

Fluid Flow Operations
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Lecture – 08

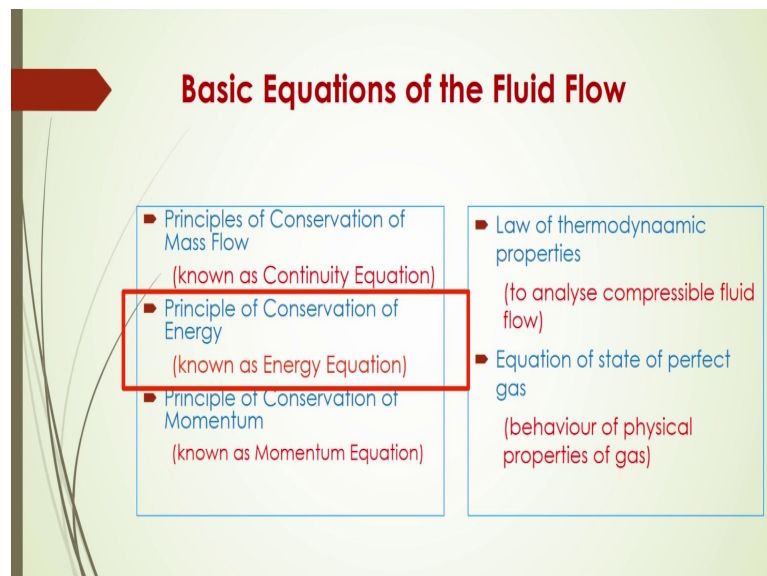
One-Dimensional Flow-Part 2

Keywords: Bernoulli's equation; Euler's equation; Dynamic and Static pressure; Stagnation pressure; Hydrostatic grade line; Energy line; Measuring instruments

Welcome to massive open online course on Fluid Flow Operations. Today, in this lecture, we will discuss about One-Dimensional Flow as part-2. In part-1, we have discussed regarding the continuity equation, and its application with different examples. And based on that we have discussed how the fluid will be behaving, whenever it will be flowing through a pipe.

Now, as per basic equations of the fluid flow, we have discussed that conservation of mass flow under that what should be the continuity equation, basically this conservation of mass flow is called the continuity equation. Whereas, other principles like principle of conservation of energy, which is called the energy equations, and the principle of conservation of momentum is called momentum equation.

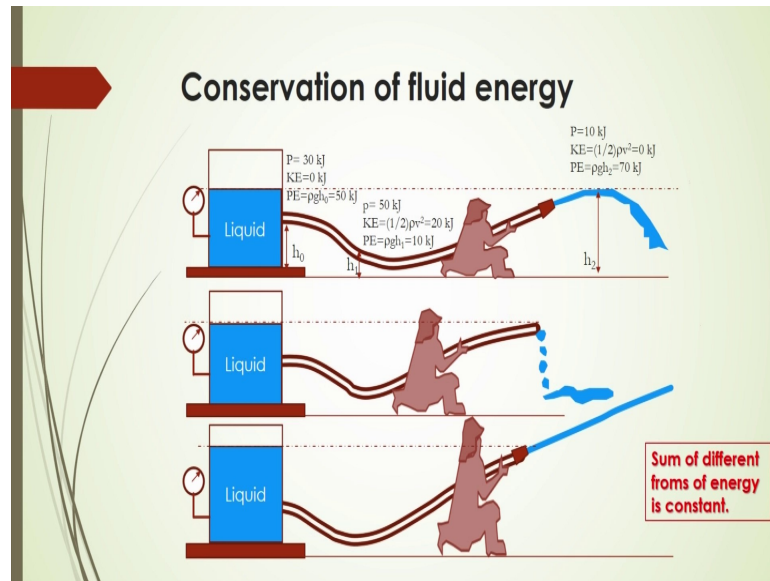
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So, and other auxiliary equations you can say that to consider along with this basic equations of the fluid flow that are law of thermodynamic properties, equation of state of perfect gas, and law of thermodynamic properties are generally analyzed compressible

fluid flow. And equation of state of perfect gas will be considered for behavior of physical properties of the gas. So, we have already discussed about the principle of conservation of mass flow. And in this lecture, we will go through some questions which are related to that energy, and also how it is derived, and also some examples are down that energy equation.

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Now, if we consider that conservation of a fluid energy, you have see the pictures. Let us see how whenever fluid is the just releasing from a tank through a pipe, and there if a pipe is not horizontal, and if it is a suppose there will be a what is that certain bent of this pipe, and also the cross section of the pipe will be changing.

And a based on this, how energy is the consuming, and during that consumption of the energy, you will see total energy will remain same, because we cannot create any energy for that. So, total energy will remain same, but only thing is that energy will be transferring from one form to another form.

Like here three energy if we consider here one is a potential energy, another is kinetic energy, and another is called pressure energy. So, a there are three types of energy. If I sum this you will see in that at any location of this pipe here shown that, the total energy will be same. So, here at a particular location let it be here at this position. So, here what should be the pressure energy at that pressure energy, and what will the kinetic energy

based on the fluid velocity, and what will be the potential energy based on the height of the fluid at viscous it will be flowing.

So, total energy of these three forms will be equal to same in all cases. So, you see here another point here, let it be potential energy 30 kilo joule and kinetic energy is 0 here, because there is no velocity at this point. Here there will be a certain potential energy, so summation of energy will be same in all locations here.


Similarly, in other locations suppose if we change the velocity at this locations the first figure that here the velocity at the outlet of this fluid from the pipe will be higher, and they are you will see that a kinetic energy will be higher, because it is coming from the pipe at a certain velocity, and this potential energy will be maximum they are. And kinetic energy at this whenever it will be releasing, and it will going downward at this maximum point there will be no velocity, so kinetic energy will be equal to 0.

Whereas, here potential energy will be pressure energy will be certain extent, and then potential energy will do the maximum, wherein the bottom most now figure here, you will see there will be some nozzle will be used at the outlet of this pipe, and cross sectional area will be very small. In that case velocity will be very high, because the liquid is coming through that nozzle, and liquid will be coming as a liquid jet, by that a position there were higher velocity, so higher kinetic energy relating to the potential energy and pressure energy there.

So, in this case we are just saying that the conservation of the fluid energy will be a satisfied, if summation of the energy will be remain same, and to be constant there. So, sum of different forms of energy is constant, this is the principles of conservation of fluid energy.

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Bernoulli's Equation (uniform Flow)



Daniel Bernoulli
(1700 – 1782)
Swiss
mathematician
and physicist

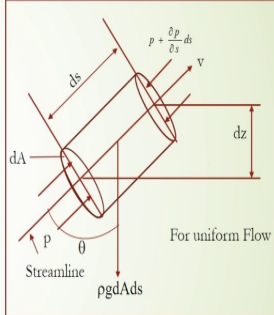
- A streamline element (a line which follows the direction of the fluid velocity) is chosen with the coordinates shown in Figure here
- Net force acting on the element:

$$F_{net} = p dA - \left(p + \frac{\partial p}{\partial s} ds \right) dA - \rho (dA ds) g \cos \theta$$

$$= - \left(\frac{\partial p}{\partial s} ds \right) dA - \rho (dA ds) g \cos \theta \quad (\text{Eq.1})$$

As per Newton's law

$$F_{net} = \text{mass} \times \text{acceleration} = \rho dA ds \frac{dv}{dt} \quad (\text{Eq.2})$$



Based on these are principle of energy, we will derive that Bernoulli's equation. This is one of the important energy equations to apply the fluid flow phenomena or to apply the fluid flow in a conduit or any other applications in real life. They are so Bernoulli's equations and this is applicable, whenever fluid will be flowing at a uniform condition.

So, in this figure if you see that a the picture here the picture in this case, if suppose if we consider that a one control volume of this fluid element, and the fluid is flowing at a velocity V. And stream line of this fluid element will be the same in the direction of the fluid velocity, and it is chosen with a coordinates are shown in figure here.

Like this here, this is in the now V direction that is here V, and it will be certain angle with the horizontal. So, there will be pipe flow there will be a flow in through the pipe, the pipe is inclined at an angle theta, and through which there the liquid will be flowing. So, in that condition if this cross section of at this point of this pipe is dA, and at this point there will be a suppose here again the same cross sectional area dA so, and that length of this fluid element is considered as ds.

And if we are considering that this is your stream line, and then these are the stream line, and the uniform velocity is there. So, this uniform velocity is V. So, what should be then of course acting on this element, and after that how energy can be calculated based on this force.

Now, what should be the net force applying in this fluid element, so how to calculate there. So, whenever fluid is flowing that at this cross section, there will be some pressure. So, there will be some pressure energy pressure will be exerting on this cross section, and at this cross section in the opposite direction this pressure will be there.

Now, this pressure at this outlet section will be some increment of the pressure along with this distance of this pipe or length of this fluid element there. So, in this case what should be the a pressure at this section; pressure at this sections, I have to calculate so in the inlet section pressure will be p into dA , and at the outlet section the pressure will be p plus dp into dA as shown in your figure also. So, this is in the direction of the streamline.

Similarly, at a time there will be a force acting a due to the gravitation. So, the gravitational a force will be m into g . So, here $m g$ is what that the $m g$ means ρ into a volume that is ρ into dA into ds into g , so it will be your gravitational force. But, gravitational force should be acting that is in the downward directions, but here if we consider the direction of the streamline flow, then we can say that a gravitational force and the streamline that is opposite to that streamline flow will be is equal to $\rho dA ds$ into $g \cos \theta$ there.

So, if we consider that a what to be the net for then, what should be the in the V direction streamline direction at the direction of the streamlined, and what will be the force acting, and a what will be the force acting that opposite to that streamline direction. So, opposite to the streamline only gravitational force is acting. So, if you look at this equation number-1, we can get p into dA first term, so p into dA is nothing but the force applying to this cross section dA . And p plus dp into dA , this is the a pressure this is the pressure which is acting at the a section two in the opposite direction of this a flow.

And if we multiply it by dA , then it will be force. So, p plus dp into dA ; dp into dA is nothing but what is that pressure increment or move pressure according to that length ds . And if you multiply this ds , so dp into ds , this will be your pressure increment, and then p plus dp into dA , this will be your pressure total pressure that is acting opposite to this flow of direction.

And then minus gravitational force in this the component of the gravitational force in this streamline that will be ρ into dA ds into $g \cos \theta$, so that will be finally minus dp

ρds into dA minus ρds into $dA ds$ into $g \cos \theta$. What is ρ is the density of the fluid, and dA is the small element of the cross sections here θ is the angle at which this pipe is inclined to this horizontal or flow is flowing at an angle θ to the horizontal.

And then what is that this is your F_{net} . Now, if we apply the Newton's law, then F_{net} will be is equal to mass into acceleration that will be is equal to $\rho dA ds$ into dv/dt . So, dv/dt is nothing but the acceleration of the fluid element. So, in this mass into acceleration that will be your F_{net} . So, $\rho dA ds$ this is your mass, and dv/dt is nothing but the acceleration.

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From Eqns. (1) and (2)

$$\frac{dv}{dt} = -\frac{1}{\rho} \frac{\partial p}{\partial s} - g \cos \theta \quad (\text{Eq. 3})$$

The velocity may change with both position and time. In one-dimensional flow it therefore becomes a function of distance and time, $v = v(s, t)$.

$$dv = \frac{\partial v}{\partial t} dt + \frac{\partial v}{\partial s} ds \quad (\text{Eq. 4})$$

The acceleration is then

$$\frac{dv}{dt} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial s} \frac{ds}{dt} = \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial s} \quad (\text{Eq. 5})$$

So, this if we equate this equation number-1 and equation number-2, then we can have this equation number-3 as dv/dt that will be is equal to minus 1 by ρds minus $g \cos \theta$, this is the first simplification. Now, we can say that from this equation this acceleration will be is equal to that means, density average of a pressure gradient, and minus this what is that a gravitational acceleration opposite to that direction of this flow stream, and it will be represented that as gravitational acceleration component, and $g \cos \theta$.

And the velocity may change with the both position and the time, so in that case a in a non-dimensional flow, it therefore becomes a function of distance and the time also. So,

if we consider that if we are dv is equal to $\frac{dv}{dt} dt + \frac{dv}{ds} ds$, then this will be your what is that velocity increment with respect to of a distance and time.

So, they acceleration will then be calculated from this equation number-4 by dividing dt , then it will be $\frac{dv}{dt}$ that will be equals to $\frac{dv}{dt} dt + \frac{dv}{ds} ds$ into dt dt will be cancelled out, and plus $\frac{dv}{ds} ds$ into ds by dt that will be is equal to simply $\frac{dv}{dt} dt + \frac{dv}{ds} ds$ by dt this is the fluid element distance, it will be by divided by ds this is nothing but distance by dt that is the velocity, so it will be v and a $\frac{dv}{ds}$. So, $\frac{dv}{dt} dt + \frac{dv}{ds} ds$, it will be your acceleration term.

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If the z axis is in the vertical direction

$$\cos \theta = \frac{dz}{ds} \quad (\text{Eq. 6})$$

So Eqn (3) becomes with Eqn (5)

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial s} = -\frac{1}{\rho} \frac{\partial p}{\partial s} - g \frac{dz}{ds} \quad (\text{Eq. 7})$$

For steady state $\frac{\partial v}{\partial t} = 0$

So

$$v \frac{dv}{ds} = -\frac{1}{\rho} \frac{dp}{ds} - g \frac{dz}{ds} \quad (\text{Eq. 8})$$

It is called **Euler's Equation** of motion for one dimensional non-viscous fluid flow.

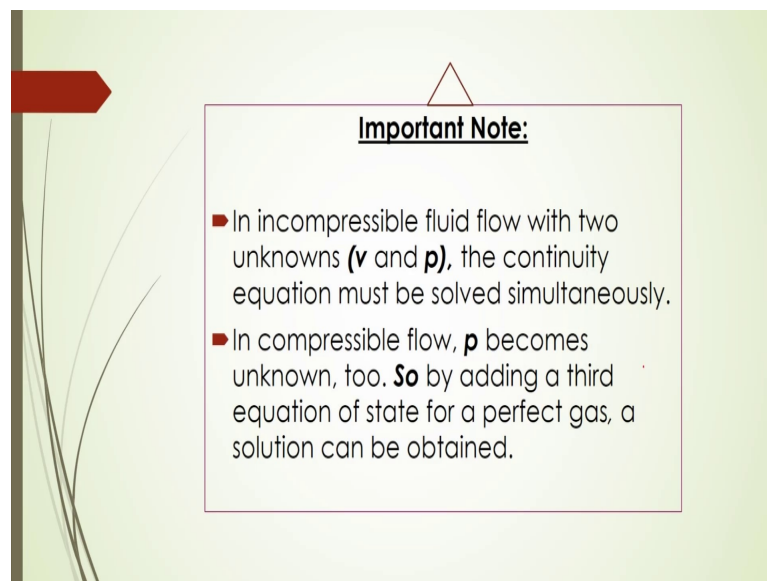
Now, if we equate this equation number-3 and 5, we can get a we can simply will be able to calculate what with the velocity distribution. Now, before going to that we have to actually calculate what should be the value for $\frac{dv}{ds}$ here.

Now, $\frac{dv}{ds}$ to calculate here interesting that to get these $\frac{dv}{ds}$ by $\frac{dv}{ds}$ equation for that over from the geometry, we can calculate here a what should be the $\cos \theta$. $\cos \theta$ in terms of ds and dz , so $\cos \theta$ will be is equal to $\frac{dz}{ds}$ as far what is that geometry here. So, it will be here this, so this mass is your what is that d what is that this mass is your ds and this mass is your d ds and this is your dz , so ds into $\cos \theta$ this will be your dz .

So, we can write dz by ds in equation number-6 as $\cos \theta$. So, equation number-3 becomes with equation number-5 $\frac{dv}{dt} + v \frac{dv}{ds}$ that will be is equal to $-\frac{1}{\rho} \frac{dp}{ds} - g \frac{dz}{ds}$. So, this is your simplified equation. Now, see $\frac{dv}{dt}$ will be equals to 0 for steady state condition. And this part $-\frac{1}{\rho} \frac{dp}{ds}$, this part will be is equal to pressure gradient, and per unit a volume and also here this $g \frac{dz}{ds}$, this will a gravitational component.

Now, total equation can be a simplified as $v \frac{dv}{ds}$ is equal to $-\frac{1}{\rho} \frac{dp}{ds} - g \frac{dz}{ds}$, because the steady state condition $\frac{dv}{dt}$ will be equals to 0. Now, this form of equation-8 equation-8 it is called Euler's equation. And Euler's the is an eminent mathematician born in Switzerland; and a he has given or he has derived this equation earlier. So, as for his name it is a represented as Euler's equation.

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Important Note:

- In incompressible fluid flow with two unknowns (**v** and **p**), the continuity equation must be solved simultaneously.
- In compressible flow, **p** becomes unknown, too. **So** by adding a third equation of state for a perfect gas, a solution can be obtained.

Now, in this case you have to remember that in incompressible fluid flow with two unknowns suppose v and p , the continuity equation you have to solve simultaneously. And for a compressible flow, this pressure becomes unknown to in that case a by adding a third equation of state for a perfect dash a solution can be obtained, because they are your density will be changing with respect to pressure also.

Again if we go to that equation number-8, and after integration of this equation number-8 with respect to that a distance of that fluid element to obtain a relationship between points a finite distance apart along with the streamlines as here $v^2 + 2 \int \frac{1}{\rho} dp + g z = \text{constant}$

integration of dp by ρ plus gz that will be is equal to constant. So, this is your equation number-8.

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Bernoulli's Equation

Eqn. (8) is now integrated with respect to s to obtain a relationship between points, a finite distance apart along the streamline as:

$$\frac{v^2}{2} + \int \frac{dp}{\rho} + gz = \text{constant} \quad \text{(Eq. 9)}$$

Integration: $v \frac{dv}{ds} = \frac{1}{\rho} \frac{dp}{ds} + g \frac{dz}{ds}$ (Eq. 8)

and for an incompressible fluid ($\rho = \text{constant}$),

$$\frac{v^2}{2} + \frac{p}{\rho} + gz = \text{constant} \quad \text{(Eq. 10)}$$

Unit: m^2/s^2 or J/kg

Multiplying each term by ρ ,

$$\frac{\rho v^2}{2} + p + \rho gz = \text{constant} \quad \text{(Eq. 11)}$$

Unit: N/m^2

Dividing each term by g ,

$$\frac{v^2}{2g} + \frac{p}{\rho g} + z = H = \text{constant} \quad \text{(Eq. 12)}$$

Unit: m

velocity head, pressure head, potential head, total head

Now, you have to integrate this. So, after integration, we can get it here this is simply v square v square by 2, and here this will be integration of dp by ρ , and a here it will be coming as the what is that g into dz , and that will be equals to constant of integration. So, finally these three term summation of these three terms will be equals to constant.

Now, and for an incompressible a fluid, where ρ will be equals to unchanged that means constant. So, in that case of v square by 2 plus this p by ρ , so in this just integration here, there will be no change of this what is that density. So, we can segregate this one by ρ , and then p dp integration, it will be only p ; so, p by ρ plus gz that will be equal to constant here.

What should be the unit for this v square by 2, it is nothing but meter square per second square or Joule per kg that means, here what will be the energy per kg that is represented by this equation number-10. Now, if we multiply each term by ρ , then we can get here ρv square plus here p plus $\rho g z$ that will be is equal to constant. So, what will happen here a ρv square by 2, this is what is called kinetic energy. This is as a you can say as pressure, and this is also p , what is the unit for ρv square by 2, this is the unit for pressure. So, it is Newton per meter square, and $\rho g z$ is what is that a potential energy, this is also that a as a pressure $\rho g z$, this is a Newton per meter square.

So, here so equation number-10, we can represent it by equation number-11 also. And here in this case if we again divide this equation number 10 by by g that is gravitational acceleration, then we can have here v square by 2g plus p by rho g plus z that will be equals to constant.

Now, v square by 2 g is the nothing but the velocity head, this is in terms of head y this is the unit of that means, distance here that is m. So, v square by 2 g is nothing but H is here. H is the head H is called head. So, v square by 2 g, it is called velocity head p by rho g, this is the pressure head, and z is the a potential head. The unit for this is terms is the m meter only, so that is why you can say this is the head. So, velocity head, pressure head, and the potential head and the summation of these three heads are called a total head, now this total head will be equals to constant. So, this is your Bernoulli's equation one form of Bernoulli's equation there.

And so these Bernoulli's equations are applied only for uniform flow in this case, other limitations also are there we will discuss later. And so this is the conservation of energy, you can say that total energy will remain constant there. So, v square by 2 g this is velocity a head plus p plus rho g plus z that will be is equal to total head will be constant. And in terms of energy you can say v square by 2 plus p by rho plus g z that will be equals the constant in equation number-10. So, this is a called the conservation of energy.

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Energy Equation for non-uniform flow

- In a real fluid flow across a large cross sectional area, the velocity distribution is not uniform
- The kinetic energy per unit weight of the fluid transferred across the section is greater than that calculated by using average velocity
- Hence true kinetic energy per unit weight can be expressed as

$$KE_{true} = \alpha \frac{v^2}{2g}$$

Then Bernoulli's Equation Takes the following form:

$$\alpha \frac{v_1^2}{2g} + \frac{p_1}{\rho g} + z_1 = \alpha \frac{v_2^2}{2g} + \frac{p_2}{\rho g} + z_2$$

α is called kinetic energy correction factor
 $\alpha = 1.03$ to 1.06 for turbulent flow
 $\alpha = 2$ for laminar flow

And if we consider that the flow is not uniform, so in that case what will happen? So, in a real fluid you will see that there is a cross section in which a through which the flow will be there, and in that case the velocity distribution will not be uniform, because the cross sectional area will change. So, in that case maybe for lower cross sections the velocity will be higher, and for a higher cross section velocity will be lower. So, in that case you will see the flow of the fluid will not be uniform in nature.

So, in that case the kinetic energy per unit weight of the fluid that will be transferred across their section will be greater than that of a calculated by using average velocity. So, if we consider on the average velocity, there will be some kinetic energy per unit weight, but if we consider that any cross sections without taking average velocity actual velocity, then the kinetic energy will be of course greater than the then that calculated by a considering the average velocity.

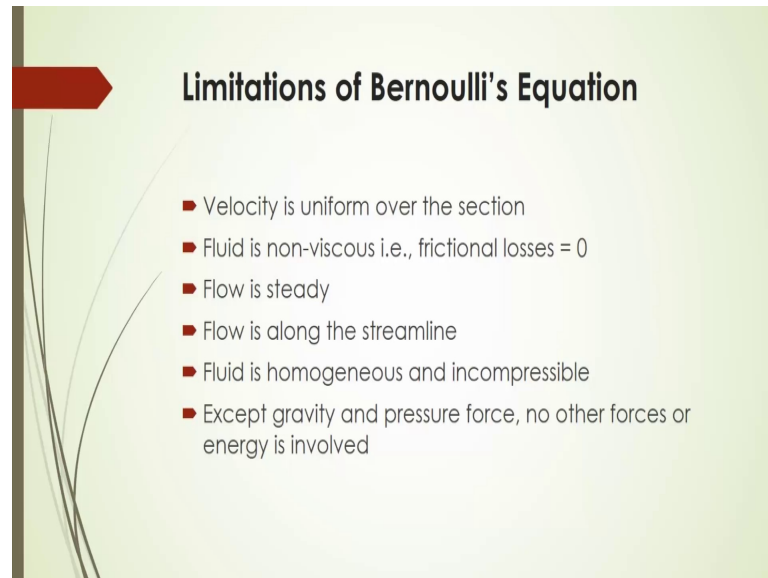
So, hence kinetic energy per unit of weight in this case can be expressed as kinetic energy through that will be α into v square by $2g$ that means you have to multiply some factor with that means, a kinetic energy that is v square by $2g$. So, this α is the multiplication factor, and this factor is called the kinetic energy correction factor. This kinetic energy correction factor will be your 1.03 to 1.06 for the turbulent flow if there is a turbulent flow, in that case you will see there we non-uniformity of the fluid flow.

And for laminar flow laminar flow generally this laminar flow almost uniform inflow, so in that case the α should be is equal to 2, finally then v square by g will be the your kinetic energy there. So, in that case you have to remember that the kinetic energy correction factor for turbulent flow will equals 1.03 to 1.06, whereas for laminar flow it will be 2. Now, finally with this correction factor the Bernoulli's equation can be expressed as α into v_1 is square by $2g$ plus p_1 by ρg plus z_1 that will is equal to α into v_2 square by $2g$ plus p_2 by ρg plus z_2 . What is this 1, 2? 1 is the cross section one, and 2 is plus another cross section there. So, through which this flow will be flowing.

So, in the cross section-1 in that case what will be the kinetic energy, and what will be the potential energy, and the pressure energy. Summation of these three at these sections will be equals to summation of this three energy that means, here kinetic energy, pressure energy, and potential energy. At these two sections the summation of this energy at these

two sections will remain same there, so that is why you are equating at these two sections the summation of these energies there.

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What will be the limitations of this Bernoulli's equations? So, generally these Bernoulli's equations are applied, when velocity is uniform over the cross sections. And where fluid is non-viscous also in that case frictional losses will be 0. So, Bernoulli's equations are applied only for those conditions. And also they are flow should be steady, and for unsteady flow Bernoulli's equations are not applied. And flow is done along the that means streamlines.

So, whenever you are considering the energy, so you have to consider the flow will be along the streamline. What should be the flow, if it is supposed not in a streamline it is vertical or some other forms, then you cannot apply this Bernoulli's equation. And in the case of fluid where homogeneous and incompressible are conditions will be there, then only you can apply the Bernoulli's equations. And also accept gravity and pressure force no other forces or energy is involved in this Bernoulli's equation. So, these are the limitations of the Bernoulli's equation that you have to remember. Whenever you are going to apply these Bernoulli's equations that you have to keep in mind those I think limitations there.

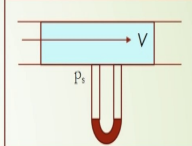
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Dynamic, Static and Stagnation Pressure

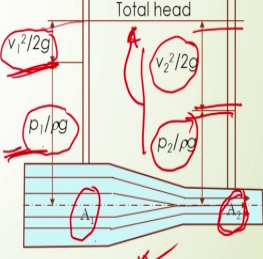
If the streamline is horizontal, then the term ρgh can be omitted giving the following:

$$\frac{\rho v^2}{2} + p_s = p_t$$

Dynamic Pressure + Static Pressure = Total or stagnation Pressure



$$\frac{v_1^2}{2g} + \frac{p_1}{\rho g} = \frac{v_2^2}{2g} + \frac{p_2}{\rho g} = \text{Total head}$$



Note: Whenever $A_1 > A_2$, then $v_1 < v_2$ and $p_1 > p_2$, where the flow channel is narrow (where the streamlines are dense), the flow velocity is large and the pressure head is low.

Now, based on this kind of a Bernoulli's equations, we can have what should be the dynamic pressure, static pressure, and stagnation pressure. So, if the stream line is horizontal, suppose flow is flowing if it is horizontal in flow, then you can say the term ρgh will be neglected, because they are both the sections the height will be same. So, in that case potential energy differences will be 0.

And in that case if we consider that Bernoulli's equations there, so it will be ρv^2 by 2 plus p plus here potential energy equal to 0, and that will be equal to total pressure here. So, ρv^2 by 2, this is also pressure energy, and this is your pressure energy. So, this one you this is called the ρv^2 by 2, it is called the dynamic pressure and p_s that is called static pressure. And this a total of this dynamic pressure and static pressure, it is called total pressure or sometimes it is called stagnation pressure.

So, if the horizontal pipe fluid is flowing, so in that case we are getting two types of pressure dynamic pressure, and static pressure. And summation of these two pressures is called the stagnation pressure. Now, again the Bernoulli's equation that is energy conservation of energy will be satisfied here. So, if we consider two sections of this pipe here, cross section-1 and cross section-2, then you can say this summation of this dynamic pressure, and the static pressure will remain same.

So, in that case here you can say in terms of head v_1^2 by 2 g plus p_1 by ρg that will be equal to v_2^2 by 2 g plus p_2 by ρg that will be is equal to total head. So,

total head a based on these what is the dynamic pressure and static pressure that will be remain same. So, if I consider here this figure what happened, you will see these two cross sections here cross section A 1 and cross section A 2.

So, in this case the velocity of this fluid will be v_1 , and velocity at this section should be v_2 . And then what will be the energy here in this case v_1 by $2g$ v_1 by $2g$ is the what is that it is called kinetic energy, and this is your p_1 by ρg , it is called pressure energy. So, total energy will be is equal to what is that this is total energy, and it can be represented by the head also.

So, p_1 by ρg is called pressure head, and v_1 square by $2g$, it is called a velocity head or yeah simply you can say it is already velocity a head we have told. So, these are two heads that is velocity head, and the pressure head a summation of this, it will be is equal to total head. And here again if you consider this cross section-2 cross section-2 here, what should be the pressure head up to this, and this one will be your velocity head. So, summation of this two head a will be equals to total head. So, this is your principle of energy. So, we can say that a total a head in this case will remain a constant.

So, very interesting that whenever if you are getting the cross section is greater than another cross section. One cross section is better, suppose here A 1 is greater than A 2. In this case of course, then we can say that v_2 will be is greater than v_1 v_2 v_2 a if A 1 is greater than A 2, then you can say v_2 will be is greater than v_1 , because here cross section is in the section A 2 will be a lower. So, higher velocity will be there, whereas cross section is higher in this section A 1, so velocity will be lower.

So, in that case v_1 is less than v_2 , and you can expect that p_1 will be is greater than p_2 there. So, where the flow channel is narrow, you can expect that the streamlines would be more dense, and the flow velocity will be large, and the pressure head will be low there. So, in this case a very interesting that since the total head will be constant. So, if velocity head increasing, then pressure head will be a decreasing.

Similarly, if pressure head is increasing, then velocity will be decreasing. So, this is the principle. So, based on these Bernoulli's equations, and the conservation of energy, we can calculate what should be the dynamic and static pressure, and also the summation of this two will be represented as the stagnation pressure.

there will be some energy loss during that operation, so that loss should be considered.

If we consider that loss head, a head loss it is called so that will be considered, then total head will remain constant. So, here at these sections there will be a certain I think a loss of head. Because, whenever fluid is flowing from this cross section to this another cross sections, there will be a certain change of kinetic energy, and also there will be a some loss of energy there. And based on that that loss if we consider that, it would be H_2 , then the total head will be H_2 plus here, what is that velocity head v^2 square by $2g$ plus pressure head plus potential head.

So, here four components 1, 2, 3, 4 say the four components of head that will give you the total head. Here at these sections also here, suddenly this higher cross sectional area to the lower cross sectional area, here also there will be a sudden change of energy that means, some loss of energy will be there. So, in that case potential energy will be here at a certain height, it will be z_3 and to be your pressure head and there will be some kinetic energy head, and there will be a loss h_3 there.

So, in that case or the real fluid case a real flow, you have to consider that there will be a certain a loss of energy, so that loss of energy are to be considered there to get the conjunct the of the energy equation. So, if there is no head loss, then simply this a summation of this velocity head, pressure head, and potential head will be a total head. And if there is a certain head loss, then you have to consider there.

Now, a you will see whenever you will supply any energy from one location to another location, there will be some usedness of some machines to just deliver this fluid to one location to another location. In that case that machine will supply some energy a to the fluid, and in that case you have to consider what should be the what is that a head due to that supply of that external machine or by devices to discharge this liquid from one location to the another location. So, they are a very interesting that for that there will be some head that head should be also should also be a considered they are in your energy equation.

So, so as far Bernoulli's equations or energy conservation of energy; so, v_1^2 square by $2g$ plus p_1 by ρg plus z_1 plus H_e , H_e is nothing but the here head a due to the supply of energy to the fluid by external device such as pump between the two section there, so

that will be considered here. And then to will be is equal to $v^2/2g + p/\rho g + z$ that is pressure head, and z that is potential and head plus h_L is the head loss here and again at section-3 the summation of this a velocity head, pressure head, potential head, and the head loss at the sections 3.

So, you will see that the total energy will be remain same that will be is equal to total a head. So, we can say from this a concept energy concept, we can generate that what should be the hydraulic grade and energy line. Now, this hydraulic grade is nothing but the you have to sum it up of only two components. One is the potential energy, and the pressure energy that means, potential head and the pressure head, summation of these two heads are will give you that what is that hydraulic grade.

And here at this location also what should be the what is that pressure head, and the potential head, pressure head, and the potential head that will give you that a hydraulic grade line. Whenever, you are adding that dynamic pressure that we see here velocity head then and the energy loss or head loss that will give you the total energy.

So, energy line without considering the head and head loss, then what should be the line there, this will be your the red line is represented by this energy line, whereas the dotted green line is represented by the hydraulic grade line here. So, from this figure you will be I think able to understand what should be the actually how this grade line, and energy line hydraulic grade line, an energy line can be generated based on this energy component.

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Example: A liquid is flowing in a horizontal pipe at a flow rate of $0.05 \text{ m}^3/\text{s}$. The length of the pipe is 200 m . Its centre line is 3 m above the datum line. The pipe tapers from 0.3 m diameter to 0.2 m . If the pressure at the longer end of the pipe is 98100 N/m^2 . Calculate the pressure on the other end. Neglect the losses. There is no head exerted by any external machine

$$\frac{V_1^2}{2g} + \frac{p_1}{\rho g} + z_1 + H_e = \frac{V_2^2}{2g} + \frac{p_2}{\rho g} + z_2 + h_2$$

Find V_1 and V_2 from continuity Equation:
 $V_1 = 0.707 \text{ m/s}; V_2 = 1.592 \text{ m/s}$
 Then, $p_2 = 99115 \text{ N/m}^2$

Handwritten notes:
 $V = \frac{Q}{A}$
 $V_1 = \frac{Q}{A_1}$
 $V_2 = \frac{Q}{A_2}$
 $A_1 = \frac{Q}{V_1}$
 $A_2 = \frac{Q}{V_2}$

And let us do an example for this Bernoulli's equations. And in this case suppose a liquid is flowing in a horizontal pipe at a flow rate of a 0.05 meter cube per second. The length of the pipe is 200 meter, and it is center is 3 meter above the datum line. The pipe appears from 0.3 meter to 0.2 meter. And if the pressure at the longer end of the pipe is here 98100 Newton per meter square, so what should be the pressure on the other end. And in this case you neglect the losses, and there is no head exerted by any external mention here.

So, simply we can apply this Bernoulli's equation here, the equation it is given here. So, in this case is there is no external machine is applied to deliver this liquid. So, we can omit this H_e term that means, here head due to the supply of energy by external machine. And h_2 there is no head loss it is there, so h_2 will be equals to 0 . If we consider these two sections of this pipe, where in the one sections the cross section I think as per the it is given that a cross sections in one cross sections the diameter is 0.3 meter, and other cross sections it is 0.2 meter.

So, based on that a if flow rate is given 0.05 meter cube per second, so V_1 you can easily calculate, and V_2 also easily calculated by flow rate divided by cross sectional area that will be your V . So, in the V_1 , it will be Q_1 by A_1 , and V_2 will be equals to Q_2 by A_2 here. So, Q_1 will be is equal to Q_2 .

So, we can say that what should be the V_1 and V_2 . So, after calculation, we are getting V_1 equals to 0.707 meter per second, and V_2 equals to 1.592 meter per second. Then after substitution of this V_1 and V_2 , then what should be the p_2 here, p_1 is given to you at these sections the p_1 is given to this one is known to you. Now, this one, this one is known to you, g is known to you, ρ is known to you, so z_1 and z_2 also known to you send then what should be the p_2 that you have to find out. So, finally p_2 is coming as 99115 Newton per meter square. So, these examples is applied just by applying the energy equations that is Bernoulli's equations there.

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Example: A pump of 11.19 kW with 80% efficiency discharging crude oil of specific gravity 0.9 to a overhead tank as shown in Figure. If losses in the whole system is 1.5 m of flowing fluid what is the discharge head by the pump?

Solution:

$$\frac{v_1^2}{2g} + \frac{p_1}{\rho g} + z_1 + H_e = \frac{v_2^2}{2g} + \frac{p_2}{\rho g} + z_2 + h_2$$

Annotations: $v_1^2 = 0$, $p_1 = 4500$, $z_1 = 4$, $H_e = ?$, $v_2^2 = 0$ since atmosphere, $p_2 = 0$, $z_2 = 25$, $h_2 = 1.5$

So, $0 + 4500/(0.9 \times 1000) + 4.0 + H_e = 0 + 0 + 25 + 1.5$

Implies $H_e = 17.5$ m

Power of pump = $11.19 \times 0.8 = 8.9484$ kW

Power = specific gravity * Q * H_e implies Q = 0.0571 m³/s

Other example suppose if we use any external devices or machine, in that case what should be that a head by a external machine based on this Bernoulli's equation here. So, in that case if we are given example of that a pump of 11.19 kilo watt with 80 percent efficiency discharging crude oil of specific gravity of 0.9 to a over a tank as shown in figure here. If losses in the whole system, is 1.5 meter of following fluid, what is the discharge head by the pump here.

So, in that case again. So, v_1^2 by $2g$ plus p_1 by ρg plus z_1 plus H_e that will be equal to v_2^2 by $2g$ plus v_2 by ρg plus z_2 plus h_2 . In this case v_1 is equal to 0 from this tank there will be a discharge of liquid by this pump here. So, v_1 will be equals to 0 here, and v_2 that you have to find out and z_1 is given to your four meter, whereas z_2 it is given to you, and here 25 meter.

And a here p_1 , what is the p_1 value is given to you that is I think you know this p_1 value it is given to you. And also power of this pump is given to you I think; it is a 11.19 kilo watt that means you are 8.9484 kilo watt that means, 80 percent efficiency. So, finally this will be your power of pump.

And power that will be this specific gravity into Q into H_e that implies that what is that Q will be is equal to this, but what is H_e here; H is what is that head a due to the supply of energy by this external machine. So, if we substitute this v_1 is equal to 0, and p_1 is like this. And what is that z_1 is 4, and here H_e is unknown to you. And here again v_2 is 0, and again p_2 is also atmospheric pressure, so p_2 is 0. And then a z_2 is given to 25, and h_2 is 1.5, it is given to you.

So, from these equations you can have what will be the H_e that means, here head loss by this external a form. Based on that power of pump, so you can calculate what should be the flow rate of this fluid. So, flow rate will be is equal to 0.0571 meter cube per second. So, this is your what is that a discharged, and also discharge head is a 17.5 meter.

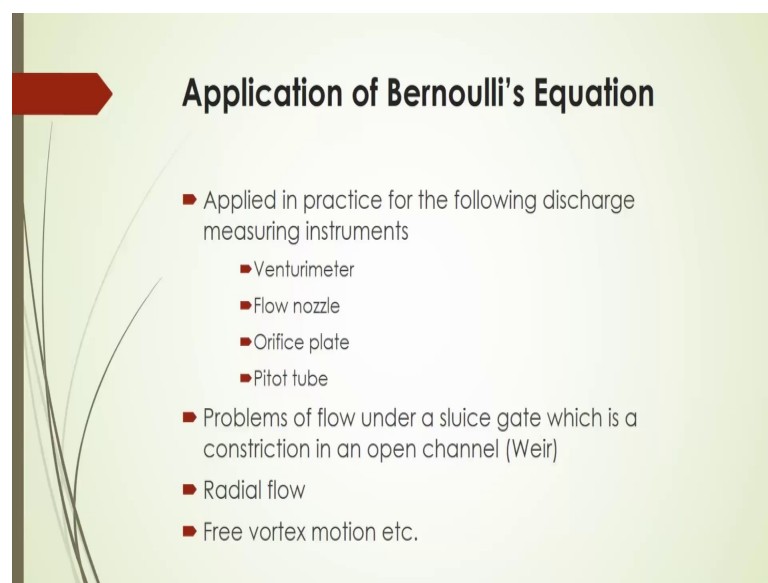
Now, application of Bernoulli's equations other application of Bernoulli's equations like a what is that it can be applied in practice for the following discharges measuring instruments like venturimeter, flow nozzle or if we split pitot tube, and problems of flow under a sluice gate a which is a constriction in an open channel that is called we are even radial flow, free vortex motion etc So, all those things in you can apply the Bernoulli's equation based on this total conservation of energy will be equals to same there.

So, we can apply in different way the application of this Bernoulli's equation based on that a Bernoulli's equation, we can developed some flow measuring instruments that is called venturimeter that venturimeter is nothing but a what is that just narrowing the channel. Whenever it will be flowing through the pipe or that means, narrowing that pipe and through which there will be a velocity change. And based on that velocity change, there will be pressure; a change by this principle of this Bernoulli's equation or energy equation.

And in that case what should be the velocity, and if you know the velocity what should be the flow rate of that, so because the flow rate is the calculated based on the velocity into cross sectional area, because the cross sectional area is known to you, so what should be the flow rate there.

And flow nozzle also there is important that a in that case if we use some nozzle, and through which that the fluid will be flowing. So, in that nozzle there will be higher velocity that, will z velocity. Once you know that that is a z velocity, and what should be that cross sectional area of the nozzle whole size if you know that whole size are, you will be able to calculate what should be the velocity at that particular nozzles that is z velocity. And if the conservation of the mass is there, if a the fluid is flowing at a certain mass, then a mass flow rate or volumetric flow rate. So, from that a through a cross sections are you will be able to calculate, what should be the velocity there.

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And by this you would be able to develop the flow nozzle, and also another important that orifice plate. Another important measuring instruments by which you would be able to calculate, what should be the velocity at a particular a location there. And also a pitot tube that also in the stream line what should be the what is the dynamic pressure, and a static pressure will be able to calculate by this a pitot tube based on this a Bernoulli's equation.

And also we have already shown earlier that some sluice gate operation there the design of that a gate, and how much pressure is a producing they are once the velocity through the river is known to you, then what will be the pressure will be developed based on this energy equations also. If you know the pressure head they are the flow, then what should be the velocity there. And the through the we are also by which you can calculate what to

the flow rate by this we are, so that flow rate can be calculated based on this conservation of energy law.

And in that case you will see that whenever fluid is flowing over that some we are, then what will be the velocity and discharge from that you will be able to calculate based on this energy law. And also radial flow whenever fluid is flowing through a certain or angular velocity, then how this vortex will be formed and at different locations based on this energy equations what will be the velocity they are once you know the pressure at different position. So, we can apply these energy equations in different instruments, we will be discussing later in that later on also in a separate module the function. And also principle of venturimeter, flow nozzle, orifice plate, pitot tube are based on this Bernoulli's equation.

And so in this lecture, we have learned the conservation of energy, how it can be derived, and how this a energy equations will give you that a hydraulic grade line, and the energy line. If is there any energy loss or if is there any head, I supplied by external machine, how to calculate even if the flow is non-uniform, then what should be that modified Bernoulli's equations there, then there you have to consider some kinetic energy correction factor.

In that case, there will have to remember that kinetic energy correction factor is the I think certain value α for this turbulent flow, and the laminar flow there. So, we have discussed here. The next lecture we will be discussing more about that this energy equations and energy principle there.

So, thanks for all today.