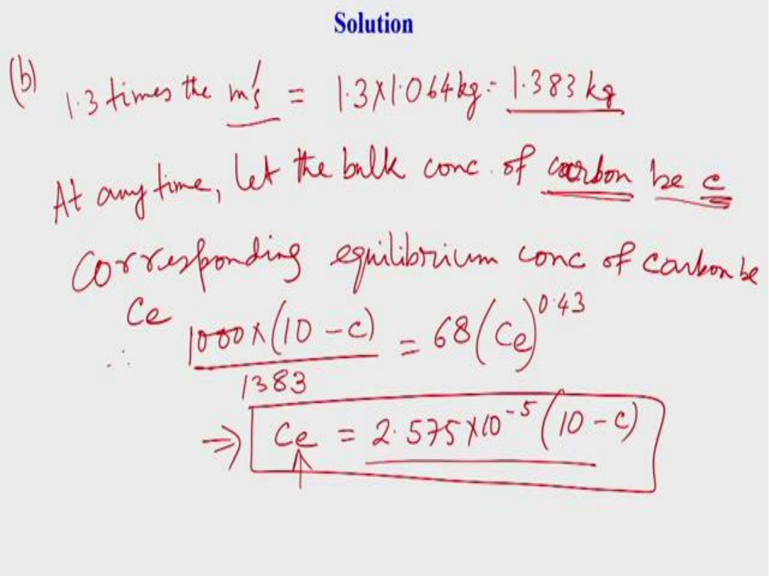


Solution

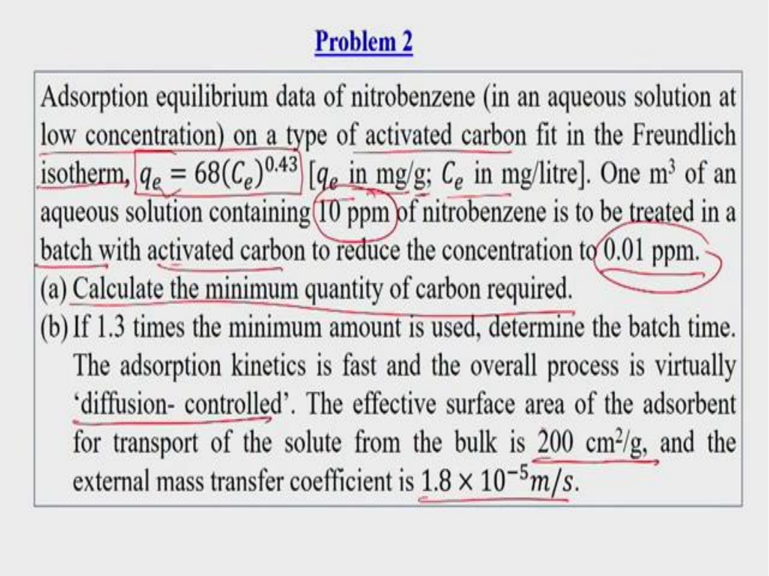
(b) 1.3 times the m_s' = $1.3 \times 1.064\,kg = \underline{1.383\,kg}$

At any time, let the bulk conc. of carbon be c

Corresponding equilibrium conc of carbon be Ce

$$\therefore \frac{1000 \times (10 - c)}{1383} = 68(C_e)^{0.43}$$

$$\Rightarrow \boxed{C_e = 2.575 \times 10^{-5}(10 - c)}$$



Problem 2

Adsorption equilibrium data of nitrobenzene (in an aqueous solution at low concentration) on a type of activated carbon fit in the Freundlich isotherm, $q_e = 68(C_e)^{0.43}$ [q_e in mg/g; C_e in mg/litre]. One m^3 of an aqueous solution containing 10 ppm of nitrobenzene is to be treated in a batch with activated carbon to reduce the concentration to 0.01 ppm.

(a) Calculate the minimum quantity of carbon required.

(b) If 1.3 times the minimum amount is used, determine the batch time. The adsorption kinetics is fast and the overall process is virtually 'diffusion- controlled'. The effective surface area of the adsorbent for transport of the solute from the bulk is 200 cm^2/g, and the external mass transfer coefficient is $1.8 \times 10^{-5} m/s$.

Then for the second case that is the problem is 1.3 times the minimum amount is used, then determine the batch time. So now we need to calculate the how much time will be required to separate the same amount. Means from 10 ppm to 0.01 ppm, how much time we will be using? But the dose actually has increased from 1.043 gram to 1.064 kg to saY1.3 times of 1.064 kg. So then we need to calculate their mass transfer coefficient value is given.

So for this case now mass, then 1.3 times the m S prime or minimum is equal to 1.3 into 1.064 kg that is coming out as 1.383 kg. So that amount will be required and the solute concentration in the exhaust feed also will be 0.01 ppm or 0.01 kg per this one what is called gram per liter, milligram per liter.

So then we can say at any time now we need to get the what is the adsorption rate actually in terms of time. We need to get this differential equation. So at any time let the bulk concentration of carbon say be C. So at any time let the bulk concentration of the carbon be C, then we can say the corresponding equilibrium concentration of carbon be C. So then we can say Freundlich equation will be like this, saY1000 into 10 minus C divided bY1383 will be equal to 68 into say Ce to the power 0.43. Or we can say from here we will be getting Ce is equal to 2.575 into 10 to the power minus 5 into 10 minus C.

So now we have this one, so now we have this relation, Ce is equal to say equilibrium concentration of the solute at any point of time will be related to 2.575 into 10 to the power minus 5 into 10 minus C where C is the bulk concentration of carbon at any time. And Ce is the equilibrium concentration of the carbon.

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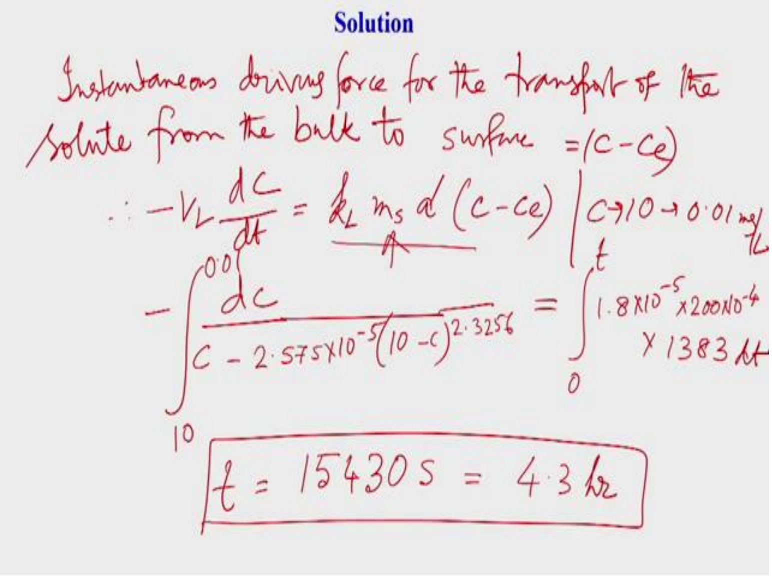

Solution

Instantaneous driving force for the transport of the solute from the bulk to surface $= (C - C_e)$

$$\therefore -V_L \frac{dC}{dt} = \underset{\uparrow}{\underline{k_L m_s a}}\,(C - C_e) \quad \Big|\ C \to 10 \to 0.01\ mg/L$$

$$-\int_{10}^{0.01} \frac{dC}{C - 2.575\times10^{-5}(10 - C)^{2.3256}} = \int_0^t 1.8\times10^{-5} \times 200\times10^{-4} \times 1383\, dt$$

$$\boxed{t = 15430\ s = 4.3\ hr}$$

Now we can say this one, this instantaneous driving force, so for the transport of the solute from the bulk to the adsorbent site actually or surface to surface will be like this: C minus Ce.

So if we write this continuity equation then will be like this: minus VL into d C by d t, that will be equal to kL into say m S into say a prime, the specific surface area into C minus Ce. So that C actually will be changing from 10 to 0.005 like this, we can say actually C is changing from 10 to 0.01 milligram per liter.

So now we need to just rearrange this equation and we need to integrate, and all the other parameters are already supplied. So we can do one thing, we will be doing this d C by say C minus Ce. Ce is given as 2.575 into 10 to the power minus 5 into 10 minus C, that is to the power 2.3256. And it is integrating with dt, that will be say we will be integrating this one from 10 to 0.01 concentration and then it will be time 0 to t. That we will be getting this one, so all the other parameters we will be putting now. This kL m S a prime we will be putting here like this, soY1.8 into 10 to the power minus 5.

Then m S actually that will be say we can say and kL 200 into 10 to the power minus 4 and into this is, sorry m S is equal to 1383. So that is we can say into dt. So from there actually we can do the integration and by any integration method we will be getting this t is equal to we can say total time will be equal to 15,430 second. That is equal to 4.3 hours. So we will be getting this one, so that we can say t is equal to, we can say time is equal to 15,430 second.

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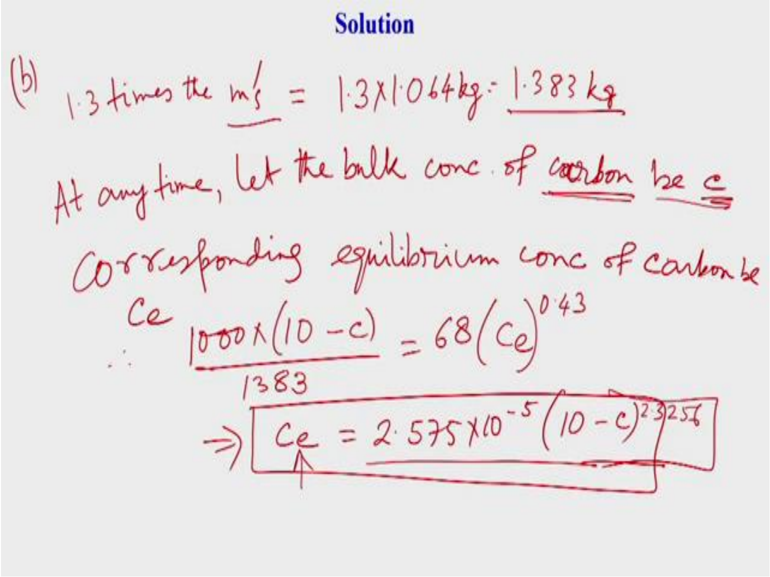


Solution

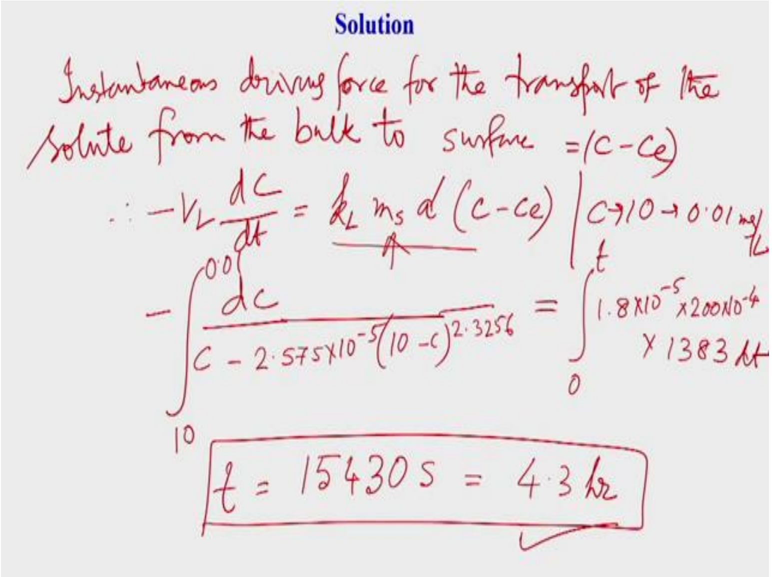
(b) 1.3 times the m_s' = $1.3 \times 1.064\,kg = \underline{1.383\,kg}$

At any time, let the bulk conc. of carbon be c

Corresponding equilibrium conc of carbon be Ce

$$\therefore \frac{1000 \times (10 - c)}{1383} = 68(C_e)^{0.43}$$

$$\Rightarrow \boxed{C_e = 2.575 \times 10^{-5}(10 - c)^{2.3256}}$$



Solution

Instantaneous driving force for the transport of the solute from the bulk to surface $= (C - C_e)$

$$\therefore -V_L \frac{dC}{dt} = k_L m_s a (C - C_e) \Big| \begin{array}{l} C \to 10 \to 0.01 \text{ mg/L} \\ t \end{array}$$

$$-\int_{10}^{0.01} \frac{dC}{C - 2.575\times10^{-5}(10-C)^{2.3256}} = \int_0^t 1.8\times10^{-5} \times 200\times10^{-4} \times 1383\, dt$$

$$\boxed{t = 15430\text{ s} = 4.3\text{ hr}}$$

So that is the point and say here actually you see in this equation it will be like this 10 minus C to the power 2.3256. So it will be coming like this. So finally this formula will be like this, C to the power, Ce is equal to 2.575 into 10 to the power minus 5 into 10 minus C to the power 2.3256.

So finally we got this total time for the adsorption of 10 ppm nitrobenzene to this 0.01 ppm nitrobenzene in 4.3 hours. Now we will be discussing about the another thing, that is multi stage cross current operation.

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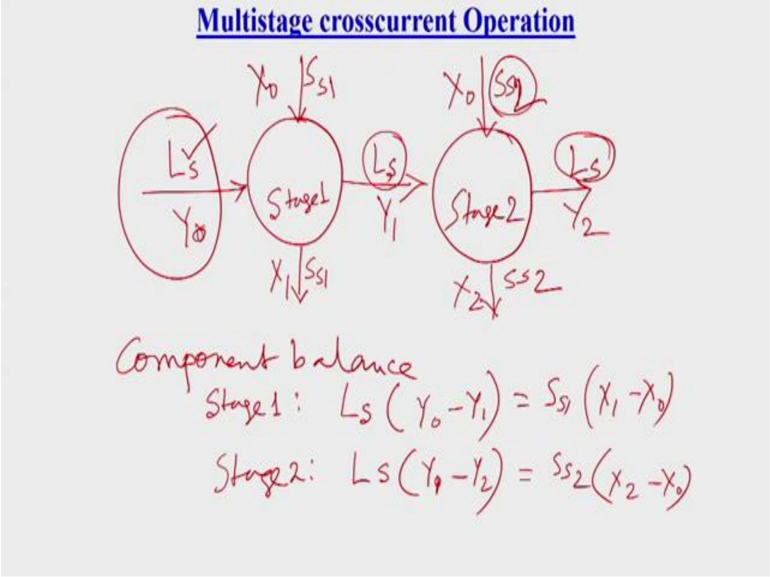

Multistage crosscurrent Operation

Component balance

Stage 1: $L_S(Y_0 - Y_1) = S_{S1}(X_1 - X_0)$

Stage 2: $L_S(Y_1 - Y_2) = S_{S2}(X_2 - X_0)$

So that is similar to the other mass transfer operations. Like this, we will start with this one. Suppose this is the stage 1, so here we will be putting this feed like this LS with this Y0 concentration of the solute and we are adding this adsorbent with X0, that may be 0. And with this concentration of the, this adsorbent is SS1. So whenever it will be separated into these two stages, one with adsorbent and other will be adsorbate, so adsorbent that is we can say it will remain constant as SS1 because this is solute free basis.

But the concentration of the solute will be on the adsorbent will be X1. And in the exhaust stream or in the whatever exhaust feed will be there, there we say the solute this flow rate will be LS with concentration of Y1. Now this LS or this feed flow rate of LS with solute concentration of Y1 will be the feed to the stage 2. That is why it is we can say this one cross current operation. So it will be fed to this stage 2 and then there also again we are putting another say adsorbent. So that is we can say, we can take SS1 or we can take SS2 whatever may be.

Any other flow rate also we can take or same flow rate also with the same solute concentration or maybe other solute concentration. Now we are allowing the system to attain the equilibrium, so here also we will be getting these two different streams. One will be the this SS1 actually will be there, so if we take this is SS2 then it will be SS2 and will be say it is X2. And this LS whatever the feed actually has entered this, only one feed has entered, so if it is solute free basis so then whenever it will be exiting the stage 2 that will be, that is also solute free basis.

And then this time the solute concentration will be Y2. So we can assume that the Y2 will be much more lower whenever it will be leaving the second adsorber. And for that actually we can say we can do the component balance. So we can do suppose in the stage 1, like this very simply we can add LS into Y0 plus SS1 into X0, will be equal to LS into Y1 plus SS1 into X1. So from there we can say just by manipulation we will be writing LS into Y0 minus Y1 is equal to SS1 into X1 minus X0.

For stage 2 like this in the same way we can write this LS into Y1 minus Y2 is equal to SS2 into X2 minus X0. So X2 minus X0, just by doing the component balance only we will be getting.

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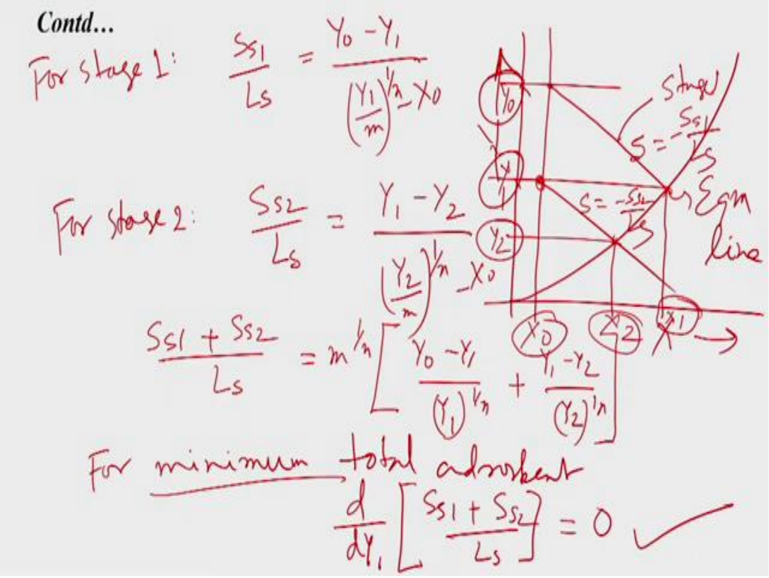

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For stage 1: $$\frac{S_{S1}}{L_S} = \frac{Y_0 - Y_1}{\left(\frac{Y_1}{m}\right)^{1/n} - X_0}$$

For stage 2: $$\frac{S_{S2}}{L_S} = \frac{Y_1 - Y_2}{\left(\frac{Y_2}{m}\right)^{1/n} - X_0}$$

$$\frac{S_{S1} + S_{S2}}{L_S} = m^{1/n}\left[\frac{Y_0 - Y_1}{(Y_1)^{1/n}} + \frac{Y_1 - Y_2}{(Y_2)^{1/n}}\right]$$

For minimum total adsorbent

$$\frac{d}{dY_1}\left[\frac{S_{S1} + S_{S2}}{L_S}\right] = 0 \checkmark$$

So for stage 1 now we will be done this one, SS1 by say LS, that will be Y0 just like previous one Y0 by Y1 divided by this whatever Y star was there actually we will be putting like say Y1 by m to the power 1 by n. Minus X0 we are keeping but we told that say in general X0 becomes 0 in most of the cases. For stage 2 SS2 by LS, we will be having this similar thing. Like we can say Y1 minus Y2 divided by Y2 by m to the power 1 by n minus X0.

So now we will be doing one thing, we will be adding these two equations. Suppose this equation and this equation and then we will be getting this SS1 plus SS2 divided by LS, that will be coming as m to the power 1 by n and whole because in all the cases in the denominator of the denominator is m to the power 1 by n. So it is coming as m to the power 1 by n. Then we will be getting this from these two. And Y1 will go and we will be getting this Y0 minus Y1 divided by Y1 to the power 1 by n plus Y1 minus Y2 by Y2 to the power 1 by n.

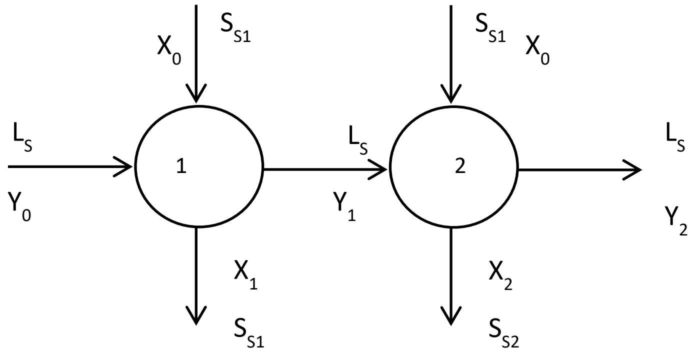
And say this time we are assuming also that this X0 will be going and for minimum total adsorbent we can say the d into dY1 into this SS1 plus SS2 by LS, that will be equal to 0. Now if we do this graphically just for two stages in the cross current operation like this, so if we plot this Y and X in these two directions, this in X axis and Y axis if we plot this one, suppose we have this equilibrium relationship. This is equilibrium line. And so let us take the first stage is like this, we can say X0 and Y0.

Now it is following this operating line. Let us take this stage 1. So it is coming and this we can say X1 and correspondingly we can say this is Y1. Now this whenever it is again it is coming to this Y1 and X0, that is for stage 2 Y1 and X0, so we can say now the starting point for the second stage is here. And again suppose this slope is slope will be equal to minus SS1 by LS, now the slope will be suppose in this case slope will be minus SS2 by LS. Now it is coming here, this we can say this one X2.

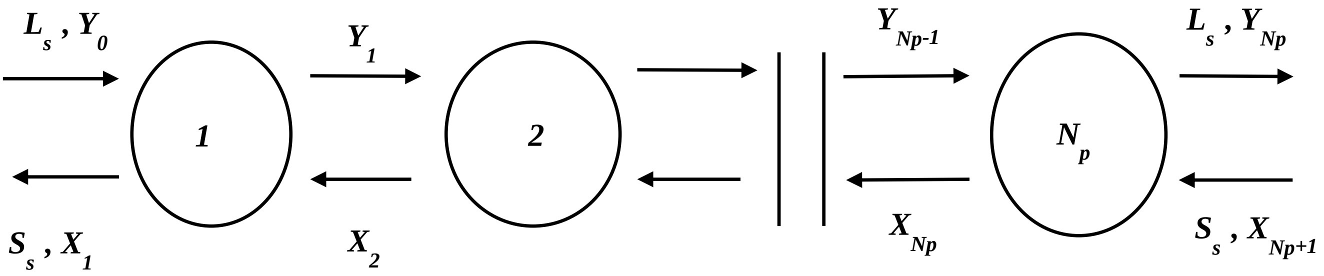
And for that case we will be getting this Y2. So this is Y2, this is Y0, this is Y1 and this is X0. That is X0 we assume that for in all the cases it will be like this a constant one for the adsorbent. So in general it is 0, then it starts from the Y axis point. Suppose then it will be, say for the first it will start from here. For the second stage it will start from here like this. So the point is that in the beginning suppose the X (amou) X1 amount of the solute actually was adsorbed on the surface.

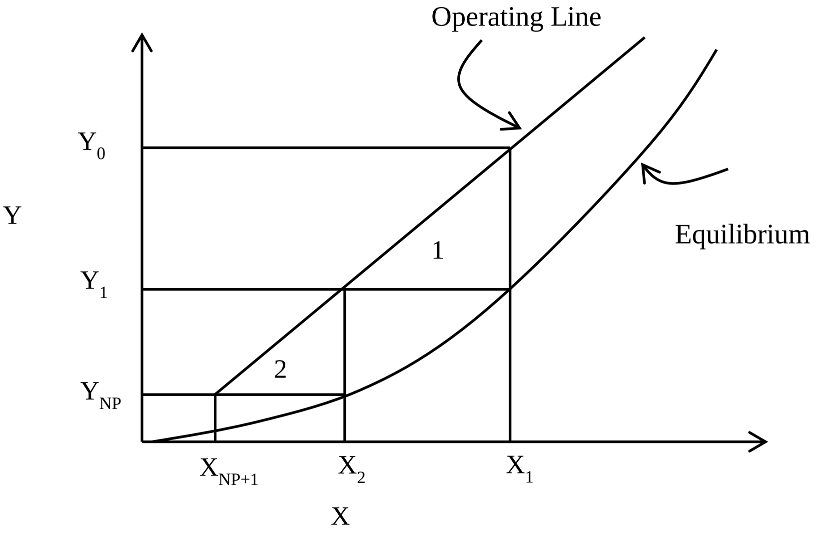
Then in the next stage the solute which is present in the feed becomes Y1, so now we say Y1 is very less and then driving force actually is decreased and then we are getting the final stage. It will be getting this Y2, so this sorry X2. So X2 amount will be separated like this from there using this adsorbent. So we are going to get all these and using this we will be able to get the what will be the minimum adsorbent flow rate. So that we will be getting… means this one SS1 and SS2 amount, how much minimum amount we will be getting just by differentiating this equation by putting this differential as 0 we will be getting.

Multistage cross current operation



Multistage countercurrent operation ‘

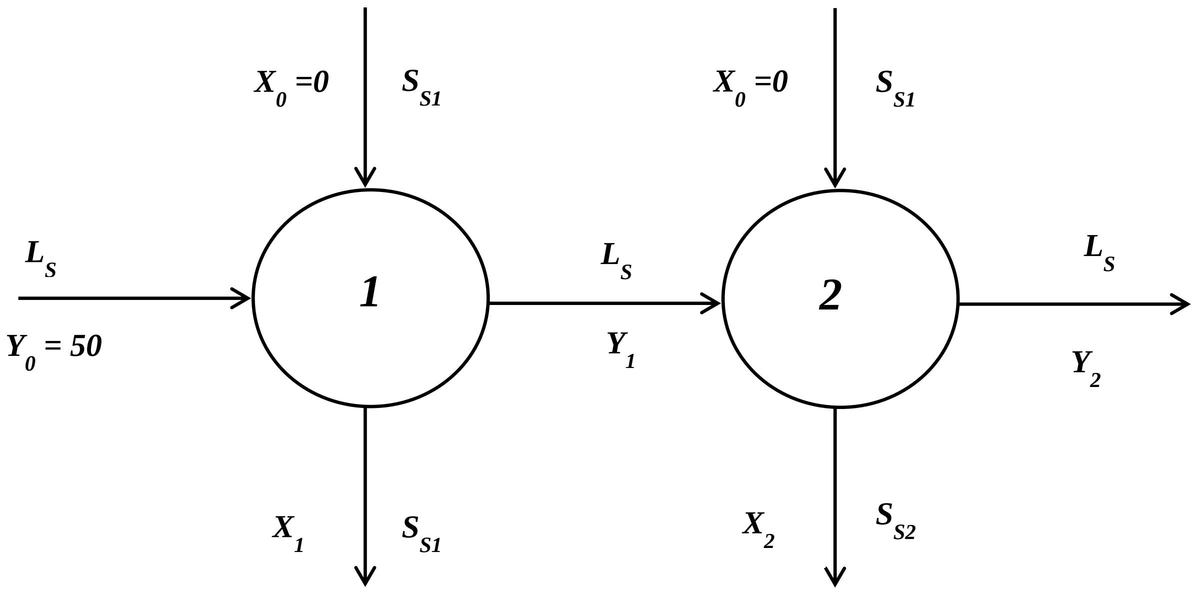




Problem 3:

Decolorization of 1000 kg of waste oil is done using 24.72 kg of adsorbent in two successive contacts. 12.36 kg clay is added to the oil and a contact time of 30.6 minutes is allowed. The adsorbent is separated and the oil is further treated with the rest of the adsorbent for 12.2 minutes. Calculate the dye concentration in the oil after two stage contact. The mass transfer coefficient (k_L) is $5.2 \times 10^{-6} m/s$. The loss of oil with clay separated after the first stage contact may be neglected. Initial dye concentration is 50 units. Density of oil is 950 kg/m^3. Adsorption equilibrium relation is $Y = 4.2 \times 10^{-4} X^*$, where Y is number of colour units/kg oil. X^* is number of colour units/kg clay in equilibrium. Effective surface area of adsorbent is 25 m^2/kg.

Solution:



Now we will be solving now one problem. This is a very simply problem. Say like decolorization of 1000 kg of waste oil is done using 24.72 kg of the adsorbent in two successive contacts. So 12.36 kg clay is added to the oil and a contact time of 30.3 minutes is allowed. This is we can say this is in the first stage. The adsorbent is separated and the oil is further treated with rest of the adsorbent for 12.2 minutes.

So out of this 24.72 kg so half is given in the first stage and half is given in the second stage. Calculate the clay concentration in the oil after two stage of contact. So we need to measure the how much amount of the dye will be there. Means maybe in some units that will be say exhausted and will be there in the oil. That is not separated during these two stages. The mass transfer coefficient kL value is 5.2 into 10 to power minus 6 meter per second. The loss of oil with clay separated after first stage contact may be neglected.

But in reality some amount of, small amount of the oil is always wasted actually during this separation. And initial dye concentration is 50 units. Whatever the unit will be no problem. And density of oil is 950 kg per meter cube and adsorption equilibrium relation is given as, suppose this is the relation Y is equal to 4.2 into 10 to power minus 4 X star, where Y is number of color units and X star is the number of, we can say this one Y is the number of color units, X star is the number of color units clay in the equilibrium.

And effective surface area of adsorbent is given as 25 meter square per kg. That is the effective surface area a prime. So now we will be, first we will draw this schematic diagram, then we will be discussing stage by stage. So we will be first completing this stage 1, then we will be going to the stage two.

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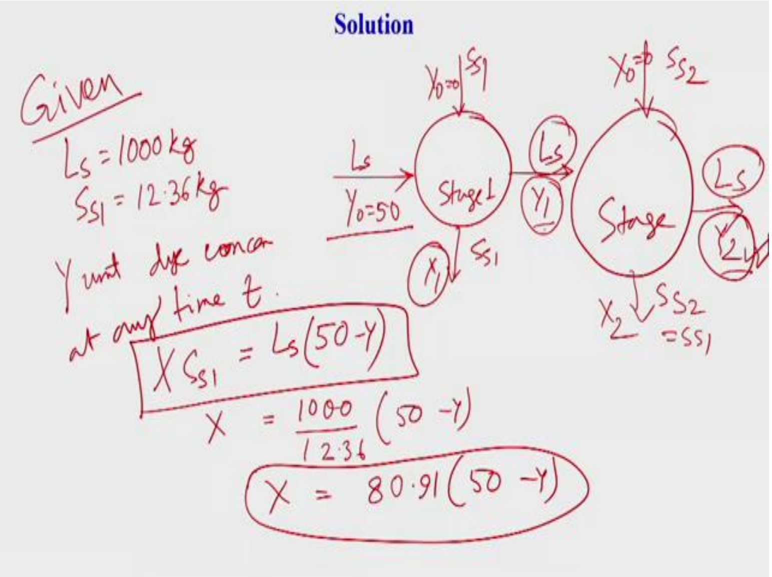

Solution

Given

$L_s = 1000\ kg$

$S_{S1} = 12.36\ kg$

Y unit dye concn at any time t.

$$\boxed{X S_{S1} = L_s(50 - Y)}$$

$$X = \frac{1000}{12.36}(50 - Y)$$

$$\boxed{X = 80.91(50 - Y)}$$

Say this is stage one, so there we have SS1 is given and X0 is actually taken as 0 here as per problem. And then we have this LS and say Y0 is say 50 unit, whatever unit is given that is we can say 50 unit. Now we need to get whatever will be Y1, we do not know. LS we have, and then how much amount of this S S, suppose this SS1, how much amount of X1 we have we need to calculate first.

Just from the component balance we will be getting this one. Let us take say whatever is given in the problem, say LS is given as 1000 kg and SS1 is given as 12.36 kg, also SS2 is also 12.36 kg. So from this first stage whatever the LS or we can say flow rate, feed flow rate with this Y1 will be entering and here also again SS2 will be adding, that is same actually. X0 is equal to 0 here. And then we have this SS2 that is equal to SS1 in this problem and we need to get X2. And from here we will be getting, LS actually we will be getting same and we will be getting Y2.

So we need to find this what is the Y2. As per the problem what is the Y2 we have? Say 50 this unit is there in the feed and then it will be reduced to Y1 then it will be reduced to Y2. So what will be the Y2? Just we will be doing the component balance from one to another stage like this. So let us take this for dynamic case. We can say take Y that is any unit actually, unit dye actually dye concentration at any time at time t.

So we can say this X into SS1 is equal to LS into 50 minus Y. So at any time we can say this is we can say we can write this one. And so that will be X is equal to LS means that is LS is equal to 1000 by SS1 is equal to 12.36 into 50 minus Y. So X is equal to 80.91 into 50 minus Y. So this is relation between X and Y.

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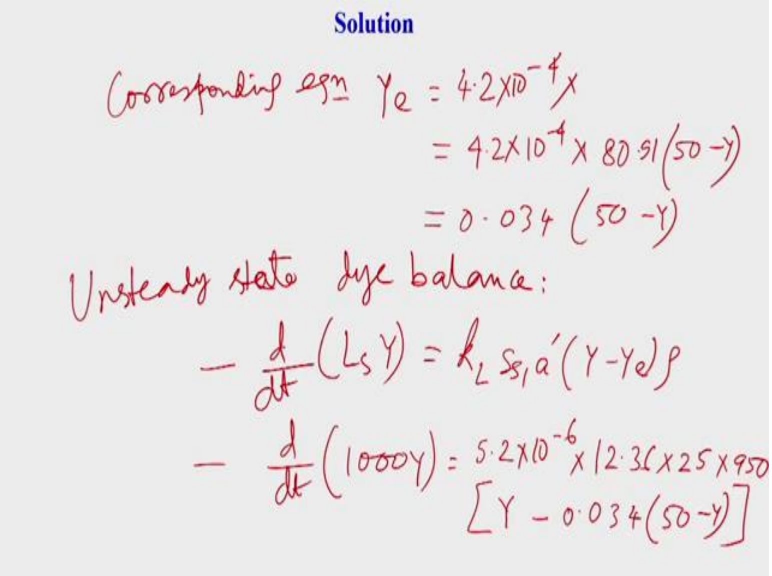


Solution

Corresponding eqn $Y_e = 4.2\times10^{-4} X$

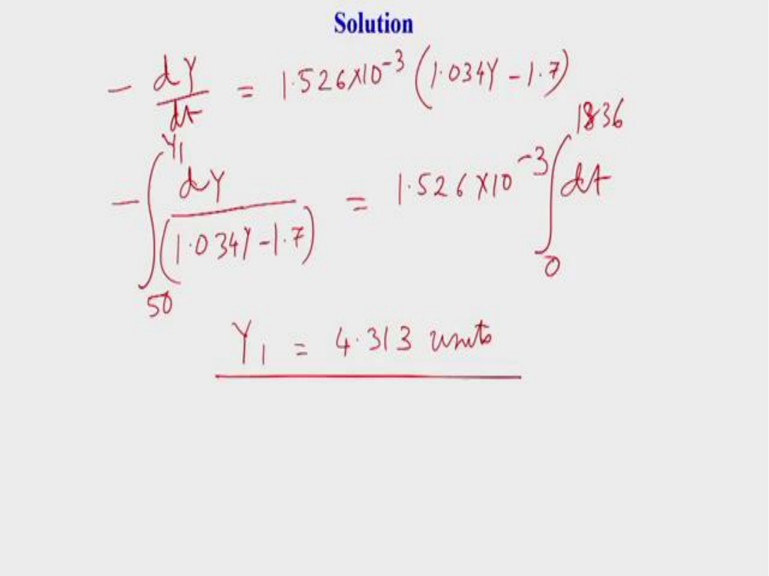
$$= 4.2\times10^{-4}\times 80.91(50-Y)$$

$$= 0.034(50-Y)$$

Unsteady state dye balance:

$$-\frac{d}{dt}(L_S Y) = k_L S_{S1} a'(Y-Y_e)\rho$$

$$-\frac{d}{dt}(1000Y) = 5.2\times10^{-6}\times 12.36\times 25\times 950\,[Y-0.034(50-Y)]$$



Solution

$$-\frac{dY}{dt} = 1.526\times10^{-3}(1.034Y-1.7)$$

$$-\int_{50}^{Y_1}\frac{dY}{(1.034Y-1.7)} = 1.526\times10^{-3}\int_{0}^{1836} dt$$

$$\underline{Y_1 = 4.313 \text{ units}}$$

Now we have this corresponding equation actually, corresponding equation that Ye is equal to, that is supplied 4.2 into 10 to the power minus 4 into X, is equal to we can say 4.2 into 10 to the power minus 4 into whatever X we obtain now, 80.91 into 50 minus Y. So it is coming out as 0.034 into 50 minus Y. Now we need to do the unsteady state, unsteady state dye balance. We will be doing this one, minus d by dt into LS into Y. That will be this kL into SS1 into a prime into say Y minus Ye into rho.

So therefore d dt of LS is equal to 1000 into Y is equal to kL is equal to is given as 5.2 into 10 to the power minus 6 into SS1 is equal to 12.36, a prime is given as 25 into Y minus Ye as 0.034 into 50 minus Y. So that is into 950 also,so from here actually we will be just by manipulating this one:

Minus d Y dt will be equal to 1.526 into 10 to the power minus 3 into 1.034 Y minus 1.7. So that we will be getting just by manipulating the previous equation and then we will be getting this at, now we need to get the limits only or we will be doing this one minus d Y bY1.034 Y minus 1.7, equal to 1.526 into 10 to power minus 3 into dt. So we will be integrating this one, so when time t is equal to 0 it is 50 in the beginning. When we need to get this time total time, actually we can say that is 1836 seconds.

What will be this concentration? That is Y1. So from here actually we will be getting this Y1 just by integrating this one, we will be getting Y1 is equal to say 4.313 units. So it is now decreasing from 50 units to 4.313 units.

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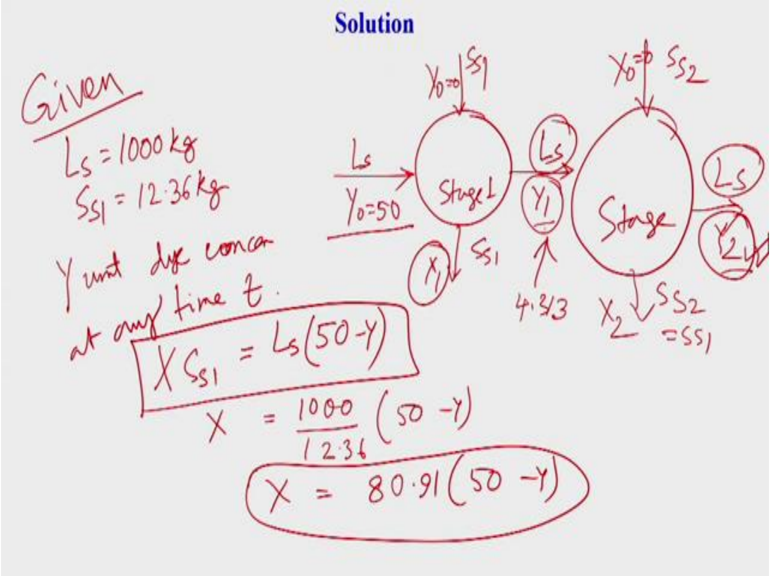

Solution

Given

$L_s = 1000$ kg

$S_{S1} = 12.36$ kg

Y unit dye concn at any time t.

$$\boxed{XS_{S1} = L_s(50-Y)}$$

$$X = \frac{1000}{12.36}(50-Y)$$

$$\boxed{X = 80.91(50-Y)}$$

So here in this graph we can say schematic diagram, it will be say 4.313. So 52, 4.313 units actually it is coming. Now the same dose is given there.

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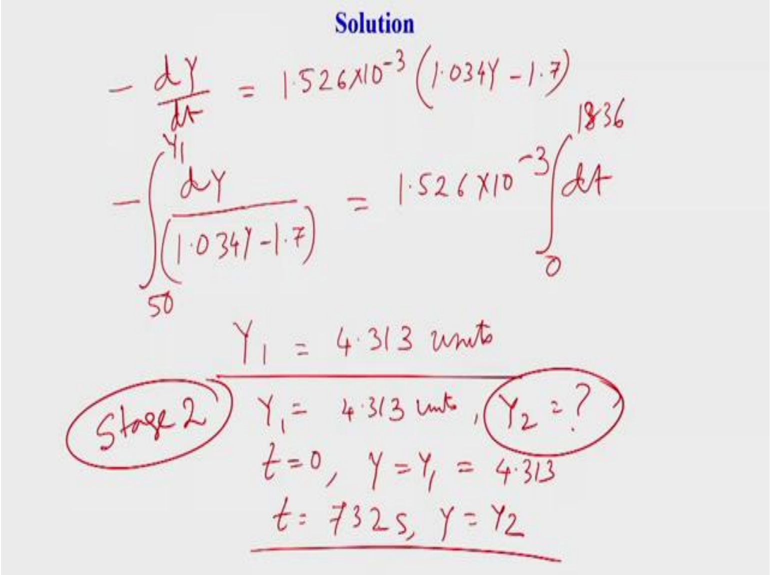

Solution

$$-\frac{dY}{dt} = 1.526\times10^{-3}(1.034Y-1.7)$$

$$-\int_{50}^{Y_1}\frac{dY}{(1.034Y-1.7)} = 1.526\times10^{-3}\int_0^{1836} dt$$

$$Y_1 = 4.313 \text{ units}$$

Stage 2

$Y_1 = 4.313$ units, $Y_2 = ?$

$t=0,\ Y=Y_1 = 4.313$

$t = 732\text{s},\ Y = Y_2$

So we have here this one say same dose. So for stage 2 so we have say Y1 is equal to 4.313 units. And we need to find out this Y2, that is the our problem, the question. And so we can say when t will be 0 then Y will be Y1, that will be 4.313. And when t will be equal to say 732 second, then what will be, Y will be equal to Y2. So this time is given then at that time what will be the Y2?

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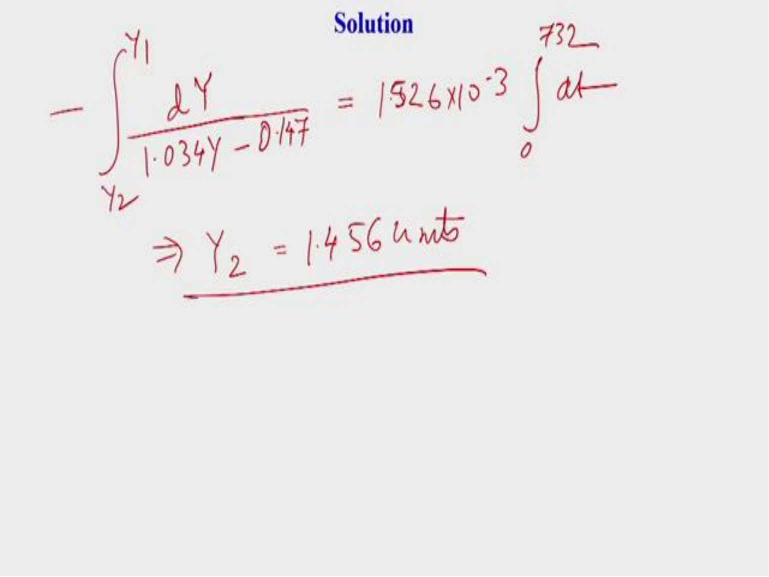
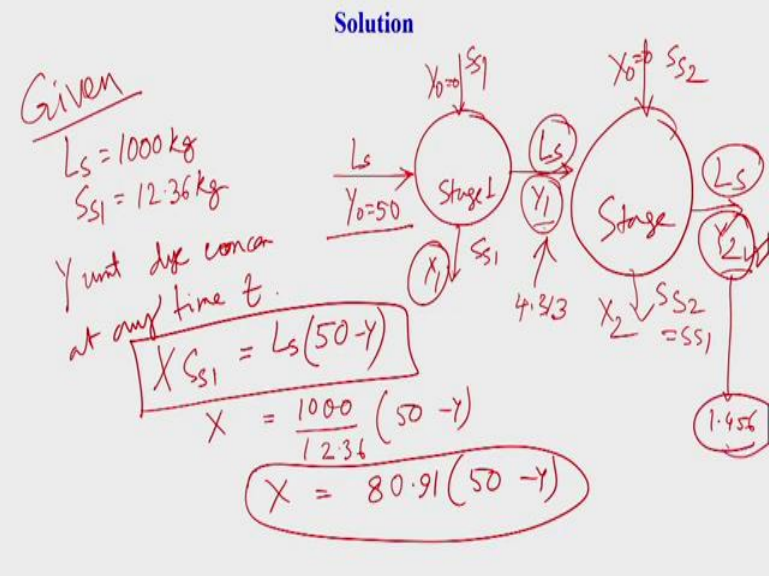


Solution

$$-\int_{Y_2}^{Y_1} \frac{dY}{1.034Y - 0.147} = 1.526\times10^{-3}\int_0^{732} dt$$

$$\Rightarrow Y_2 = 1.456 \text{ units}$$



Solution

Given

$L_s = 1000$ kg

$S_{S1} = 12.36$ kg

Y unit dye concn at any time t.

$$X S_{S1} = L_s(50-Y)$$

$$X = \frac{1000}{12.36}(50-Y)$$

$$X = 80.91(50-Y)$$

Just in that same equation, so we will be doing this the same equation we will be putting this one. Say Y2 to Y1 dY bY1.034 Y minus 0.147 is equal to saY1.526 into 10 to the power minus 3 into 0 to say 732 into dt. So from here actually we will be getting this is we can say from there we will be getting, everything is known, we will be getting this Y2. That Y2 will be equal to say, this is we can say Y2 will be equal to 1.456 units. So now we have this, we can say this at the end we have these 1.456 units.

So this Y2 that is will be equal to 1.456 units. So that is the question also. So how much amount of this solute actually will be there after the second stage?

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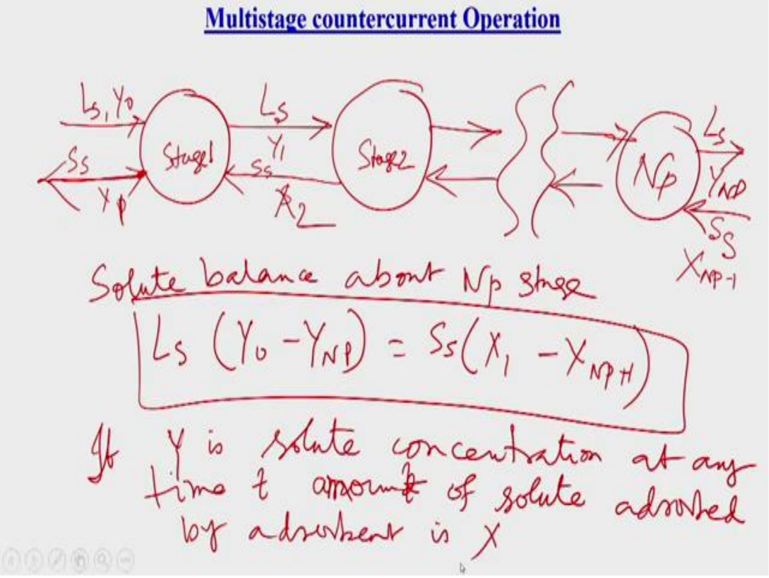

Multistage countercurrent Operation

Solute balance about Np stage

$$\boxed{L_s(Y_0 - Y_{NP}) = S_s(X_1 - X_{NP+1})}$$

If Y is solute concentration at any time t amount of solute adsorbed by adsorbent is X

Now we have multistage countercurrent operation. This is like the similar type of operation actually whatever we have calculated for other mass transfer operations also. This will be like this. Say this is stage one, and we are putting this LS with concentration Y0 and we are adding adsorbent with say S S with concentration let us take X0. And then from here this feed will be exiting with Y1 and it will be feeding to say stage 2. And from there whatever the adsorbent actually is coming from this stage 2 will be added to the stage 1.

So we can say S S with X2. So that way it will be continuing and if we say there are so many stages and we have this suppose stage Np then we can say from this stage we will be getting this LS and with this YNP. And in the beginning, whatever the adsorbent that is added, this is S S and with concentration XNP minus 1. So the solute balance about Np stage will be like this: LS into Y0 minus YNP will be equal to S S into X1 minus XNP plus 1.

So we can say this one the solute balance will be there for the Np stage. And if we say if Y is solute concentration at any time t, so amount of solute adsorbed by adsorbent is X.

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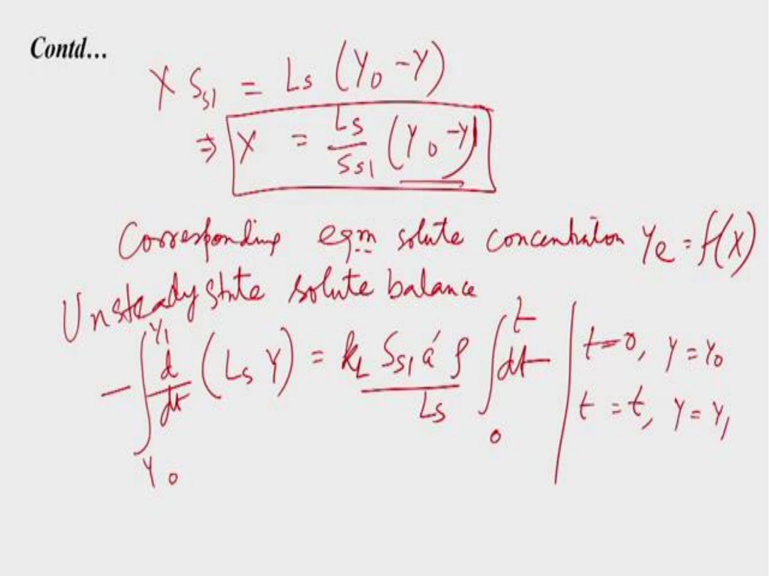

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$$X S_{S1} = L_s (Y_0 - Y)$$

$$\Rightarrow \boxed{X = \frac{L_s}{S_{S1}} (Y_0 - Y)}$$

Corresponding eq$\underline{m}$ solute concentration $Y_e = f(X)$

Unsteady state solute balance

$$-\int_{Y_0}^{Y_1} \frac{d}{dt}(L_s Y) = \frac{k_L S_{S1} a' \rho}{L_s} \int_0^t dt \quad \left| \begin{array}{l} t=0,\ Y=Y_0 \\ t=t,\ Y=Y_1 \end{array} \right.$$

So in that case we can say this X into SS1 will be equal to LS into Y0 minus Y1, Y0 minus Y. So that will be, from there actually we can say that X will be LS by SS1 into Y0 minus Y. So that is solute concentration will be varying with this Y0 and Y. And say corresponding equilibrium we can say equilibrium solute concentration Ye that will be the function of say X.

So we can do one unsteady state. Say solute balance like we can say this minus d dt of LS into Y that will be equal to kL into say SS1 into a prime into rho divided by LS into dt. So that we will be now integrating this one, suppose this Y0 to Y1, then it will be like 0 to t. Then from there actually we will be we have this suppose boundary condition when t is equal to 0, we can say Y will be equal to Y0 when t equal to t or whatever the desired time we have then Y will be equal to Y1.

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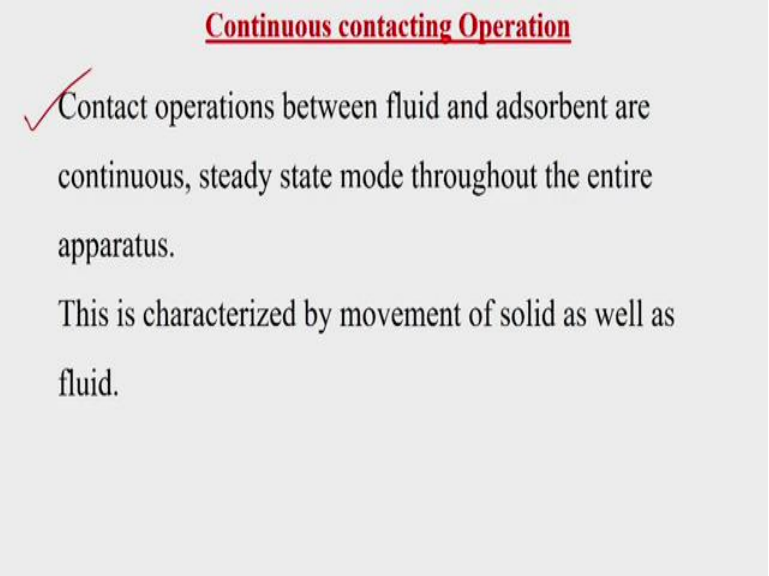

Continuous contacting Operation

✓Contact operations between fluid and adsorbent are continuous, steady state mode throughout the entire apparatus.

This is characterized by movement of solid as well as fluid.

So now we have this the next operation that is continuous contacting operation, that is contact operations between this fluid and adsorbent are continuous, then steady state mode throughout the entire apparatus. Then this is characterized by movement of the solid as well as this fluid. We will be discussing in detail about this continuous mode of operation in case of this we can say in the next class we will be discussing in detail about that in the continuous mode of operation. And there we will be discussing about the breakthrough point in detail.

We will be discussing in detail about this continuous mode of operation in the next class. And there we will be discussing about the breakthrough point in detail.

References

1. R. E. Treybal “Mass Transfer Operations”, McGraw –Hill International Edition, 3rd Ed., 1981.

2. W. L. McCabe, J. Smith and P. Harriot, "Unit Operations of Chemical Engineering", 6th Ed., McGraw – Hill International Edition, 2001.

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