Fluid Flow Operations
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Lecture – 09
One Dimensional Flow- Part 3

Keywords: Momentum equation; Linear momentum; Jet force; Jet pump; Propeller efficiency; Angular momentum;

Hello everybody. Welcome to this massive open online course on Fluid Flow Operations. In this lecture we will discuss One Dimensional Flow as part 3. Here we will discuss the principle of conservation of momentum.

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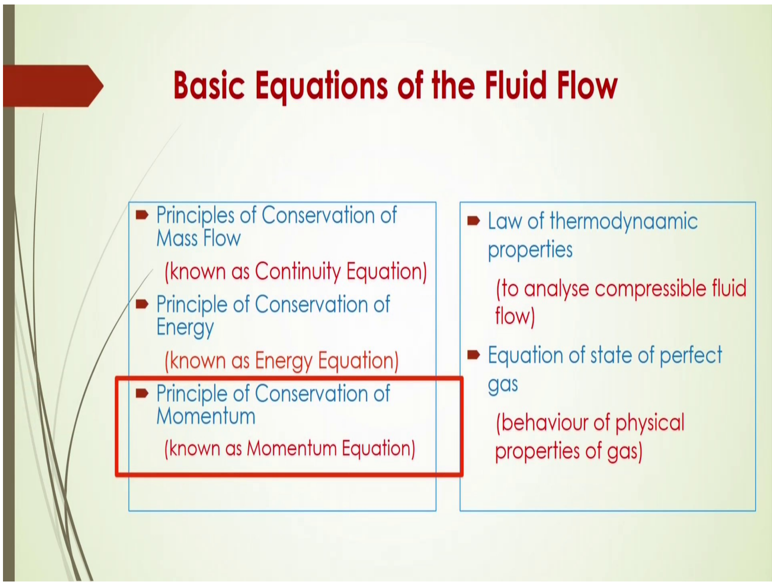

In the previous lectures we have discussed other basic equations of fluid flow like mass conservation and also energy conservation equations. In this lecture we will try to cover this principles of conservation of momentum which is called momentum equation.

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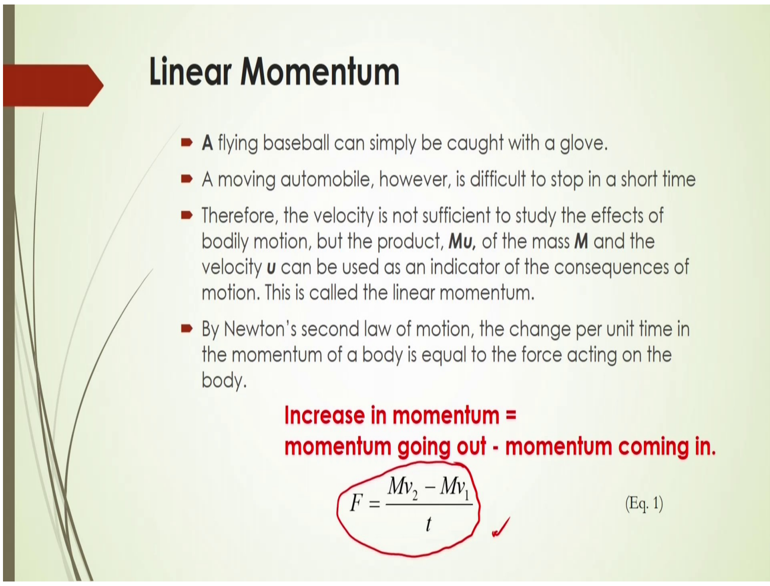

And in this case of course, you will see when this momentum will be existing. Let us see that if any a flying baseball if it is caught by you, then what will happen? There will be a certain change of momentum on your hand when your baseball will fall in your hand. Even you will see whenever we are driving that automobile. So, when it will be moving and if you see if you suddenly stop that automobile it will be very difficult to stop at a very short time. So, in that case due to its momentum the automobile cannot be stop immediately whenever you stop your start there.

So, there will have some momentum like which you can go for that for it while. Therefore, in this case to represent this momentum you will see you cannot only consider the velocity there this momentum effect of this other factor that will give you this momentum. Now, like mass if suppose heavily loaded that automobile then what should be the momentum how it can be or produced you will see a heavily loaded automobile you will have more momentum, if it has the same velocity.

And also if suppose the lighter body we will have the same velocity it will have less momentum. So, how can we represent to this momentum there? If we multiply this mass of the body by its velocity, it will be considered as the momentum. But what should be the change of momentum with a short period of time, if suppose velocity if you are changing suddenly from original to another one. So, there will be some change.

Now, if we have the velocity at initially it will be let it be what is that initially it will be v 1 and after the certain time if the velocity it changing to v 2 then what should be the velocity change v 2 minus v 1. And in that case mass will not be changing for the same mass of body, if we change this velocity from v 1 to v 2 then we will have the change of momentum. Initially the momentum which is Mv 1 and finally, the momentum due to the velocity will be Mv 2. So, the change of momentum will be Mv 2 minus Mv 1.

Now, if this momentum change with respect to time with a short period of time t then change of momentum or you can say increase of momentum or decrease of momentum based on the velocity that will be equals to momentum going out and the momentum going coming in. What should be the increment per unit time? So, in this case we can say that the velocity it will not be the sufficient to represents this body motion, but product of this mass and velocity can be used as an indicator of the consequences of the motion of the body and this is called the linear momentum.

By Newton's second law of motion the change per unit time in the momentum of a body which will be equal to the force acting on the body. So, the momentum increase of momentum can be represented by the force acting on the body. So, force will be is equal to then Mv 2 minus Mv 1 divided by t as it is given in this equation here, and the slide. So, momentum change will be equals to F that will be equal to Mv 2 minus Mv 1 by time t. So, in this way we can easily calculate the change of momentum by this equation. If you know the velocity and the mass then we can easily calculate the momentum. So, this is linear momentum because there is no change of mass of the body whenever it will be changing its velocity from one to another.

If suppose any momentum is having that mass and velocity both will be changing then it will be non-linear momentum. Like suppose any liquid is thrusting on a inclined surface in that case you will see that liquid which is thrusting on the inclined surface would be divided into two parts, one part will go downward along with the surface another will go up again along with the surface of the body.

So, in that case we will see the mass will be divided into two parts. Similarly, the velocity at which the fluid mass will be thrusting on the solid particle or any solid surfaces the velocity will also will be changing accordingly. So, in that case both mass and velocity will be changing and due to which we can say that momentum also will be changing accordingly. So, in that case it will be non-linear because both will be changing there.

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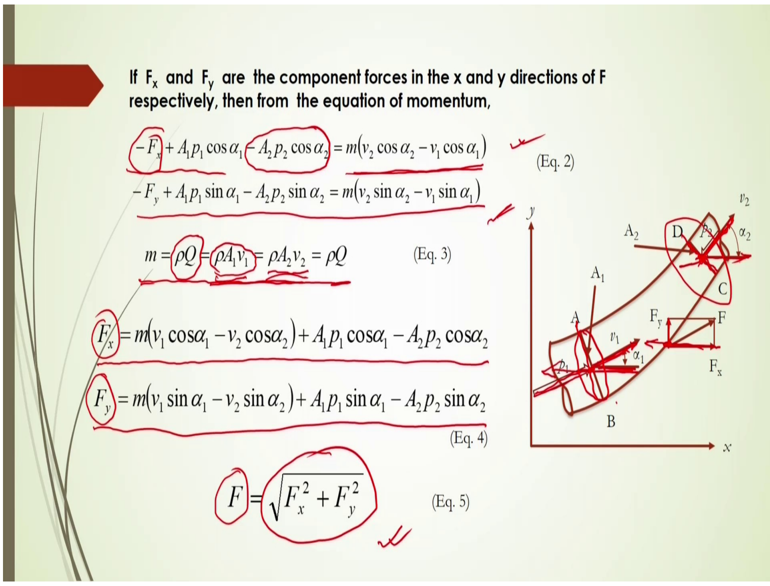

And also, you will see some other cases there were some angular velocity change and because of which there will be a momentum change in the angular direction.

So, will be discussing here, now all the cases and also where the momentum change will be applied with examples we will discuss here. Now, let us see one derivation for this how to actually represent this moment here. Let us considered this fluid element as shown in figure. Here see, the fluid is moving through this control volume through this pipe that is bend. And here and due to this moving of the fluid through this conduit. There will be a certain change of pressure from this section to this section A B section to D C section.

And in this s sections the velocity of the fluid is v 1 and at section 2 that is in the this sections the velocity or the fluid is v 2. And cross sectional area both the sections are A 1, A 2 respectively and due to this moment of fluid at this velocity is you will see there will be a sudden momentum excreting on the surface of this conduit. And equally the same momentum will be excreted by the surface so that there will be a balance of this two forces. And of course, you have to balance this two forces if it is working in the several directions, then you have to consider the balance in the several directions.

Now, if we are considering this things suppose x and y directions here in the two dimensional cases, if suppose F x and F y are the component of forces in the x and y directions of force a respectively than from the equation of momentum we can write here this minus F x at this moment here. Suppose this is the moment come force acting over this. So, here this x direction this is y direction in the x direction what will be the force acting on that or here what will be the x direction forcing acting on that. So, it will be minus F x in this direction the momentum will be acting opposite to the fluid motion here by this surface.

So, minus F x it will be acting in this direction. Minus F x plus or what will be the other forces acting over the surface here, it will be what is that A 1 p 1, p 1 is the pressure at this point into cos alpha. Why? Because this element is this in the x direction, the fluid motion is they are at the direction which you will be making an angle of alpha 1 with respect to horizontal axis.

So, in that case plus A 1 p 1 it will be acting, that is A 1 p 1 it is the force and in the x direction it will be cos of alpha 1. And again in this surface here the fluid motion will be in the direction of alpha 2 with the horizontal then we can say the force acting in this direction will be A 2 p 2. Now, in the x direction what will be the force over here it will be A 2 p 2 into cos alpha 2.

So, this A 2 p 2 cos alpha 2 will be in the negative direction, ok. So, what will be the force effective force here? Minus F of x plus A 1 p 1 cos alpha 1 minus A 2 p 2 cos alpha 2. That will be is equal to what? It is that mass into velocity change that will be your momentum change. What will be the momentum change here? Mass into velocity change. Mass is what here? In this sections mass will be equals to what? That is the same and here at the section what will be the mass is to be same, because mass conservation will be following their mass flow rate. So, this will be mass flow rate.

And then v 2 in the x direction what will be the v 2? Simply v into cos alpha 2. Here v 1 in the x direction what would be the velocity that will be v 1 cos alpha 1? So, this will be acting in this direction negative of x direction here also it will be acting here in this direction x direction. So, in the x direction positive x direction. So, velocity change will be v 2 cos alpha 2 minus v 1 cos alpha 1. So, this will be your momentum change for unit time.

So, in this case you can easily balance on that by this equation number 2. Similar in the y direction if you considered this momentum change it will be minus F y again plus A 1 p 1 cos sin alpha 1 minus A 2 p 2 sin alpha 2 that will be is equals to m into v 2 sin alpha 2 minus v 1 sin alpha 1. So, in this case if we simply by these two equations by taking the mass conservation equation mass conservation equation here like, mass will be m will be is equal to rho Q rho is the density of the fluid, Q is the volumetric flow rate of the fluid.

So, rho Q will be is equal to mass flow rate and then rho A 1 v 1 that will be equals to what is that? Mass flow rate and here rho A 2 v 2 in this sections then it will be your flow out at this D C section. So, that will be is equal to rho into Q. So, here we can say that mass conservation law that m should be calculated like this.

Now, if we substitute this value in this above momentum equations here. So, we can simplify it F of x will be is equal to this and F y will is equal to this. And from this F x and F y what should be the effective force or resultant force? Will be acting by this solid surface to balance this fluid momentum here. So, this will be is equal to what? F will be is equal to root over F of x square plus F of y square. So, this will be your resultant force. So, what force will be acting? Based on the momentum equation we can easily calculate in this way.

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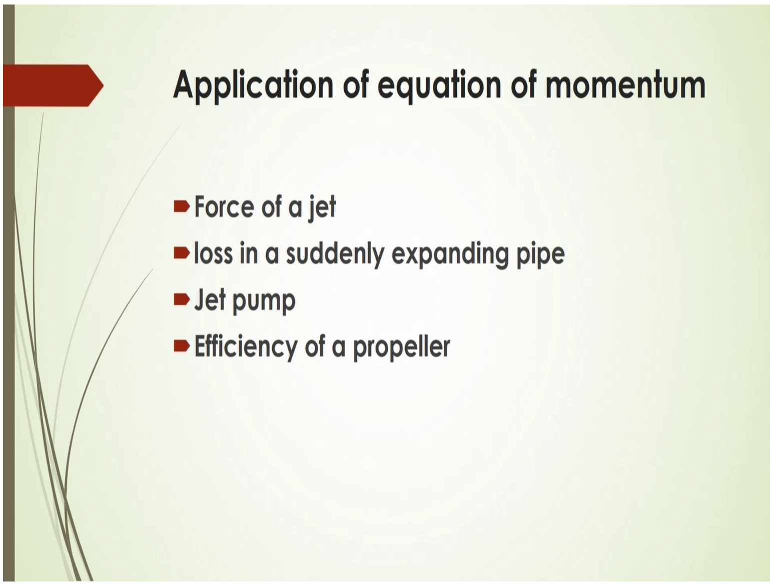

Now, let us have some applications of the equation of momentum here. Now, if you consider that what should be the force of a jet, if any jet is emerging on a solid surface then what should be the force acting on that? Solid surface by the jet and also how these solid surface will be balancing by exactly equal amount of momentum on the fluid.

Similarly, other application like you can apply this momentum equation to calculate the loss in a suddenly any expanding pipe. Sometimes in the real fluid real operation we having some operations in such way that you have to supply some fluid from the narrow tube to the coarser tube or coarser tube the narrow tube.

So, in that case if we supply some liquid from the narrow tube to the suddenly expanding pipe in that case you will see there will be some loss of energy. And you have to calculate that loss of energy by this momentum balance equation.

Again, how this momentum of force will be calculated by this momentum equation for the jet pump and also what should be the efficiency of the propeller that can be calculated by this equation of momentum.

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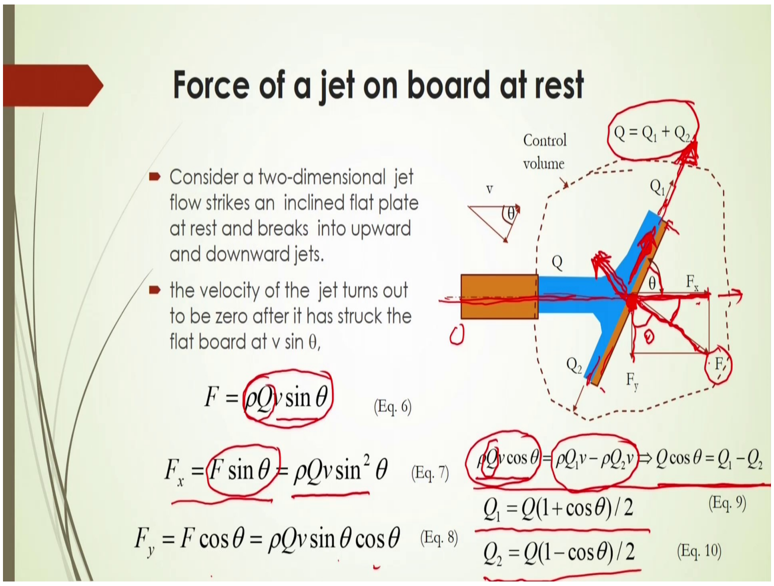

Let us considered this force of a jet on board at rest. Here see this you local at this one this is called a board flat board, and in this flat board the liquid is coming from a source from the pipe or from your jet. And it will be coming and it will be thrusting on this board at a certain velocity at a certain flow rate. And, whenever it will thrust or push on this board the liquid will be divide into two parts as a Q 1 and Q 2. Similarly, the velocity also of this fluid will be changing in this directions.

And if you apply this velocity here or flow rate in this direction, accordingly the force will be active over this body point here and the force acting on this body will be in this direction perpendicular to this board. Now, this perpendicular to this board you have to have this force component whenever it will be excreting on the fluid mass. So, that you have to calculate that will be your force, that will be F that you have to calculate it.

Now, how to calculate this? You have to do the momentum balance here. So, in this case consider two dimensional jet that will strikes on the inclined, flat, plate or board you can say at rest and it breaks into the upward and downward jets. Now, in this case the velocity of the jet turns out to be 0 at this point, after it has struck the flat board at this point at a velocity of v into sin theta. Why? Because the velocity in this direction is if it is considered in this direction then this will be your x direction but perpendicular to this direction what should be the velocity that will be is equal to v sin theta there.

Now, then what should be the F there? This force will be nothing but this force this in this direction this fluid will be acting and opposite directions as per Newton's third law the same force equal same force in the opposite directions will be applied by this flat plate. So, this F should be is equal to what is the velocity or what is the flow rate that is Q of the fluid. So, rho into Q that will be your, what is that? Mass and into velocity v into sin theta this will be your momentum change. This will be your velocity. So, momentum will be is equal to what that is the rho. What is that? Q v sin theta, ok. So, mass into velocity. So, this will be your momentum. Now, if it is coming from the 0 velocity then momentum we have to change will be is equal to rho Q v sin theta minus here 0. So, it will be equals to rho Q v sin theta.

Similarly, this force balance to be done in x and y direction in the x and y direction the components of the, this fluid momentum or this will be F, this F component will be divided into two parts in the x direction and y direction. So, in the x direction the components will be F x and in now, this y direction it will be F y. So, F x will be is equal to F into sin theta because this is your theta. So, F y is equal to F cos theta; and this will be your what is that? Sin theta. So, F x will be is equal to F sin theta that will be is equal to if you substitute here the value of a we can get rho Q v sin square theta.

Similarly, F y in the y direction it will be coming as F into cos theta that mean rho Q v sin theta into cos theta. Now, interesting that what should be the momentum change in this particular direction here, what will be the momentum change? Now, it will be rho Q v sin theta because v is acting over here in this direction. So, v sin theta it will be your v cos theta. So, rho into Q into in this direction it will be v cos theta, in this direction what will be the momentum. So, this momentum will be given by this flow of fluid at flow rate Q.

Now, this will be effective in this direction. So, in this direction what would be that v into cos theta if you see that it will be your rho Q v cos theta. So, that will act in this direction, where the component of flow rate will be flowing upward along this flat plate. Now, what were the effective then momentum, what is acting in this direction? Now, if we consider that flow rate is divided into two parts Q 1 and Q 2. Now, in this Q 2 what will be the fluid part? This is Q, this is Q 1 in this direction this will be your Q 1 fluid mass. So, rho Q 1 v in this direction it will be there minus rho Q 2 v in this direction

So, this will be your what is that? Change of momentum whenever it will be changing in the divided into two parts. Because this moment here the energy it will be the same there this because this one and this one ok. So, this will be subtracted this will be divided. And then ultimately from this balance you can say that Q into cos theta that will be is equal to Q 1 minus Q 2.

Now, from this momentum equation and also from the mass conservation of equation that means, Q will be is equals to Q 1 plus Q 2 then you can have after solving these two equations what should be the Q 1. Q 1 will be is equal to Q into 1 plus cos theta divided by 2 whereas, Q 2 will be is equal to Q into 1 minus cos theta divided by 2. So, by this equation 9 and 10 we can easily calculate from this momentum balance equation what should be the Q 1 and what should be the Q 2. Once we know the flow rate of the fluid at which it will strikes on the flat surface which will be inclined at an angle theta.

So, by this equation you can calculate the thrust or force acting by the liquid jet on the board. But this is applied only the condition that board will be at rest.

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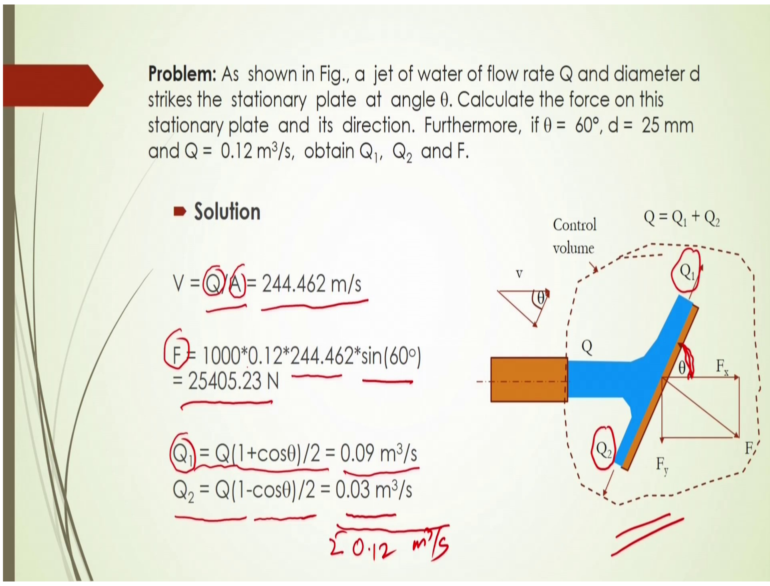

Let us do an example for this. As show in this figure here, now a jet of water of flow rate is Q and diameter d that strikes a stationary plate at an angle theta here. Now, calculate the force on this stationary plate and its direction so that the theta is measured and we should be found at 60 degree and diameter of this jet is given 25 millimeter, and flow rate of the jet is given 0.1 meter cube per second. And then find out what should be the Q 1 and what should be the Q 2, and also what should be the amount of force acting by this fluid and to balance this how much force is acting oppositely by this plate. So, that will be your F.

So, first of all we have to calculate what should be the velocity of the jet. So, velocity of the jet you can calculate from the flow rate and the cross sectional area of the jet. So, here the cross sectional area is given to you A and the flow rate is Q. So, Q by A that will be your jet velocity. Now, F is as for that previous slides that we have shown that the F should be is equal to rho Q, V into sin theta simply you just substitute here, rho is given to you Q is given to you and what is that rho Q v, v now you have calculated here. And what is that? Sin theta, theta is 60 degree. So, finally, after calculation you are getting the a value 25405.23 Newton.

Similarly, we can have from those equations of 9 and 10 what should be the component of this flow rate that is Q 1. So, Q 1 will be is equal to than substitution of this theta and Q value in this equation then you can get 0.09 meter cube per second and Q 2 will be is equal to Q into one minus cos theta by 2 that is equals to 0.03 meter cube per second. See here the summation of this mass will flow the conservation of mass equation. So, in this case this will be summation that will be equals to what 0.12 meter cube per second.

So, in this way we can solve the problem whenever any liquid jet is thrusting or striking on a incline rest board or plate.

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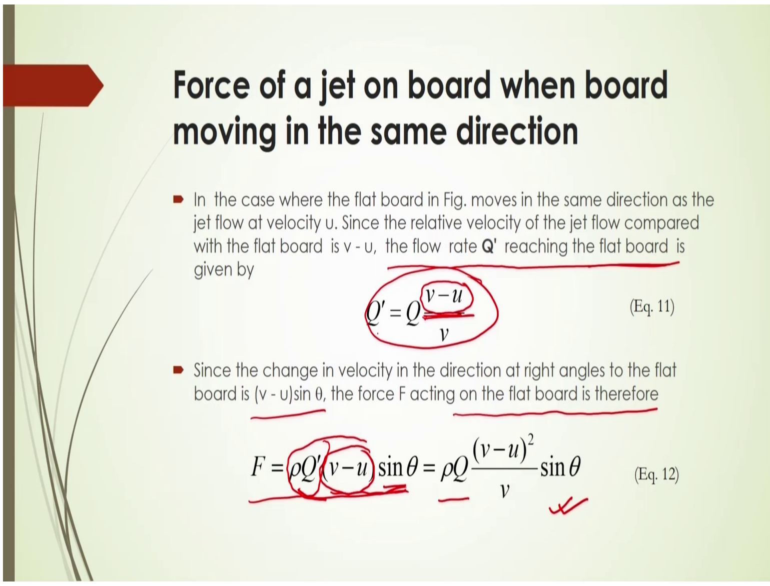

Now, the case where the board now is not in the rest in condition, it will be moving in the same direction at some other velocity. Now, in the case where the flat board it is shown in the figure here, in the same direction if it is the flat board is a flowing in the same direction at new velocity. Then what should be the effective mass that acts on the surface of the board? So, that you have to calculate.

So, here in this case fluid velocity is u v, and also the board velocity or flat plate you can say the velocity if it is u then from that from this equation, equation number 11 you can calculate what should be the effective flow rate reaching the flat board discuss the effective or relative velocity is v minus u. So, Q dashed divided by v minus you that will be is equal to Q by v, from that equation you can easily calculate what should be the Q dashed here.

So, Q dashed will be equals to here Q into v minus u divided by v. So, Q dashed is the effective flow rate or you can say that resultant flow rate that will be reaching on the moving flat board. Since the change in the velocity in the direction at the right angles to the flat board is which will be as v minus u v sin theta, then the force acting on the flat board according to the balance of the momentum equation, you can see that simply.

What will be the effective mass here? That will be reaching that rho into Q dashed. And what will be the relative velocity? That will be v minus u into sin theta. So, this will be your simply what is that? If you substitute this Q dashed then it will be rho into Q into v minus u square by v into sin theta. So, from this equation number 12, we can calculate what should be the force if the board is moving in the same direction at a certain velocity.

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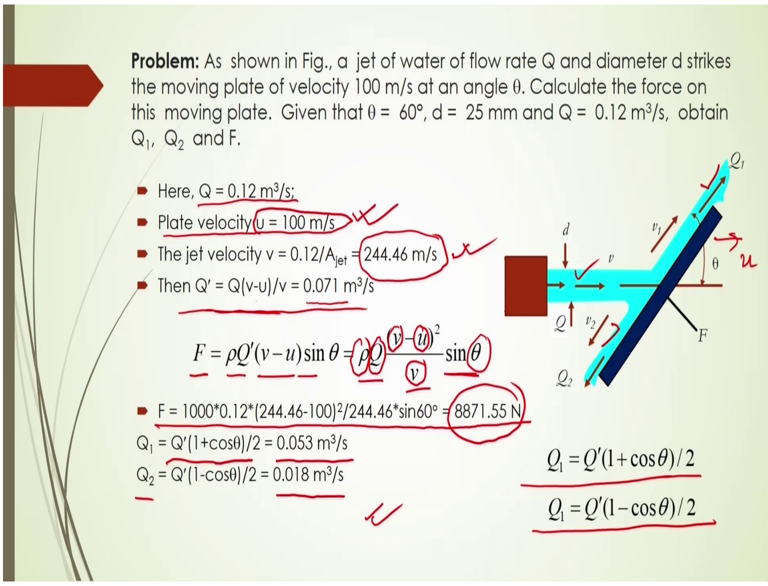

Let us do an example for this. As shown in this figure here, here this is the jet and this jet is dividing into two parts whenever it will be approaching to this board and we should be moving in the same direction at a velocity u. Now, in this case the jet of water of flow rate Q and diameter d that strikes the moving plate or velocity 100 meter per second at an angle theta. Now, calculate the force on this moving plate in that case theta again it is given 60 degree, d is 25 millimeter and Q is 0.12 meter cube per second. So, what is the difference from this?

Previous problem is that we have not considered that the flat board is moving at a certain velocity but here in this case flat is moving at a certain velocity that is 100 meter per second. So, the fluid jet is moving from the conservation mass conservation equation that will be is equal to u that is equal to 244.46 that is your jet velocity whereas, plate flat plate velocity is what? u that is 100 meter per second.

So, what would be the effective mass that will be approaching to this moving plate? That will be is equal to Q into B minus u by v as per equation number 11. So, this will be 0.071 meter cube per second. Similarly, if we calculate this F based on the equation 12 the F will be is equal to F will be is equal to rho into Q dashed into v minus u into sin theta that will be equals to rho Q into v minus u square by v after substitution of this Q dashed here finally, it will be coming rho Q u v minus u square by v into sin theta.

Now, theta you know, Q you know, rho also it is given to you, v is known to you, u is known to you now, v is known to everything is known to you. So, finally, after substitution of these values here we can easily calculate what should be the F, and this F will be is equal to 8871.55 Newton. And Q 1 again it will be Q dashed into 1 plus cos theta by 2 that will be 0.053 and Q 2 will be is equal to 0.018 here. So, based on this equation you can calculate what will be the components of the mass of the jet that will be moving along with the surface of this moving plate.

So, this is your example. So, you can easily now understand how to calculate the force acting on the moving plate by the liquid jet and also that jet will be balanced by the a moving plate by exerting the same amount of force on that and that will be calculated by the momentum equation here.

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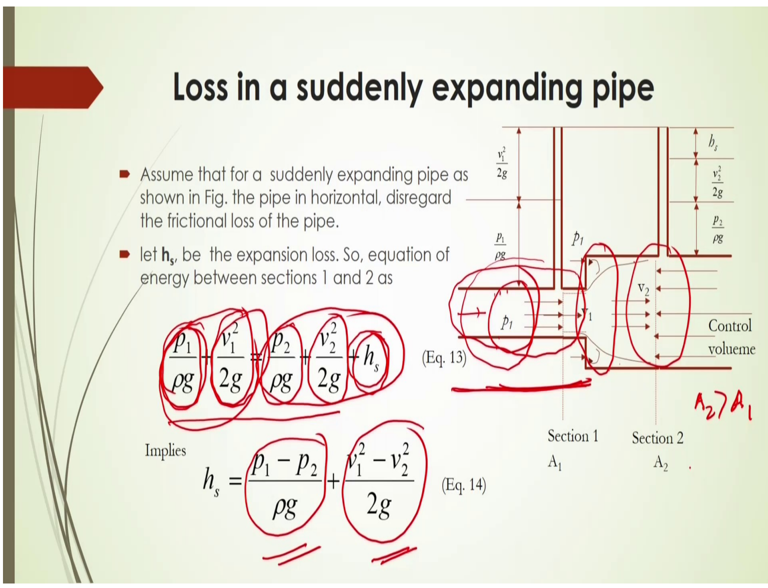

Another example of momentum equation is that loss in a suddenly expanding pipe. Here see in this diagram this is the pipe section, whose cross sectional be sudden their pressure exerting in this pipe whenever fluid is flowing at a certain velocity that will be v 1, and at the sections here see the diameter of this pipe will be higher than this pipe.

So, in this case section 1 and section 2 the cross sectional area will be difference. So, in this case A 2 is greater than A 1, that means, here this pipe is suddenly expanding from cross section A 1 to A 2 smaller to higher cross sectional area. So, in that case you will see whenever fluid will be moving from these sections the liquid will be changing its velocity from v 1 to v 2 that means, the same amount of fluid mass will be acting over the sections because of the mass conservation. So, there will be change of momentum. So, based on that change of momentum equation or momentum we can easily calculate what should be the force acting over there due to the change of cross section.

Now, whenever the cross section will be changing there will be loss of energy, there will be sudden loss of energy. What should be that loss of energy? First of all we have to do energy balance over this two cross sections area. Already we have learn how to apply the Bernoulli's equations that is energy equation. So, if we apply the Bernoulli's equation over these two cross sections of this pipe. So, you can have this at the sections what should be the tube pressure at the sections what will be the velocity and pressure there. So, if we have that at cross section 1 and 2 then p 1 rho g plus v 1 square by 2 g that will be is equal to as for this Bernoulli's equation here. So, p 2 by rho g plus v 2 by rho g.

So, this is up to this if there is no cross section change. But since there is a cross sections then you can easily sudden expansion of this because of which you will see some changes of energy where you will not get exactly the same energy what is supplied by the sections here.

So, there will be some loss that loss of course, to be considered here. So, that is why this h system that is loss of energy that you are considering here. So, this is actually head term p 1 by rho g, this is head, this is head. So, in this section 1 this is smaller diameter pipe the total head is this one and the larger sections total head will be this one by considering the loss of energy there or loss of head there. So, which will be implying that h s will be is equal to this plus this. This is pressure energy this is kinetic energy here.

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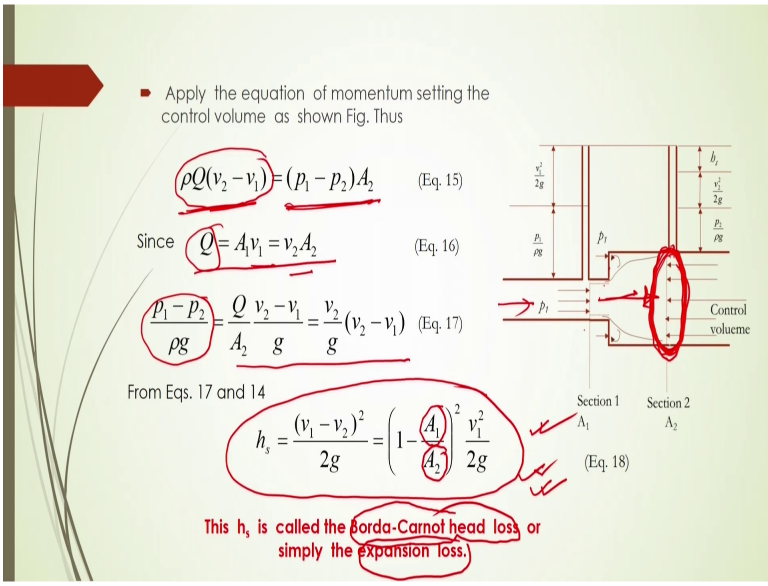

So, this loss energy how to calculate. Now, from the momentum equation based on this control volume here, so we can do the balance of momentum here. So, in this case what should be the momentum change? Whenever velocity it changing that will be rho Q into v 2 minus v 1 and here the force is acting it will be p 1 minus p 2 into A 2 here. Because this momentum when we have this momentum change that will be acting over this cross sections, because of this sudden expansion this momentum it will be thrusting over this cross section here.

Now, at this cross sections what will be the force to balance those momentum change? That force will be is equal to what would be the pressure change here p 2 minus p 1 or p 1 minus p 2 into cross sectional area that will be A 2 here. Here you have to consider this cross sectional area A 2 because that momentum is acting over this cross section of A 2. So, that is why you have to consider this the pressure change into cross sectional area.

So, again from this mass conservation equation what should be this velocity v 1 and v 2, from this volumetric flow rate and then what is that after substitution of this v 1 and v 2 and we can get this equation here p 1 minus p 2 that will be is equal to this. And so by, this equation number 15 and 14 we can simply calculate what should be the head loss whenever there will be a sudden expansion of this pipe. So, by this equation number 18 we can easily calculate.

In this case we have to know the cross sections of this section 1 and cross sectional area of section 2. And also if you know the velocity at which the fluid is moving from this smaller diameter pipe. So, if it is v 1 then you can easily calculate what would be the energy or head loss over there.

So, this h s is called the Borda-Carnot head loss or simply the expansion loss. So, you have to remember this that Borda-Carnot head loss or which is called the expansion loss that you can easily calculate. Once you know the velocity through the pipe at this cross section 1 and also the cross sectional area of these two sections.

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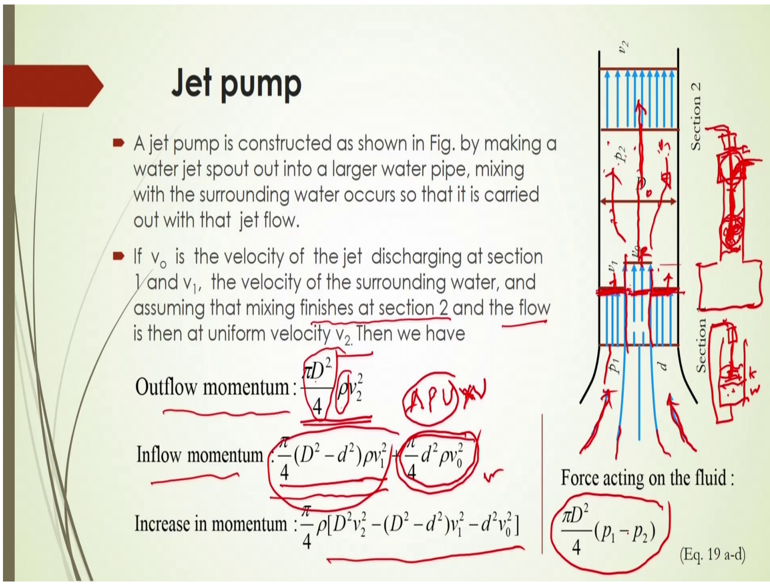

Another example of momentum equation like jet pump. See here, this is one jet pump, in this case one jet will be produced by changing the cross section of the pipe at this location here, just the liquid will be passing through this smaller cross sections and then it will be moving over this through this pipe.

Now, in this case here whenever liquid is coming from the other cross sections and if it is passing through this smaller cross sectional area there will be a formation of jet. And jet whenever it will be plunging that will be plunging over a pool of liquid or some other surfaces you will see there will be a certain what is that momentum change and that momentum change we will be actually applied for a sudden operation like, suppose fluid mixing is there.

If suppose in any reactor I want to actually mix the fluid uniformly throughout the cross sections. In that case, if we use jet here instead of stirrer if we supply the liquid continuously through a nozzle in the nozzle you will see through the nozzle whenever liquid the liquid is coming the liquid will having the velocity that will be higher than the other portion.

So, at high liquid jet whenever it will be plunging over the pool of the liquid or whole mass in the reactor then you will see that the liquid jet that liquid momentum will be distributing over the throughout whole reactor. And that energy distribution will mix whole fluid element in the reactor. So, to get the mixer, to get the mixing of this fluid element we can apply this jet pump.

Sometimes to supply the gas by sucking to this liquid jet. Here suppose any liquid jet is coming in a pan of liquid in a pan of liquid, and this jet is a applying here through a nozzle like this and here one gap and here one this provision is make like that through this is called ejector system or on that gas suction chamber. Whenever liquid jet is coming through this suction chamber and if it is plunging in the pool of the liquid here you will see the jet will automatically suck this gas and it will carry through this liquid jet and it will be plunging into the pool of the liquid here. So, this gas is plunging into the liquid jet here. So, and gas will be entrapping or entraining into the liquid here. Now, this gas will be mixing over here.

Again, another example suppose if any in any column suppose jet is applying, liquid jet is applying in this column where this column there will be a mixture of immiscible liquids like kerosene and water, kerosene and water. So, in this case if you want to mix this kerosene and water you have to apply this jet of liquid, jet of liquid continuously through the jet then you can get the formation of jet by mixing of this liquid by breaking it surface.

So, for mixing and the formation of droplet, formation of bubble you can apply this jet pump, ok. Now, what should be the force applying by this jet to mix this fluid in the reactor or some other applications? So, you have to calculate by this momentum equation.

Now, if v 0 is the velocity of the jet that discharging here discharging, at a section 1 here the velocity of the surrounding water is there and assuming that mixing finishes at section 2 here, there will be a mixing, there will be a mixing of this fluid at this section 2. And the flow is then uniform velocity v 2, it will moving further then outflow momentum will be is equal to cross sectional area into density into v 2 into v 2. That means, here cross sectional area into density into v 2 square this will be your mass into v, so if it than into v this will be your momentum. So, according to that the in the section 2 the momentum outflow momentum will be is equal to pi D square by 4 this is cross sectional area into rho into v 2 a square.

Similarly, inflow momentum will be is equal to pi by 4 into d square minus d square here this will be your total cross sectional area. So, this is a small d. So, small d after subtracting this small d cross sectional area of these two, then remaining portion will be the cross section of this part section 1. So, this will be a cross section and this will be flowing at the velocity u 1. So, this will be rho into v 1 or v 1. So, plus and then whenever it will be moving this section then it will be your what is that pi by 4 d square into rho v v 0 square. So, this is will be a total momentum in flow.

So, momentum at this sections and at this section this will be given by this and momentum from this section it will be why this. So, total momentum over this here it will be is equal to this two. An increasing momentum that will be subtracting this two then you will get the increasing momentum.

Now, this force acting on the fluid by the jet that will be is equal to here. What is the pressure? Pi D square by 4 p 1 minus p 2, this will be your force acting on this.

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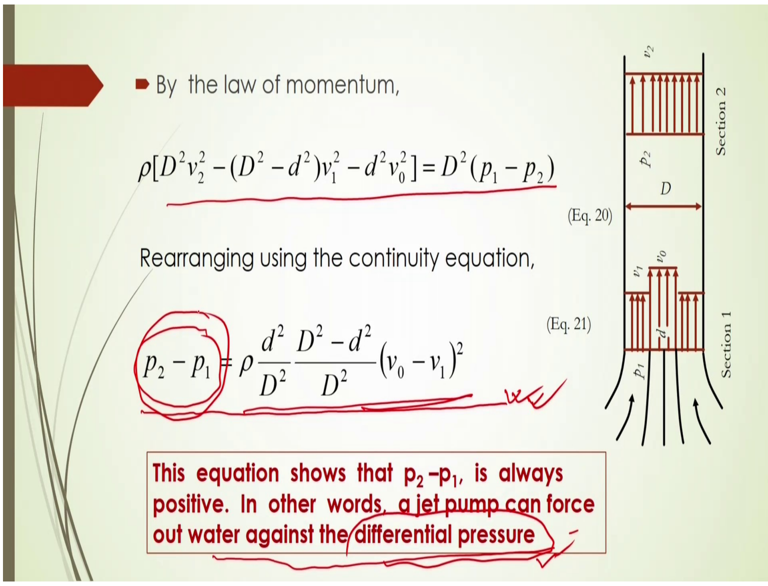

Now, balancing these two equations we can get by law of momentum simply by balancing this two. So, we can have after rearrangement p 2 minus p 1 will be is equal to what. So, from this jet what will be the pressure creating by the jet pump that will be is equal to p 2 minus p 1. So, this will be your as per equation number 21.

So, this equation shows that pressure difference by the jet pump whenever it will be supplying or any applying for mixing purpose just allowing it through a small nozzle then the pressure difference can be calculated.

So, this equation shows p 2 minus p 1 always positive, since the pressure p 2 will be higher than the p 1 here. So, in other words the jet pump can force out water against the differential pressure. So, very interesting that main principle is that jet pump can force out the water against the differential pressure. So, this is the law of jet pump. So, what should be the differential pressure that you can calculate by this equation number 21?

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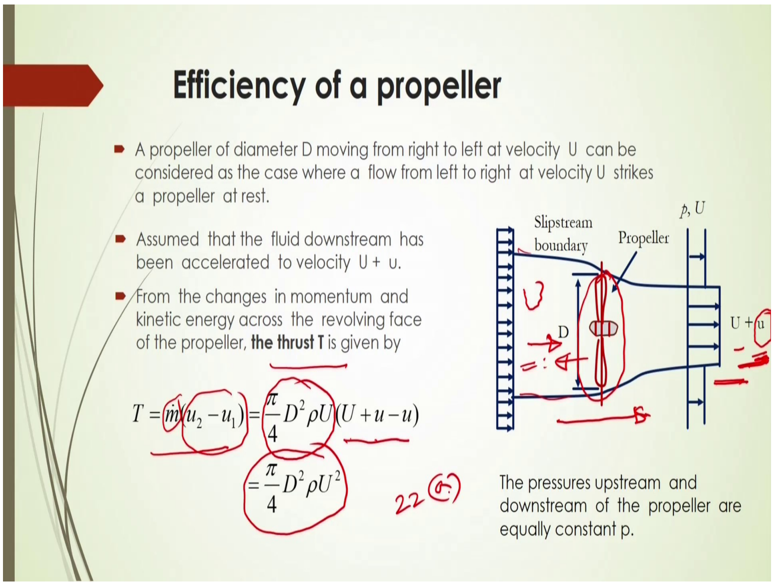

Now, another example of momentum equation that is called if to how to calculate the efficiency of a propeller. Now, this is one propeller like Aeroplan here. So, one it is propeller and this by this movement of this propeller the liquid will be moving from this section to this section.

Now, at this sections if velocity is u and whenever propeller is applying some liquid will be throwing from this sections to this section, and the fluid will get some acceleration and getting the velocity higher than this. So, that higher velocity that means, increment of velocity is let it be small u then at this sections the velocity will be u plus capital U plus u So, a propeller of diameter D moving from right to left at velocity u can be considered as the case higher a flow from left to right at velocity u strikes a propeller at rest. So, in this case fluid will be forcing in this direction where propeller will be forcing in this direction. So, this balance will be obtained by this momentum equation.

Now, first of all you have to calculate what will be the fluid velocity at this section that will be U plus u, U plus u and here this is U there will be sudden increment of this velocity that will be by u. So, from the changes in momentum and the kinetic energy across the revolving forces by this propeller and we can calculate the thrust T and then what will be the efficiency of that propeller also.

Now, in this case what would be the thrust that acting that will be calculated by the momentum exchange or momentum change that will be m dot into u 2 minus u 1. So, that will be is equal to what? M dot that is mass flow rate that will be is equal to pi by 4 D square rho U at this sections into increase of velocity that will be U plus u minus u. This will be cancelled out and then finally, we can get what is that pi by 4 D square rho U square.

Now, the pressures upstream and the downstream should be constant that will be p that you have to remember. So, in this case we are calculating how the liquid will thrust on the propeller and propeller oppositely will also keep the force to balance on this.

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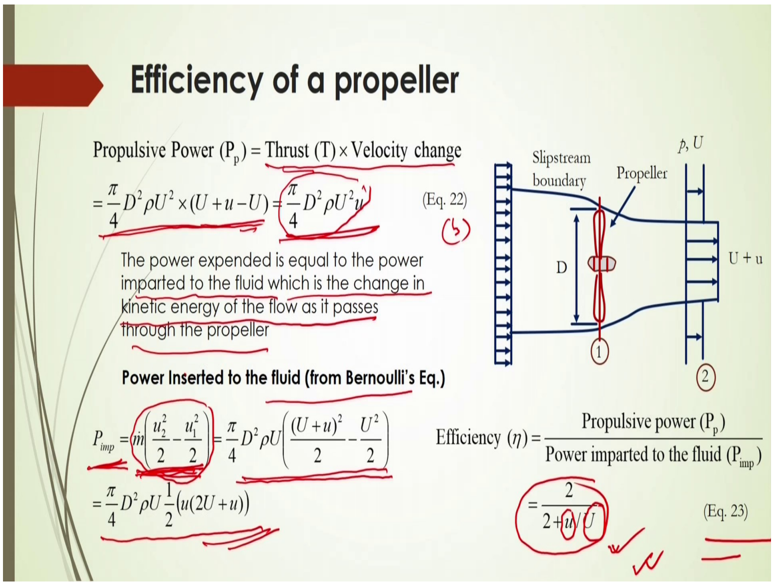

Now, after that you have to calculate based on this thrust what should be the propulsive power that given by this propeller. Now, power is calculated by thrust into velocity change. Now, this thrust is already calculated by equation number here, this suppose this 22 a, this is 22 a, and here this is 22 b. So, from this you can then from 22 a if you substitute the value of thrust here and a velocity change is nothing but what is that, here it will be U plus u minus u. So, finally, it will be coming as pi by 4 D square into rho U square into small u.

The power expanded is equal to the power important to the fluid which is the change in the kinetic energy of the flow as it passes through the propeller. So, this is the important point that you have to remember by this you have to calculate this efficiency of the propeller.

Now, power inserted to the fluid is calculated based on the Bernoulli's equation that will be is equal to p imparted that will be m into that is velocity change this is because pressure is remain constant and this is horizontal. So, what is that potential, head also will be neglected and the pressure head will be neglected on the velocity head will be calculated here.

So, based on that we can calculate what will be the power this will be then this is m in dot u 2 square by 2 minus u 1 square by 2. So, this will be here. It will be of course, after substitution of this then you can get this value and finally, it will be by this equation.

Then efficiency will be is what? What? Propulsive power of on power imparted to the fluid here. So, after substitution of this power propulsive power and this here p imparted by this equation and after simplification you can get what should be the efficiency of the propeller that can be calculated by this equation here that is 2 by 2 plus u by U capital.

So, very interesting that if you know the you can known only the increment of the velocity whenever fluid is stream is thrusting by the impeller and the velocity is having the velocity increment of small u. Then what will be the increment? Small u. And the what will be the initial velocity of that fluid from which you can easily calculate what was the efficiency of the propeller by the simple equation, equation number 23.

So, we have applied this momentum equation in the several application like a efficiency of the propeller to calculate the differential pressure of the jet pump and also what is that other application that we have discussed here. So, I think we now know how to calculate to how to use the momentum equation, how to use the mass conservation of equation and also how to use the energy equation. So, this 3 equations the basic equations the fluid flow and we shall applied in the various fluid flow operations and also for designing various devices based on this principle. And also, the designing of that like here jet pump propeller all those things a.

I think this one directional fluid mass is I think now we can have the idea the energy equations, momentum equations and the mass conservatives into apply in the various fluid operations in the practical some process industries, ok. And so for further I think you can follow some other books also.

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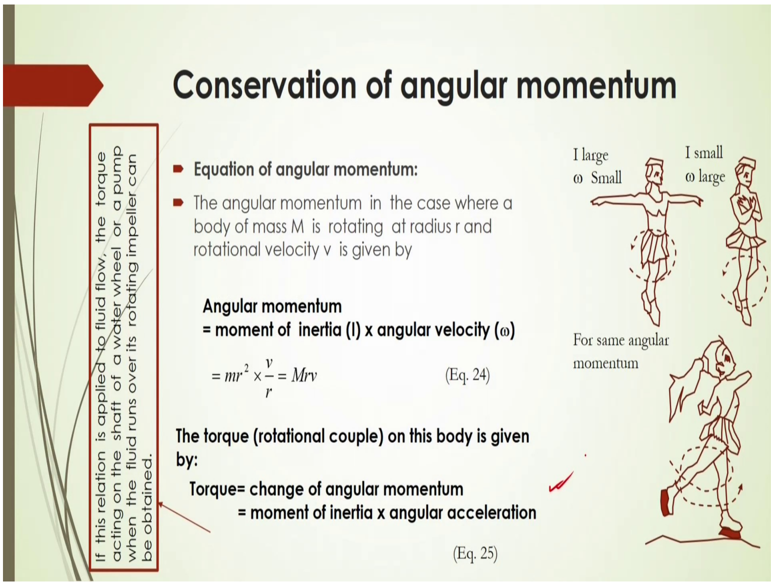

In this case angular momentum also, you can calculate based on this here. So, this slides you can I think go through that how to actual use that conservation of angular momentum here.

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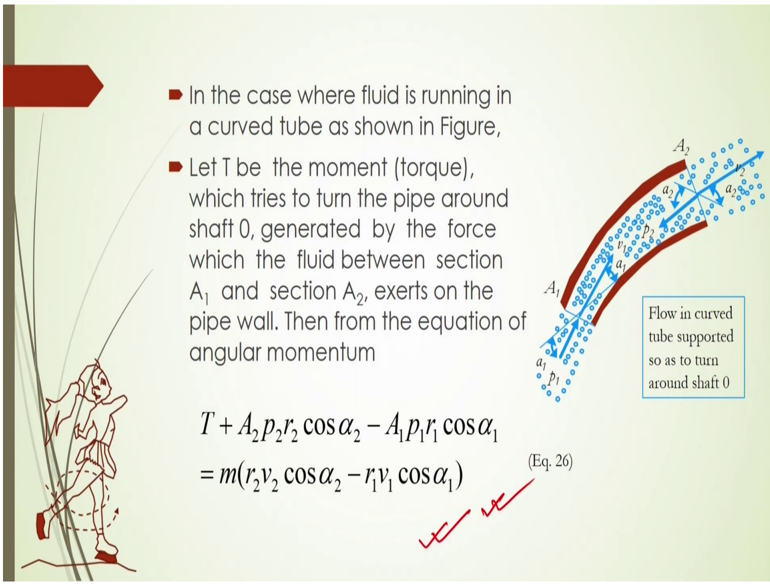

So, and also these are the some application of this angular momentum. And here from this equations also whenever fluid is flowing through this can do it at a certain angle at a with a certain angular momentum, then what should be the force acting over there you can easily calculate from this.

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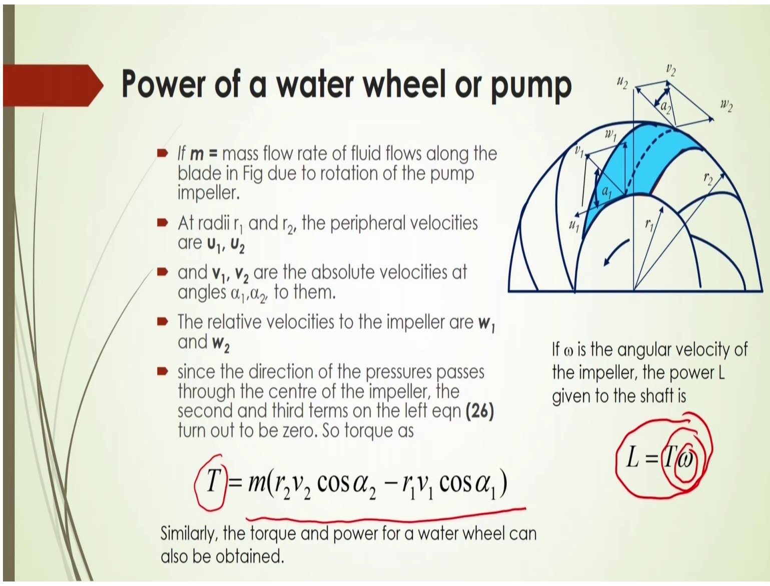

And what should be the power? There will be if there is a wheel or pump there that also you can apply the momentum equation here. And based on the if you are considering here that what should be the torque whenever fluid is actually discharging by the pump, then how what amount of actually torque is applied by this wheel of the pump then you can calculate from this equation.

And if you know the angular velocity of the impeller the power L you can calculate from this equation torque into angular momentum here. So, w is the angular velocity. So, here this T into omega that we are angular momentum and then you can calculate this a power based on this.

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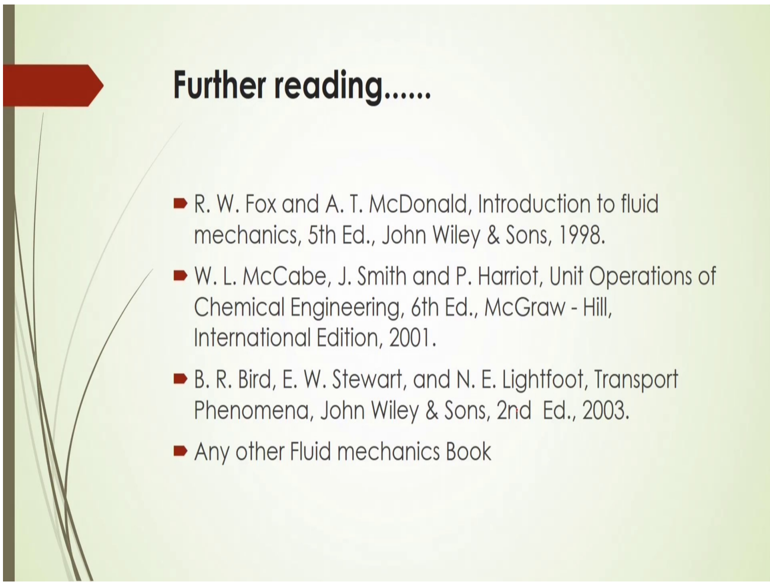

So, for further reading I should actually suggest this other text books of this, here given in this slides. So, thank you for a giving attention for this lecture.

Thank you.