Mass Transfer Operations II
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Lecture 20
Various models and applications: design aspect

Now, we will be discussing on various models and application, there actually will be discussing the design aspect.

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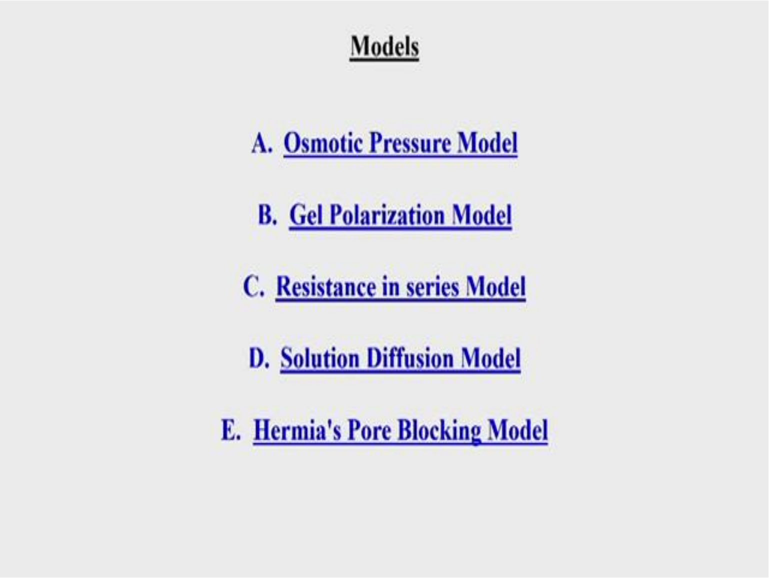

In membrane operations we generally discuss the few models like Osmotic Pressure Model then Gel Polarization Model then Resistance in series Model, Solution diffusion Model, Hermia's Pore Blocking Model there are so many other models also. So we will be discussing a few of these, the main objective of this models is that the effect of concentration polarization can be quantified using these models and flux decline profile then concentration of the solute in the permeate can be predicted or we can say this one if we want to design a membrane module or membrane reactor then we need to do the theoretical studies.

First we will be discussing on Osmotic Pressure Model. The definition is saying that we will be considering this resistance due to for the separation in terms of Osmotic Pressure because we know that for the as per the darcy's law the driving force is del P minus del pie there with time this del pie goes on increasing, where the solute concentration is more than Osmotic pressure is very less but were the solute concentrations are less their contribution of the Osmotic pressure is there. So, we will be discussing now this Osmotic Pressure Model.

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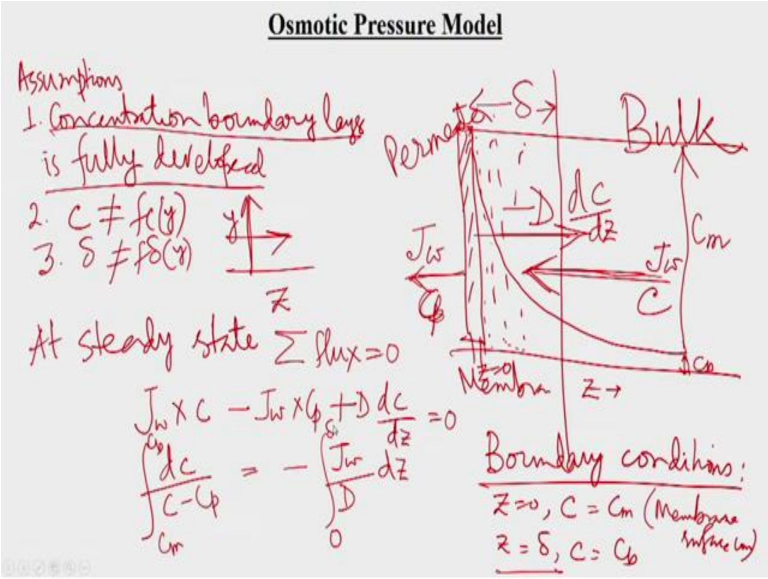

So first we need to do the assumption like this as say assumptions. One is that we can say this concentration boundary layer is fully developed and C actually is not the function of C in y axis we will be discussing what is that in y axis and delta that is also not the function of we can say delta in the y direction mean if we say this is we can say this in Z direction suppose flow fluid is flowing and y is this we can say this in the radial direction.

Let us take this membrane like this material so then say flow of the permeate will be like a in the beginning, the solute as well as solvent will be moving towards this membrane and then we can say Jw actually will be getting that whatever the permeate flux and concentration of the solute will be Cp and say that is which will defuse back to this bulk suppose this part is bulk and this part is permeate side and this is membrane.

So, which will go back to this bulk that is we can say this one D dc dz so that amount actually will be going back to the bulk again and say we can say this one say we assume that the concentration boundary layer is fully developed so we can say the concentration profile will be like this and say if we say Z is in this direction, so the bulk concentration will be like this which we can say this one concentration of the solute in the bulk in the beginning.

So, this is we can say bulk and whenever this entirely developed system, so then we can say the total concentration will be that is we can say this one Cm that is membrane surface concentration, this Cm will be like this membrane surface concentration so we can see the solute will be deposited like here and then say this way whatever this we can say this one the thickness of this solute will be developed like this we can say that is delta.

So, now we can say this one field is directing towards this membrane surface and then permeate flux is obtained and solute concentration in the permeate is Cp and we can say this one back diffusion of the solute to the bulk, so at the steady state we can say that is minus dc dz. So, now we can say this one at steady state we can say whatever the flux values are entering and which is exiting so we can say from there we can say this summation of the flux value will be 0.

Then say what is entering suppose this flux is there with concentration at any time we can say this one ec, so Jw into C and what is exiting that is we can say this one Jw into Cp and which is going back minus dc dz so minus minus dc dz, D dc dz so that will be equal to 0. So this is at the steady state we can say this one this is say entering and these 2 are exiting like this.

So from there Jw into C minus Jw into Cp plus D into dc dz is equal to 0. So, from here we can say dc by C minus Cp is equal to minus Jw by D into dz, so that is we can say just by we can say this one manipulation, now we say if we say if we take this boundary conditions if we say for this type of same membrane operations, suppose Z is equal to 0, so here actually Z is equal to 0 that time we can say C will be Cm means this is the membrane surface concentration, membrane surface concentration.

And when this is fully developed say we can say this one Z is equal to delta there C will be this bulk concentration means after this delta whatever the concentration is there means delta thickness is developed due to this membrane operation and that is why in the membrane surface the solute concentration is (())(08:32) that we say membrane surface concentration and at the bulk when there is no effect of this we can say the thickness delta there C will be Cb. Now, we will be doing these one we will be integrating this one from Cm to Cb and if we integrating this one m Cm means Z will be 0 to delta.

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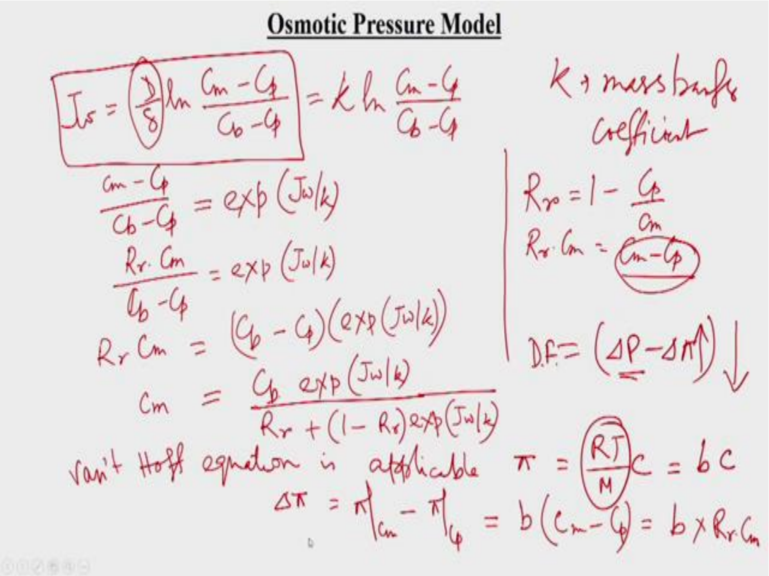

So, from here will be getting say if after integration we will be getting these one Jw will be equal to D by delta into ln Cm minus Cp divided by Cb minus Cp. So, that actually we are getting just by integrating the previous equation from here we will be getting this if we integrate this equation we will be getting this Jw or we can say ln Cm minus Cp by Cb minus Cp that will be Jw by D into delta from there just after manipulation we will be getting Jw is equal to D by delta.

And this D by delta we know this is mass transfer coefficient, so k ln Cm minus Cp by Cb minus Cp, so this is we can say k is the mass transfer coefficient. Now, we will be doing just manipulation in this equation like this Cm minus Cp by Cb minus Cp that will be equal to say Jw by k the say it will be like exponential of Jw by k just keeping this one and taking the antilogarithm.

So, now we know another thing just we can say we will take Rr that is real retention is equal to 1 minus Cp by Cm, so from there we can say this one Rr into Cm is equal to Cm minus Cp, so here we can put 1 thing, so we can put this Cm minus Cp H Rr into Cm, so they are actually we can say put here that is Rr into Cm by Cb minus Cp is equal to exponential Jw by k.

So, Rr into Cm is equal to say Cb this is Rr into Cm will be getting as like Cb minus Cp into exponential Jw by k and then if we manipulate this, this equation then we will be getting Cm is equal to Cm is equal to we can say Cb into exponential Jw by k divided by Rr plus 1 minus

Rr into exponential Jw by k, so just by manipulating these 1 we will be getting this Cm, Cm is equal to or we explained this one in the beginning that Cm actually membrane surface concentration cannot be measure directly, so there we can measure indirectly.

So from this equation we can say the Cm or we can say membrane surface concentration can be measure using this known parameters. And here we also explain that say if the solute molecular rate of the solute is very small then Osmotic Pressure has the contribution and we know this as per this van't Hoff equation this with the progress of this membrane separation process the Osmotic pressure going on is going on increasing.

So, here we can say this one this van't Hoff equation is applicable here, so there pie is equal to say RT by M into C that we can take as this term b into C and so then delta pie will be means with time what will be happen that will be pie that is Cm minus pie will be at Cp. So, that difference will give the osmotic pressure gradient, so with time what will happen the resultant driving force will go on decreasing as this driving force is delta P minus delta pie as this this delta pie is going on increasing so this driving force we can say this one driving force will go on decreasing.

Because you see delta P whatever the transmembrane pressure drop is the supplied that is that will remain constant for the entire process. So, the effective driving force will go on decreasing. So that is nothing but we can say this one b into Cm minus Cp. So, that is we can say this one again we can get this one Cm minus Cp we can take is equal to b into Rr into Cm, so we have this delta pie is equal to this one.

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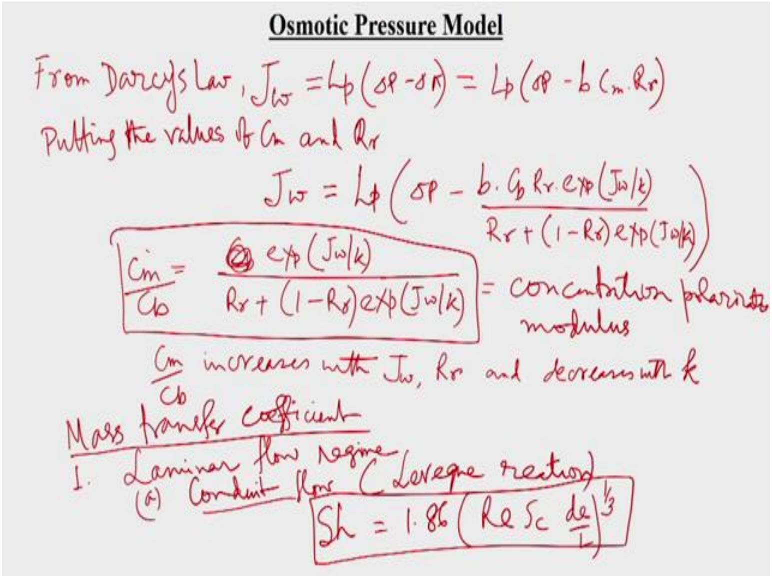

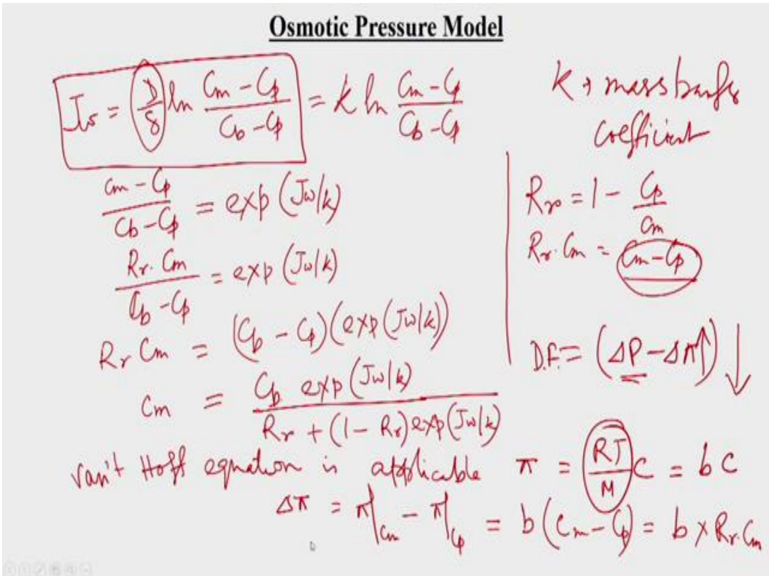

Now, we can say this we know from Darcy's law we know that Jw this is we can say the water flux is equal to these Lp into del P minus del pie, so that will be nothing but Lp into del P minus b into Cm into Rr. So, now we can say this one will putting these values of Cm Rr in this equation suppose putting the values of Cm and Rr. So we can get this Jw this water flux or permeate flux value will be Lp into del P minus b into Cm into R that is we can say b into Cm whatever we have obtained from this pervious equation Cm is equal to Cb into exponential Jw by k by Rr plus 1 minus Rr into exponential Jw by k will be putting here b into Cb into Rr into exponential Jw by k divided by Rr plus 1 minus Rr into exponential Jw by k.

So, from here actually we can say and Jw is equal to then we have Lp into del P minus b into Cb into Rr into exponential Jw by k by Rr plus 1 minus Rr into exponential J by k, where b is taken as this RT by M so that is the we can say this one expression for Jw and we can say now we can do one thing that Cm is equal to from these we can say just by manipulation we can get again Cm is equal to Cb into exponential we can say Jw by k divided Rr plus 1 minus Rr into exponential Jw by k and from there this Cm by Cb will be this one, that is we can this one exponential Jw by k by Rr plus 1 minus Rr into exponential k.

These Cm by Cb that is we can say this ratio is called as that concentration polarization modulus, concentration polarization modulus. So, Cm by Cb that we get that can give 1 measure of the Osmotically control system like if we say that C if Cm by Cb is very high then we can say that membranes surface concentration of the solute is much more higher compare to this lower one or there we can say this one flux value will go on decreasing or we can say this one Rr value will go on decreasing and we can say and with k value it will go on increasing.

So, we can say Cm value Cm by Cb, so that will we can say this one that increases with Jw this Rr and decreases with we can say this one k value. Now in case we need understand this Mass Transfer coefficient values for this case actually this whatever the k value actually we need to get this is very simple we can say this Mass Transfer consideration we need to this one explain mass transfer coefficient consideration.

So, suppose if we say if it is in laminar flow region then so we can say this one if it is conduit flow. So, there we can say this one Leveque relation will be we need to follow like we know this one which is Sherwood number is equal to 1.86 into Reynolds smith and de by L to the power 1 third, where d is the equivalent diameter of the channel. So, from there we can say this one using we need to follow this correlation means were laminar flow region say Reynolds number is less than 2100 is in the conduit flow.

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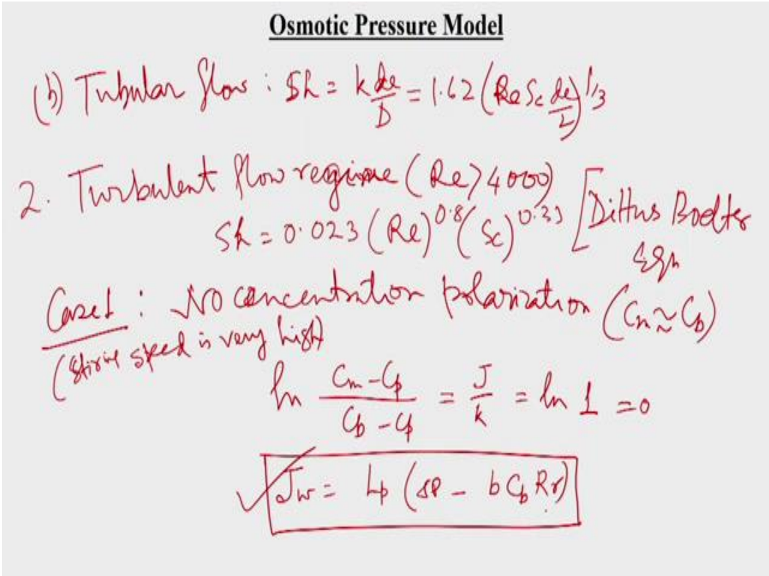

So, if it is in the say in laminar flow if it is in the we can say Tubular flow condition, so Reynolds number will be, the Sherwood number will be equal to k de by D that is equal to 1.62 into Reynolds smith and de by L to the power 1 third. So with it is we can say this Turbulent flow regime if it is Turbulent flow regime then suppose Reynolds number is greater than 4000.

So, then we have this Sherwood number is equal to 0.023 and Reynolds number to the power 0.8 and smith number to the power 0.33, so that is like Dittus boelter type equation. So, now actually we will be considering some simple cases like we will take this first step like we will be discussing now case 1 were we can say this one no concentration polarization.

Suppose there is a condition where we can say case 1 no concentration polarization. So, there we can say this no concentration so Cm will be almost equal to Cb means say the solutes are not deposited on the membrane surface and in the entire membrane separation process the solute concentration remains almost same at the membrane surface.

It can happen we the starting speed is very high, so this is we can say this one starting speed is high. So, in that case we have this ln Cm minus Cp by Cb minus Cp will be equal to Jw by k that will be equal to say if we put Cm is equal to Cb then it will be like 1, so that time we can say this one 0. So, in that case we can say this one this Jw then that case will be Jw is equal to Lp into del P minus b into Cb into this one Rr, so in that case whenever this there is no concentration polarization that time Jw will be Lp into del P minus Cb into Rr.

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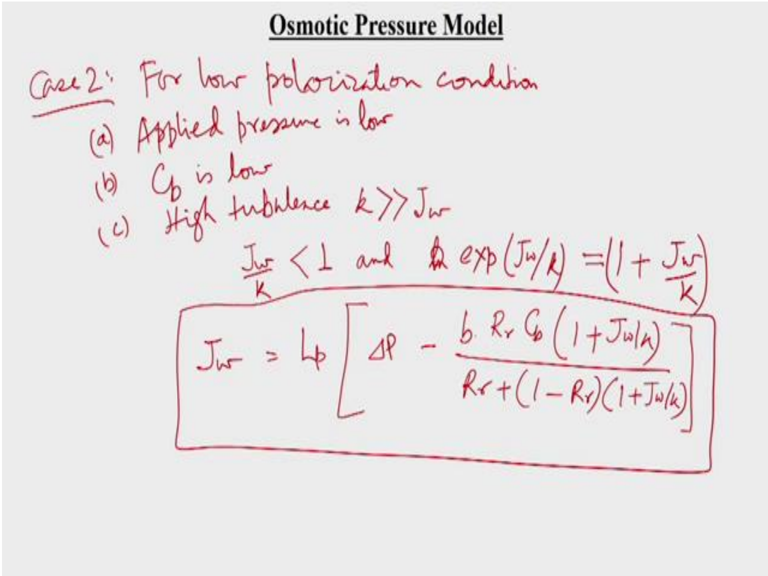

In another case suppose if we say case 2 where we can say this low polarization modulus with the low polarization condition suppose for low polarization condition suppose applied pressure is low then we can say Cb is low or we can say this one bulk concentration or feed concentration is very low and say high turbulence.

So, in that case if turbulence is very high means this k will be this one very much greater than Jw or we can say this one mass transfer coefficient value will be very high, so for that case this we can say in that case the ratio Jw by k will be lower than 1 and in that case say ln, exponential Jw by k that is we can say this one can be expanded as 1 plus Jw by k simply we can write this one.

Now, we will be putting this one in this expression for Jw. So, in that case Jw is equal to Lp into del P minus what about the expression for del pie that was there in terms of b into Rr into Cb into exponential Jw by k will be converting now into 1 plus Jw by k divided by say Rr plus 1 minus Rr into exponential Jw by k will be replaced by 1 plus Jw by k. So, now we can say this one, so these will be the expression for these flux permeate flux value.

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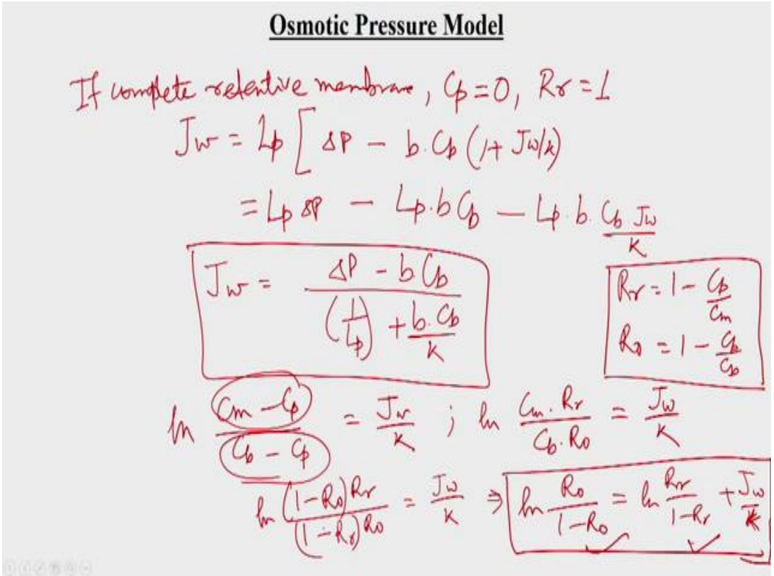

Now, if complete retentive membrane is there suppose, if this complete retentive membrane means we can say this one, so that time whatever the feed is there no solute will be passing through the membrane the entire solute is retented, then Cp will be equal to say 0 and say this one Rr will be 1 minus Cp by Cm that will be 1 minus 0 means Rr will be 1, so in that case this Jw will be Jw will be say will take Lp into del P minus b into Cb into 1 plus Jw by k.

So, in that case will be that will be say we can say Lp into del P minus Lp into b into Cb into just by just we need to do the manipulation b into Cb minus Lp into b into Cb into Jw by k, so from there we can say this Jw will be equal to just after manipulation will be del P minus b into Cb divided by 1 by Lp plus b into Cb by k.

So, this is the expression for permeate flux if we can say Rr is 1 but this is not realistic, so realistic approach is not that Rr should be 1 it should be sometimes less then 1 or we can say the 0.999 like this but Cp in most of the cases Cp will be near to 0 but be not be 0 and in this case we can say we can write these way ln Cm minus Cp by Cb minus Cp is equal to Jw by k that we got from this we can say this one from the very basic we can say this one flux analysis.

So, then we will be just putting these Cm minus Cp value there in terms of Rr, so it will be like this ln Cm into Rr divided by Cb into we can say R 0 that will be coming as Jw by k because we know that Rr is equal to 1 minus Cp by Cm whereas (())(31:19) is equal to 1 minus Cp by Cb from these 2 equations actually we will be getting that ln Cm into Rr by Cb into Rr just by getting these 2 values actually from this equation we will be getting like this or in terms of 1 minus R 0 or 1 minus Rr because Cp by Cm and Cp by Cb.

So, here also we can say from here we will be getting in terms of say Cp by Cm is equal to 1 minus Rr and Cb by Cp by Cb is equal to 1 minus Ro, so from here we can do the manipulation like this ln into 1 minus Ro into Rr divided by 1 minus Rr into Ro is equal to Jw by k. So, from here actually we can say ln Ro by 1 minus Ro is equal to ln Rr by 1 minus Rr plus Jw by k. So, now we have this linear expression for (())(32:42) real retention flux value and Mass transfer coefficient.

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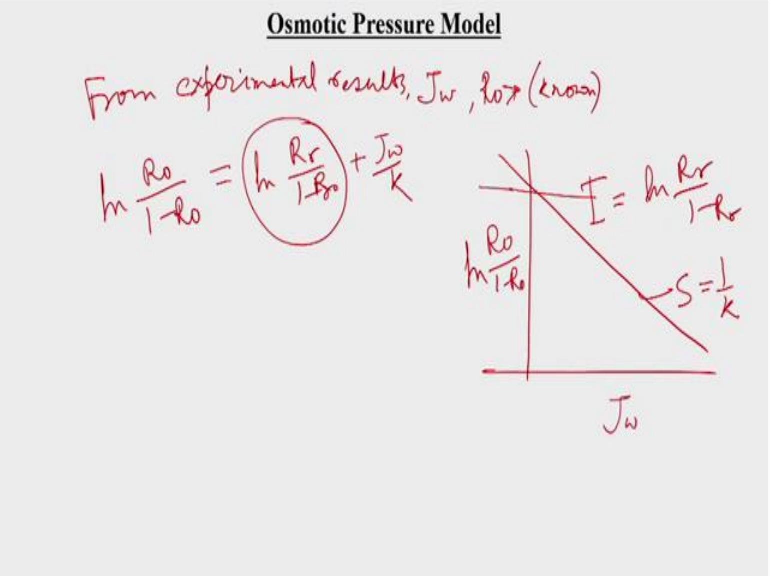

So, now we can do one thing say this from experimental results, suppose this from results we can get this Jw and Ro, so we can get say we can get this one these will be known. Now, we will be plotting this we can say this one ln Ro by 1 minus Ro with Jw. So, we will be getting these, suppose the expression is ln Ro by 1 minus Ro that is y is equal to ln Rr by 1 minus Rr plus Jw by k.

So, as we will be plotting this one, so we can say this one intercept actual will be getting this is nothing but this is intercept will be see that is we can say from here we will be getting I is equal to ln Rr by 1 minus Rr and this slope, slope will be 1 by k. So, we will be getting this intercept and slope and from there we will be getting what will be the real retention value an what will be the Mass transfer coefficient the inverse of this slope will get this will be given it is Mass Transfer coefficient values.

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Prob. 1:

An enzyme is being concentrated in an ultrafiltration module with feed in cross flow. Under the given flow condition, mass transfer co-efficient at membrane surface is estimated to be 3×10^{-5} m/sec. Bulk concentration is 0.3 mass%. If the water flux is 0.41 m^3/m^2h, calculate polarization modulus and concentration of enzyme on membrane surface. The membrane has a distribution of pore and 95% rejection of solute is achieved. If diffusivity of enzyme is 8×10^{-7} cm^2/sec, calculate the thickness of mass transfer film.

Now, we will be solving a simple problem like this, an enzyme is being concentrated in an ultrafiltration module with feed in cross flow sell. Under the given flow condition, mass transfer co-efficient at membrane surface is estimated to be 3 into 10 to the power minus 5 meter per second. Bulk concentration is 0.3 mass percent. If the water flux is 0.41 meter cube per meter square hour, calculate polarization modulus that is Cm by Cb and the concentration of the enzyme on the membrane surface that is Cm .

The membrane has a distribution of pore as distribution of pore and 95 percent rejection that is we can say this one retention is equal to 0.95. If the diffusivity of enzyme is 8 into 10 to the power minus 7 centimeter square per second, calculate the thickness of the mass transfer film. This we can say delta. So, we need to get Cm by Cb then Cm then rejection is given then we need to get delta. So, this values we need to get from the this one we need to solve this problem.

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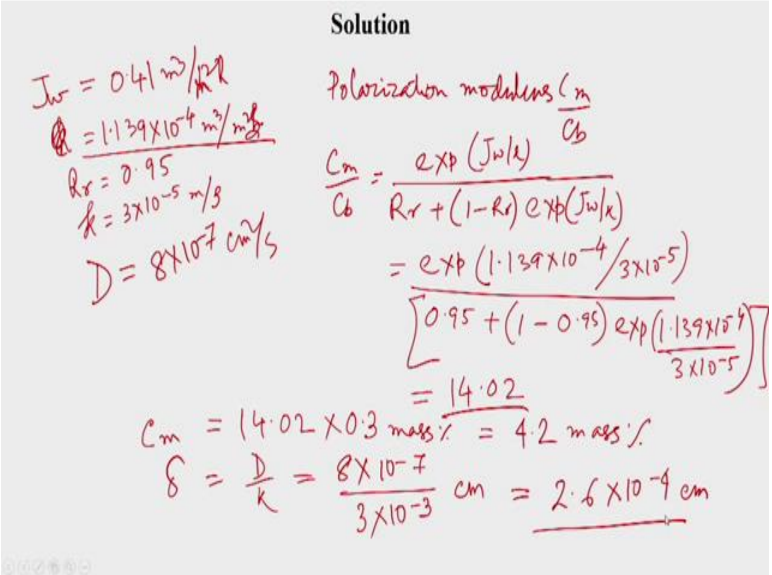

Now, we will be solving this one very simply we can solve this one whatever the given this one values are there we will be writing now in the beginning say suppose Jw that is given as 0.41-meter cube per meter square per meter square hour, then h is given as 1.139 into 10 to the power minus 4-meter cube per meter square hour, then Rr is given as 0.95 and mass transfer coefficient is given as 3 into 10 to the power minus 5 meter per this second.

Now, we need to find first say polarization modulus that is Cm by Cb, so Cm by Cb is equal to exponential Jw by k by Rr plus 1 minus Rr into exponential Jw by k. So, that is given as exponential Jw is equal to 0.41. So Jw is equal to actually this is 1.139 into 10 to the power minus 4 by k is equal to 3 into 10 to the power minus 5 divided by Rr is equal to 0.95 plus 1 minus 0.95 into exponential 1.139 into 10 to the power minus 4 divided by 3 into 10 to the power minus 5.

So, it is coming out as say the value is 14.02. So, we can say the Cm by Cb is equal to 14.02 or we can say this one the solute concentration increase in this membrane unit is done by 14 times. Now, we need to get the Cm is equal to say we can say this Cm is equal to say 14.02 into that Cb is equal to 0.3 mass percent we can say 0.3 mass percent, so it is coming out as 4.2 mass percent.

Now, we say delta, delta is equal to D by k. So, that is we can say this D value actually is given as 8 into 10 to the power minus 7, D is given as 8 into 10 to the power minus 7 centimeter square per second, so that is we can say 8 into 10 to the power minus 7 by 3 into

10 to the power minus 5, 10 to the power minus 3 in term of meter we can say 10 to the power minus 3 it will be like this in centimetre.

So, it is coming out as 2.6 into 10 to the power minus 4 centimeter. So, thickness is 2.6 into 10 to the power minus 4 centimeter this one thickness will be generated. Next, we will be discussing about the Gel polarization model.

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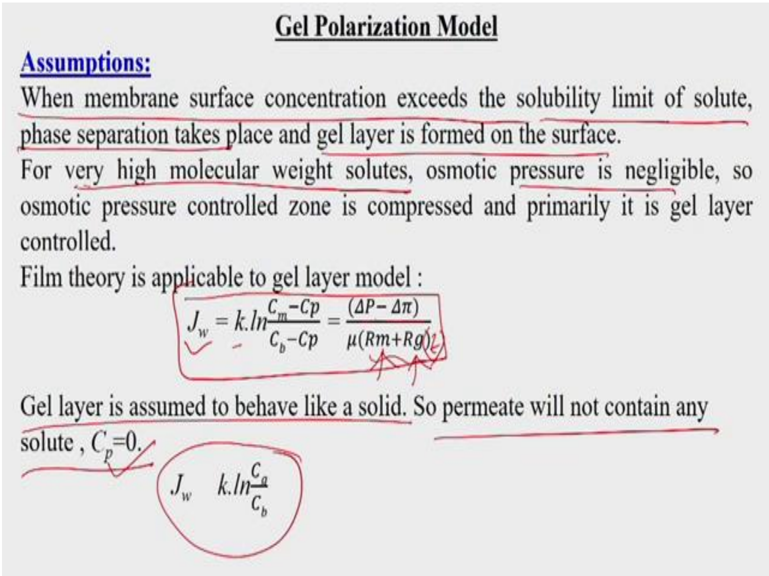

So, in Gel polarization model there are assumptions also, when this membrane surface concentration exceeds the solubility limit of the solute, then phase separation takes place and gel layer is actually is formed on the membrane surface. So, for very high molecular weight solute, osmotic pressure is negligible because we see we know that the as per this van't Hoff this equation these molecular weight is there at the denominator so we can say the osmotic pressure value will be very less.

So, osmotic pressure controlled zone is compressed and we can say this primarily it is gel layer controlled. So, there are we can say this one we can you this film theory like this say Jw will be equal to k into ln into Cm minus Cp by Cb minus Cp that is nothing but delta P minus delta pie that is driving force by mu into Rm plus Rg were this Rm actually we can say this one membrane surface concentration and Rg is equal to the gel layer resistance.

Because, you see in in the case of this high molecular rate compounds there it is tendency of gel formation over the membrane surface and that is why the resistance value is going on increases actually with time that Rg that is the function of we can this one time, so gel layer is assumed to behave like a solid so we can say this one that is why it is given additional resistance like membrane resistance that is the intrinsic property of the membrane, so it gives the resistance for the transport of the solvent through this and when this gel layer will be formed over this membrane surface that is we can say this one it is like dynamically formed membrane and it gives the additional resistance.

So, permeate will not contain any solutes so we can assume that it is giving a new we can say the layer and the solutes will be trapped on this that is why say no solute will be permeating through this membrane that is why Cp is assumed to be as 0. So, then we have this Jw is equal to k into ln into Cg by Cb.

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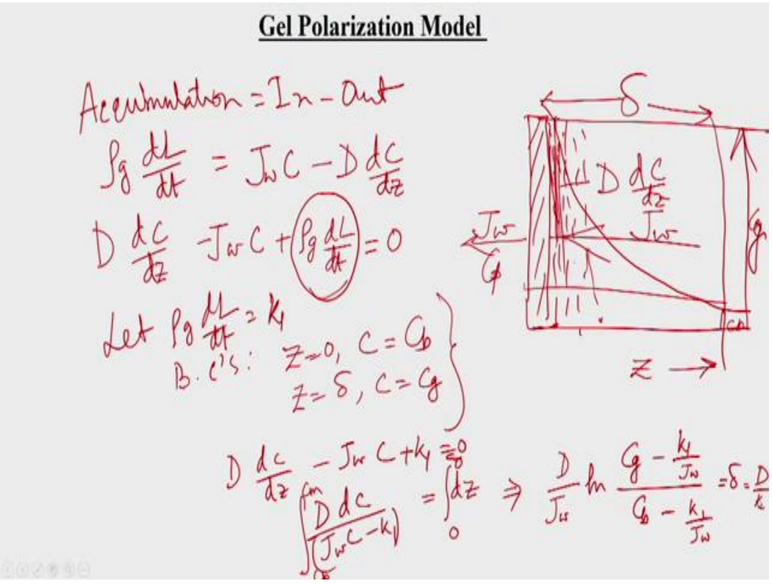

Now, we will be solving this one say for this gel polarization model will be deriving this equation for this permeate flux value, suppose we have this membrane again and Jw is this we can say this one flux of the we can say the solvent is there towards this membrane and say Jw whatever out of this solvent is this and Cp that is we assume that it is will be equal to 0 but for the timing we ae keeping this one and the back diffusion that I told this one that is D dc dz will be there and here in this Z direction suppose this gel layer or thick layer actually will be formed in the membrane surface and then say concentration profile will be like this say here say the bulk concentration will be Cb.

And whatever the maximum concentration will be that is we can say this one Cg when we can say gel concentration is such a concentration where we should get 0 flux value or we can say this one there will be no permeate flux, so that will be there. So, in that case delta will be here also we can say this one up to what distance we can say the effect of the solute actually is there effect of the membrane is there so that delta we can say this one this is the thickness of the say gel layer after that actually there you know effect of the membrane surface.

So, here actually we can say this one in the gel layer suppose in this gel layer here the solutes are deposited we can say this one in gel layer so we can say accumulation will be is equal to in minus out. So, in that case accumulation will be like this rho g into dL dt is equal to we can say this one in it will be equal to Jw into C and say out will be say whatever is there say we can say means it is going out D dc dz or we can say this one if we keep this one in the same side then it is like this D into dc dz minus Jw into C plus rho g into dL dt that will be equal to 0.

So, let take this one rho g into dL dt is equal to say a constant parameter because for the timing because whenever we will be doing the integration across this Z so we assume that this accumulation will not be we can say altering, so we can keep this one as this constant so let rho g into dL dt that is equal to say let us say k1 then boundary conditions will be like this say were Z is equal to 0 so C will be Cb and when Z is equal to delta we can say this one C will be Cg.

So, here we can say we have taken this Z actually in the opposite direction compare to the osmotic pressure model but in that case we assume that Z was actually 0 then membrane concentration was the solute concentration or dynamic concentration that is the we can say this one reverse this one rotation is adopted here we can assume the same thing also but for simplicity actually we have assumed that this boundary conditions the reverse also can be true.

Like this in that case D into dc by dz minus Jw into C plus k1 will be equal to 0 or we can say D into dc by Jw into C minus k1 that is equal to dz. Now, we will be doing the integration from 0 to delta and say it will be like Cb to Cm, if we take the reverse one also there will no problem so from here we will be getting D by Jw into ln Cg minus k1 by Jw by Cb minus k1 by Jw that is equal to delta is equal to say we can say D by k.

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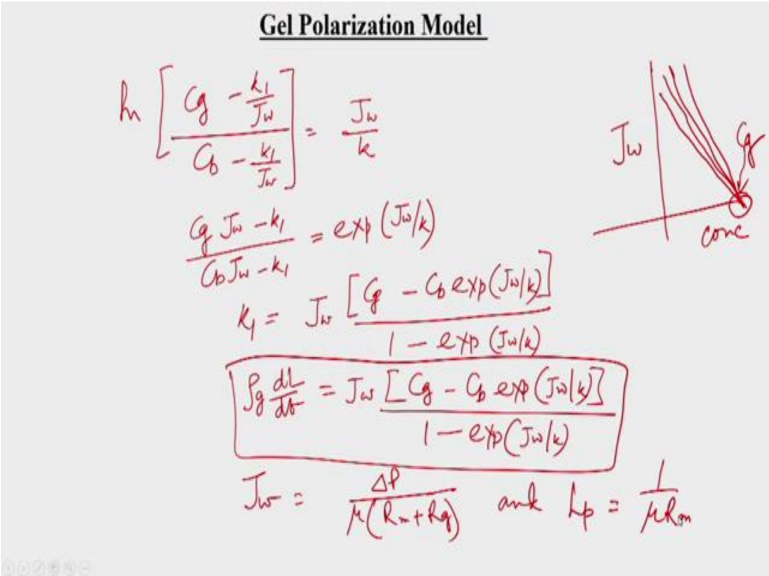

Now, you see for the gel polarization model, so if we say the concentration of the solute with this we can say we can say this one flux value Jw, so we have like this it is decreasing an then it is touching some point that is we can say this one where we can say this some were this we can say Jw will be 0 like this in from different we can say this one k values so for different k values we will get one particular point, so corresponding concentration actually will be giving this Cg.

Or gel concentration is a concentration where we can say the flux value will be 0, or when the complete gel will be formed then solvent concentration will be solvent flux or we can say the permeate flux value will be 0, that is why the we will be getting 1 for particular point this one what is called Cg or gel concentration.

Now, this ln Cg minus k1 by Jw by Cb minus k1 by Jw is equal to say Jw by k just by manipulation and by adjustment we can say this one we will be getting these Cg into Jw minus k1 by Cb into Jw minus k1 that will be like is equal to exponential Jw by k and from there actually we will be getting this we can say just by manipulation we can say k1 will be equal to Jw into Cg minus Cb into exponential Jw by k divided by 1 minus exponential Jw by k.

And then this k1 is equal to say rho g into say dL dt that will be is equal to Jw into Cg minus Cb into exponential Jw by k divided by 1 minus exponential Jw by k, so from here we can say, so now we have this expression and say we can say we have also this Jw is equal to del P by say mu into Rm plus Rg that we told that say because gel layer resistance also place an important role in case of gel polarization model means were the solutes are with bigger molecular weight. And say here and we can say this one Lp membrane permeability will be like this 1 by mu into Rm.

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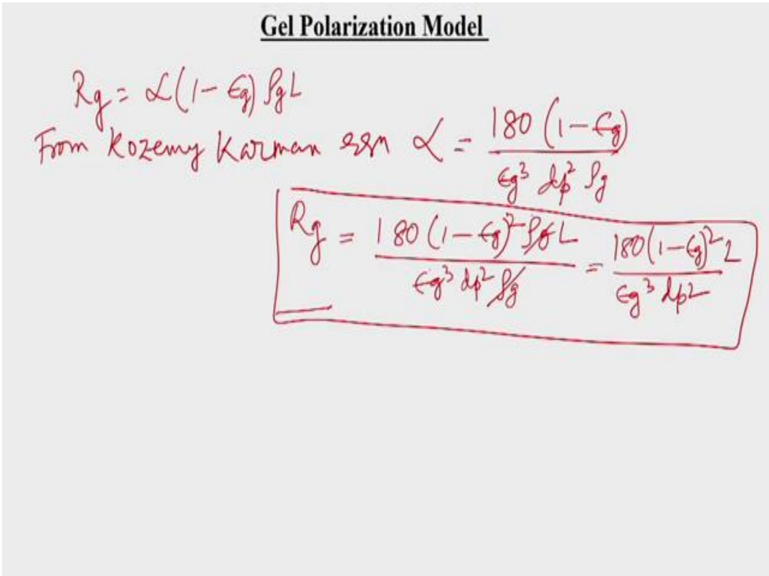

And Rg that will be obtain by say we can say this one Rg will be equal to we can say alpha into 1 minus epsilon g into rho g into L. So, that we know this one this gel layer resistance will be like this whatever the void fraction is there and then alpha is this we can say specific gel layer resistance. So, from kozemy karman equation we will be able to get we can say this one alpha from kozeny karman equation.

So, we will be getting this alpha is equal to this 180 into 1 minus epsilon g divided by epsilon g cube into dp square into rho g and say now we can say this one Rg if we put this alpha value in this Rg we will be getting 180 into 1 minus epsilon g into again 1 minus epsilon g that is square into rho g into L divided by epsilon g cube epsilon g cube and dp square and rho g that we go that will be equal to we can say ultimately 180 into 1 minus epsilon g square into L divided by say epsilon g cube into dp square.

So, that Rg actually we will now get as 180 into 1 minus epsilon square into L by epsilon cube into dp square. So, from there we will be getting, now we have got this Rg value so and Rm we will be getting from this one 1 by mu into Lp, so we can sy this one permeate flux value can be obtained from the generally expression of like this from del P by mu into Rm plus Rg.

So, from here we can we will be able to get this permeate flux value.

In the next class, we will be discussing the resistance in series model.