

Mass Transfer Operations II
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Lecture No. 21
Various models and applications design aspects

So we were discussing on the gel polarization model, now we will be solving one problem on gel polarization model and how to calculate the gel layer thickness or pressure drop for the gel layer controlled separation process.

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Problem 2

An aqueous macromolecular solution is being ultrafiltered under cross-flow over an asymmetric membrane of average pore size 20 nm (tortuosity, $\tau = 1.7$), open area of 35% and a permselective skin thickness of 0.45 μm . A plate and frame module of 7 mm channel depth is used.

The following data are given: Reynolds number of the liquid, $Re = 6500$, bulk solute concentration, $C_b = 1.3\%$; solution viscosity, $\mu_l = 9.5 \times 10^{-3} \text{ Ns/m}^2$, density = 1030 kg/m^3 , solvent (water) viscosity, $\mu_s = 8.8 \times 10^{-4} \text{ Ns/m}^2$; diffusivity of solute, $D = 3.5 \times 10^{-7} \text{ cm}^2/\text{s}$, $\Delta P = 3 \text{ bar}$; temperature = 27°C , molecular weight = 5000, rejection, $R_r = 0.99$, $C_g = 12\%$, diameter of the gel particle, $d_p = 40 \text{ nm}$, average porosity of gel is 50%. Mass transfer coefficient can be calculated from correlation: $Sh = 0.025(Re)^{0.75}(Sc)^{1/3}$. Take channel depth as characteristic length in mass transfer correlation.

Calculate (a) gel layer thickness (b) pressure drop (ΔP) at which gel layer formation on the membrane starts.

The problem is like that an aqueous macromolecular solution is being ultra-filtered under cross-flow over an asymmetric membrane of average pore size of 20 nanometer and tortuosity value is 1.7 and open area of 35 percent and permselective skin thickness of 0.45 micron. So a plate and frame module of 7 millimetre channel depth is used, the following data are given like this for this solving this problem, Reynolds number of the liquid which is flowing through this channel is the Reynolds number is 6500

And bulk solution solute concentration is 1.3 percent, solid viscosity is a solution viscosity is 9.5×10^{-3} newton second per meter square, density is $1030 \text{ kg per meter cube}$ and solvent viscosity is 8.8×10^{-4} Newton second per meter square and solute diffusivity is 3.5×10^{-7} centimeter square per second and pressure drop is 3 bar, temperature is 27 degree Celsius and molecular weight is 5000 and real retention is 0.99 and gel layer concentration is 12 percent and diameter of the gel particle that is average diameter d_p is equal to 44 nanometer.

And average velocity of the gel is 0.5 and mass transfer coefficient can be calculated from this correlation, the Sherwood number is equal to 0.025 into Reynolds number to the power 0.75 and Smith number to the power one third and we need to take this channel depth as the characteristic length in mass transfer correlation, means in this zone. Now we need to calculate this gel layer thickness and pressure drop at which gel layer formation on the membrane starts. So we need to solve this problem.

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Solution

$$R_m = \frac{32 \tau l_m}{\epsilon d^2} = \frac{32 \times 1.7 \times 0.45 \times 10^{-6}}{0.35 \times (20 \times 10^{-9})^2} \Delta P = 3 \times 10^5 \text{ Pa}$$

$$R_m = 1.748 \times 10^{11} \text{ m}^{-1} \quad R_m = ? \quad R_g = ? \quad d_p = 40 \times 10^{-9} \text{ m} \quad L = ?$$

$$S_c = \frac{\mu}{\rho D} = \frac{9.5 \times 10^{-3}}{1030 \times 3.5 \times 10^{-11}} = 2.635 \times 10^5$$

$$Sh = 0.025 (6500)^{0.75} (2.635 \times 10^5)^{1/3} = 1150 = \frac{k_l l}{D}$$

$$\therefore k_l = \frac{1150 \times 3.5 \times 10^{-11}}{7 \times 10^{-3}} \text{ m/s} = 5.8 \times 10^{-6} \text{ m/s}$$

$T = 1.7$
 $l_m = 0.45 \times 10^{-6} \text{ m}$
 $\epsilon = 0.35$
 $d = 20 \times 10^{-9} \text{ m}$
 $Re = 6500$
 $S_c = \frac{\mu}{\rho D}$
 $Sh = ?$
 $k_l = 9.5 \times 10^{-3} \text{ N s/m}^2$
 $\rho = 1030 \text{ kg/m}^3$
 $D = 3.5 \times 10^{-11} \text{ m}^2/\text{s}$
 $R_r = 0.99$
 $C_g = 0.12$
 $C_b = 0.013$
 $C_p = ?$

So in the beginning will be this one mentioning which are already given in the problem, . So this tortuosity is given like say tau is given as 1.7 then thickness of the membrane is given as 0.45 into 10 to the power minus 6 meter then epsilon is given as 0.35 of this membrane, then d is diameter of the pore is given as 20 into 10 to the power minus 9 meter, then Reynolds number is given as 6500, then Smith number actually we need to calculate that is mu by rho into d.

These are supplied will be calculating from there. Sherwood number will be getting from this correlation is given like 0.025 into Reynolds number to the power 0.75 and Smith number to the power one-third. So that will be getting this one, we do not know now, then mu L is equal to 9.5 into 10 to the power minus 3 Newton second per meter square then density is 1030 kg per meter cube, then diffusivity value is given as 3.5 to 10 to the power minus 11 meter square per second, then real retention is given as 0.99, Cg is given as 12 percent and bulk concentration is given as 1.3 percent.

So this C_p we need to calculate from this R_r value will be calculating C_p and then ΔP that is given as say 3 bar that is equal to 3×10^5 pascal and Δp_i we need to calculate, we will be calculating this one, then R_m will be calculating we do not know, then R_g that also we will be calculating we do not know, these diameter of this particles which are deposited on the membrane surface that is given as 40×10^{-9} meter and we need to calculate then first problem is given as what will be the thickness this L or thickness of this gel layer. .

So first we will be calculating from these actually will be calculating this suppose we will be getting membrane resistance say the formula actually will be say R_m , that is we can see this membrane resistance the formula will be also again it we will be deriving this one 32τ into l_m by say ϵ into d^2 , so that will be like $32 \times 1.7 \times 0.45 \times 10^{-6}$ divided by $0.35 \times 20 \times 10^{-9}$ to the power 2.

So that is coming out as in the meter inverse it will be so that is coming out as 1.748×10^{11} meter inverse. So now we have this R_m actually we are getting as 1.748×10^{11} meter inverse, and then so we can calculate the Smith number that is equal to μ by ρ into d , so μ is given as 9.5×10^{-3} divided by density is 1030 into d is equal to 3.5×10^{-11} , so that is coming out as say 2.635×10^5 .

So now we have the Smith number we have and we can calculate this Sherwood number that is the correlation for Sherwood number in this high turbulence zone say at 6500 Reynolds number that is 0.025 into Reynolds number to the power say 66500 to the power 0.75 and smith number 2.635×10^5 to the power 5 to the power one-third.

So it is coming out as say 1150 this is Sherwood number is equal to say we know kl by D . So from here so we can say now therefore we can say k will be this mass transfer coefficient we can calculate from here is equal to say 1150 into D is equal to 3.5×10^{-11} by l , this length is given as the 7×10^{-3} , that is we can say meter per second. So it is coming out as say 5.8×10^{-6} meter per second. So now so we have now got this mass transfer coefficient also we have now.

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Solution

$$R_r = 1 - \frac{C_p}{C_g} = 1 - \frac{C_p}{0.12} = 0.99 \Rightarrow C_p = 1.2 \times 10^{-3} \text{ kg/m}^3$$

$$\therefore \Delta \pi = \frac{(C_g - C_p)RT}{M} = \frac{(0.12 - 1.2 \times 10^{-3}) \times 0.0821 \times 300 \times 1000}{5000}$$

$$\Delta \pi = 0.585 \text{ atm} = 0.593 \text{ bar} = 0.593 \times 10^5 \text{ Pa}$$

$$K \ln \frac{C_g - C_p}{C_b - C_p} = \frac{(\Delta P - \Delta \pi)}{K(R_m + R_g)}$$

$$5.8 \times 10^{-6} \ln \frac{0.12 - 1.2 \times 10^{-3}}{0.013 - 1.2 \times 10^{-3}} = \frac{(3 \times 10^5 - 0.593 \times 10^5)}{8.8 \times 10^{-4} (1.748 \times 10^{11} + R_g)}$$

$$R_g = 2.022 \times 10^{13} \text{ m}^{-1}$$

Now we will be calculating say we know this R_r is equal to 1 minus C_p by say here actually, give say C_g . So now because you see C_g and C_m both are same in this case we are assuming then that is equal to you can say 1 minus C_p by 0.12 is equal to 0.99 from here will be getting C_p that is equal to C_p is equal to 1.2 into 10 to the power minus 3 say kg per meter cube. So now we have this C_p also we have got.

Now we will be calculating osmotic pressure gradient. So once we know the C_p value this one then we will be getting $\Delta \pi$ that is is equal to we can say this gradient in the concentration that we can see C_g minus C_p by M into R into T . So that is coming out at C_g is equal to 0.12 minus C_p is equal to say 1.2 into 10 to the power minus 3 into R , R value will be say 0.0821, T will be say 27 degree celsius, so it is 300 kelvin divided by 5000 into 1000.

So it is coming out as say 0.585 atmosphere. So it is equal to into 1.023 so it will be like 0.593 bar is equal to say 0.593 into 10 to the power 5 Pascal. So we have now $\Delta \pi$ also. So we will be calculating now we will be putting this one in the gel polarization model that is we can say that we will be putting this say $\ln K \ln$ say C_g minus C_p by C_b minus C_p that is equal ΔP minus $\Delta \pi$ by μ into R_m plus R_g . So here most of the terms are known except these R_g .

So now we will be putting this K is equal to say whatever we got these 5.8 into 10 power minus 6 \ln C_g is equal to 0.12 minus 1.2 into 10 to the minus 3 divided by C_b is equal to 0.013 minus C_p that is same 1.2 into 10 to power minus 3 is equal to ΔP is equal to 3 into 10 to the power 5 minus 0.593 into 10 to the power 5 by μ is equal to 8.8 into 10 to the

power minus 4 into R_m we got this one as 1.748 into 10 to the power 11 plus R_g . So from this equation, we will be getting this R_g just by calculating this we will be getting R_g is equal to 2.0 to 2 into 10 to the power 13 meter inverse. So now we have got this R_g we have got this Δp , so that we have got.

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Solution

$$R_g(L) = \frac{180(1-\epsilon_g)^2 L}{\epsilon_g^3 d_p^2} \Rightarrow L = \frac{2.022 \times 10^{13} \times (0.5)^2 (40 \times 10^{-9})^2}{180(1-0.5)^2}$$

$$L = 8.98 \times 10^{-5} \text{ m}$$

(a) Gel layer thickness, $L = 8.98 \times 10^{-5} \text{ m}$ (Ans)

(b) Δp , $R_g = 0$, $\epsilon_g = 0.12$ [Gel layer starts]

$$k \ln \frac{C - C_g}{C - C_p} = \frac{(\Delta p - \Delta p_0)}{h(R_m + R_g)}$$

Now we will be getting this using this we can say kozeny karmann equation, so we will be we know this one from there from kozeny karmann equation we have this R_g the function of time that is we can say 180 into 1 minus epsilon g to the power 2 into L, real the thickness by epsilon g cube, that is for gel and d_p square the particle size of the gel that square.

So that will be equal to you can say, we can say this 180 from there, we have R_g we have and epsilon g we have then d_p we have from there we can say L is equal to whatever the R_g value is now we have got 2.0 to 2 into 10 to the power 13 and into epsilon g that is we can say 0.5 that is that is cube into d_p is 40 into 10 to the power minus 9 square, so L will be R_g epsilon g cube d_p square divided by 180 into 1 minus 0.5 square that is, so from there actually will be getting L is equal to say 8.98 into 10 to the power minus 5 meter.

So we can say the thickness of gel layer actually we can say 8.98 into 10 to power minus 5 meter. So so gel layer thickness L is equal to 8.98 into 10 to the power minus 5 meter, so this is one answer. Now the second question is given problem is given like this pressure drop at which gel layer formation on the membrane starts. So for that we can say means whenever when we need this minimum Δp where the gel layer starts so that time R_g will be 0 and C_g is given like 0.12, so that gel layer starts.

So this is the condition R_g will be 0 so we will be putting this one in this equation to get whatever the ΔP we will be getting because this $\Delta \pi$ will be the same for the same system, so we can say this using the same formula like this K into $\ln C_g$ minus again we will be putting this K into $\ln C_g$ minus C_p by C_v minus C_p will be equal to ΔP minus $\Delta \pi$ by μ into R_m plus R_g . So there actually we say R_g is equal to now it is 0 so we can say only we need to find what will be that in this is what will be the ΔP .

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Solution

$$5.8 \times 10^{-6} \ln \frac{0.12 - 1.2 \times 10^{-3}}{0.013 - 1.2 \times 10^{-3}} = \frac{\Delta P - 0.593 \times 10^5}{8.8 \times 10^{-4} (1.748 \times 10^{11} + 0)}$$

$$\Rightarrow \Delta P = 0.614 \text{ bar}$$

And so for that case K will be like $5.8 \times 10^{-6} \ln C_g$ that is 0.12 minus 1.2×10^{-3} divided by 0.013 minus 1.2×10^{-3} that is equal to ΔP we need to find and $\Delta \pi$ is already there 0.593×10^5 divided by μ is equal to 8.8×10^{-4} into μ into R_m plus R_g R_m is 1.748×10^{11} plus R_g is equal to 0.

So this is the condition from here actually we will be getting whatever ΔP so that is ΔP will be 0.614 bar, so this is the pressure, so transmembrane pressure this one at which gel layer formation on the membrane starts. So whenever we have this $(0.41) 0.614$ bar pressure that time we can say the gel layer starts forming.

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Resistance in series Model

According to this model, flux decline is due to the combined effects of (i) fouling of the membrane (reversible and/or irreversible pore blocking) and (ii) concentration polarization over the membrane surface.

The resistances against solvent flux are:

- membrane hydraulic resistance, (R_m)
- reversible resistance due to pore blocking, (R_{pb})
- gel-type deposited layer resistance (R_g)
- resistance due to osmotic pressure build-up ($\Delta\pi \uparrow \rightarrow (\Delta P - \Delta\pi) \downarrow$)

$J_0 = L_p \Delta P = \Delta P / \mu R_m$

where, J_0 = Pure water flux ($\pi = 0$)

L_p = membrane hydraulic permeability

ΔP = transmembrane pressure difference

So now we will be discussing about this resistance in series model so then the third model will be like this resistance in series model. So this is your so we can say for any membrane separation operation the as the process progresses then this suppose the solutes some solutes will be (stay) stucked inside the membrane pores and some layers will be formed on the membrane surface and that is why we can see the with time the throughput will go on decreasing and that time several resistances like say due to pore blocking or gel formation or we can say due to osmotic pressure build up the resistance values will go on increasing.

And membrane hydraulic resistance actually will remain constant because this is the intrinsic property of the membrane, but you see other this one resistance actually will go on increasing. That is why the throughput will go on decreasing with time so we can say according to this model that is resistance in series model this flux decline is due to the combined effect of fouling of the membrane that is you may be reversible or irreversible then concentration polarization over the membrane surface.

So these are resistances like membrane hydraulic resistance that will that is R_m that will remain constant for the entire membrane operation because this is the property of the membrane but reversible due to this pore blocking so R_{pb} so that it will go on (decrea) increasing with time and gel layer type formation so due to this R_g that will also will increase and this resistance due to osmotic pressure build up say we can say this $\Delta\pi$ will go on increasing that is why we can say the ΔP minus $\Delta\pi$ the defective value will go on decreasing or resistance value this we can say driving force will go on decreasing, .

So in that case actually we will be trying to cope correlate all the resistances here like this in the beginning. So whenever the pure water flux on time t is equal to 0 so the that fewer water flux is a multiple of membrane permeability and driving the what is called a pressure gradient where this we can see this for pure water $\Delta \pi$ is equal to 0, that is equal to say ΔP by μ into R_m , but with time we can see this L_p is the membrane permeability and ΔP is the transmembrane pressure differential pressure gradient.

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Resistance in series Model

$$J_w = \frac{1}{A} \frac{dV}{dt} = \frac{\Delta P}{\mu (R_m + R_g + R_p)}$$

$R_g \rightarrow$ Kozeny Karman

$$R_m = \frac{32 \tau l_m}{\Delta \cdot 3}$$

$R_g(t) = \frac{180 \epsilon_n (1-\epsilon)^2 L}{\epsilon^3 \Delta P^2}$

$\epsilon_n = \frac{\text{distance of pore length}}{\text{thickness of membrane}}$

$R_m = \frac{1}{\mu L_p}$

For 'dead end filtration' / batch cell

$$J_w = \frac{\Delta P}{\mu (R_m + R_g(t))} \quad \checkmark R_p \text{ may be neglected}$$

Resistance in series Model

$$\frac{1}{A} \frac{dV}{dt} = \frac{\Delta P}{\mu (R_m + R_g)}$$

$$= \frac{\Delta P}{\mu (R_m + \alpha \frac{C_b V}{A})}$$

$$\frac{1}{A} \frac{dV}{dt} = \frac{\Delta P}{\mu R_m (1 + \frac{\alpha C_b V}{A R_m})}$$

$$\frac{1}{A} \frac{dV}{dt} = \frac{J_0}{(1 + \frac{\alpha C_b V}{A R_m})}$$

Mass balance in the gel layer:

$$L A (1-\epsilon) \rho_g = C_b V$$

$$L = \frac{C_b V}{A (1-\epsilon) \rho_g}$$

$$R_g = \alpha (1-\epsilon) \rho_g L$$

$$= \alpha (1-\epsilon) \rho_g \frac{C_b V}{A (1-\epsilon) \rho_g}$$

$$R_g = \frac{\alpha C_b V}{A}$$

So with time so we will be getting the correlations like this now, we will be trying to correlate the all the resistance values and we will be trying to find out the other resistance values experimentally also and when we can, then tabulate the permeate flux or through put value or output of this process, with we can say the solute concentration and transmembrane

pressure gradient and osmotic pressure development. So like this so we know these as from this Darcy's law that this J_w is equal to so that is we can say $\frac{1}{A} \frac{dV}{dt}$, that flux that is nothing but $\frac{\Delta P}{\mu (R_m + R_g + R_{pore\ blocking})}$, so this R_p .

So this out of these we can say this R_g can be obtained from kozeny karmann equation; kozeny karmann equation we can get R_g value and then like this R_g , it is a function of time that is nothing but say $\frac{180}{\epsilon^3} \frac{L}{d_p^2}$ that is a thickness by ϵ^3 into d_p^2 square. So from there will be getting this R_g value, so that that we can get this R_m value that this one in the previous problem we this one we wrote this one that R_m value can be obtained by this equation $\frac{32 \tau L_m}{\mu}$ by μ into say L_p .

So that also we can say R_m also we can get this one, sorry this $\frac{32 \tau L_m}{\mu}$ by ϵ into ϵ into d^3 , so that where L_m actually we can say this one L_m is the thickness membrane thickness and τ is the tortuosity, this tortuosity actually we can say the tortuosity is the ratio between this distance, distance of pore divided by thickness, thickness of membrane, so distance of the pore so if we say this the paths of the membrane is not straight if it is it is not following a straight route then distance of the pore will be pore pore distance of this pore length divided by the thickness of the membrane.

So the pore length is not straight suppose if it is suppose this is the membrane surface and if it is like straight then this distance actually same as the thickness then we can say that that will be 1 but if this distance of this like this, it is not straight if it is a tortuous or it is not it is inclined like this way, then this distance divided by thickness, so this thickness, so that is why we can say in most of the cases in membrane applications the tortuosity values are always greater than 1 and it can go up to say 3.5 means if it is very highly zigzag route like this it may be like that also so then we can say the tortuosity value will be very high.

So so now we can get this R_m also we will be getting from this equation or sometimes we can get R_m from this we can say equation that is R_m is equal to equation $\frac{1}{\mu L_p}$, if we know this we can say viscosity of this solvent as well as the permeability of the membrane from there also we can get membrane resistance and then we can say then we will be getting like this for say for dead, dead end filtration or we can say batch cell.

So we can use this J_w is equal to $\frac{\Delta P}{\mu (R_m + R_g)}$, there is a function of time so there we can say R_p may be neglected, so R_p maybe neglected here and we can say this J_w is equal to $\frac{1}{A} \frac{dV}{dt}$ that will be equal to say $\frac{\Delta P}{\mu (R_m + R_g)}$ that

we will be getting from this we can say solute balance like this will be doing this supposed this for R_g we can get also from the solute balance we can do this one will do the mass balance, mass balance this one in the gel layer so we will be doing like this suppose length of this thickness of this gel area of the gel gel energy gel area into $1 - \epsilon_g$ means say that how much amount of this porosity is there in the gel into ρ_g that will be nothing but C_b into V .

So that is we can say this how much amount of the solid concentration is there in the bulk into V . So from there we can say this L is equal to we will be getting like C_b into V divided by A into $1 - \epsilon_g$ into ρ_g , and we know this is R_g is equal to we can say α into $1 - \epsilon_g$ into ρ_g into L that is equal to say α into $1 - \epsilon_g$ into ρ_g , L is equal to C_b into V divided by A into $1 - \epsilon_g$ into ρ_g .

So that that will be like this $1 - \epsilon_g$ is ϵ_g then ρ_g ρ_g will be cancelled out and α into C_b into V by A , so that is R_g . So now we can write here say 1 by A is equal to say ΔP divided by μ into R_m plus R_g is equal to we can say α into C_b into V by A . So from there 1 by A say we can say if we take out this that is equal to say ΔP by μ into R_m if we take out then 1 plus α into C_b into V by A into R_m we will be getting that is 1 by A dv this dt . So from here we can say this is equal to say 1 by A ΔP by μ into R_m that is nothing but we can say J_0 or in the beginning that is 1 , then 1 plus α into C_b into V by A into R_m . So now we are getting this 1 by A dv/dt is equal to J_0 , so this is initial flux value J_0 plus 1 plus α into C_b into V by A into R_m .

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Resistance in series Model

Initial condition, $t=0, V=0$

$$\int_0^V \left(1 + \frac{\alpha C_b V}{A R_m}\right) dV = A \int_0^t \left(1 + \frac{\alpha C_b V^2}{2 A R_m}\right) dt$$

$$\textcircled{1} + \frac{\alpha C_b V}{2 A R_m} = \frac{A J_0 t}{V}$$

$S = \frac{\alpha C_b}{2 A R_m}$

So now we will be taking this initial condition like when t is equal to 0 we can say V will be equal to 0 then we will be doing this integration for that of this manipulating this equation like we can say $1 + \alpha C_b$ into V by A into R_m into dv is equal to A into integration dt into $V + \alpha C_b$ into V^2 by 2 into A into R_m . So that is we can say whenever we can say V is equal to 0 that is at time t is equal to 0 and when time t is equal to t that time it will be V , . So from here we can say $1 + \alpha C_b$ into V by $2 A R_m$ is equal to A into say J_0 into t by V .

So now we have this equation like this Y is equal to MX plus C , we will be putting say t by V versus say V , so we will be getting this straight line equation say Y is equal to say MX plus C . So these that is why we are putting this t by V versus V , then we will be getting this and then slope actually will be getting this one is nothing but slope will be αC_b by $2 A$ into R_m , .

So from here actually we will be getting this slope we will be getting from there we can get to say R_m is known and C_b is known area is known say α we will be getting so using these resistances in series model, we will be able to calculate all the resistance values and then we will add all these resistance values from there we will be able to calculate the permeate flux values.

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Problem 3

A feed containing 27.216 kg protein (mol. Wt. =1510) in a 7560 liter solution is to be concentrated to 1512 liter by batch ultrafiltration at 25°C using a ceramic membrane module. The initial flux is $2.26 \times 10^{-5} \text{ m}^3/\text{m}^2\text{s}$ at a pressure differential of $304 \times 10^3 \text{ Pa}$ across the membrane.

Calculate the membrane area to be provided if the job is to be done in 3 h. Any reduction in driving force due to osmotic pressure is to be taken into account. Complete retention of solute can be assumed. Concentration polarization and membrane fouling may be ignored.

So we will be now solving one problem the problem is that a feed containing 27.216 kg protein that molecular weight is 1510 in a 7560 litre solution is to be concentrated to 1512 litre by batch ultra-filtration at 25 degree Celsius using a ceramic membrane module. The

initial flux value is 2.26×10^{-5} meter cube per meter square second at a pressure differential of 304 this bar 304×10^3 bar, 304×10^3 bar, 304×10^3 to the power 3 Pascal.

Now we need to calculate the membrane area to be provided if the job is to be done in 3 hours. The time is 3 hours any reduction in the driving force due to osmotic pressure is to be taken into consideration. So we cannot neglect that osmotic pressure, the gradient or we cannot ignore that how much osmotic pressure is developed during this separation process in the duration of 3 hours. Complete retention of the solute can be assumed so R_r will be 1 so we can C_p will be 0, concentration polarization and membrane fouling may be ignored, so in that case we can say maybe concentration polarization is not taking place or membrane fouling is not significant.

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Solution

Let V be the volume of solution in the cell at any time t and solution conc. is $\frac{n}{V}$ where n is mass

$$\Delta \pi = \frac{RTn}{MV}$$

$$J_w = -\frac{1}{A} \frac{dV}{dt} = L_p \left(\Delta P - \frac{RTn}{MV} \right) = L_p \Delta P \left(1 - \frac{RTn}{MV \Delta P} \right)$$

$$-\int_{V_0}^V \frac{V dV}{V - \frac{RTn}{M \Delta P}} = L_p \Delta P \cdot A \int_0^t dt \quad \left| \begin{array}{l} t=0, V=V_0 \\ t=t, V=V \end{array} \right.$$

So now we will be solving this problem, so we will start with this we can say the solving this equation by deriving the equation for we can say the how much volume actually is varying with time that we can do this one. So let V be the volume of solution in the cell at any time t and solution concentration will be concentration is n by V where n is mass. So that is the concentration is n by V . Now we have this osmotic pressure that we cannot ignore that is we are keeping this one as RT , this n by M into V , and we know this J_w that is equal to say minus 1 by A dV/dt or we can take 1 by A dV/dt also. For that case we need to take the say boundary conditions accordingly, that is equal to L_p into ΔP minus $\Delta \pi$ that is RT n by M into V .

So now we can take this one we can take out this delta pi from here delta P from here. So L_p into delta p is $1 - \frac{RTn}{MV}$ into delta P. So now we will be doing the manipulation here so we can say we will be doing this V into dV just by manipulating this equation we can say from here from these two expressions we can say we will be manipulating this one by V minus $\frac{RTn}{M}$ into delta P just taking out V from here, the V is the variable here so we are putting this one is equal to $\frac{dV}{V}$, so $A dt$ we can say the L_p then $\frac{dP}{dt}$ into A into dt . So from there actually we are getting this and now we are putting the boundary conditions like this when time t is equal to 0 so V will be V_0 and so that is why we say time is equal to 0 then V will be V_0 when time will be t when time will be t so then V_0 will be V will be V final volume. So this t will be like this then final volume will be V .

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Solution

$$(V_0 - V) + \frac{nRT}{M \Delta P} \ln \frac{V_0 - \frac{RTn}{M \Delta P}}{V - \frac{RTn}{M \Delta P}} = L_p \Delta P \cdot A \cdot t$$

$$2.26 \times 10^{-5} = L_p \Delta P \left(1 - \frac{27216 \times 0.0821 \times 298}{1510 \times 7560 \times 3} \right)$$

$$L_p \Delta P = \frac{2.26 \times 10^{-5}}{0.98} = 2.305 \times 10^{-5} \text{ m}^3/\text{m}^2\text{s}$$

$$(7560 - 1512) + \frac{27216 \times 0.0821 \times 298}{1510 \times 3} \ln \frac{7560 - \frac{27216 \times 0.0821 \times 298}{1510 \times 3}}{1512 - \frac{27216 \times 0.0821 \times 298}{1510 \times 3}}$$

$$A = 25.294 \text{ mL}$$

Now from here we will be getting this just by putting the boundary conditions we will be getting V_0 minus V plus $\frac{nRT}{M}$ into delta P into $\ln \frac{V_0 - \frac{RTn}{M}}{V - \frac{RTn}{M}}$ into delta P that is coming out as this L_p then $\frac{dP}{dt}$ into A into t . So from there actually we know all this we can say all these values except this L_p and $\frac{dP}{dt}$ minus value we do not have so, we can do one thing, we will be putting now 2.26 into 10 to the power minus 5 is equal to L_p into $\frac{dP}{dt}$ into $1 - \frac{27216}{1510} \times 0.0821$ and into 298 , divided by 1510 into 7560 into 3 .

So from there L_p into $\frac{dP}{dt}$ that will be obtained as 2.26 into 10 to the power minus 5 by 0.98 that is coming out at meter cube per meter square second so 2.305 into 10 to the power minus 5 meter cube per meter square second, so we can say now if we put this L_p into $\frac{dP}{dt}$ in this above equation so we will be getting like this we will be putting this in this above equation

$V_0 - V$ so we will be putting this one $7560 - 1512 + 27216$ into 0.0821 into 298 divided by 1510 into $3 \ln 77560 - 27216$ into 0.0821 into 298 into 298 divided by 1510 , 1510 into 3 divided by 1512 minus the same thing 27216 into 0.0821 into 298 divided by 1510 that same value this this part 151 into 3 .

So from here actually we will be getting this here you see this area is equal to L_p into ΔP is obtained as 2.305 into 10 to the power minus 5 into area into time is equal to say 3 hours into 3600 second. So here you see this one area we will be getting as say A will be equal to say here A is equal to 25.294 meter square, so this area actually is required for this separation. So using this we can say resistance in series model so we will be able to calculate all the resistances offered during the membrane separation processes.

So thank you.

In the next class we will be discussing about this electric field enhanced membrane separation processes.