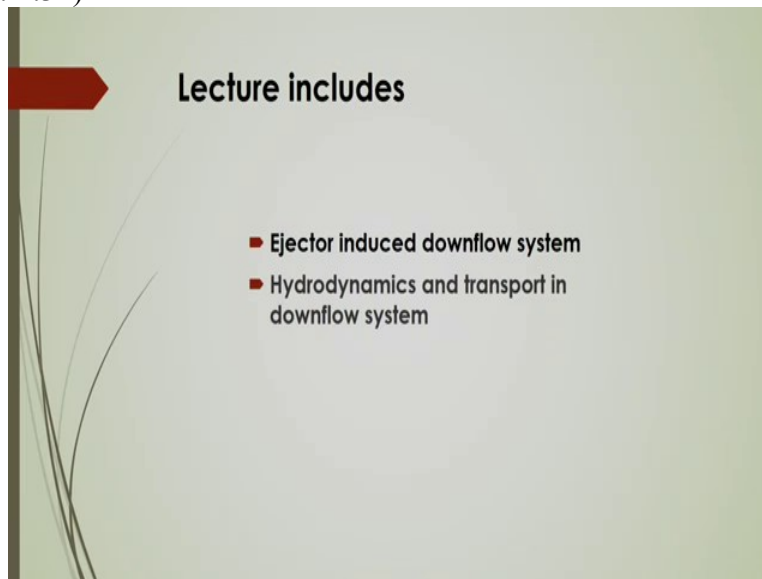


Chemical Process Intensification
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Module 8: Interfacial area based processes PI
Lecture 8.3: Hydrodynamics and Transport in Downflow System

Welcome to massive open online course on Chemical Process Intensification. So, we were discussing in the module 8 regarding interfacial **area-based** process intensification. In the previous lecture, we have discussed something about the different configurations of the ejector systems and their applications and also how that ejectors is working for the generation of micron size bubble as well as the droplet for that creation of the interfacial area for the mass transfer in chemical engineering processes. So, in this lecture, we will discuss something more about that ejector system and how that ejector system is working in the downflow system and what are there different hydrodynamic characteristics that we will discuss in this lecture.

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What is downflow system? (Recap)

- Units for carrying out reactions and mass transfer operations in which a gas, made up of one or several reactive components comes into a contact or reacts with a liquid.
- They are characterized by the fact that the gas is dispersed in the continuous liquid phase in the form of bubble against its buoyancy effect by downward liquid momentum.

So, this lecture will be included the subject on Hydrodynamics and transport in downflow systems and in that case, we have discussed something about what are the functions of that downflow systems that working in a particular chemical engineering processes for their intensification. So as per the downflow system, it is generally considered a **unit** for carrying out the reactions and mass transfer operations in which a gas that will be made up of one or several reactive components, that comes into a contact or reacts with a liquid, that liquid may be in continuous or batch wise in mode.

In that case they are characterized by the fact that the gases dispersed in the continuous liquid phase in the form of dispersed phase of bubbles against its buoyancy which will be that creating more interfacial area as well as residence time in the liquid medium and during that formation of the bubbles, it will be you know flowing downward due to the liquid movement and these things already have been described in details in the previous lecture.

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How to disperse the gas downwardly in the liquid? (recap)

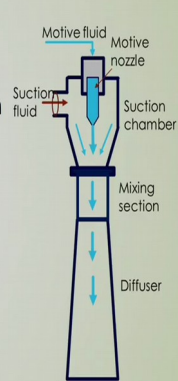
- By entraining gas by downward liquid jet.
- Sucking the gas by liquid jet (**Ejector system**) and then its plunging into the pool of liquid.
- Breaking the surface of the pool of liquid and entrapping the gas into the liquid as a bubble
- Then dragging downward by downward liquid momentum.

So, we will discuss that what are that actually ejector system, how it is being used for that downflow system. In that case, this ejector system will suck the gas from the atmosphere or from any other sources by sucking it by the liquid jet and then it is pumping into the pool of the liquid. And in that case, the breaking of the surface of the pool of liquid and entrapping the gas into the liquid as a bubble and in that case, the dragging downward by the downward liquid momentum would be the more important phenomena where that you can get more residence time in that particular downflow system.

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(.....Recap)

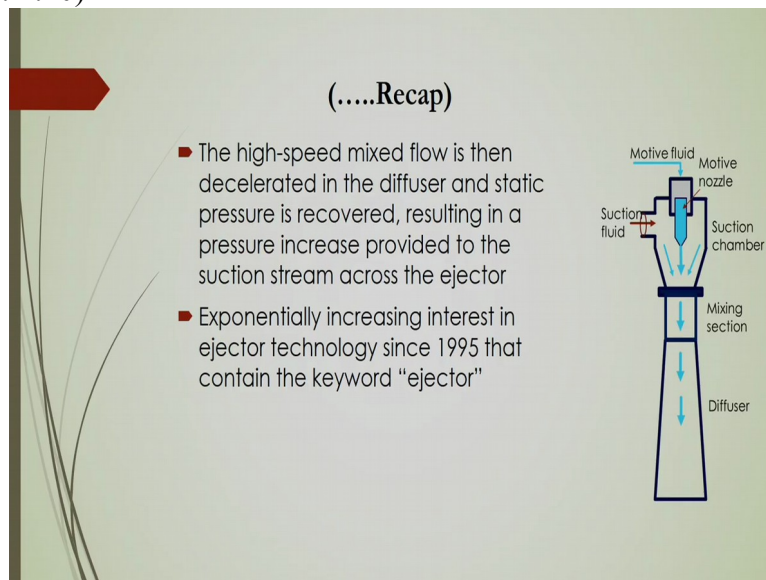
- The motive fluid is expanded through a usually converging-diverging (though sometimes converging-only) nozzle to high velocity and low pressure.
- This high velocity and low pressure is used to entrain the suction fluid through the suction nozzle
- The motive and suction fluids are then mixed in the mixing section



The diagram illustrates the internal structure of an ejector system. It consists of several key components: a 'Motive nozzle' at the top where 'Motive fluid' enters; a 'Suction chamber' where 'Suction fluid' is drawn in; a 'Mixing section' where the two fluids combine; and a 'Diffuser' at the bottom where the mixed fluid exits. Arrows indicate the downward flow of the motive fluid, the inward flow of the suction fluid, and the downward flow of the mixed fluid through the diffuser.

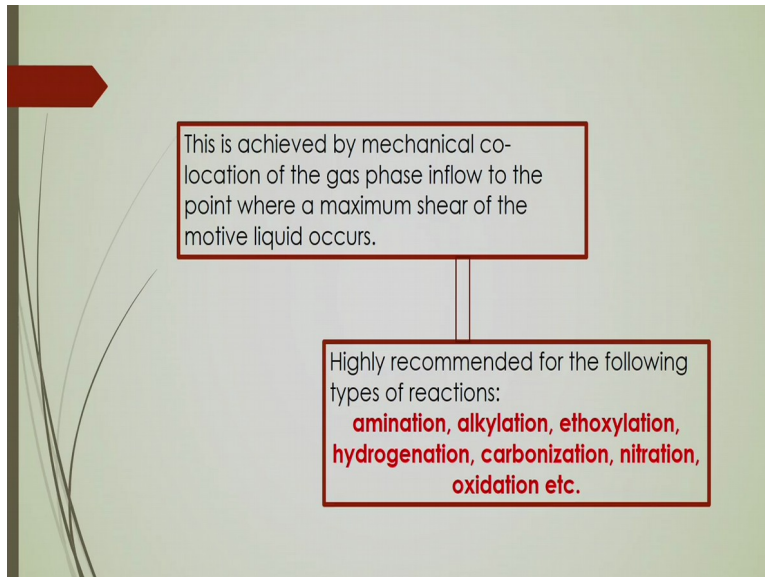
And also, we have discussed that the motive fluid whenever it will be expanded through a usually converging diverging section of that ejector, sometimes that nozzle will be giving the high velocity and because of which there will be high kinetic energy and formation of the low pressure somewhere as per that Bernoulli's equation and then the high velocity and the low-pressure will be used to entrain the suction fluid through the suction nozzle. And the motive and suction fluids are then mixed in the mixing section. And the motive and suction fluids are then mixed in the mixing section.

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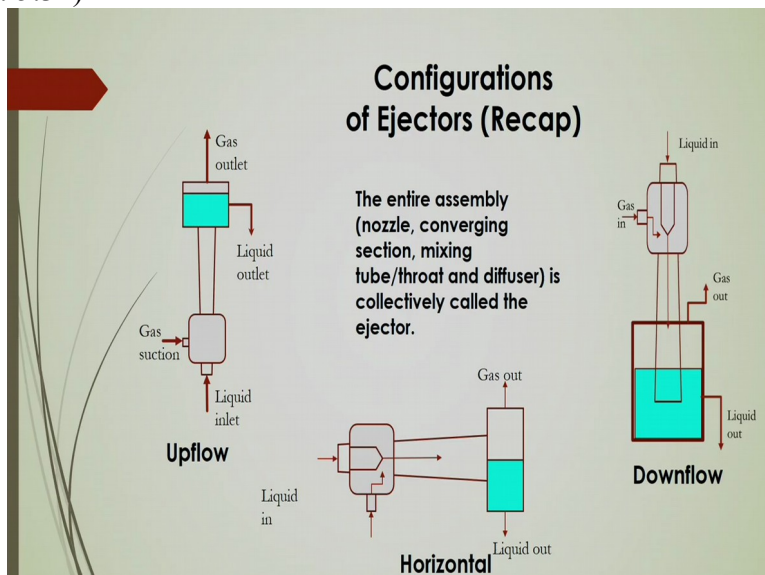
And also, this high-speed mixed flow is then decelerated in the diffuser and static pressure where it will be recovered and then it will result in a pressure that will increase to the suction stream across the ejector and then exponentially whenever it will be increasing interest in the ejector technology that can be utilized for that particular chemical engineering process for the intensification. And it is happening since 1995 as a keyword of that ejector system and its applications.

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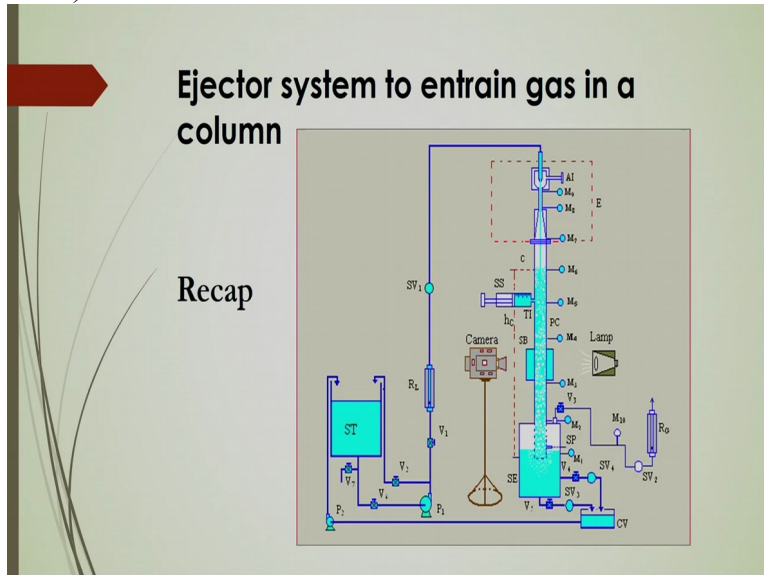
So, this is basically achieved by the mechanical co-location of the gas phase inflow to the point where a maximum shear of the motive liquid occurs **and, in that case,** highly momentum you know that forces will be actually used to recommend for the following types of reactions like that amination, alkylation, ethoxylation, hydrogenation, carbonization, nitration, oxidation, et cetera. There are several other applications in the chemical process based on this ejector systems where that generation of the interfacial area just by dispersing the liquid opposite to its buoyancy force in the case of downflow system.

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And **also**, we have discussed the different configurations like upflow, downflow and horizontal, that is in details in previous lectures we have described.

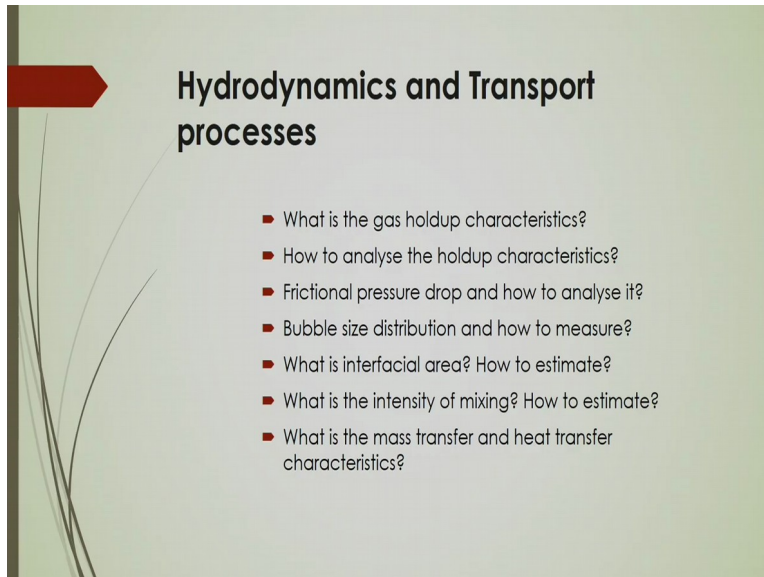
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Now we will discuss that hydrodynamics in that downflow system where the ejector is being used for the entraining gas in the column. See this setup here in the slides that how this liquid is coming inside through the jet and whenever jet is coming to the column, it will suck the gas from this location and then it will be dispersed in the liquid by entraining that gas as a dispersed phase of bubbles.

And due to this downward movement of the liquid through the column and then it will come to the separator and in the separator, it will be separated. And it will be coming out from the outlet of this system here.

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Hydrodynamics and Transport processes

- What is the gas holdup characteristics?
- How to analyse the holdup characteristics?
- Frictional pressure drop and how to analyse it?
- Bubble size distribution and how to measure?
- What is interfacial area? How to estimate?
- What is the intensity of mixing? How to estimate?
- What is the mass transfer and heat transfer characteristics?

So, in this case what are the different hydrodynamic characteristics whenever this downflow of the liquid whenever it will be entraining that gas as a dispersed phase of bubbles and in that case what are the different mechanisms and what are the different hydrodynamic characteristics? The mechanism we have already discussed but what are the hydrodynamic and transport characteristics for this system? Now in this case, you have to know that what will be the hold-up characteristics, gas hold-up characteristics and also how to analyze that hold-up characteristics in the downflow system and also what are the different frictional pressure drop and how to analyze it?

Bubble size distribution or drop size distribution and how to measure this drop size or bubble size in this particular system? And what is the interfacial area? How to estimate those interfacial area that is created by generating that bubbles or droplets in this particular downflow systems by ejector system? And also mixing is the one important factor. In this case, whenever jet will be plunging into the pool of the liquid and forms bubbles and dragging it downward against its buoyancy, in that case you will see there will be intense mixing of that gas liquid in a mixing zone.

So, in that case, you have to know what is that degree of mixing or what extent of mixing happening inside the column whenever jet will be plunging these gases or entrapping that gases or entrainment of that gases into the column. And based on that, what should be the mass transfer

and heat transfer characteristics, that also to be known. **So**, these are the different hydrodynamic and transport phenomena of this downflow system for the process intensification.

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Volume fraction or holdup of phase

- The volume fraction of the dispersed phase is defined as
$$\alpha_d = \frac{V_d}{V_t} \quad \text{Or } A_d/A \quad (1)$$
- The volume fraction of the continuous phase
$$\alpha_c = \frac{V_c}{V_t} \quad \text{Or } A_c/A \quad (2)$$

where V_d is the volume of the dispersed phase, V_c is the volume of the continuous phase in the total volume $V_t (= V_d + V_c)$

The volume fraction of the dispersed phase is often referred to as holdup

Now since this volume fraction or hold-up of the phase is one of the important hydrodynamic characteristics there, because it governs the mass transfer phenomena and heat transfer phenomena. If you increase the hold-up or volume fraction of the particular phases in that systems, you will see there will be increase of interfacial area because if you increase the gas hold-up, suppose gas-liquid mixer in the downflow system, then in this case, this interfacial area, specific interfacial area directly related to that gas hold-up.

And **also**, if you have that size of that gas bubbles, then you can say what will be the amount of that interfacial area based on that size of that bubbles. **So**, this gas hold-up and size of the bubbles are those most important factors for the creation of the bubble and also the interfacial area there. So in this case, you have to know what actually, how this gas hold-up will be defined and how it can be measured. So, this volume fraction of the dispersed phase is called hold-up. **Generally**, it is called hold-up. Now if it is a particular gas phase volume fraction then it will be called as gas hold-up.

So, this gas hold-up will be defined as this volume of that gas divided by the total volume of gas and liquid. So, in general, it can be defined as that if suppose there is a mixture of discrete phase and continuous phase, then you can consider that discrete phase as d and continuous phase as

that c. So, in that case, if we consider that volume of the discrete phase, discrete phase in that case you can say that **those phases** will be dispersed in the continuous phase, that will be discrete phase.

So, this discrete phase, volume of the discrete phase out of the total volume of discrete and continuous phase, **those fractions** will be considered as a volume fraction of the dispersed phase or discrete phase. So, this is generally denoted by alpha and d here in this case, it is denoted for that dispersed phase. So,

$$\alpha_d = \frac{V_d}{V_t}$$

What is that V_d ? V_d is the volume of the dispersed phase and T is the total volume of the dispersed phase and continuous phase. It can be defined either way like here, this cross-sectional area of the dispersed phase that is occupied in the system or in a particular vessel or in a conduit you can say and out of total cross-sectional area.

So, this volume fraction of the dispersed phase can also be defined by this equation here like A_d divided by A. A_d is the cross-sectional area of dispersed phase and capital A, that is total cross-sectional area of the vessel or that conduit in which this gas and liquid mixture will be there. And the volume fraction of the continuous phase of course will be if it is binary system, that remaining portion will be the that volume fraction of the continuous phase. So, that volume fraction of the continuous phase can be defined as the volume of that continuous phase out of total volume of discrete and continuous phase.

So, it will be defined as

$$\alpha_c = \frac{V_c}{V_t}$$

where V_c is the volume of continuous phase and V_t is the total volume of the continuous and discrete phase. It is also defined as like this, A_c by A. A_c is called that cross-sectional area of the continuous phase and A is called the total cross-sectional area of the gas and liquid phase where it will be flowing in a continuous phase. So, in this case, V_d is the volume of the dispersed phase and V_c is the volume of the continuous phase in the total volume of V_d and V_c .

So, in this case volume fraction of the dispersed is often referred to as the hold-up. So, you have to remember that hold-up means volume fraction of the particular phases. So, either maybe you know that continuous or you know dispersed phase.

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For gas-liquid two-phase flow, Gas is dispersed and liquid is continuous

Gas holdup (α_g)

$$\alpha_g = \frac{\text{Volume of gas}}{\text{Volume of gas + volume of liquid}} = \frac{V_g}{V_g + V_l}$$

Liquid holdup (α_l)

$$\alpha_l = \frac{\text{Volume of Liquid}}{\text{Volume of gas + volume of liquid}} = \frac{V_l}{V_g + V_l}$$

$$\alpha_g + \alpha_l = 1$$

Now for the gas liquid two-phase flow, the gas is dispersed and liquid is continuous. In that case, this gas hold-up will be defined as that alpha g, this is alpha g, that means here gas hold-up for the you know gas here. So, α_g for here gas. So, it will be simply volume of gas out of volume of gas plus volume of liquid. So, it will be like this,

$$\alpha_g = \frac{\text{Volume of gas}}{\text{Volume of gas + volume of liquid}} = \frac{V_g}{V_g + V_l}$$

V_l is the volume here. And liquid hold-up will be is equal to α_l that will be is equal to volume of liquid by volume of gas plus volume of liquid.

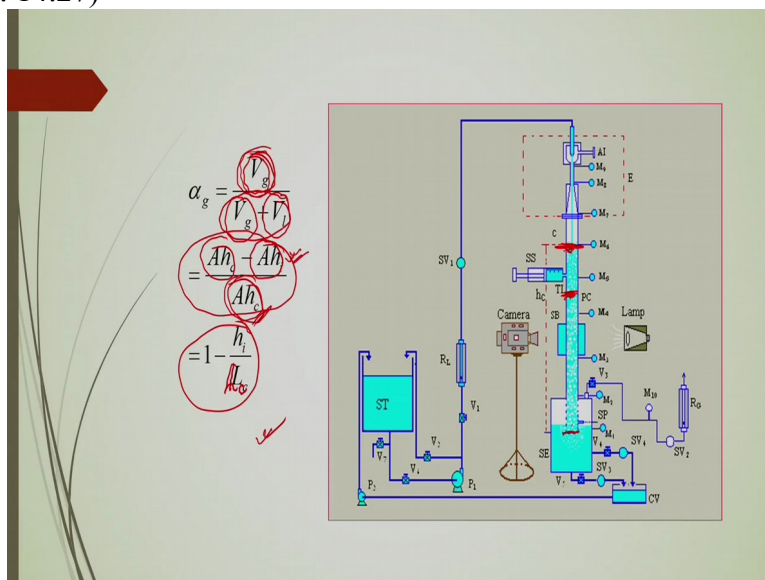
$$\alpha_l = \frac{\text{Volume of Liquid}}{\text{Volume of gas + volume of liquid}} = \frac{V_l}{V_g + V_l}$$

Here it will be defined like this. But these **two-phase**, whenever we are considering the binary phase, then α_g and α_l , So, in that case, the summation of this α_g and α_l will be equals to 1 for the binary system.

Whereas, if you are considering that the three-phase system of gas, liquid and solid, So, in that case, what will be the volume fraction of the gas? Out of total volume, what will be the volume fraction of liquid? Out of total volume, what will be the volume fraction of solid out of total volume? So, in that case, you can get separately that α_g , α_l and α_s . α_s is the volume fraction of the solid. So, α_g , α_l and α_s , that will be is equal to 1. So, that is for three-phase system.

So, here in this case, if you consider this figure, if you consider that is one column where you can consider that countercurrent operation or co-current operation, if you consider that co-current operation, from the bottom gas will be flowing and liquid also will be flowing, So, in that case the gas will be coming out at this outlet and liquid will be that outlet from this location. So, this will be a co-current upflow whereas the downflow system, where liquid will be coming out from the top and gas is also coming in from the top. So, that will be co-current from the top to bottom. So, this is called downward movement of the gas flow and liquid flow. So, this is called downflow system.

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So, in this case, how you will measure that gas hold-up? Very interesting that if you suddenly stop that operation, okay, let us consider this first. Here suppose without any operation, without

any the gas liquid flow rate, what will happen? In a column, if you consider this is the column **and**, in the column, if you consider that initially the liquid phase, initially liquid phase will be a certain height like this. So, this is your initial liquid phase. Okay. Now if you allow that liquid jet to be plunged on this pool of the liquid, you will see the gas will be entrained and accordingly you will see after entraining that gas, the mixture of this gas liquid volume will increase and it will go up to a certain level.

So, this up to this, it is called mixture volume. So, this mixture volume it will come at a certain gas and liquid operation velocity or operational flow rate. So, in that case you will see at a particular jet velocity here in this case, you will see that gas liquid mixture will be coming up to a certain height of this column. Now if you add that particular condition, if you take this measurement of height, then if you can calculate what will be the volume of that gas-liquid mixture, that easily you can calculate what should be the cross-sectional area of the column and what will be the height of this here, height of this gas-liquid mixture, then you can easily calculate what should be the volume of this gas liquid mixture.

This is simple, suppose the gas liquid mixture height is h_c , then you can calculate what is the A into h_c . That will be your total volume of gas liquid mixture. Now very interesting that if you suddenly stop your operation of this liquid, what will happen? You will see this gas-liquid mixture height gradually decreased and it will come to a certain height where you can get the only clear liquid height. That means that after a certain time, all the gas which was entrained into the liquid by the liquid jet, it will be **degassing** or disengagement of the gases and gradually whenever disengagement will happen, the gradually that liquid level it will come down to a certain height here.

So, this will be your clear liquid height, there, there will be no entrained of the gas into the liquid. So, that will be clear liquid height. So, we now know that at a particular operating condition, we are getting the total gas liquid height and after a certain time, after stopping that operation, you can get the clear liquid height. Now at this clear liquid height, what will be the volume of that liquid that is inside the column? That you can calculate easily. What will be the cross-section? That is A into the height of this clear liquid height. Let it be here, h_i .

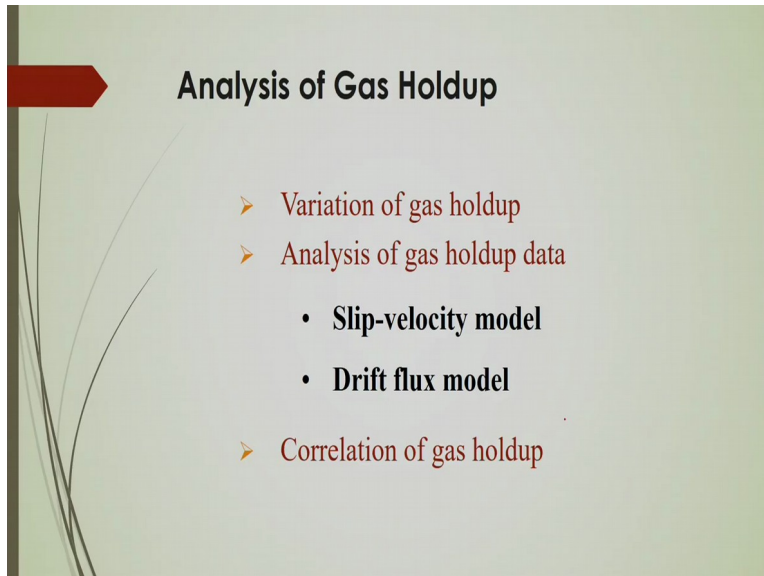
So, this clear liquid height and this clear liquid volume, if you calculate and if you subtract from this total volume of this gas liquid mixture volume, then you can easily obtain the how much volume of that gas is entrained into the liquid. So, that entrained gas volume will be called as volume of the gas that has been entrained. Now this gas volume fraction, the volume fraction of this volume can be calculated by this volume of this gas divided by the total volume of gas liquid mixture volume.

So, that is why, we can have this, this is your volume of gas entrained and this is your volume of gas entrained plus volume of liquid here. So, this will be your total gas liquid volume. And this is your what? What is this? This is the volume of gas entrained. Now this can be calculated, this volume of gas can be calculated, this total gas liquid volume mixture minus what will be that? Clear liquid volume mixture. And out of these, you can calculate this total volume of gas liquid mixture here. So, from this, you know that division of this gas liquid you know from this gas liquid mixture, what will be the gas volume?

That you can calculate A into h_c minus A into h_l . Then if you divide it by the total volume fraction, then you can obtain that gas volume fraction there. So, it can be easily calculated as that if you cancel A then you can simplify it by this 1 minus h_l by h_c here. h_c means total gas liquid height here. So, here, you can calculate the gas hold-up in this downflow system, it is simply 1 minus h_l by h_c . What is h_c ? h_c is the gas liquid mixture height during the operation at that particular condition and what will be the h_l ?

h_l is the initial or after degassing the all gases, what will be the clear liquid height? So, from this calculation, you can easily measure what will be the volume fraction but this volume fraction of the gas will be overall gas volume fraction. It will not give you that locational gas volume fraction or you can say that axial gas volume fraction you cannot get here. It is simply you know that overall gas volume fraction. And also you will not get this radial gas hold-up, this gas hold-up maybe you know that changing with that radial position but if your column is very narrow, if it is less than 5 centimeter you can expect that radial gas hold-up distribution will be that negligible and in that case only overall gas hold-up will be the enough to consider there.

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Now how to analyze this gas hold-up? Whenever you are measuring this gas hold-up, you will see there will be a different effect of parameters on this, the effect of variables on this gas hold-up. Maybe this gas hold-up will be changing with respect to that nozzle diameter, this gas hold-up will be changing with respect to physical properties of the system. You can say that this gas hold-up will be depending on the how this entrainment is happened and this entrainment, what will be the minimum entrainment velocity and what will be the minimum entrainment velocity beyond which that you can expect that gas hold-up?

This is the things. And after getting that overall gas hold-up, you can analyze that gas hold-up to predict based on that experimental data by a certain model. So, there are actually 2 models are available to analyze this gas hold-up data. That is called slip-velocity model, another is called drift flux model. And also, since this gas hold-up depending on different variables, maybe you know that more than 2, 3, even 5 also. There are several variables at a time that will be affecting on that gas hold-up. So, in that case, this complex you know that dependency of these variables on this gas hold-up effect, then you can actually predict this gas hold-up based on these different variables just by making a **general empirical correlation**.

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Slip Velocity Model

The slip velocity of bubbles relative to surrounding liquid is defined as

$$V_s = \pm[V_g - V_l] = \pm\left[\frac{V_{sg}}{\epsilon_g} - \frac{V_{sl}}{1 - \epsilon_g}\right]$$

To account the interaction between bubbles within the column, the following relationship is used for slip velocity

$$V_s = V_b f(\epsilon_g)$$

where V_b is the bubble rise velocity and $f(\epsilon_g)$ shows the effect of the interaction of neighbouring bubbles

So, we can analyze this gas hold-up. First of all, this slip velocity model, then drift flux model we will discuss also. Now as per slip velocity model, we can say that this slip velocity of the bubbles that will be relative to the surrounding liquid that is defined as that is how that is slip is happening. Means what that whenever gas is flowing against the buoyancy, even you know that along with that liquid, you will see that there will be a slip between that gas and liquid. Now if gas and liquid both have the same velocity, then there will be a slip velocity, it will be that 0 but if there is that opposite of that gas and liquid velocity, in that case you can get that more slip.

That means there will be certain slip that you can obtain by that subtracting or adding that gas and liquid velocity. So, V_s will be defined like this, V_s will be is equal to plus minus V_g minus V_l

$$V_s = \pm[V_g - V_l] = \pm\left[\frac{V_{sg}}{\epsilon_g} - \frac{V_{sl}}{1 - \epsilon_g}\right]$$

and what is that V_g ? V_g is the actual velocity of the gas and V_l is called actual liquid velocity. Whereas this actual gas velocity will be defined as that V_{sg} by ϵ_g or alpha g you can say. V_{sg} is called superficial gas velocity and ϵ_g is called that you know gas hold-up in that gas liquid mixture. So, this will give you that actual velocity.

This actual velocity is greater than the superficial gas velocity and actual liquid velocity is will be is equal to V_{sl} divided by ϵ_l . That ϵ_l is nothing but 1 minus ϵ_g . So, if you know that gas hold-up, then you can easily calculate what should be the you know that actual velocity of the gas and liquid. If you know the actual gas velocity of that gas and liquid, then you can easily calculate what should be the slip velocity. For the downflow system, you have to consider that this negative sign and for that upflow system, that will be a positive sign here.

So, to account the interaction between bubbles with the column, the following relationship can be obtained as here given. So, V_s will be a function of you know that gas hold-up, that is slip velocity will be a function of gas hold-up

$$V_S = V_b f(\epsilon_g)$$

because the slip velocity is actually the relative velocity of the actual gas and liquid velocity, those who are related to the gas hold-up. So, slip velocity can be related by this equation where in this case, one parameter, it is called V_b , this V_b is called bubble rise velocity.

And function of ϵ_g , $f(\epsilon_g)$ will give you that certain function of the gas hold-up in that particular system. So, this V_b will give you the bubble rise velocity and if you know the bubble rise velocity from this slip velocity, you can calculate what will be the function of that gas hold-up there. Either way, if you know the you know that slip velocity from that experiment, actual velocity of gas and liquid and if you can calculate the slip velocity then you can directly relate with that certain gas hold-up function. So, that gas hold-up function may be either in a **straight-line** form or some other you know that nonlinear equation may be there.

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Downflow vs upflow: Slip velocity

$$V_s = \lambda \varepsilon_g^\alpha D_R^\beta$$

$$V_b = \lambda D_R^\beta$$

$$D_R = d_n / d_c$$

$V_b = 0.73 - 1.80$
 $\alpha = 3.05,$
 $\beta = 0.58,$
 $\lambda = 4.51$

Davidson & Harrison (1966)	Bubbly Upflow	$V_s = V_b (1 - \varepsilon_g)^n$	$V_b = 0.79, m = 1.0, n = -1;$
Zahradnik et al., (1997)	Bubbly Upflow		$V_b = 1.0, m = 1.0, n = -1;$
	Bubbly Downflow		$V_b = 0.03, m = 1/3, n = 1.0, m = 1, n = -2.28;$

So, there are several you know functions will be there for that function of the gas hold-up. For the special case of downflow system, you will see that this slip velocity will be a function of gas hold-up like this here.

$$V_s = \lambda \varepsilon_g^\alpha D_R^\beta$$

$$V_b = \lambda D_R^\beta$$

$$D_R = d_n / d_c$$

It is given that V_s will be is equal to

$$V_s = \lambda \varepsilon_g^\alpha D_R^\beta$$

where D_R is called that ratio of nozzle diameter to the column diameter and in this case, V_b will be defined as λD_R^β . Here λ is one parameter which will be obtained from the experimental data.

So, in this case, downflow case, this V_b can be obtained within this range of velocity of 0.73 to 1.80 that is meter per second, while alpha is the coefficient that will be 3.05 and beta will be is equal to 0.58 whereas lambda, that is parameter will be is equal to 4.51. So, for this downflow system you can obtain this slip velocity based on this equation. Once you know this slip velocity, then what will be the ϵ_g ? You can easily calculate that ϵ_g .

And other way that upflow system, you can compare with this downflow system, then you will see that this V_s will be is equal to V_b into 1 minus Epsilon g to the power m to the Power n where for that upflow system, you will see that V_b will be is equal to 0.79, m is equal to 1.0 and n is equal to minus 1. And for again upflow system, another correlation it is developed based on that Harrison experimental data, then they got that this V_b is equal to 1.0 and m will be 1.0 and n is equal to minus 1. Whereas in case of bubbly downflow, this V_b should be is equal to 0.03 and m is equal to 1 by 3, n is equal to 1.0 and m is equal to 1, n is equal to minus (2 point) you know 2.28 as per Zahradnik et al., 1997.

So, here in this case, this is actually depending on the you know that how you were defining that gas hold-up function. So, in case of Majumder et al., that is published in you know that 2006 that here in this case, you will see that as per this function, this V_b should be within this range where alpha beta will be defined like this. But in case of Zahradnik case, they obtained they have actually suggested this function of this slip velocity like this and according to their model, this V_b will be like this, it is given here, 0.03 whereas m and n are 1 by 3 and 1.0 respectively there for the downflow system.

So, we can get that upflow and downflow system in the separate way for analyzing that what should be the bubble rise velocity in the downflow system and what will be the upflow system based on this slip velocity model.

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Zuber -Findlay Model

To analyze the gas holdup data by drift flux model the average gas velocity is plotted against the gas-liquid mixture velocity, V_m , as given by the following equation;

$$\bar{V}_g = V_{sg} / \epsilon_g = C_0 V_m + V_d$$

where

C_0 is the distribution parameter, which accounts for effect of non- uniform flow and

V_d weighted average drift velocity, accounts for the effect of local relative velocity.

Zuber and Findlay (1965)

Another model, it is very important to analyze that gas hold-up, how to predict that gas hold-up based on this model. It is called the drift flux model.

$$\bar{V}_g = V_{sg} / \epsilon_g = C_0 V_m + V_d$$

In this case, you will see whenever gas will be flowing as a dispersed phase of bubbles, you will see that the one gas bubbles will sometimes push another bubble. Maybe the surrounding gas bubbles also will be drifted by the bottom bubbles or surrounding bubbles. So, in this way one bubbles will interact with the other bubbles.

So, whenever they will be moving, the you know that near bubbles will push another near bubble to that you know that moving up, even going downwards, so, there will be interaction and there will be pushing or you can say drift one bubble **by another bubble**. So, this is called drift. Drift velocity will obtain by another bubble. So, in that case what will be the normal rise velocity of the bubbles? That will be enhanced by the drift of another bubbles. This happened for the you know that droplet also. So, based on that drift rate or interaction of the bubbles that what will be the change of bubble velocity and what will be the flux of that velocity change?

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Zuber -Findlay Model

To analyze the gas holdup data by drift flux model the average gas velocity is plotted against the gas-liquid mixture velocity, V_m , as given by the following equation:

$$\bar{V}_g = V_{sg} / \epsilon_g = C_0 V_m + V_d$$

where

C_0 is the distribution parameter, which accounts for effect of non-uniform flow and

V_d weighted average drift velocity, accounts for the effect of local relative velocity.

Zuber and Findlay (1965)

Zuber and Findlay 1965, they have developed this model to analyze that gas hold-up based on that interaction of the gas bubbles and also the drift, that means relative velocity of the gas bubbles whenever it will be flowing in a certain reactor of gas liquid system. So, in that case, V_g will be actually considered as the actual gas rise velocity and it will be defined as V_{sg} by Epsilon g. What is that V_{sg} ? V_{sg} is the superficial gas velocity and Epsilon g is the gas hold-up in that particular system. So, it will be a function of you know gas liquid mixture velocity and according to that model, this function maybe you know that linear function and maybe nonlinear function.

So, according to that, Zuber and Findlay and considering that interaction of the gas bubbles and also velocity distribution of the gas inside the reactor, they suggested that this actual velocity of the gas will be as a function of mixture velocity and it will be a linear function. So, based on that linear function, they have defined that this V_g will be is equal to what? That it will be C_0 into V_m plus V_d .

$$\bar{V}_g = V_{sg} / \epsilon_g = C_0 V_m + V_d$$

This V_d will be representing that weighted average drift velocity of that discrete phases which will accounts for the effect of local relative velocity of the gas bubbles there.

And this C_0 is the coefficient of that drift flux model, this C_0 is the distribution parameter which accounts for the effect of non-uniform flow. If there is a non-uniform flow, you will see there will be interaction of the bubbles will increase. That means your radial directions, axial direction, random motion of the bubbles and interaction of the bubbles will be increased. So, it will actually give you the degree of that interactions based on that distribution of the velocity whether it will be uniform velocity or non-uniform.

Generally, for uniform velocity, you will see the interaction of the bubbles will be very less. So, in that case, you will see that drift velocity will be relatively lower than that you know that non-uniform flow case. So, C_0 is the distribution parameter and it will account the effect of non-uniform flow. Now how will you estimate that C_0 and V_d ? What will be the value for that? It may be for upflow system, it may be for downflow system. That will be different. And of course, it will give you that uniformity, non-uniformity, degree of uniformity, non-uniformity and also the drift parameters or drift velocity in the system.

So, it can be obtained from the experimental data. First of all, you have to do experiment for the estimation of gas hold-up. Once you know the gas hold-up, you can have the what is that? Actual velocity of the gas velocity there. So, if you know the gas hold-up, then what will be the superficial gas velocity? That can be calculated from your gas flow rate divided by the cross-sectional area. So, once you know that superficial gas velocity and gas hold-up, you can easily calculate what should be the you know actual velocity of the gas.

Now this will be is equal to you know that C_0 into mixture velocity. Mixture velocity is the summation of superficial gas velocity and superficial liquid velocity. So, once you know that superficial gas velocity and liquid velocity and the gas hold-up, you have to draw a you know that graph based on your experimental data like this here, V_{sg} plus V_{sl} , this will be in your X axis, that is simply V_m and in the Y axis you can have this V_g , that means your actual gas velocity, that will be simply here, V_{sg} divided by ϵ_g .

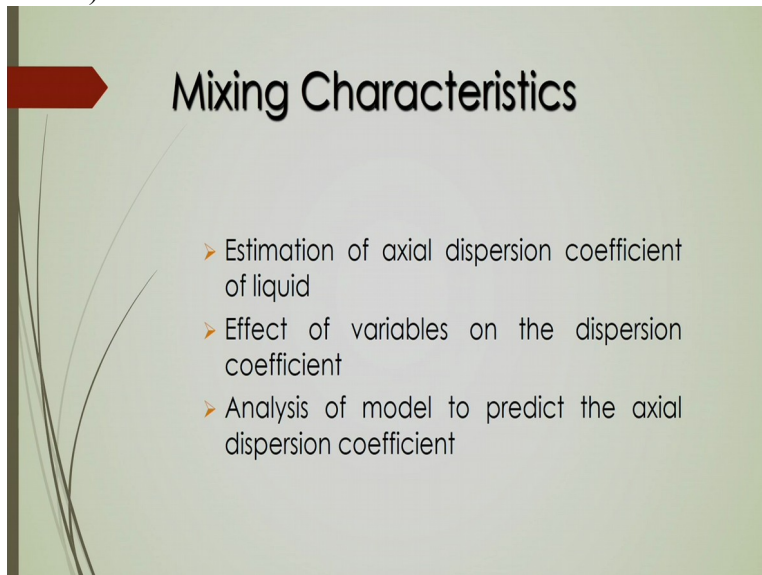
Now you will see whenever you are plotting this V_g versus V_m , you can get this data like this. This type of data you can expect. This type of profile you can expect from your experimental data. So, if you add this data here, you will see there will be a straight line. Now if you extend this straight line, you will see there will be intercept of the straight line to this Y axis. So, this

given this equation to calculate that V_d . So, different investigators, they have developed different correlations for C_0 and V_d . You can get the different value of C_0 and V_d from these correlations.

Whereas for downflow system, this Majumder et al in 2006, they have developed this correlation for this C_0 and V_d based on their experimental data and you can calculate this C_0 and V_d from these correlations here. So, once you know that C_0 and V_d based on your experimental operating conditions, then you can easily get what should be the C_0 , what should be the V_d , what should be the velocity distribution coefficient and what should be the drift flux velocity. Once you know that velocity, then you can easily calculate what will be the average velocity.

Now without doing experiment if you change the operating variables accordingly you can get what should be the actual velocity inside the column. So, based on this drift flux model of Zuber and Findlay 1965 you can assess, you can analyze the you know that gas hold-up from this equation.

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Mixing Characteristics

- Estimation of axial dispersion coefficient of liquid
- Effect of variables on the dispersion coefficient
- Analysis of model to predict the axial dispersion coefficient

Next, another you know that important hydrodynamic characteristic, it is called mixing characteristics. So, in that case, what will be that degree of gas liquid mixing or gas liquid solid mixing there inside the column, that is very important because **these mixing characteristics** will give you that what will be the performance of that particular mass transfer or heat transfer for the particular chemical engineering operation. So, in this case, you have to estimate degree of mixing or intensity of the mixing by certain method and also what are the different you know

that variables that is affecting on the dispersion coefficient and also what will be the model to analyze that axial dispersion there.

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Estimation of axial dispersion coefficient of liquid

Mixing Characteristics has been analyzed by axial dispersion model (ADM)

$$\frac{\partial C}{\partial t} + E_z \frac{\partial^2 C}{\partial Z^2} - \frac{V_L}{1 - \epsilon_g} \frac{\partial C}{\partial Z}$$

Solution of ADM [Levenspiel & Smith (1957)] with open - open boundary condition

$$C_\theta = \frac{1}{2\sqrt{\pi\theta/Pe}} \exp\left[-\frac{(1-\theta)^2 Pe}{4\theta}\right] \quad \text{BC: } \begin{matrix} Z = -\infty; \frac{\partial C}{\partial Z} = 0 \\ Z = +\infty; \frac{\partial C}{\partial Z} = 0 \end{matrix}$$

The slide also features a graph of concentration C versus axial distance Z, showing a bell-shaped curve. Handwritten red annotations include circles around the ADM equation and the solution formula, and a checkmark next to the boundary conditions.

Now estimation of the actual dispersion coefficient of the liquid. You can estimate these mixing characteristics in gas liquid mixer in the downflow system by tracer techniques by axial dispersion model. Now in that case, based on that axial dispersion model, it is simply that how that profile of the concentration will happen in this case. Now what is that C here? C is actually that concentration of the tracers. Suppose in your column if you suppose insert some tracer like non-reacting you know chemical compounds, like air water system.

If you add some sodium chloride solution like 10 percent of the total volume you supply, you will see with respect to time the concentration of that tracer particles will be changing inside the column and that you can analyze just by taking that sample and analyzing its concentration of the tracer by instrument like conductivity meter or something. If you analyze that concentration, you will see that the change of concentration with respect to time, that will be certain you know fraction. Like this if you plot this concentration of tracer versus time, you can have this type of profile.

Now this profile can be predicted by certain model just by stochastic model, from the basic material balance. That material balance will give you this you know ultimate that model equation. It is called axial dispersion model.

$$\frac{\partial C}{\partial t} = E_z \frac{\partial^2 C}{\partial Z^2} - \frac{V_L}{1 - \epsilon_G} \frac{\partial C}{\partial Z}$$

Now based on this axial dispersion model, what you are getting? That how this concentration will be changing with respect to time and also parallelly if there is a flow, then according to that flow, how this concentration will be changing axially. And this is your, what is that? Diffusion of the concentration. So, based on this axial dispersion model, you can analyze what should be the dispersion coefficient that is denoted by E_z here and based on that gas liquid flow and at that particular operating conditions if you fit this concentration profile, experimental concentration profile with respect to time with this concentration profile by this axial dispersion model, you can easily calculate what will be the E_z .

Otherwise, you can solve this equation to get that concentration profile and matching with that experimental data, then you can easily assess what should be the degree of mixing there. Now solution of this axial dispersion model as per Levenspiel in 1957 with open open boundary condition, it is given like this.

$$Z = -\infty; \frac{\partial C}{\partial Z} = 0$$

$$Z = +\infty; \frac{\partial C}{\partial Z} = 0$$

That is C_0 , C_0 is simply C by C_0 what will be the initial condition of the you know, concentration of the tracer and C is the concentration at any time.

$$C_0 = \frac{1}{2\sqrt{\pi\theta/Pe}} \exp\left[-\frac{(1-\theta)^2 Pe}{4\theta}\right]$$

So, C by C_0 will be C theta and here, this theta is called that dimensionless parameter of the time, that is t by t_m .

t_m is called mean residence time, that is theta. So, theta will be is equal to t by t_m . t_m is called mean residence time. That means how long that fluid will residing inside the column. And Peclet number, that is Pe , Pe is defined as here, that means V_L by E_z . Here V means axial velocity of the

liquid, L is the what is that? Length of the gas liquid mixture and E_z is the dispersion coefficient. So, based on this solution of this axial dispersion model and if you feed this solution with the experimental data of this C versus time, then you can get this parameter of you know that Peclet number and the t_m value.

Now how to solve this or how to feed this experimental data with this axial dispersion model? Either by numerically or by some other method. That is called momentum method.

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where

$$Pe = \frac{V_s Z}{(1 - \epsilon_G) E_z} = \frac{\text{Transport by convection}}{\text{Transport by dispersion}} \quad \checkmark$$

$Pe \rightarrow \infty \Rightarrow \text{Plug flow}$
 $Pe \rightarrow 0 \Rightarrow \text{Completely mixed flow}$

$\sigma_\theta^2 = \frac{\sigma^2}{t_m^2} = \frac{2}{Pe} + \frac{8}{Pe^2}$ Levenspiel & Smith (1957)

$$\sigma^2 = \frac{\sum (t_i - t_m) C(t_i) \Delta t_i}{\sum C(t_i) \Delta t_i} \quad \theta = \frac{t}{t_m} \quad t_m = \frac{\sum t_i C(t_i) \Delta t_i}{\sum C(t_i) \Delta t_i}$$

Based on that moment method, you can get this solution of this method like this. Here, from the first moment, you can get the mean residence time. It will be defined as this here. If you know the concentration with respect to time, then you can calculate t_m from this equation

$$t_m = \frac{\sum t_i C(t_i) \Delta t_i}{\sum C(t_i) \Delta t_i}$$

and then theta will be is equal to t by t_m

$$\theta = \frac{t}{t_m}$$

and from the second moment of this equation, you can have this variance of the distribution of this concentration from this equation, based on this open open boundary condition and this Sigma square that is called variance that you can calculate from your experimental data as.

$$\sigma^2 = \frac{\sum (t_i - t_m)C(t_i)\Delta t_i}{\sum C(t_i)\Delta t_i}$$

Once you know that Sigma square, and t_m value, then you can easily calculate what will be the sigma theta square. This sigma theta square will be a function of pecelet number.

$$\sigma_\theta^2 = \frac{\sigma^2}{t_m^2} = \frac{2}{Pe} + \frac{8}{Pe^2}$$

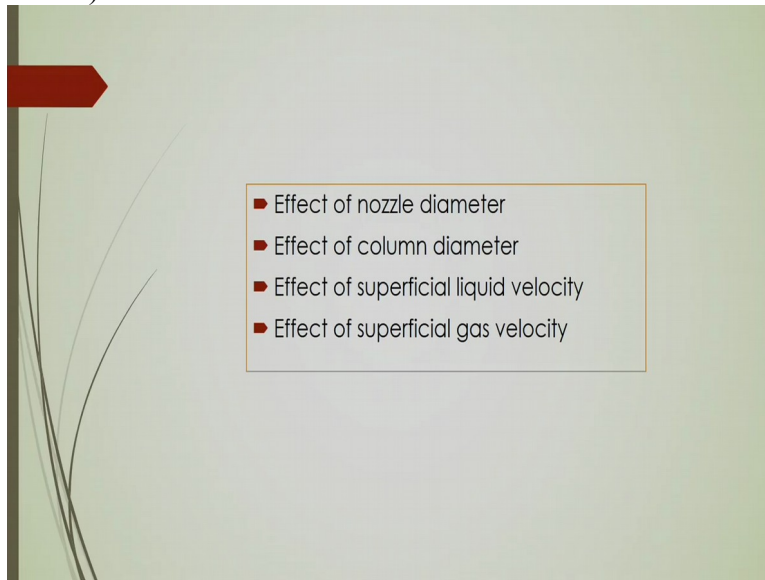
The Peclet number is simply defined as here,

$$Pe = \left(\frac{V_{sl}Z}{(1 - \epsilon_G)E_z} \right) = \frac{\text{Transport by convection}}{\text{Transport by dispersion}}$$

V_{sl} is the superficial liquid velocity, Z is the that means here axial length of you know that effective length of gas liquid mixture where the tracer is input and from where the sample of that tracer is collected, the length of these 2 points.

And the physical significance of this Peclet number is the transport by convection, by transport by dispersion. If Peclet number is going to infinity, then you can say that plug flow, there will be uniform that flow whereas if Peclet number is tends to 0, that means you can say that gas liquid mixing will be there, complete mixing. So, based on this, you can get the degree of mixing in between based on your Peclet number data. Now this is the way to find out that Peclet number. Once you know that Peclet number you can access it by dispersion number also. 1 by Peclet number it is called dispersion number. If dispersion number is going to infinity, then you can say the more mixing. If dispersion number is going to 0, then you can say that plug flow phenomena will be there.

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Then now what are the different effects of these variables on this dispersion coefficient? So, you can say that if you increase the nozzle diameter, that means kinetic energy will be less. That means your jet velocity will be less, less kinetic energy distribution, in that case dispersion will be less. Whereas if you increase the column diameter, you can say that there will be a mixing, more mixing inside the column because the gas liquid mixture will get more circulation inside the bed and more residence time will be inside the bed. So, in that case, you can get more mixing there and more dispersion there.

If you increase the superficial liquid velocity, you can get more mixing because their energy distribution will be high. Even if you increase the superficial gas velocity also, there will be you know more dispersion of that liquid inside the bed because there will be that internal circulation, even the random motion of the bubbles will disperse the fluid element there in more way. In that case, increase the gas velocity, increase the gas hold-up and if you increase the gas hold-up, you can get that more liquid velocity inside the bed, more axial liquid velocity and because of which you can get the more kinetic energy, more momentum transfer, more dispersion there.

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Model Analysis

Velocity distribution model

Basis:

The mixing of liquid in homogeneous system occurs under the combined action of variation of velocity and the motion of gas bubbles in the liquid [Taylor Model (1953)].

Now you have to analyze these mixing characteristics by certain model. It is called velocity distribution model. Based on that velocity distribution model, what is the basis? The mixing of the liquid in the homogeneous system that will occur under the combined action of the variation of the velocity and the motion of the gas bubbles in the liquid. So, this is actually based on the concept of Taylor, that is given in 1953 that they told that this mixing will happen only under the action of variation of the velocity and the motion of the gas bubbles. Based on that concept, this velocity distribution model is developed to analyze the mixing characteristics.

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According to Aris (1956)

$$E_z = \frac{d_c^2 V_0^2}{k D_m} + D_m$$

where

D_m represents molecular diffusion coefficient
 V_0 represents local velocity and
 k determines the form of velocity distribution.

Now according to the Aris 1956, they told that this axial dispersion coefficient, E_z will be a function of that axial velocity of the liquid as well as what is that, column diameter. Now if you functionalize this, then you can have this equation based on that Aris, that will be is equal to

$$E_z = \frac{d_c^2 V_0^2}{k D_b} + D_b$$

What is that d_c ? The column diameter, V_0 is the axial liquid velocity, and K , K is called that you know velocity distribution that will give you form of velocity distribution.

And the D_m , D_m is actually the molecular diffusion coefficient. In this case, if you are considering that bubble is moving, so, here instead of D_m , you have to consider that bubble diffusion there or bubble movement. So, bubble dispersion coefficient in that case you can get.

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The dispersion coefficient due to bubble motion, D_b is uniform in the column and the contribution of D_b to the overall process is much larger than that of molecular diffusion

$$E_z = \frac{d_c^2 V_0^2}{k D_b} + D_b$$

D_b and k depends on the operating variables geometry of the gas distributor

$$V_0 = V_{10} - V_{1w}$$

$$V_{10} = -V_{1w} = V_1 \quad V_0 = 2V_1$$

$$V_1 = 17.69 V_{sg}^{0.016} V_j^{1.266} d_n^{2.467} d_c^{-1.894}$$

So, we are defining in that way. The dispersion coefficient due to the bubble motion, that is D_b is uniform in the column and the contribution of the D_b to the overall process is such, in that case it will be much larger than that of molecular diffusion. So, according to that, we are just replacing that D_m by D_b here and D_b and K here depends on the operating variables, that geometry of the

gas distributor here also. And V_0 is the axial you know that velocity, in this downflow system this will be is equal to

$$V_0 = V_{l0} - V_{lw}$$

where V_{l0} is the liquid velocity, axial liquid velocity, and here lw is called the liquid velocity of the wall.

Now this V_{l0} will be is equal to minus V_{lw} according to downflow system and it will be is equal to V_l . So, ultimate V_0 it will come as 2 into V_l .

$$V_0 = 2V_l$$

So, this V_l is the axial liquid velocity.

$$V_l = 17.69 V_{sg}^{0.016} V_j^{1.266} d_n^{2.467} d_c^{-1.894}$$

So, this axial liquid velocity depending on the gas hold-up. That gas hold-up also depending on the superficial gas velocity, jet velocity, nozzle diameter, even column diameter. So, according to that, we can get these correlations to find out that axial liquid velocity at the axis there.

So, once you know that axial liquid velocity in the downflow system, if you substitute here and then draw a graph here like this here. In this case, d_c square V_0 square in the X axis, then sorry. Then we can okay define this way. If we consider that here, this will be as X and then we can write here E_z , that will be is equal to

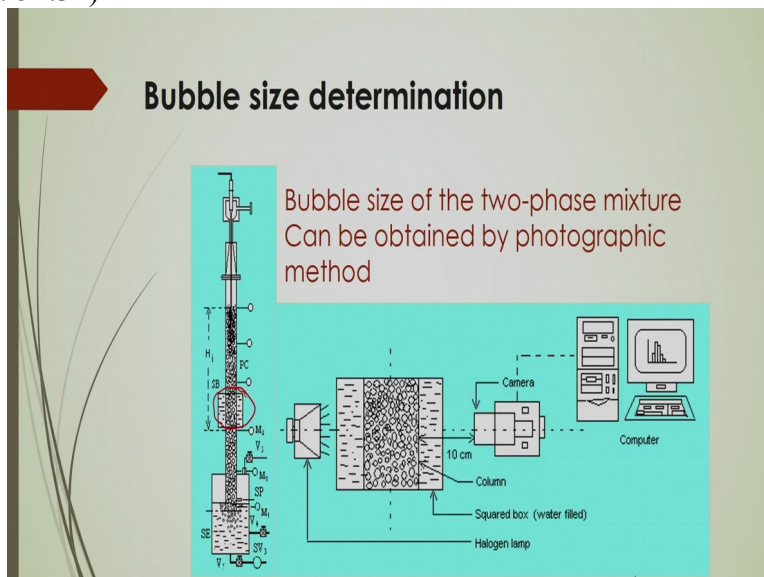
$$E_z = \frac{X}{kD_b} + D_b$$

So, if you plot this as E_z versus your X, then you can get this type of straight-line there. Whereas this intercept will give you the D_b and slope will give you, you can get from this slope as 1 by $K D_b$. So, once you know this slope and intercept, you can easily calculate what should be the K

and D_b . So, once you know this K and D_b , then you can say what should be the velocity distribution and what will be the dispersion coefficient of bubble motion.

So, if you know this dispersion coefficient of bubble motion, from which you can calculate, you can predict that what will be the actual dispersion coefficient of the liquid inside the column. So, this is based on the bubble movement, how you can analyze the axial dispersion coefficient of the liquid inside the column.

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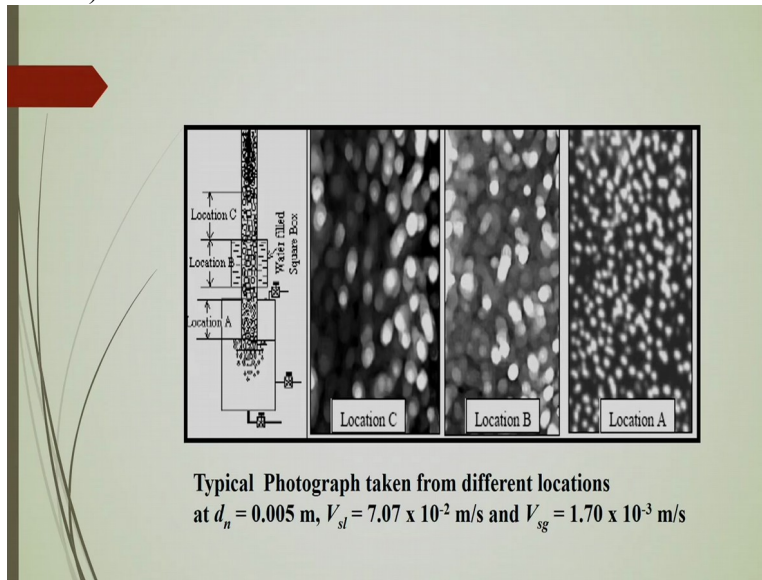


Next is another important hydrodynamic characteristic, bubble size determination. Bubble size distribution, you can estimate this bubble size in the column whenever liquid jet is entraining, that bubble as the dispersed phase of bubbles and it will be moving downwards. You can capture that bubble by a camera, high-speed camera. Once you get this snapshot of this bubble, you can analyze it by suitable image analysis software to find out the size of the bubble. Also shape of the bubbles. So, that is also important here.

It is shown here in this picture that from this location, we are collecting that bubble image by camera. After that, it will be analyzed in the computer by image analysis software. So, after getting this you can find out that interfacial area of this you know that bubble manually, that means surface area of that bubble manually. Once you know the surface area, then you can say that πD will be equal to surface area, that is perimeter. Once you know that perimeter or surface πR^2 , then from which you can calculate what is the radius of that bubbles.

Like this, if you know that more than 500 bubbles manually taking and then radius of that bubbles, average or mean radius of the bubbles or mean diameter of the bubbles, then if you segregate that bubble diameter in a bin, in a class, then within a certain range of that bubble class, you can have that distribution of the bubbles with its frequency.

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Like this here, you will see that at different locations, you will see that these are the bubble picture and each bubbles, you will see sometimes that will not be the uniform in size. Some bubbles will be elliptical, some bubbles will be elongated, some bubbles will be spherical, so there are different shapes of the bubbles at different locations you will see and in that case, you have to concentrate on the mean bubble size. And before going to that mean bubble size, you have to analyze individual bubbles whether it will be you know that spherical in shape or not.

If it is not spherical in shape, you have to convert it to the equivalent spherical shape. If you are getting that elliptical bubble, you have to consider this elliptical bubble equivalent to the spherical bubble, just by considering that equivalent bubble diameter and equal to the volume of that elliptical bubble.

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Equivalent spherical bubble diameter

$$d_{be} = \sqrt[3]{d_{b,max}^2 d_{b,min}}$$

Bubble number flux $\dot{N}(Z) = \frac{\text{Gas volumetric flowrate}}{\text{Bubble volume} \times \text{Cross-sectional area}}$

$$= \frac{Q_g}{\frac{\pi}{6} d_{vs}^3 \times \frac{\pi}{4} d_c^2} = \frac{24 Q_g}{d_{vs}^3 \times d_c^2}$$

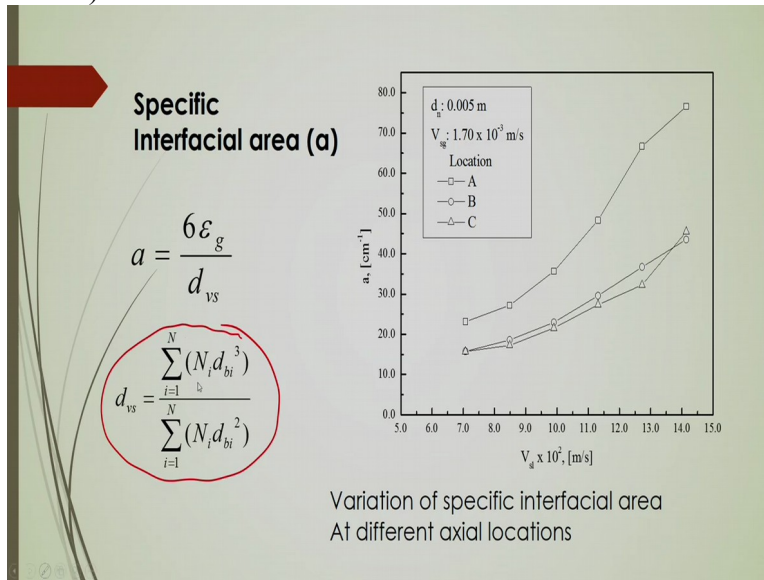
So, just by calculating that elliptical bubble here in this case equivalent bubble diameter how to calculate here by this equation, if you know the major axis and minor axis, that maximum diameter and minimum diameter for that elliptical bubble, then you can easily calculate what should be the equivalent bubble diameter for individual.

$$d_{be} = \sqrt[3]{d_{b,max}^2 d_{b,min}}$$

Once you know that equivalent bubble diameter for 500 bubbles or more than that, you can have the mean bubble diameter.

That mean bubble diameter may be different type of mean, Sauter mean bubble diameter. In this regard, you have to remember the most acceptable or suitable bubble mean diameter there. This is called Sauter mean bubble diameter.

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Equivalent spherical bubble diameter

$$d_{be} = \sqrt[3]{d_{b,max}^2 d_{b,min}}$$

Bubble number flux $\dot{N}(Z) = \frac{\text{Gas volumetric flowrate}}{\text{Bubble volume} \times \text{Cross-sectional area}}$

$$= \frac{Q_g}{\frac{\pi}{6} d_{vs}^3 \times \frac{\pi}{4} d_c^2} = \frac{24 Q_g}{d_{vs}^3 \times d_c^2}$$

That Sauter mean bubble diameter it is defined like this here. This is called Sauter mean bubble diameter.

$$d_{vs} = \frac{\sum_{i=1}^N (N_i d_{bi}^3)}{\sum_{i=1}^N (N_i d_{bi}^2)}$$

This is here N is the Nth number of bubbles and d_{bi} is the individual bubble diameter. If you are considering number of bubbles 500, then accordingly you can get that d_{vs} , that is volume to

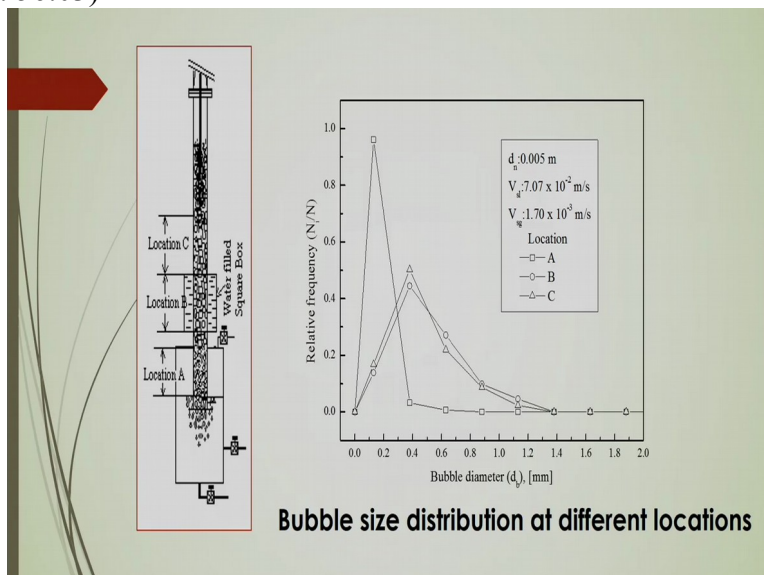
surface mean diameter, it is called Sauter mean bubble diameter. So, once you know that sauter mean bubble diameter and also based on that **Sauter** mean bubble diameter, you can calculate what will be the number of bubbles are produced in that particular column at a **particular operating condition**. That number of bubbles you can calculate based on this. It will be denoted by number flux, bubble number flux.

$$\dot{N}(Z) = \frac{\text{Gas volumetric flowrate}}{\text{Bubble volume} \times \text{Cross-sectional area}}$$

$$= \frac{Q_g}{\frac{\pi}{6} d_{vs}^3 \times \frac{\pi}{4} d_c^2} = \frac{24 Q_g}{d_{vs}^3 \times d_c^2}$$

This gas volumetric flow rate divided by bubble volume into cross-sectional area. So, you can define in this way. And finally you can calculate the bubble number flux based on the Sauter mean bubble diameter. So, once you know this bubble number flux for that individual bubbles, you know that how many bubbles will be that in this, how many bubbles will be within this range of bubble size, how many number of bubbles will be within this range of bubble size, how many number of bubbles will be within this bubble size range out of that total 500 bubbles.

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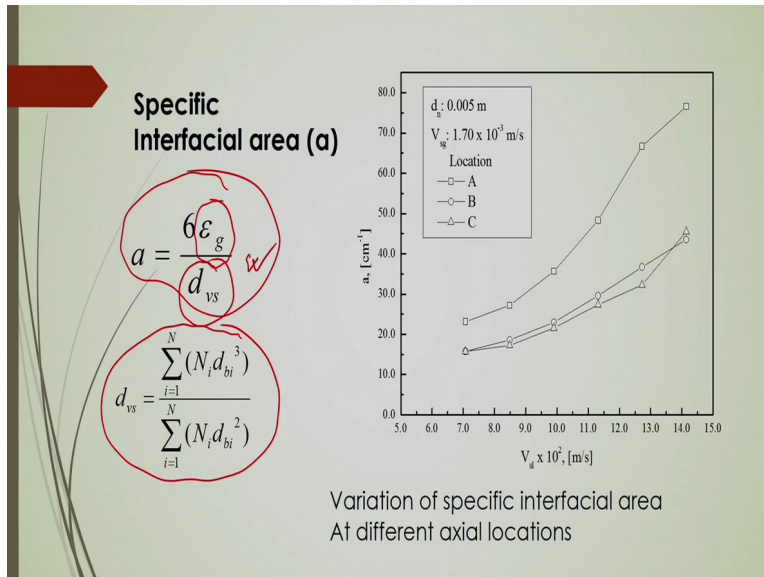


After knowing all those, this bubble number flux, you can draw that profile based on that bubble number, bubble diameter with this bubble number flux. And if you know the total number, then bubble number flux divided by total number, you can get that bubble frequency there. So, bubble

frequency versus bubble diameter if you plot it, you will get this type of distribution. This is called bubble size distribution here at different locations. So, this bubble size distribution, the spreading of this bubble size distribution will be changing according to the operating conditions.

If you are getting more spread, that means your bubble size range will be large. Whereas if you are getting the narrow size distribution, that means your most of the bubbles will be within that very smaller size range. So, in that case, you can assess what would be the bubble size distribution in that particular.

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Once you know that bubble size, that is Sauter mean bubble diameter, you can obtain what would be the specific interfacial area because this specific interfacial area is nothing but the total surface area divided by the total volume of the system. So, it is called specific interfacial area. It depends on the gas hold-up as well as the size of the bubble. So, once you know that gas hold-up, that is Epsilon g and the bubble size that is as a mean bubble diameter, that is D_{vs} , you can calculate from this equation what should be the specific interfacial area.

$$a = \frac{6\varepsilon_g}{d_{vs}}$$

This specific interfacial area is very important for the mass transfer characteristics because overall mass transfer coefficient depending on this specific interfacial area. And mass transfer depends on that overall mass transfer coefficient. The mass transfer coefficient is a function of this specific interfacial area. So, you can assess what will be the mass transfer in the column between gas and liquid based on this interfacial area.

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Mass Transfer

■ Complete mixing model

$$\frac{dC_{l,t}}{dt} = k_l a (C_{l,e} - C_{l,t})$$
$$\frac{C_{l,t} - C_{l,0}}{C_{l,e} - C_{l,0}} = 1 - e^{-k_l a t}$$

(at $t = 0$, $C_{l,t} = C_{l,0}$ and at $t = t$, $C_{l,t} = C_{l,t}$)

At steady state

$$k_l a = \frac{u_{sl}}{(1 - \epsilon_g)L_c} \left(\frac{C_l - C_0}{C_{l,e} - C_l} \right)$$
$$k_l a = 1.80 u_{sg}^{0.92}$$

And see here, this mass transfer happens based on this equation and if you have that concentration of this compound transferring from one medium to the another medium and how that mass transfer is happened and this the equation and this coefficient is called the overall mass transfer coefficient, this overall mass transfer coefficient is nothing but a function of specific interfacial area. So, this specific interfacial area, once you know that, you can calculate overall mass transfer coefficient and this overall mass transfer coefficient depending on the gas velocity inside the column. So, that, for the downflow system, you can easily predict what should be the mass transfer coefficient once you know that superficial gas velocity by this equation here.

$$k_l a = 1.80 u_{sg}^{0.92}$$

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Heat Transfer

- Wall-to-bed (by measurement of the energy input)

$$h = \frac{\dot{q}}{A(T_s - T_b)} = \frac{IV}{A\Delta T}$$

where
h is the heat transfer coefficient (kW/(m²K)), q is the heat rate supplied by electric power or by steam which is equal to the product of electric current, I, in Ampere and voltage, V in unit Volt, A is the heat transfer area (m²) and ΔT is the temperature difference between the heat source (T_s) and the bulk media (T_b).

Again you can calculate the heat transfer characteristics in this downflow system. That heat transfer coefficient you can calculate by this equation that what will be the heat flux.

$$h = \frac{\dot{q}}{A(T_s - T_b)} = \frac{IV}{A\Delta T}$$

And if you know the T_s and T_b here, what is that here? Here H, H is called the heat transfer coefficient, Q is the heat supply rate and this may be by you know that electric power or by other system and I is called here ampere or current here and V is called voltage and A is called heat transfer area and the Delta T that is T_s minus T_b , the temperature difference between that heat sources, that is T_s and the bulk media T_b in the medium. So, once you know that T_s and T_b , then you can easily calculate what should be the heat transfer coefficient for your supply rate of heat in that case.

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Overall and volumetric heat transfer coefficient

$$\dot{q} = h_v V_R \Delta T_{lm}$$
$$\Delta t_{lm} = \frac{T_{g,in} - T_{g,out}}{\ln \left(\frac{T_{g,in} - \bar{T}_l}{T_{g,out} - \bar{T}_l} \right)}$$

V_R is the total internal volume of the reactor and h_v is the volumetric heat transfer coefficient based on that volume

And this heat transfer, you know that can be calculated from this equation.

$$\dot{q} = h_v V_R \Delta T_{lm}$$

If in this case, if you are getting that nonlinear distribution of the heat inside the column. So, overall and volumetric heat transfer coefficient, then you can calculate from this equation here. This will be your overall volumetric heat transfer coefficient. And here, Delta T_m is called the T_m , it is called logarithmic mean heat transfer coefficient, it is defined, logarithmic mean heat transfer difference, that is temperature difference it is called. So, this can be obtained from this equation.

$$\Delta t_{lm} = \frac{T_{g,in} - T_{g,out}}{\ln \left(\frac{T_{g,in} - \bar{T}_l}{T_{g,out} - \bar{T}_l} \right)}$$

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Heat Transfer coefficient

$$Nu \propto \left(\frac{\rho_l \varepsilon^{1/3} d_c^{4/3} \bar{t}^{1/3}}{\mu_l} \right)^{3/4} Pr^{1/3}$$

$$\varepsilon = K_m \left(\frac{1}{2} u_j^2 \right) / \bar{t}$$

$$\bar{t} = \frac{H_c \varepsilon_g}{u_{rel}}$$

$$K_m = 3.04 Re_{ln}^{-0.465} A_R^{0.408} H_{rn}^{-0.25} Su_n^{0.92} Mo^{0.339}$$

$$Nu = \lambda Pr^{1/3}$$

$$\lambda = 1.110 \frac{Su_n^{0.23} Mo^{0.085} A_R^{0.408} Re_{ln}^{0.634}}{H_m^{0.0625}} \times \left(\frac{H \varepsilon_g}{u_{rel} \bar{t}} \right)^{0.08} \left(\frac{d_c \bar{t}^{0.08}}{u_j^{0.5} d_n^{0.69} H^{0.063}} \right)$$

And heat transfer coefficient is a function of Prandtl number. From this equation you can calculate what should be the Nusselt number and from the Nusselt number you can easily calculate what should be the heat transfer coefficient. And this heat transfer coefficient actually depending on that Prandtl number and also the how the mixing is happened inside the column, that intensity of the mixing, that is mixing coefficient, that is calculated based on this equation.

$$Nu \propto \left(\frac{\rho_l \varepsilon^{1/3} d_c^{4/3} \bar{t}^{1/3}}{\mu_l} \right)^{3/4} Pr^{1/3}$$

And once you know that mixing coefficient, you can calculate what will be the energy dissipation in that downflow system by the liquid jet and once you know that energy dissipation, you can easily calculate what should be that you know Nusselt number 1 that heat transfer coefficient based on this equation here.

Because this Nusselt number that is hd/K , this is the Nusselt number is nothing but hd/K . What is d here? d is called diameter of the column, h is called the heat transfer coefficient and K is the thermal conductivity of the fluid here. So, this hd by K is a function of Prandtl number and also other physical properties of the system and energy dissipation. This energy dissipation, you can calculate from the mixing coefficient there. So, heat transfer phenomena also you can obtain from this downflow system.

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Further reading.....

- S. K. Majumder, Hydrodynamics and Transport Processes of Inverse Bubbly Flow, 1st ed. Elsevier, Amsterdam (2016)
- Opletal et al., 2018, Gas suction and mass transfer in gas-liquid up-flow ejector loop reactors. Effect of nozzle and ejector geometry, Chemical Engineering Journal 353 (2018) 436–452
- Balamurugan et al., 2006, Hydrodynamic Characteristics Of Gas-liquid Ejectors, Chemical Engineering Research and Design, 84(A12): 1166–1179

So, for these hydrodynamic and transport processes like mass transfer and heat transfer coefficient, you can get more information from this my book here, published by Elsevier. It is totally regarding the downflow system of this gas liquid system and also its application based on this mass transfer and heat transfer, it is given in more detail. You can go through this book for getting more information, even for better understanding of this system, for the process intensification of gas liquid system for getting more interfacial area, more mass transfer, more heat transfer.

Also, you can follow other books also, other reference also, journal papers also you can follow to get the more information. So, I think we have discussed more about that interfacial area generation and how that intensification of the chemical engineering process based on the generation of the interfacial area in this module. So, we are actually going to finish this module for this interfacial area-based process intensification. In the next lecture onward, we will try to discuss something more about that process intensification process in the distillation system. So, thank you for this lecture.