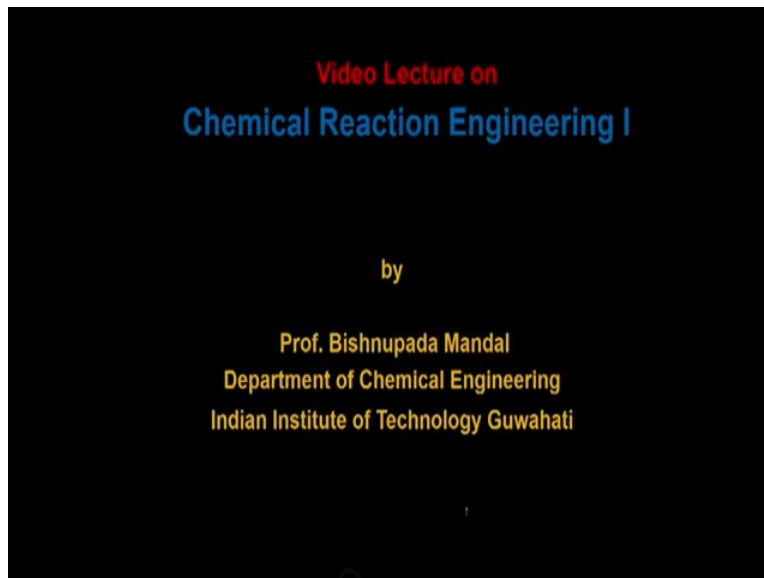


**Chemical Reaction Engineering 1**  
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**Indian Institute of Technology Guwahati**  
**Lecture 28 - RTD Measurement and Moments of RTD**

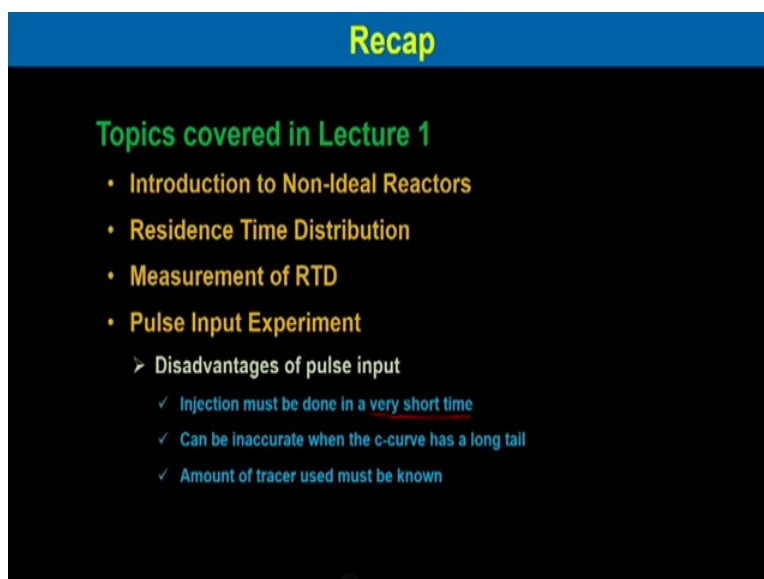
Welcome to the second lecture of module 5. In this module, we are discussing residence time distribution, that is non-ideal reactors and the reactor design for the non-ideal reactors.

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So before going to this lecture let us have brief recap on our previous lecture.

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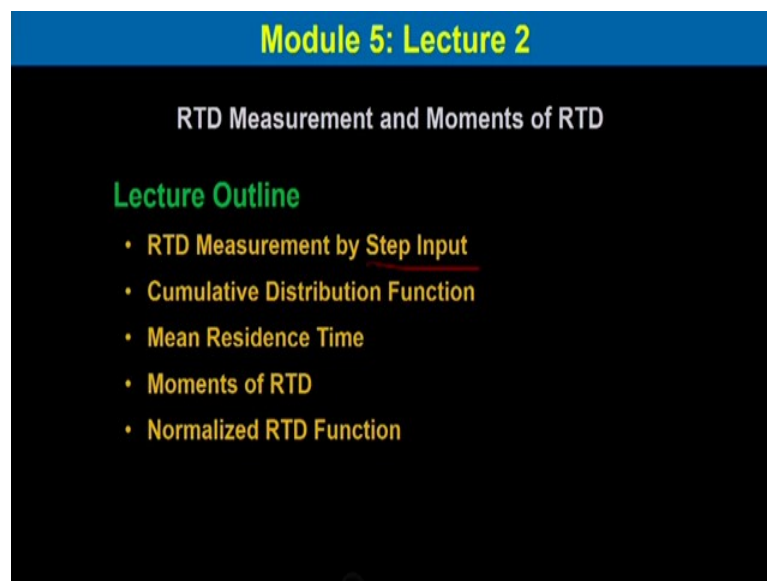


In the last lecture, we have covered introduction to non-ideal reactors and we have seen for the non-ideal reactors how the residence time distribution varies from one reactor to the other reactor due to non-ideality in the system. We have looked into the measurements of RTD, how we can measure the experimentally the residence time distribution function. And then one of the experiments we have considered is the pulse input experiments.

So in these cases, we have seen there are certain disadvantages of pulse input measurements for to determine the residence time distribution. And there we can see the injection must be done at a very short time which is difficult in case of the pulse injection. So that may lead to some error in the measurements. So it can be inaccurate when the c-curve has a long tail. So we have seen when there is a the tracer if it stays for a longer time in the reactor, so the distribution of the concentration versus time curve, we could see there is long tail and then we have to truncate the long tail to complete the calculations.

So it can lead to inaccurate results. Then the amount of tracer used must be known. So the amount of tracer we inject and that should come out, so that has to be balanced.

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So in this lecture, we will consider RTD measurements, different moments of RTD. The lecture outline would be we will see how we can measure the residence time distribution by step input. Earlier we have done pulse input, we will see in the step input method. Then we will see the cumulative distribution function and we will calculate the first moments of RTD that is the mean residence time and then other moments of RTD. Then we will see the

normalized RTD function, and is its important to calculate normalized RTD function for the comparison of different RTDs of the reactor.

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**Measurement of RTD by Step Input**

**Step Input**

- Consider a step input of **tracer concentration** in a system with a **constant volumetric flow rate**.
- Constant rate of tracer addition to a feed initiated at  $t = 0$ .
- Before this time no tracer was added to the feed.
- Symbolically:

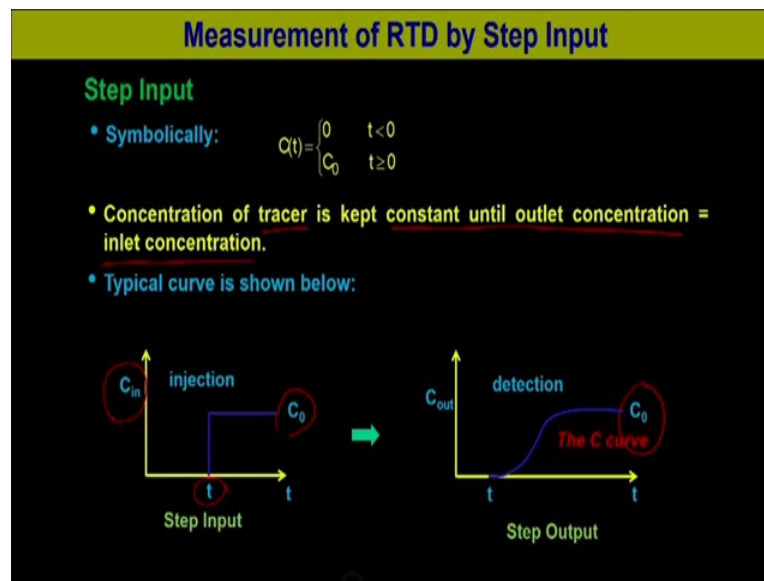
$$C(t) = \begin{cases} 0 & t < 0 \\ C_0 & t \geq 0 \end{cases}$$

*$C_0$  = initial tracer concentration.*

So let us start with measurement of RTD by step input. So in this case, we consider tracer concentration to be given as a step input in a system with a constant volumetric flow rate. So the volumetric flow rate of the reactants should be constant and we give a step input of the tracer. The constant rate of tracer addition to a feed initiated at  $t=0$ . So before this time no tracer was added to the feed. So the tracer is added at  $t=0$  and before that there was no tracer addition into the feed.

$$C(t) = \begin{cases} 0 & t < 0 \\ C_0 & t \geq 0 \end{cases}$$

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Now this is we have got from the tracer input experiments. And the concentration of the tracer is kept constant until outlet concentration is equal to the inlet concentration. So outlet tracer concentration should be equal to the inlet tracer concentration. So if that is happened, the typical curve we could see for the tracer input. This is a tracer inlet concentration with respect to time and this is the step injection, so step input at  $t$  and then we will get  $C_0$  which is injected over here.

And then output we can see  $C_{out}$  curve. So this is the response curve or when you detect it. So this is the step output and the concentration would be  $C_{out}$ . So inlet concentration is  $C_0$  and outlet concentration is  $C_0$ . So the concentration of the tracer we should keep constant until we get the outlet concentration  $C_0$ . So this is the  $C$  curve.

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**Measurement of RTD by Step Input**

- Since the concentration is constant with time,  $C_0$ , we can take it outside the integral sign:
- Then:  
$$C_{out} = C_0 \int_0^t E(t) dt$$
- Dividing by  $C_0$  yields:  
$$\left(\frac{C(t)}{C_0}\right)_{step} = \int_0^t E(t) dt = F(t) \quad \rightarrow F(t) = \left(\frac{C(t)}{C_0}\right)_{step}$$
- Differentiating:  
$$E(t) = \frac{d}{dt} \left(\frac{C(t)}{C_0}\right)_{step}$$

Now since the concentration is constant with time that is  $C_0$ , we can take it outside the

integral sign  $\left(\frac{C(t)}{C_0}\right)_{step} = \int_0^t E(t) dt = F(t)$

So this  $F(t)$  is the cumulative distribution function. So  $F(t) = \left(\frac{C(t)}{C_0}\right)_{step}$ . So from here we

can write  $F(t) = \left(\frac{C(t)}{C_0}\right)_{step}$ . So this is the cumulative distribution function

$F(t) = \left(\frac{C(t)}{C_0}\right)_{step}$ . So if we differentiate we would obtain  $E(t) = \frac{d}{dt} \left(\frac{C(t)}{C_0}\right)_{step}$ . So

differentiating this relation we can obtain  $E(t)$ . And again integrating the  $E(t)$  curve we can get the  $F(t)$

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**Advantages and Disadvantages of Step Input**

**Advantages**

- The positive step is usually easy to carry out experimentally than the pulse test.
- An additional advantage is that the total amount of tracer in the feed over the period of test does not have to be known as it does in the pulse test.

**Disadvantages**

- First problem: It is difficult to maintain a constant tracer concentration in the feed.
- Second problem: Obtaining the RTD from step test involves differentiation of the data and presents more serious drawback to the technique, since differentiation of data can lead to large errors.
- Third problem: Large amount of tracer required for this test. If the tracer is very expensive, a pulse test is better.

So the advantages and disadvantages of the step input. So advantages are in case of the positive step when we give is usually easy to carry out experimentally than the pulse test. So here we need not to maintain the time which is very short. So it is injected over a period of time. An additional advantage is that the total amount of tracer in the feed over the period of the test does not have to be known. So as it is we have seen in case of the pulse test, we need to know how much tracer is injected and whether that is coming out with the effluent schemes.

So that mass balance has to be confirmed in case of the pulse tracer injection test. But in case of a step input that total amount of the tracer we need not to be known for the calculation.

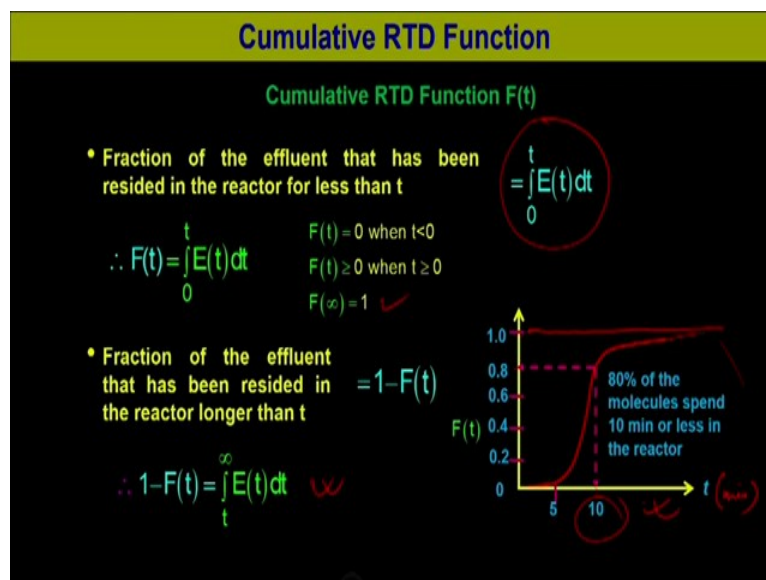
The disadvantage for this case, there are some problems related to this step input that is it is difficult to maintain a constant tracer concentration in the feed. So throughout the step injection it is very very difficult to maintain the concentration  $C_0$ . So that is one of the difficult drawback in this case of the step input.

The second problem which we face is obtaining the RTD from the step test involved, differentiation of the data and this presents more serious drawback to the technique, since differentiation of the data sometimes can lead to large error if the data obtained are not so accurate. So that we have seen when we analyze the kinetic data, in case of the batch reactor kinetic data we have seen there are two methods of analyses. One is integral method another is a differential method.

In case of the differential method we have seen if the accuracy of the data is not there or the data are scattered differential method may not give accurate results. It gives a large error in sometimes. So that is the second problem similar to that in case of the step input.

Now the third problem in this case is that the large amount of tracer is required for this test because we have to continuously add the tracer for a period of time to the feed. So the period of time is such that when the outlet concentration matches with the inlet tracer concentration if that matches, so we have to give large quantity of the tracer along with the feed. So if the tracer is very expensive then this method is difficult to use. Then in that case, pulse test is the better method to be adopted.

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So RTD distribution function we have seen that the cumulative distribution function

$F(t) = \int_0^t E(t) dt$ . So that is a cumulative the total amount of effluent that has been inside the

reactor  $\int_0^t E(t) dt$ . So that is why this part is called the cumulative distribution function. So

therefore  $F(t) = \int_0^t E(t) dt$ . And we can see that  $F(t) = 0$   $t < 0$ .

So which can be seen  $F(t)$  versus  $t$  curve, this is cumulative distribution function curve and when  $F(t)$  would be greater than equal to 0 when  $t$  is equal to greater than equal to 0, so you

could see the distribution curve  $F(t)$  distribution curve with respect to time. And  $F(\infty) = 1$  which is over here, so at  $t = \infty$ . So at  $t = \infty$  and  $F(\infty) = 1$  So the fractions which is resided between the time  $t$  to infinity, the fraction of the effluent that has been resided in the reactor longer than  $t$  that is  $t$  to infinity would be always  $1 - F(t)$ . So that is equal to  $1 - F(t)$ .

So therefore  $1 - F(t) = \int_t^{\infty} E(t) dt$ . So that is the fraction of the effluent which has been resided from  $t$  to along a time that is  $t$  to infinity. So this is you can see from here, the 80 percent if you see at time 10 minute or 10 second whatever, so if we say it is minute, so then 80 percent of the materials you could see spent 10 minutes or less in the reactor. So the cumulative distribution function you will get  $F(t)$  would be equal to 0.8.

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### Mean Residence Time

Mean Residence Time,  $t_m$

- For an ideal reactor, the space time or average residence time,  $\tau$  is defined as  $V/v_0$ .
- The mean residence time,  $t_m$  is equal to  $\tau$  in either ideal or nonideal reactors.
- The mean value of the variable is equal to the first moment of the RTD function,  $E(t)$ .

$$t_m = \frac{\int_0^{\infty} tE(t) dt}{\int_0^{\infty} E(t) dt} = \frac{\int_0^{\infty} tE(t) dt}{1} = \tau \qquad \frac{V}{v_0} = \tau = t_m$$

- By calculating  $t_m$ , the reactor  $V$  can be determined from a tracer experiment.

Now mean residence time, so mean residence time  $t_m$ , we define with  $t_m$ . For an ideal

reactor, the space-time or the average residence time  $\frac{V}{v_0} = \tau$ . So  $V$  is the reactor volume and small  $v$  is the volumetric flow rate. So for ideal reactor the space-time or average residence

time  $\frac{V}{v_0} = \tau$ . The mean residence time  $t_m$  is equal to tau in either ideal or non ideal reactors.

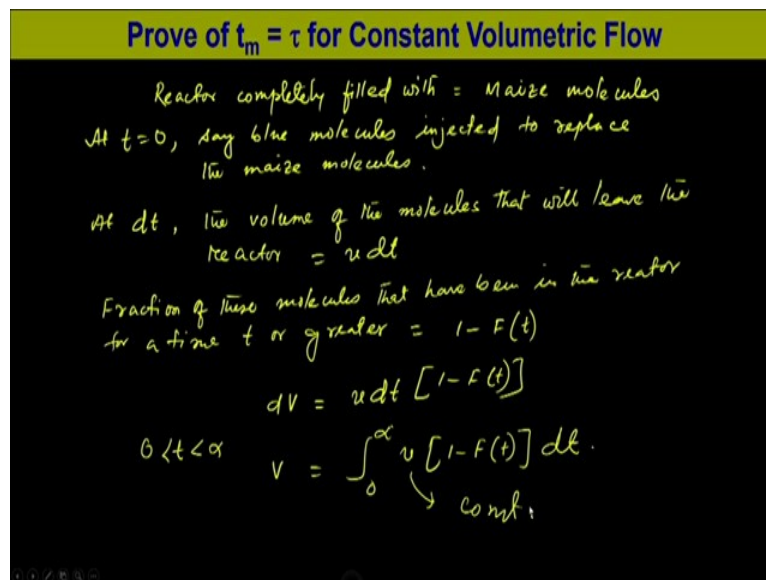


So the mean residence time would be equal to tau. The mean value of the variable is equal to the first moment of the RTD function  $E(t)$ . So this can be calculated

$$t_m = \frac{\int_0^{\infty} tE(t) dt}{\int_0^{\infty} E(t) dt} = \int_0^{\infty} tE(t) dt = \tau$$

$\frac{V}{v_0} = \tau = t_m$ . So if we can calculate  $t_m$ , the reactor volume can be determined from the tracer experiment.

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Now we need to prove that  $t_m$  and tau for constant volumetric flow rate  $\tau = t_m$ . Now let us consider the following situation, the reactor initially completely filled with, so reactor completely filled with maize molecules. Now at time  $t=0$ , we start injecting other kind of molecules into the reactor say blue molecule injected to replace the maize molecules.

So initially the reactor volume is  $V$  which is filled with occupied with the maize molecules and now at time  $dt$ , say at time  $dt$  the volume of the molecules that will leave the reactor would be equal to  $v dt$ . So the fraction of the molecules that have been in the reactor greater

than this time  $t$ . so fraction of the material or fraction of these molecules that have been in the reactor for a time  $t$  or greater, this would be  $1 - F(t)$ .

So because only maize molecules have been in the reactor for a time  $t$  or greater, the volume of the maize molecules that is  $dV = vdt [1 - F(t)]$ . Now if we sum up all the volume which is replaced, so between the time  $t$  greater than zero and less than infinity, so we can write

integrate this relation would-be  $V = \int_0^{\infty} v [1 - F(t)] dt$ .

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**Prove of  $t_m = \tau$  for Constant Volumetric Flow**

$$V = \int_0^{\infty} v [1 - F(t)] dt.$$

↓  
const.

$$= v \int_0^{\infty} [1 - F(t)] dt.$$

Integration by parts

$$\int x dy = xy - \int y dx$$

$$\frac{V}{v} = t [1 - F(t)] \Big|_0^{\infty} + \int_0^{\infty} t dF.$$

Now since this volumetric flow rate is constant, so we can write  $V = \int_0^{\infty} v [1 - F(t)] dt$ . Now

$V = \int_0^{\infty} v [1 - F(t)] dt$ . So if we use the integration by parts, so if we use integration by parts

that is  $\int x dy = xy - \int y dx$ . So then we can write from here the  $\frac{V}{v} = t [1 - F(t)] \Big|_0^{\infty} + \int_0^{\infty} t dF$

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**Prove of  $t_m = \tau$  for Constant Volumetric Flow**

$$\frac{V}{v} = t [1 - F(t)] \Big|_0^{\infty} + \int_0^{\infty} t dF$$

At  $t = 0$ ,  $F(t) = 0$   
 At  $t = \infty$ ,  $1 - F(t) = 0$

$$\frac{V}{v} = \tau = \int_0^{\infty} t dF \Rightarrow \tau = \int_0^{\infty} t E(t) dt$$

$$dF = E(t) dt$$

So we have this relation by integration of the parts by dividing the volume, volumetric flow

rate so we had  $\frac{V}{v} = t [1 - F(t)]_0^\infty + \int_0^1 t dF$ . Now at  $t = 0, F(t) = 0$  and at  $t = \infty, 1 - F(t) = 0$ .

Then the first term on the right-hand side would be zero, so this term would be zero. So we

can write  $\frac{V}{v} = \tau = \int_0^1 t dF$ .

So however we know that  $dF = E(t)dt$ . So if we substitute over here, we can get from here

$$\tau = \int_0^1 t E(t) dt$$

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**Prove of  $t_m = \tau$  for Constant Volumetric Flow**

$$Z = \int_0^\infty t E(t) dt$$

$$t_m = \frac{\int_0^\infty t E(t) dt}{\int_0^\infty E(t) dt} = 1$$

$$= \int_0^\infty t E(t) dt$$

$\tau = t_m$       $\frac{V}{v} = t_m$

*This result is true only for a closed system.*

$\Rightarrow V = v t_m$

So we have  $\tau = \int_0^\infty t E(t) dt$  and from the definition of  $t_m = \frac{\int_0^\infty t E(t) dt}{\int_0^\infty E(t) dt} = 1$ . So we can write  $\tau = t_m$ .

$$= \int_0^\infty t E(t) dt$$

So this result is true only for a closed system and no dispersion across the boundary. Then the

exact volume reactor volume can be calculated from  $V = vt_m$ . As  $\tau = \frac{V}{v} = t_m$ , so from here we can write  $V = vt_m$

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**Other Moments of RTD**

Variance,  $\sigma^2$

- It is very common to compare RTDs by their moments instead of comparing their entire distributions.
- Three moments are normally used.
- The first moment is the mean residence time,  $t_m$ .
- The second moment is the variance or square of standard deviation taken about the mean.
- It is defined by:
 

$$\sigma^2 = \int_0^{\infty} (t - t_m)^2 E(t) dt$$
- The magnitude of this moment indicates the spread of distribution.
- Greater the value of this moment, greater the distribution.

So we will consider the other moments of the RTDs, the one of the other moments is the variance that is the Sigma Square. So it is very common compare RTDs by their moment instead of comparing their entire distribution. So three moments are normally used for to compare the RTDs. One of them is the first moment which we have done defined earlier that is the mean residence time  $t_m$ . The second moment is the variance or the square of standard deviation taken about the mean.

It is defined this variance  $\sigma^2 = \int_0^{\infty} (t - t_m)^2 E(t) dt$ . So each terms have their usual meaning and significance. The magnitude of these moments indicates the spread of this distribution. So magnitude of this Sigma Square will indicate the spread, how the spread of the distribution is. Greater the value of this moment, the greater is the distribution. So as large the value we can obtain for Sigma Square the distribution would be that large.

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**Other Moments of RTD**

**Skewness,  $s^3$**

- The third moment is the Skewness and is taken about the mean.
- It is defined by:  
$$s^3 = \frac{1}{\sigma^3/2} \int_0^{\infty} (t - t_m)^3 E(t) dt$$
- The magnitude of this moment measures the extent that a distribution is skewed in one direction or another in reference to the mean.
- For complete description of a distribution, all moments must be determined.
- Practically, these three moments are usually sufficient for a reasonable characterization of RTD.

Now the third moment which is known as the skewness and defined with  $s^3$ . The third moment and it is taken about the mean. The third moment, the skewness is taken about the

mean. So it is defined with  $s^3 = \frac{1}{\sigma^3/2} \int_0^{\infty} (t - t_m)^3 E(t) dt$ . The magnitude of this moment measures the extent of distribution is skewed in one direction or in the other direction with respect to the mean. So for complete description of the distribution all these moments must be determined.

So if we wanted to know the complete description of the distribution of the RTDs or residence time distribution of the non-ideal reactors, then all these moments have to be calculated or have to be determined. That means mean, a variance and skewness. And practically these three moments are usually sufficient for a reasonable characterization of RTDs.

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**Example (H. Scott Fogler)**

A pulse of tracer was injected into a reactor, and the effluent concentration was measured as a function of time. The resulting data are given in the below Table.

t min	0	1	2	3	4	5	6	7	8	9	10	12	14
C g/m <sup>3</sup>	0	1	5	8	10	8	6	4	3	2.2	1.5	0.6	0

Calculate the mean residence time and variance

So let us take an example from H. Scott Fogler, it is given as an example over there. A pulse of tracer as we have considered in the first lecture was injected into a reactor and the effluent concentration was measured as a function of time. The resulting data are given in the below table that is the concentration, time versus concentration of the tracer experiments. And we need to calculate the mean residence time and variance.

So we generated the table for calculating the exit age distribution or the RTD distribution function  $E(t)$  curve. So that data we will use over here and then we will follow to calculate the other parameters to calculate the mean residence time and the variance for the same example.

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**Example (H. Scott Fogler)**

A pulse of tracer was injected into a reactor, and the effluent concentration was measured as a function of time. The resulting data are given in the below Table.


t min	0	1	2	3	4	5	6	7	8	9	10	12	14
C g/m <sup>3</sup>	0	1	5	8	10	8	6	4	3	2.2	1.5	0.6	0

Calculate the mean residence time and variance

**Solution**

$$t_m = \int_0^{\infty} t E(t) dt$$

$$\sigma^2 = \int_0^{\infty} (t - t_m)^2 E(t) dt$$

$$t_m = \int_0^{\infty} t E(t) dt = \int_0^{10} t E(t) dt + \int_{10}^{14} t E(t) dt$$


So the solution for this problem is that we know the the mean residence time equation which

is  $t_m$ . So  $t_m = \int_0^{\infty} tE(t)dt$ . So if we plot a curve  $t, t_m$  versus or  $tE(t)$ , so this will give  $tE(t)$

versus  $t$ , so if we plot that curve so area under the curve would give  $t_m$ . So once this mean is

calculated we can calculate  $\sigma^2 = \int_0^{\infty} (t - t_m)^2 E(t)dt$ . Now for the problem which is given over

here, the  $t_m$  can be calculated as  $\int_0^{\infty} tE(t)dt$ . We can see to separate into two part, one is up to

the 10 minutes, 0 to 10,  $\int_0^{10} tE(t)dt + \int_{10}^{14} tE(t)dt$ .



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**Example (H. Scott Fogler)**

A pulse of tracer was injected into a reactor, and the effluent concentration was measured as a function of time. The resulting data are given in the below Table.

t min	0	1	2	3	4	5	6	7	8	9	10	12	14
C g/m <sup>3</sup>	0	1	5	8	10	8	6	4	3	2.2	1.5	0.6	0

Calculate the mean residence time and variance

$t$	$C(t)$	$tC(t)$	$t^2C(t)$	$(t - \bar{t})$	$(t - \bar{t})^2$	$(t - \bar{t})^2 C(t)$
0	0	0	0	-5.15	0	0
1	1	0.02	0.02	-4.15	0.34	0.02
2	5	0.10	0.20	-3.15	0.992	0.4
3	8	0.16	0.48	-2.15	0.74	1.44
4	10	0.20	0.80	-1.15	0.265	3.2
5	8	0.16	0.80	-0.15	0.004	4.0
6	6	0.12	0.72	0.85	0.087	4.32
7	4	0.08	0.56	1.85	0.274	3.92
8	3	0.06	0.48	2.85	0.487	3.84
9	2.2	0.044	0.40	3.85	0.652	3.56
10	1.5	0.03	0.30	4.85	0.706	3.0
12	0.6	0.012	0.14	6.85	0.563	1.73
14	0	0	0	8.85	0	0

So if we do these calculations from the graph, we can obtain this table. So time is given over here which is written over here and then we have  $C(t)$ , we calculated  $E(t)$  earlier in the problem lecture 1. So  $E(t)$  we had different time we can calculate. Then once we know  $E(t)$  we can calculate  $tE(t)$ . Now we have to calculate  $t_m$ , so that we can calculate  $t - t_m$  and then the next column. And this  $t^2E(t)$  can be calculated from here, so first column and the third column, using that last column can be calculated.

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**Example (H. Scott Fogler)**

A pulse of tracer was injected into a reactor, and the effluent concentration was measured as a function of time. The resulting data are given in the below Table.

t min	0	1	2	3	4	5	6	7	8	9	10	12	14
C g/m <sup>3</sup>	0	1	5	8	10	8	6	4	3	2.2	1.5	0.6	0

Calculate the mean residence time and variance

**Solution**

$$t_m = \int_0^h f(x) dx = \frac{h_1}{3} (f_1 + 4f_2 + 2f_3 + 4f_4 + \dots + 4f_{n-1} + f_n) + \frac{h_2}{3} (f_{n+1} + 4f_{n+2} + f_{n+3})$$

$$t_m = \frac{1}{3} [1 \times 0] + 4 \times 0.02 + 2 \times 0.2 + 4 \times 0.48 + \frac{2}{3} [0.3 + 4 \times 0.14 + 0] = 5.15 \text{ min.}$$

So now let us calculate  $t_m$ , so  $t_m$  if you see the Fogler's appendix at the end of the book, method of calculating the integral under the curve or the area under the curve can be obtained using different methods which are given, Simpson's one-third and other method which we already discussed in our last lecture. So it can be similar method can be followed over here.

So  $t_m = \int_0^h f(x) dx = \frac{h_1}{3} (f_1 + 4f_2 + 2f_3 + 4f_4 + \dots + 4f_{n-1} + f_n) + \frac{h_2}{3} (f_{n+1} + 4f_{n+2} + f_{n+3})$ . So if we use, we can calculate

$$t_m = \int_0^h f(x) dx = \frac{h_1}{3} (f_1 + 4f_2 + 2f_3 + 4f_4 + \dots + 4f_{n-1} + f_n) + \frac{h_2}{3} (f_{n+1} + 4f_{n+2} + f_{n+3})$$

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### Example (H. Scott Fogler)

A pulse of tracer was injected into a reactor, and the effluent concentration was measured as a function of time. The resulting data are given in the below Table.

t min	0	1	2	3	4	5	6	7	8	9	10	12	14
C g/m <sup>3</sup>	0	1	5	8	10	8	6	4	3	2.2	1.5	0.6	0

Calculate the mean residence time and variance

t	C(t)	E(t)	(t - t <sub>m</sub> )	(t - t <sub>m</sub> ) <sup>2</sup>	(t - t <sub>m</sub> ) <sup>2</sup> E(t)	t <sup>2</sup> E(t)
0	0	0	0	-5.15	0	0
1	1	0.02	0.02	-4.15	0.34	0.02
2	5	0.10	0.20	-3.15	0.992	0.4
3	8	0.16	0.48	-2.15	0.74	1.44
4	10	0.20	0.80	-1.15	0.265	3.2
5	8	0.16	0.80	-0.15	0.004	4.0
6	6	0.12	0.72	0.85	0.087	4.32
7	4	0.08	0.56	1.85	0.274	3.92
8	3	0.06	0.48	2.85	0.487	3.84
9	2.2	0.044	0.40	3.85	0.652	3.56
10	1.5	0.03	0.30	4.85	0.706	3.0
12	0.6	0.012	0.14	6.85	0.563	1.73
14	0	0	0	8.85	0	0

So once we calculate the area under the curve  $t_m$ , so this we can calculate that is 5.15 minutes. So then we can calculate  $(t - t_m)$  would-be minus 5.15 and so on putting the t m values we can calculate this column. And then once we know this we can squared it and multiplied with E t so that we can get this data.

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### Example (H. Scott Fogler)

A pulse of tracer was injected into a reactor, and the effluent concentration was measured as a function of time. The resulting data are given in the below Table.

t min	0	1	2	3	4	5	6	7	8	9	10	12	14
C g/m <sup>3</sup>	0	1	5	8	10	8	6	4	3	2.2	1.5	0.6	0

Calculate the mean residence time and variance

**Solution**

So now the  $t_m$  curve is basically if you plot that  $tE(t)$  versus t it would look like this. So basically the curve would be, so the we need to calculate the area under this curve. So which is calculated that is area is equal to  $t_m = 5.15$  minute.

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**Example (H. Scott Fogler)**

A pulse of tracer was injected into a reactor, and the effluent concentration was measured as a function of time. The resulting data are given in the below Table.

t min	0	1	2	3	4	5	6	7	8	9	10	12	14
C g/m <sup>3</sup>	0	1	5	8	10	8	6	4	3	2.2	1.5	0.6	0

Calculate the mean residence time and variance

**Solution**

Now we can also plot the values for  $E(t)$  and  $t - t_m$ , square versus time. So it is from 0 to 1 and time is say 0 to say here 15 minute. If we look into the distribution, so it would be like this say up to 14 minute and this is 5 minute and say this is 10, 0. So if we plot this versus time, typical curve we will obtain from the data is like this. So we need to calculate the area under this curve to calculate the Sigma Square.

So another curve we need that is  $t^2 E(t)$  versus  $t$  and it would be from 0 to say 15 minute and this is say about 5 and the plot would look like this. So the area under this curve would be

need to be calculated so that we can calculate the integral 0 to infinity  $\int_0^{\infty} t E(t) dt$  and the area under this two curve is basically  $\sigma^2$ .

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**Example (H. Scott Fogler)**

A pulse of tracer was injected into a reactor, and the effluent concentration was measured as a function of time. The resulting data are given in the below Table.

t min	0	1	2	3	4	5	6	7	8	9	10	12	14
C g/m <sup>3</sup>	0	1	5	8	10	8	6	4	3	2.2	1.5	0.6	0

Calculate the mean residence time and variance

**Solution**

$$\begin{aligned} \sigma^2 &= \int_0^{\infty} (t - t_m)^2 E(t) dt \\ &= \int_0^{\infty} t^2 E(t) dt - 2t_m \int_0^{\infty} t E(t) dt + t_m^2 \int_0^{\infty} E(t) dt \\ &= \int_0^{\infty} t^2 E(t) dt - 2t_m^2 + t_m^2 \\ \sigma^2 &= \int_0^{\infty} t^2 E(t) dt - t_m^2 \end{aligned}$$

So Sigma Square we can write from the definition of  $\sigma^2 = \int_0^{\infty} (t - t_m)^2 E(t) dt$ . Now if we expand this, so this is a minus b, the whole square, so a square minus 2 a b plus b square. So

we can write this would be  $\int_0^{\infty} t^2 E(t) dt - 2t_m \int_0^{\infty} t E(t) dt + t_m^2 \int_0^{\infty} E(t) dt$ . So we can write this

would be equal to integral 0 to infinity t square E t dt minus twice t m because this is equal to 1 plus this is equal to 1. So this would be t m square. So basically Sigma square would be equal to integral 0 to infinity t square E t dt minus so this is t m square, so this is t into E t dt.

So this would be is equal to t m, so this would be t m square. So therefore

$= \int_0^{\infty} t^2 E(t) dt - 2t_m^2 + t_m^2$ . So this  $\sigma^2 = \int_0^{\infty} t^2 E(t) dt - t_m^2$ . So this can be calculated as we have

explained before and then we can calculate the Sigma Square.

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**Example (H. Scott Fogler)**

A pulse of tracer was injected into a reactor, and the effluent concentration was measured as a function of time. The resulting data are given in the below Table.

t min	0	1	2	3	4	5	6	7	8	9	10	12	14
C g/m <sup>3</sup>	0	1	5	8	10	8	6	4	3	2.2	1.5	0.6	0

Calculate the mean residence time and variance

**Solution**

$$\int_0^{\infty} t E(t) dt = \int_0^{10} t^2 E(t) dt + \int_{10}^{14} t^2 E(t) dt$$

$$= 32.71 \text{ min}^2$$

$$\sigma^2 = \int_0^{\infty} t^2 E(t) dt - t_m^2 = 32.71 \text{ min}^2 - (5.15 \text{ min})^2 = 6.19 \text{ min}^2$$

$$\Rightarrow \sigma = 2.49 \text{ min}$$

So now if we consider to calculate the term  $\int_0^{\infty} t^2 E(t) dt = \int_0^{10} t^2 E(t) dt + \int_{10}^{14} t^2 E(t) dt = 32.71 \text{ min}^2$

So the  $\sigma^2 = \int_0^{\infty} t^2 E(t) dt - t_m^2 = 32.71 \text{ min}^2 - (5.15 \text{ min})^2 = 6.14 \text{ min}^2$ . So we can calculate, this is the standard deviation, square of standard deviation. So Sigma we can calculate  $\sigma = 2.49 \text{ min}$ . So this way we can calculate the mean residence time and the variance.

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## Normalized RTD Function $E(\theta)$

$$\theta = \frac{t}{\tau}$$

dimensionless parameter.

Dimensionless RTD function  $E(\theta)$

$$E(\theta) = \tau E(t)$$

$\theta$  = number of reactor volumes of fluid based on entrance condition that have flowed through the reactor in time  $t$ .

Now one more thing is the use of normalized RTD function. In many cases, we have seen the normalized RTD function is used, what is normalized RTD function? This is the dimension-

less time. That means we define parameter  $\theta = \frac{t}{\tau}$ . So then we can this is the dimensionless parameter, so the dimensionless RTD function  $E(\theta)$ , we can define as  $E(\theta) = \tau E(t)$ . So the quantity  $\theta$  it represents the number of reactor volume of fluid based on entrance condition, that have flowed through the reactor in time  $t$ .

So this is the dimensionless parameter. It represents the number of reactor volume of fluid based on entrance condition that have flowed through the reactor in time  $t$ . So and based on that we can define the dimensionless RTD function  $E(\theta) = \tau E(t)$

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### Why We Use Normalized RTD

**□ Purpose**

- The flow performance inside the reactors of different sizes can be compared directly.
- Example:
  - If normalized function  $E(\theta)$  is used, all mixed CSTR have numerically the same RTD.
  - If  $E(t)$  is used, numerical values of  $E(t)$  can vary substantially for different CSTR.

$\rightarrow E(t) = \frac{1}{\tau} e^{-t/\tau}$   
 $\Rightarrow E(\theta) = \tau E(t) = e^{-\theta}$

$v_1 \text{ \& } v_2 \quad \int_0^{\infty} E(\theta) d\theta = 1$

So why this dimensionless function  $E(t)$  we use? So the purpose of using the normalized RTD function is the flow performance inside the reactor of different sizes can be compared directly. So we can directly compare among the different size of the reactors, their RTD distribution. For example, if we take normalized  $E(\theta)$  is used we could see all mixed at CSTR that mixed flow reactor have numerically the same RTD. But if we used  $E(t)$  instead of  $E(\theta)$  numerical values of  $E(t)$  can vary substantially for different CSTR. We can later we

could able to see that for all perfectly mixed flow reactor  $E(t) = \frac{1}{\tau} e^{-\frac{t}{\tau}}$ . And from here we can write  $E(\theta) = \tau E(t) = e^{-\theta}$

So from these two relations we can see for a particular time t or the identical time t, the  $E(t)$  this  $E(t)$  could be quite different for different reactors having different volumetric flow rates. Whereas the  $E(\theta)$  would be same for say volume  $v_1$  and  $v_2$ . Later we could also prove that integral 0 to infinity E theta d theta could be also 1. So in this case of normalized RTD, the

four fractions from  $\int_0^{\infty} E(\theta) d\theta = 1$ . This is because eventually all the materials have to come out, so overall function have to be 1.



So thank you very much for attending this lecture and we will continue our discussion of non-ideal reactors and then the residence time distribution of a reactor design, we will continue in our next lecture.