Fluid Flow Operations
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Lecture – 07
One-dimensional flow – Part 1

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Welcome, to massive open online course on Fluid Flow Operations. In this lecture will discuss about the One-dimensional flow as a Part 1. In the previous lecture, we have discussed about what is that circulation, rotation and also what should be the relationship between circulation, rotation and also vorticity? All these things.

Now, in this lecture have for a particular one-dimensional flow how the fluid will be behaving and also what should be the continuity equation for one-dimensional flow whenever fluid is flowing through a certain conduit.

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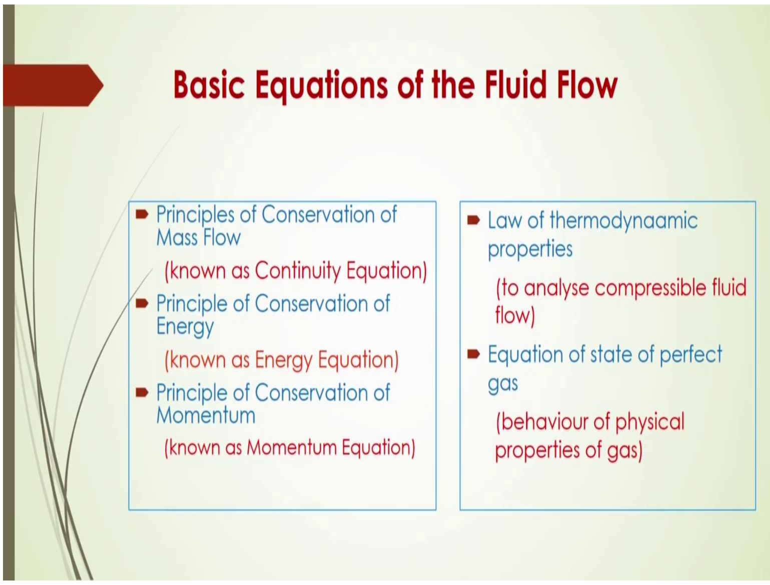

Let us see that first that what are the basic equations of the fluid flow. So, we say that the representation of the fluid flow includes this following basic equations a by which you can actually represent the fluid flow phenomena and also how the fluid actually a will be a facing different forces and how these forces will be a calculated based on this principles.

Generally, main three basic equations of the fluid flow are there one is called principles of a conservation of mass flow which is known as continuity equation and another one is called principle of conservation of energy and this is called as the energy equation and the finally, that principles of conservation of momentum. Here this is called the momentum equation.

So, by this basic three main equations you can express the fluid flow operations. Except these three equations, some other supplementary equations are required to solve those equations in this case like a law of thermodynamic properties like to analyze the compressible fluid flow along with this a basic equations. And, another is a to represent the gases or flow of gases they are of course, you have to know the physical properties of the gases because whenever right to be flowing through the conduit at a certain pressure and temperature, its physical properties will be changed.

So, in that case a equation of state for that particular a pressure and temperature to be represented and along with this basic equations and with this equation of state of that gases you have to express the phenomenon of this gas flow there. There are two types one is compressible, another is incompressible fluid; already we have discussed in the earlier lectures, there other several types of a fluids also there.

So, mainly compressible and incompressible fluids though compressible means those will a density will be changing with respect to pressure and temperature and it is called compressible like gases and incompressible where the density will be remain constant. So, it will be a compressible like water and other liquid also. And, the except these of course, compressible and incompressible fluid are there are other several different types of fluids are there. Now we have already already discussed in the previous or earlier lectures they are like a viscous liquid, non-viscous liquid, even you will see that there are Newtonian and non-Newtonian liquid.

So, whenever you are going to express all those the phenomena of these fluids whenever it will be flowing, these basic equations along with this a physical properties of the liquid or a fluid you can say that gas liquid and other type of liquid you have to consider there.

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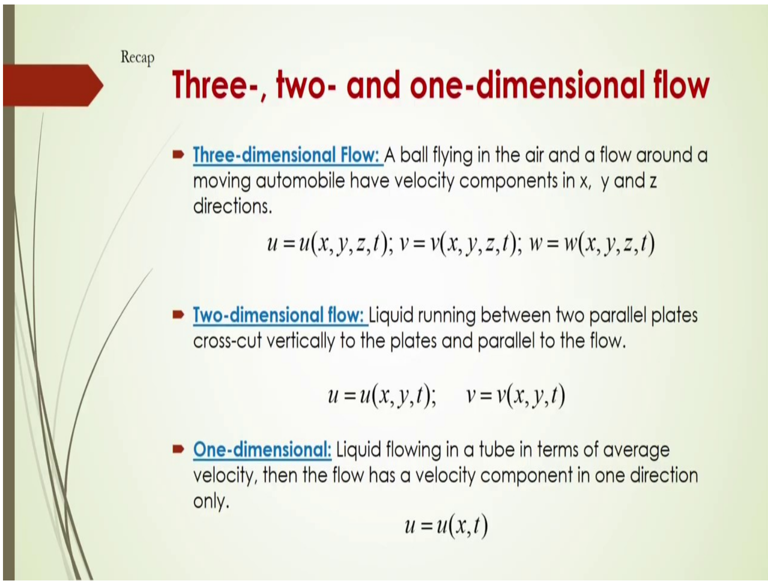

Now, if we a recap on this a one-dimensional, two-dimensional and three-dimensional flow that we have already discussed in the previous lectures that what is that one-dimensional flow, how it is actually being are represented, then they are of what should be the two-dimensional flow and how this three-dimensional flow also to be represented.

So, in that case one-dimensional flow there we have discuss that the liquid flowing in a tube in terms of average velocity of the flow has a velocity component in one direction. So, those type of flow will be called as one-dimensional flow and that one-dimensional flow will be represented by u is equal u as a function of only one dimensions like if it is in the x direction it will be only x as a function of x.

And, parallelly you have to consider that the time also this is also depends on time. If it is the steady state operation, then there will be no time consideration, but if you are if you are considering that there will be a variation of this any fluid a parameters in the particular direction, then you have to consider time also. So, in that case if you are considering that velocity of the fluid in the one direction, then it will be u as a function of u and t along with time.

So, this is one important and then another is called two-dimensional flow like liquid running between two parallel plates in that case only two-dimensions are considered and there will be what is that cross cut vertically to the plates and parallel to the flow. So, in that case the velocity of the fluid will be represented as a function of our special coordinates like x, y and also the time. So, it will be u as a function of x, y and t.

Similarly, in the y directional flow a for that particular fluid, then it will be represented by v and then v also will be represented by x, y, z because this u and v both will be changing especially; that means, here with respect to of a what is that location or you can say that the distance also. So, that is x, y and z. So, in the x direction at a particular location are from this a origin to a particular distance what will be the velocity of course, this velocity will be changing and v also will be changing in the y direction.

Similarly, for a three-dimensional flow or if suppose a ball that is flying in the air that I have given this example so, earlier in the lecture that the air, then you will see that flow around a moving ball in the sky what will happen? You will see there will be a velocity components in x, y and z direction. So, in that case the velocity of that means, here around velocity of air around that a particular object then the velocity of that a particular fluid will be represented by u, v and w in the x, y and z direction that is three-dimensions. So, they are of course, all these three-dimensions solve a function also again on the x, y, z and also with respect to time. So, three-dimensions, two-dimensions and one-dimensional flow here.

Now, whenever you are considering to express the what is that continuity equation that is called conservation of mass, we have to consider in one-dimension, two-dimension, three-dimensions also. So, we have to first derive what should be the mass conservation equation and that will be called as continuity equation.

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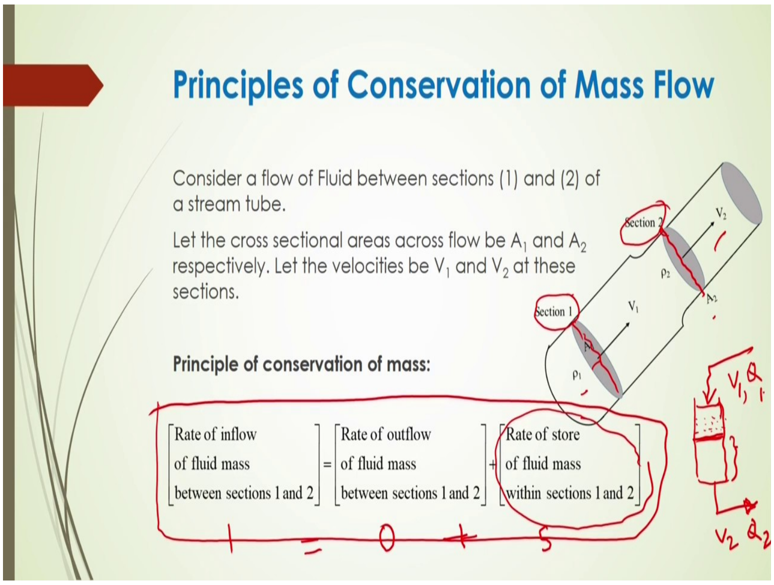

What is the principle of that conservation of mass flow? Suppose, if you consider the flow of fluid between two sections are here as shown in your figure in the slides that if you are considering the fluid between two sections here, section 1 and section 2; that means, we here the sections and here the sections and the fluid will be flowing through the sections. Let the cross section area of these sections will be as A 1 and A 2 and in this case if we consider that velocity be V 1 and V 2 through this a sections A 1 and A 2 respectively.

Then, the conservation of this a mass of flow mass flow can be represented as that the rate of inflow of fluid mass between sections 1 and 2, how much that is the liquid rate of inflow of fluid mass? This between sections 1 and 2 and rate of all flow of fluid mass between section and 1, 2 and also what should be the rate of store of fluid mass within the sections 1 and 2.

Now, if we relate all those things like what is the inflow rate of fluid between the sections and if it is considered as inflow I here and also the outflow as O here and the store of the liquid with in this the sections that if it is S, then you can simply say that I will be equals to O plus S here like this; that means, in flow rate of inflow of fluid mass this is of course, with respect to time not like that at steady state will consider that at steady state condition is also.

If suppose the rate of inflow of fluid mass between sections 1 and 2 that will be is equal to rate of outflow of fluid mass between section 1 and 2 plus rate of store of fluid mass within sections 1 and 2. You see suppose if any from any suppose any what is that pan from any pan from the top; the if liquid is supplied at a particular rate suppose V 1 velocity or volumetric flow rate suppose Q 1 and the fluid is out like this from this a pan, it will be suppose are V 2 and here the volumetric flow rate is Q 2. Here the cross sections of this pan will be same.

So, in this case if suppose this are the flow rate Q 1 and Q 2 Q 2 are same, then you say you will say that there will be no storage of water inside the in the that will there will be no increasing of that storage of liquid in the a pan inside the pan there we know that will not happen here. So, in this case see very interesting that if suppose Q 1 is greater than Q 2 volumetric flow rate of inflow is greater than Q 2, then you will see with respect to time the height of the liquid level inside the pan will increase. So, that is called a with respect to time how the liquid will be storing inside the pan or in a conduit.

But, if Q 1 and Q 2 both are same, then there will be no in there will no increase of this level of liquid inside the columns. So, in it is called steady state then there will be no storage of fluid mass within the sections 1 and 2. So, that is why, if we are having this storage then you have to consider here and if you are not having this then to be simply 0.

So, the rate of conservation that is principal of conservation of mass will the that will be represented by this equation. So, rate of inflow of fluid mass between sections 1 and 2 that will be equal to rate of outflow of a fluid mass between section 1 and 2 plus rate of store of fluid mass within sections 1 and 2.

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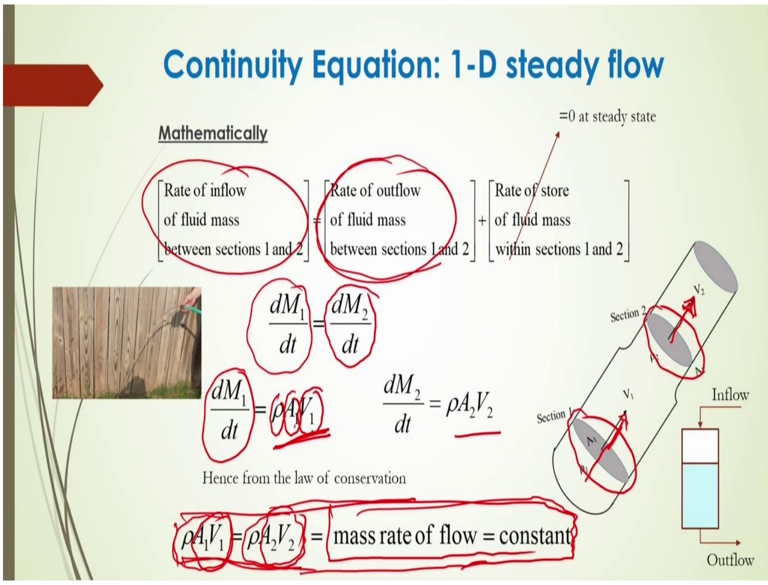

Now, if suppose there will be no storage of liquid inside the pan or any conduit, then simply you can say that will be steady state operation with respect to time there will be no storage of liquid inside the tank.

So, it will be steady state. So, this will be coming that only the inflow and outflow will be same there. So, this is called the continuity equation this is called continuity equation in the one-dimension or one-dimensional steady state flow. So, mathematically you can simply say that inflow will be is equal to outflow that is rate of inflow of fluid mass between sections 1 and 2 will be equals to rate of outflow of fluid mass between section 1 and 2. So, this one this is in this is inflow and this is outflow.

Now, how we can represent this rate of inflow of fluid mass? Suppose, here this is the section 1, this is here the liquid is in liquid in and liquid out here; that means, here inflow this is outflow. Now, in this case what will be the inflow rate here? If we considered that mass of the fluid as same, then rate of change of mass that is rate of inflow of fluid mass that will be is equal to dM 1 by dt. Similarly, at the outlet the rate of outflow of the fluid mass will be is equal to dM 2 by dt ok.

Now, what should be that dM 1by dt, the what is the rate of inflow of fluid mass? How mass? Mass you have to consider in terms of velocity here. So, what will happen? Mass can be mass will be equals to density into volume. Now, density is density of the fluid is rho and volume will be is equal to cross sectional area if it is A 1 then velocity if it is V

1, then A into V 1 that will be 0, that will be your volume per second or per time that will be is equal to then what will be the rate of mass inflow there. And, similarly rate of mass outflow will be is equal to similarly rho into A 2 into V 2.

Now, as for this equation at steady state condition then simply we can equate this 2 rate of mass between sections 1 and 2. So, hence from this law of conservation we can write simply that rho into A 1 into V 1 that will be is equal to rho into A 2 into V 2. This is your ah; that means, called mass rate of flow that will be is equal to constant here. So, this is as per continuity equation very interesting that though the rate of inflow of fluid mass always will be same with the out flow rate of fluid mass, ok. So, mass rate of flow will be constant this is the mass conservation equation here and is called as continuity equation.

Now, very interesting that if we see that this product of this rho A 1 into V 1 that will be is equal to rho A 2 V 2. Now, for incompressible flow this rho will be constant rho may not be 0, rho will be constant. So, in that case rho from the both sides we can omit or cancel it. So, it will be A1 V 1 will be is equal to A 2 V 2 simply A 1 V 1 will be is equal to here A 1 V 1 that will be is equal to A 2 V 2. What does it mean? The product of the cross-sectional area and the velocity through this cross sections will remain same.

So, here suppose if any pipe a cross section is higher than this cross sections what will happen? Then as per conservation of flow since it will be remain constant then mass flow rate should be remain a constant, then where the cross section will be higher then velocity should be lower. Whereas, cross section will be lower velocity be higher that is where that is why we are getting that different velocity of whenever the fluid in flowing a through the, a narrow tube and the coarser tube.

So, in that case the a larger diameter tube the velocity will be lower whereas, the narrow tube the velocity will be higher if you are supplying the same flow rate of liquid through this pipes.

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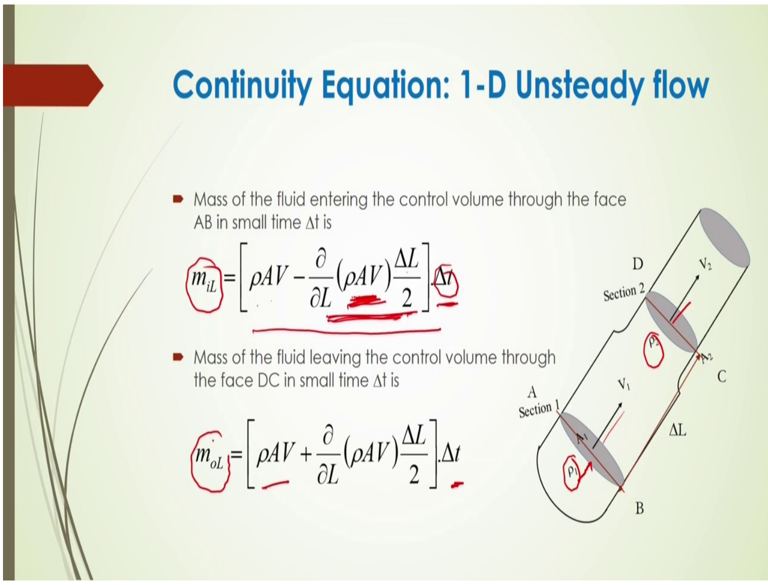

Another interesting point that a as far this continuity equation if it is unsteady of flow then how we can represent this continuity equation. Let us the consider that figure again here this A 1 is the cross sectional area and through which the velocity of the fluid is V 1 and the cross sectional area in the section 2, it will be A 2 and the velocity through this cross sections will be V 2. And, density at this concept cross section it will be rho 1 and here it will be rho 2.

And, the this case if we consider that the mass of the fluid that enters through this control volume through this sections 1 and within this sections 1 and 2 or you can say that face AV here AV in the small time delta t then we can write this equation m iL; that means, here mass flow rate that is inlet a that will be is equal to what how to represent this here this is given rho into AV minus rho dau dau L into rho AV into delta L by 2 into dt.

What is this here? Very interesting that before going to this sections there will be mass flow rate rho AV whenever it to be coming out from this sections there we change of that mass with respect to length. So, it will be dau dau L into rho AV this is mass per unit time mass per unit time, then if you multiply by delta t then within a that particular time what should be the mass and since the length of this control volume is delta L then average length will be is equal to delta L by 2 and you will get then finally, this will be your what is that change of mass with a small time of delta t. So, it will be m iL.

Similarly, mass of fluid that leaving the control volume through the face DC in small time delta t it will be represented by this here the initial liquid it will come rho AV into delta t plus rho into dau dau L into rho AV into delta L by 2 at times the this at this sections that into delta t. So, here this will be your outflow mass flow rate.

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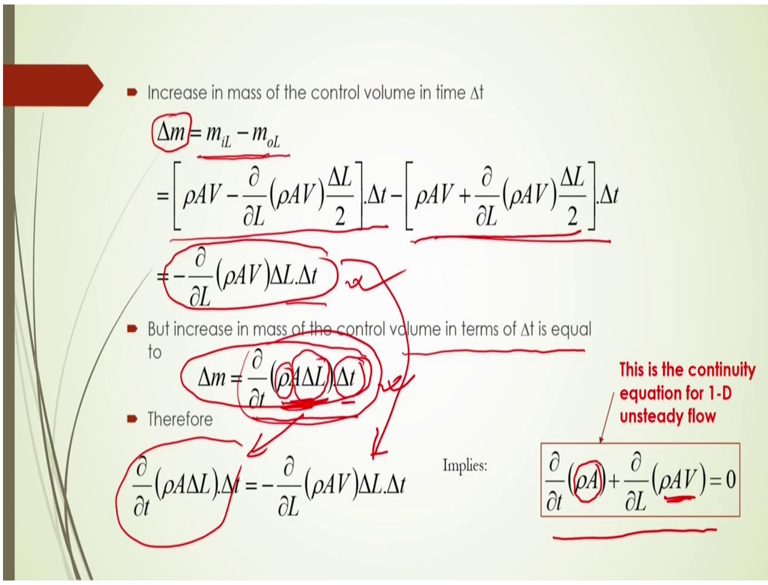

Similarly, we can write then what will be the then rate of increase of the mass of this control volume within this short time delta t.

So, increase of mass a will be delta m. So, this delta m will be is equal to what is the inflow and what is the outflow? If we subtract this in outflow from this inflow we can get the increment of the mass flow rate there. If there is a storage in a suppose a fluid conduit whenever fluid will be a moving a with a inflow will be higher than the outflow from then there will be what is that increment of this fluid, but that increment with respect to time. So, that is why this delta m with respect to time within a short period time it will be m iL minus m oL.

So, we can substitute this m iL from this equation here and then you just a substituted and then m oL then you substitute this m oL here in this form then finally, after simplification you can get this minus dau dau L rho AV into delta L delta t.

So, this is your increment of the mass increment of the mass element within a short period of delta t, but increase in mass of the control volume in terms of delta t is equal to what, that increment will be is equal to what delta will is equal to rho dau dau t that is simply rho A into delta L into delta t that control volume this is your control volume element and this is your rho this the density. So, density into volume this will be your mass then this mass per unit time rho dau by dau t and within a short period delta t it will be represented at this increment of this mass.

Now, therefore, a we can equate these two things this the expression with this expression ok. Since both are same delta m then therefore, we can write simply this will be is equal to this will be is equal to; that means, this one here this is this one and this is nothing, but this one if we equate this, then we can represent then what will be the continuity equation for the one-dimensional unsteady flow and it is coming that simply here dau dau t rho a plus dau dau L into rho AV is equal to 0. So, this is your continuity equation for unsteady flow.

In this case you will see that this rho A rho A is changing with respect to time. So, since if A is changing then of course, the if rho and A both will be changing according to then what will be the mass here it will be changing with respect to time and here also it will be change.

Now, if rho is not a changing for a compress the incompressible flow then you can simply say that dau A by dau t that will be is equal to dau dau L into AV that will be equals to 0. But, A if suppose that conduit or pipe if there is a fluid is flowing through the pipe with uniform cross sections then you can say that A will also be taken out from this equation then simply dau dau by dau t into rho and dau dau L rho V. So, this rho and V both will be changing, but if there is a uniform cross sections then there will be no change of it, but you there is a cross section change them of course, you have to consider here.

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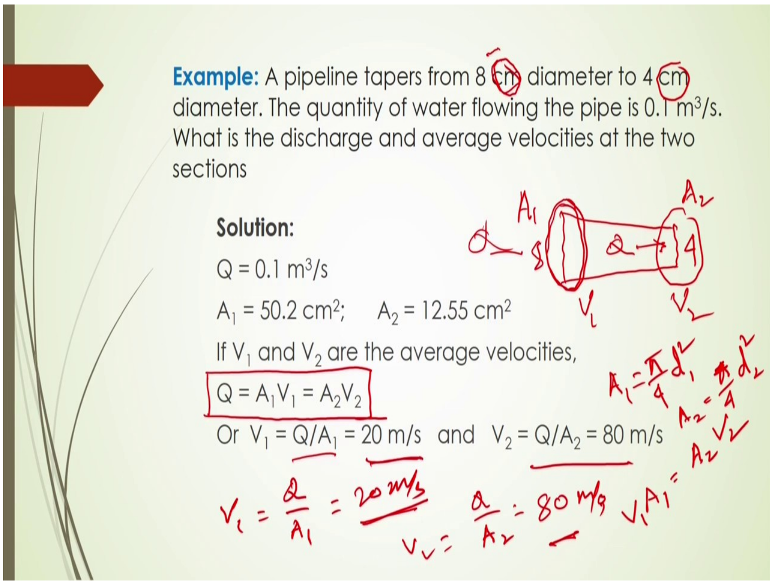

Let us do an example for this continuity equation for understanding. Now, if pipelines tapers from a 8 centimeter diameter to 4 centimeter diameter, then the quantity of water that flowing through the pipe it is given to you like 0.1 meter cube per second. So, within this flow rate of water if it is flowing through the pipe and that pipe is tapers from 8 centimeters to the 4 centimeter like this here this is 8 centimeter and this pipe is tapered to the 4 centimeter and here 8 centimeter.

Now, see here cross sectional area is varying at this location and at this location; here 8 centimeter and 4 centimeter. So, it is obvious as for conservation equation that velocity of course, will be different. Here some velocity will be there and here velocity will be some there and here cross sectional area let it be A 1 here it will be A 2. And, since it is water it is compressible fluid sorry incompressible fluid then you can say that a simply here A 1 as per mass conservation of equation or continuity equation simply A 1 will be is equal to a A 1 V 1 A 1 V 1 will be is equal to A 2 V 2 that is here and that should be is equal to volumetric flow rate. So, Q will be is equal to what A 1 V 1 equal to A 2 V 2.

Now, question is that from this conservation equation then what should be the discharge and the average velocities at these two sections. If you know this volumetric flow rate then simply you can calculate what should be the velocity at this section since you know the cross sectional area because cross sectional area you can calculate a from this diameter of this pipe simply cross sectional area A 1 will be is equal to A 1 will be is equal to what simply pi by 4 into d 1 square and a 2 will be is equal to similarly pi by 4 into d 2 square.

So, this is your cross sectional area. So, if you substitute this d 1 and d 2 you can simply get this A 1 and A 2. Similarly, how can I then calculate this V 1? V 1 is since we know that Q will be is equal to A 1 into V 1. So, V 1 will be is equal to Q by A 1. Q you know that that is 0.1 meter cube per second and A 1 is calculated by this equation, then simply you can calculate what should be the V 1 that will be 20 meter per second, but before going to calculate this be careful the all the dimensions to be considered in a particular system, not like that mixed system will be there.

Here suppose m MK system and here SI system. So, always should be same system like SI system here in this is 8 centimeter you have here it is given flow rate is given meter cube per second. So, you have to you have to consider this centimeter or you have to convert the centimeter to meter, then after substitution of this d one here you have to calculate A 1 and A 2; after that you can divide it Q by A 1 then you will get 20 meter per second.

Similarly, since this conservation of equations follows here. So, Q will be is equal to again A 2 V 2 the same flow rate in this sections also. What will be the flow rate here in the sections that flow rate of course; will be in this section also. So, in the sections what should be the V 2, say V 2 will be again then Q 2 sorry Q by A 2. So, it will be again as per substitution of this A 2 value here it will come 80 meter per second 80 meter per second whereas; here in this case it will be 20 meter per second.

So, interesting that interesting that here V 1 is very less whereas, V 2 is very high; that means here see smaller cross sections will give you the higher velocity and lots of cross sections will give you the lower velocity. So, this is the example that the follows this continuity equation here.

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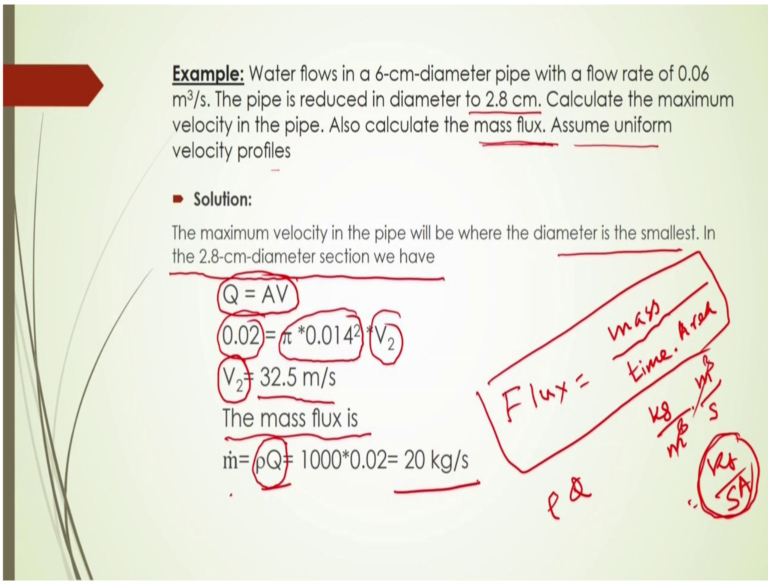

Similarly, if suppose any water flows in a 6 centimeter diameter pipe with a flow rate of 0.06 meter cube per seconds the pipe is reduced to the diameter to 2.8 centimeter here; 6 centimeter to 2.8 centimeter. Here also you can get the same the velocity at this section will be a lower whereas, velocity at this sections we have. So, in this case calculate the maximum velocity in the pipe. Also calculate the maximum mass flux assume that uniform velocity profiles will be there. So, in this case maximum velocity in the pipe will be there where cross sections will be lowest for the diameter is the smallest. So, in that case 2.8 centimeter diameter sections will give you the maximum velocity.

So, as per this Q is equal to AV formula then you can simply substitute this Q value as 0.02 and cross sectional area this pi r square you can say into V 2 or you can say pi d square by 4 also; either one and if then V 2 will be is equal to what 32.5 meter per second and the mass flux will be is equals to m dot that will be is equal to rho into Q mass flux; that means, here mass flux unit time per unit cross sectional area that is called flux. So, flux is defined as flux is defined as mass per time per area. So, this is your flux.

So, here this flux mass flux to be calculated as rho into Q what is that how it is coming; that means, a dimensions rho dimensions of rho is kg per meter cube and what is that and Q is here rho into Q is the volumetric flow rate. So, this will be meter cube per second meter cube per second. So, it will be what is that here meter cube meter cube cancel that will be is equal to kg per second that will be your mass flow rate.

Now, this masses per unit cross section of that. So, this will be your if it is per unit cross section area if it is 1, then you can simply calculate what will be the a flux. So, mass flux for; that means, per unit area here it will be is equal to m dot per unit area that will be rho into Q that will be is equal to 20 kg per second.

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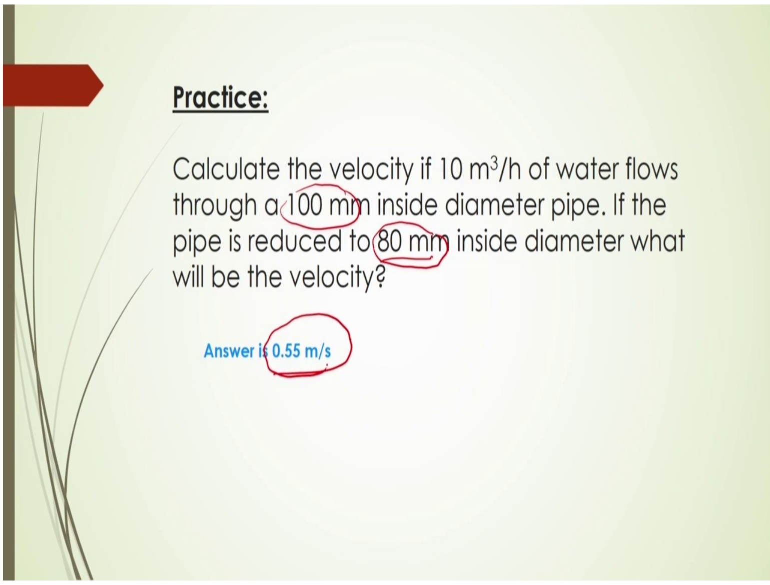

Now, you can practice also one example here the calculate the velocity if 10 meter cube per hour of water flows through a 100 millimeter inside diameter pipe if the pipe is reduced to if reduced to 80 millimeter here inside diameter then what will be the velocity there? The same way you can try once here 10 meter cube per hour you have to convert it to meter cube per second by dividing 3600. And 100 millimeter to be converted to meter; that means, you have to divided by 1000s, then you have to convert into meter and here also you have to convert into meter.

Then you have find out the cross sectional area in this for this a diameter and also for this diameter and from those part then what should be the velocity you can simply calculated here and final you can get this answer is 0.55. Try once yourself.

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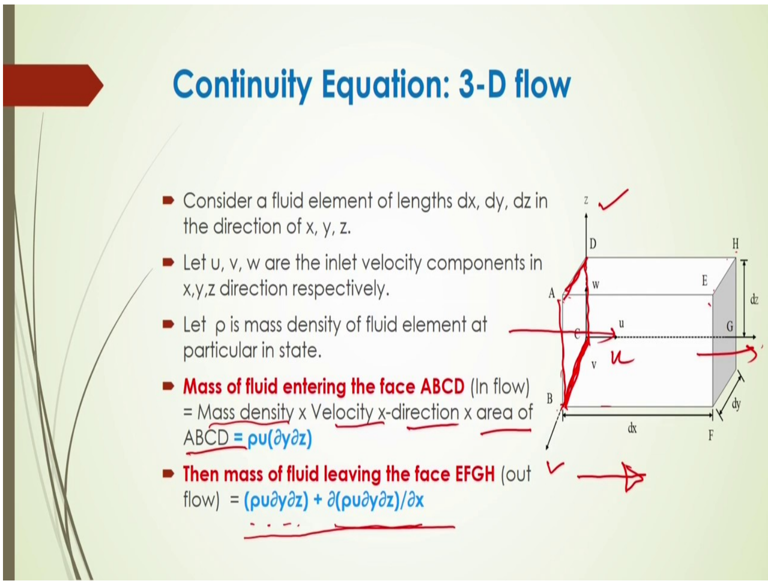

Now, continuity equation in three-dimensional flow in this case consider a fluid element of lengths dx dy and dz as per diagram here shown. Here if we considered this fluid element three-dimensional fluid element and it will have this space like this ABCD and here EF GH this two sections this cross sections here are also another cross sections here; AE HD and is a cross section BC GF that these are the cross sections of this fluid element here, now rectangular fluid element we are considering here.

So, in this case in if we consider this face we will see there will be a certain length of this rectangle here what is this what is the length of this will be dz sorry even in this direction it will be this is z directions, and this is y direction, and this is x direction. So, this what will be the length of this will be dy and this will be your dz and what is that this will be your d y and this will be your dz. So, this what will be the cross section of this area it will be what I think simply dy dz.

Now, question is that suppose that if a velocity in this suppose u, v and w are the inlet velocity components in this x, y and z directions and the if we considered the mass density of this fluid element is rho at a particular in state then, what will be the fluid entering to the particular faces? Consider this faces first like AB CD.

Suppose, the if we considered here the direction of the flow in the x so, the fluid will be crossing this cross sections and it will be leaving out from this cross sections. So, in this case mass of fluid entering the face ABCD that is called inflow of this mass that will be calculated as mass density into velocity in the x direction into area of that face ABCD. So, this will come as mass density is rho velocity is supposed u in this x direction u, then area will be is equal to what is that length into breadth that will be dy dz. So, you can say rho u rho y rho z here.

So, that mass of fluid that leaves the face EFGH that is called outflow will be represented by this here rho into u to dy dz or del y dau y dau z plus you see whenever it to be flowing out from the sections there will be a change of mass flow rate or velocity through this sections that is why that velocity in the x direction how it will be changing that will be change with respect to x. So, it will be dau rho u dau y dau z divided by dau x.

So, in that case finally, the mass will be change with respect to x here it will be dau rho u dou y dau z divided by dau x. So, this is your mass of fluid that leaves this face at this particular x direction.

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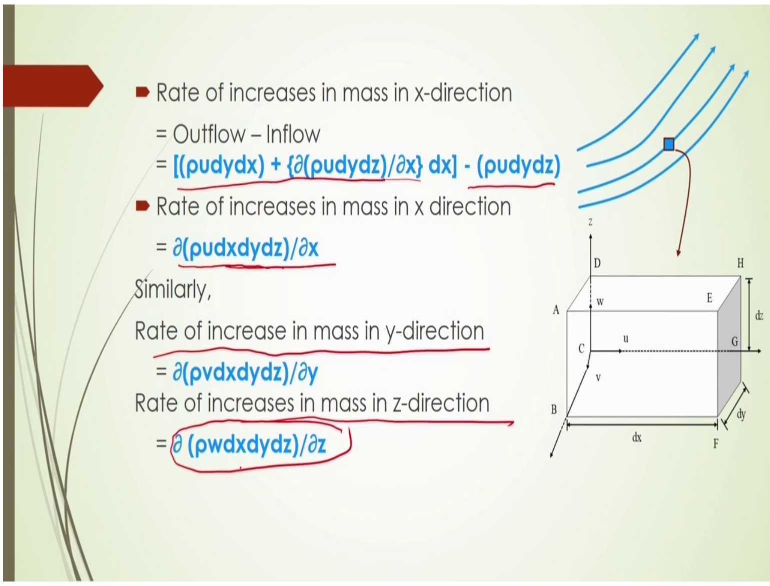

Then, if we know this inflow and outflow the in the similar expression that we have already discussed then rate of increases in mass in the x direction then it will be out flow minus inflow. What will happen then, if you substitute this outflow as this one that we have shown here this is outflow and this is your inflow that we have given here this is as in flow after substitution and the simplification you can say this will come like this dau rho u dx dy dz by dau z here.

And, similarly rate of increase in mass in y direction it will be simply dau rho v dx dy dz by dau y by dx dy dz we have multiplied here because this is a per unit volume. So, you have to multiplied by volume also. So, it will be coming like this. So, dx dy dz. So, this one rate of increase in mass in z direction, then it will be what is that similarly dau rho w dx dy dz by dau z.

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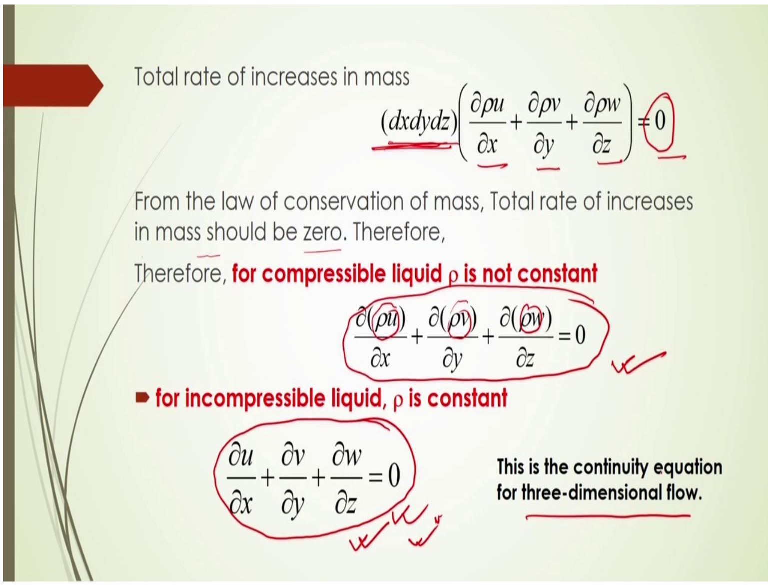

And, total rate of increase in mass then it will come dx dy dz after simplification into dau rho u by dau x plus dau rho v by dau by plus dau rho w by dau z that will be equals to what? 0, because there is no there is no storage of mass of fluid in this particular section that is called at steady state operation we are considering as per mass conservation law that simply you can write this.

Now, from the law of conservation of mass then total rate of increase in mass should be 0, that is why we have written here it will be 0. The after simplification we can get this dividing by this a volume dx dy dz we can get this equation. Now, this equation should be applied for compressible and incompressible, but for incompressible fluid this rho should be constant rho should be constant. So, that is why after cancellation of this rho dividing by rho in the both sides then it will be coming like this.

So, for incompressible flow since rho is constant we can get simply dau u dau x plus dau v dau y plus dau w dau z equals to 0; whereas, for compressible flow you cannot segregate this rho you rho from this rho u here derivation are you cannot segregate because this rho will be changing with respect to x there and y and z also. So, you have to consider this rho u here and derivation.

So, this is applicable only for this compressible flow whereas, for incompressible flow the rho can be omit out and then it will be like this. So, this is called three-dimensional continuity equation.

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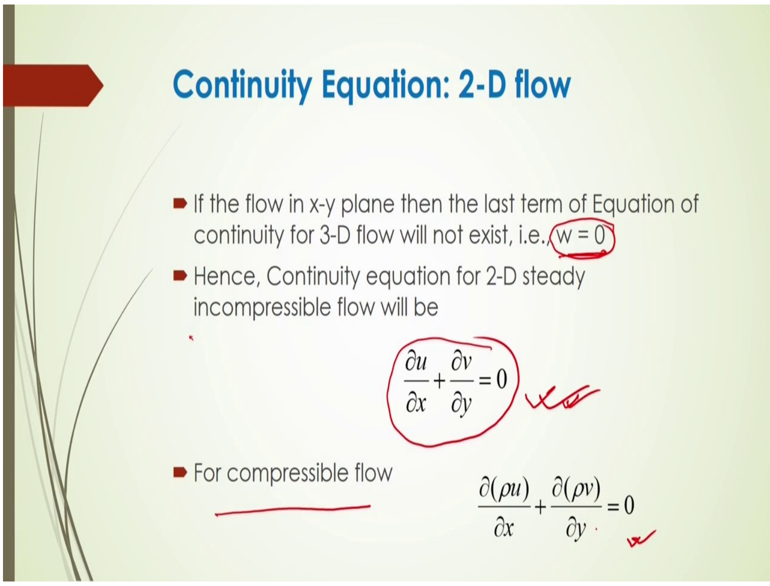

Now, what should be that two-dimensional? You can easily derive this two-dimensional continuity equation from this three-dimensional flow. If you know this dau u dau x plus dau v dau y plus dau w dau z for two-dimensional flow this if u w will be is equals to 0, suppose in the z direction there will be no flow, then in that case w will be equals to 0 and hence dau w dau z will be equals to 0 then simply it will be dau u dau x plus dau v dau y will be is equals to 0.

So, this is your two-dimensional continuity equation and this is for incompressible fluid and for compressible fluid, then it will be a rho u by dau x dau rho v dau y that will be is equals to 0. Here again then you have to consider that are density.

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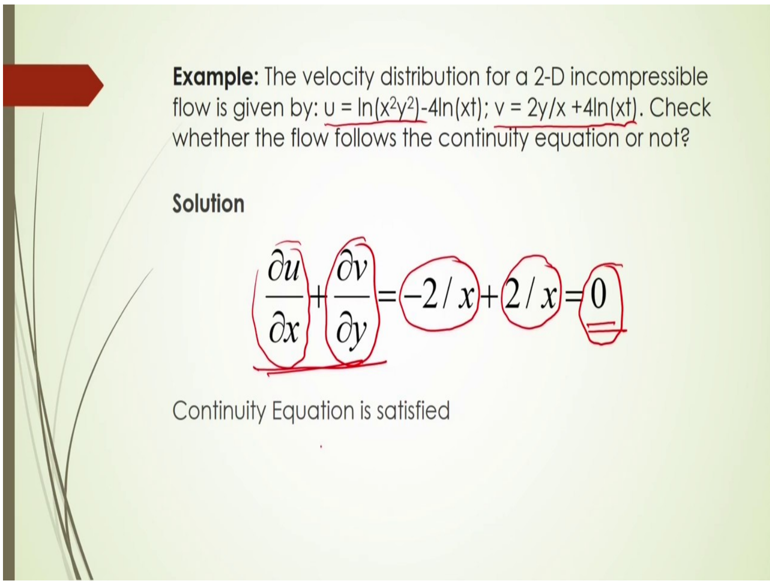

And, let us do an example here let it be here that this velocity distribution for a two-dimensional incompressible flow is given by suppose the U will be is equals to what is that ln x square y square minus 4 into ln xt are then V will be is equal to 2y by x plus 4 ln xt. So, this is your example.

And, in this case you have to check whether this the velocity distribution in the u and v direction whether combination of this velocity distribution will follow this continuity equation or not. So, in this case simply you have to calculate for your to find out what would be the dau u by dau x from this u is equal to ln x square y square minus 4 ln xt then you will get it will be minus 2x by 2 by x similarly dau v by dau y you can calculate it from the velocity distribution in the y direction then you can get it again plus 2 by x. So, summation of this to what it will become? It will be come in 0.

So, as per two-dimensional continuity equation you can have this as per your example it will be 0, based on your velocity distribution. So, this velocity distribution will follow this continuity equation.

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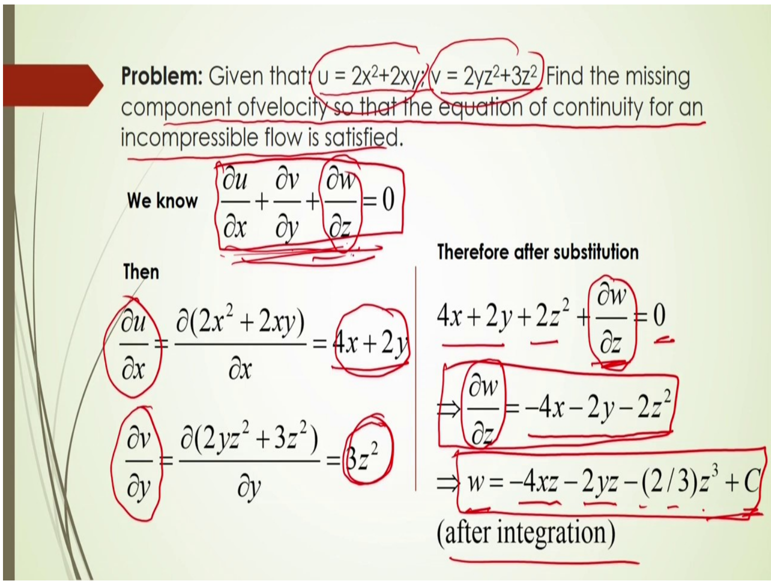

Another examples, whether this a three-dimensional fluid follow the continuity equation or not. Let it be given that the in the x direction the velocity distribution is U and this a in the y direction the velocity distribution is V. Now, find the missing component of velocity so that the equation of continuity for the incompressible fluid will be satisfied.

So, in that case very interesting that we know that the three-dimensional continuity equation in that case it should be equals to 0 summation of this velocity gradient in all directions will be 0. Then in this case we are getting only two-dimensional distribution velocity distribution u and v, but z direction is not there.

So, to find out the z direction and to follow the what is the continuity equation so, we have to considered that there will be certain velocity in the z direction. So, in that case z direction velocity let it be w. So, dau u dau x plus dau v dau y and plus dau w by dau z should be equals to 0. In this case u since it is given so, dau u by dau x will be is equals to what after derivation it will come 4x plus 2y whereas, d is given to you accordingly u if you do the derivative of this v with respect to y you can guess dau v dau y then it will you can get finally, 3z square.

Now, you know this then two-dimensions here u and v directions what will be the velocity gradient remaining one is required dau w dau z it is not known to you. So, from this continuity equation you can simply after substitution of this three component this one this one that is dau u dau x dau v dau y and remaining dau w by dau z to be remain same here you have to find out is not known to you. So, since these are known this is known and this is unknown and equal to of course, it will be equals to 0, and after simplification then dau w dau z that will come minus 4x minus 2y minus 2z square here.

So, from this equation if you integrate it, what will happen? Then you can get it w will be is equal to minus 4xz minus 2yz minus 2 by 3z cube some plus constant here. So, after integration you can get this w. So, w that is the velocity distribution in the z directions. So, velocity distribution in the z direction this will be your final equation. Now, if we considered this final equation of this velocity distribution in the z direction and u and v and reverse calculation of this dau u dau x and dau v dau y and dau w dau z you will see you can you can get it as that is simply 0.

So, that is why continuity equation will follow and as well as you can find out the remaining velocity component if your at this two-dimensional velocity distribution known to you.

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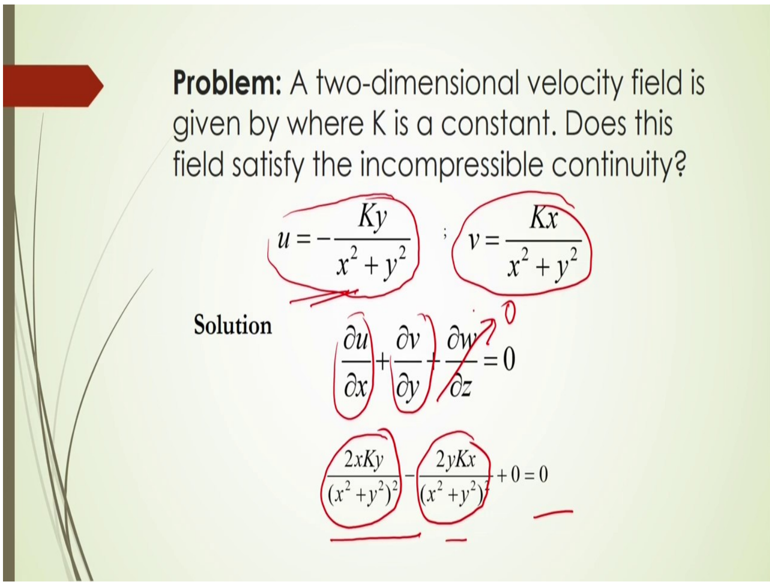

Another example suppose a two-dimensional velocity field is given to you where in this case it is given u is equal to minus Ky by x square plus y square and v is equals to Kx by x square plus y square, where K is a constant. So, in this case does this field velocity a satisfy the incompressible continuity a equation or not.

So, in this case same way is the simply a calculate what should be the two-dimensional a continuity equation in that case w, dw will be is equals to 0. So, only d u y dau u dau x and dau v dau y you have to calculate dau dau u dau x from this equations what will be the after derivation it will come like this 2x Ky by x square plus y square whole square and whereas, dau v dau y it will be coming as to 2y Kx by x square plus y square whole square

So, this two terms are equal. So, it will be canceled out final it will come 0. So, that is why these two velocity field will follow this continuity equation.

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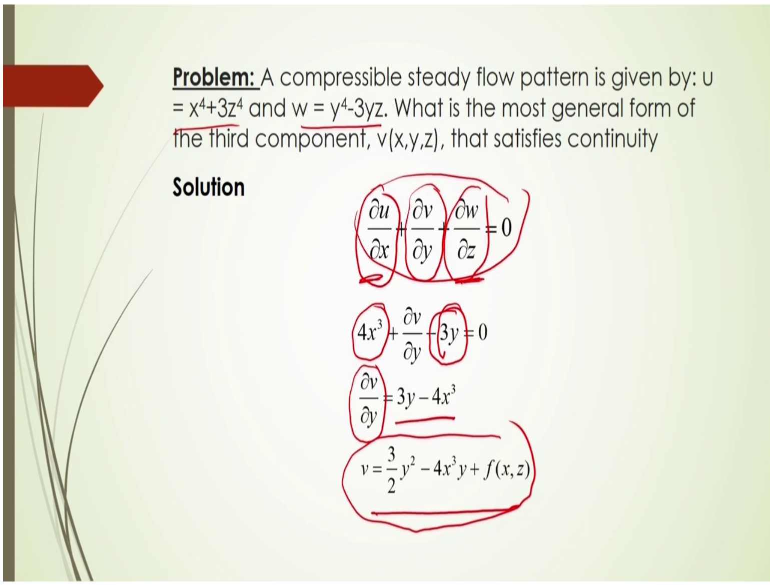

Another example a compressible steady flow pattern is given by u will be equals to this and w is equals to this here in this case in the x direction and y direction velocity component is given to you. What should be the y directional velocity component or velocity distribution equation that are you have to find out from the continuity equation. So, in this case the continuity equation follow, here this one is given to you and this one is given to you. So, what should be this one?

So, this one from the velocity distribution x direction you can find out this will be your what 4x cube and this will be your what minus 3y so, remaining in this dau v dau y. So, dau v dau y that will be is equal to 3y minus 4x cube and after integration you can get it.

So, simply this is also same way you can calculate what should be the continuity equation and finally, you can get 0 and what will be the velocity distribution in the y direction. So, this is the example you can.

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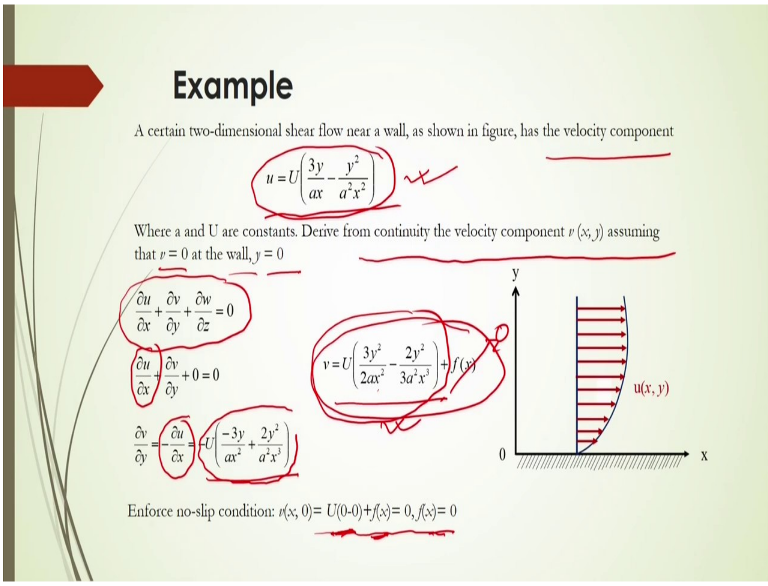

Another example like is certain two-dimensional shear flow near a valve as shown in figure here and has the velocity component like u is equal to U into a 3y by ax minus y square by a square x square; a is a constant and u also is constant here. So, derive from the continuity the velocity component in the y direction assuming that u will v will be equals to 0 at the valve y is equals to 0.

So, in this case again that continuity equation the three-dimensions will follow and the you have to calculate the dau u dau x from this equations u is equals to this. What it is coming? It will be coming minus u into this like this and then what is that in the z direction it is a 0. So, what to be the in the U directions that you have to find out. From this finally, you can get this dau v by dau y that will be is equal to minus dau u by dau x that will be is equals to this and then what is that finally, after integration you can get v will be is equals to this.

Now, a what should be that v at what is that given v is equals to at x and y is equal to 0. So, in this case U at that a U 0; that means, here at 0 minus 0 in that case y if your substituting this y is equals to 0 and v is equals to 0 in that case simply you can get it u will be equals to here 0 and fx will be is equals to some value. So, finally, you can get this v x 0 will be equals to what is that 0 and f x will be equals to 0. So, final equation will be equals to here f x to be 0. So, finally, v will be is equals to what this one. So, this velocity component.

Now, this velocity component, very interesting; that here it is coming U into 3y square by 2ax square minus 2y square by 3a square x cube. So, this is totally different from this u. Now, again if you do this derivative of U with respect to x and this v with respect to y and sum it up you will get 0. So, that is y this continuity equation will satisfied. And, so, very interesting that from the continuity equation if you know the velocity field in particular directions then remaining directions what should be the velocity component you can find out from the continuity. Only thing is that the gradient of the velocity x, y and z directions summation there summations will be 0.

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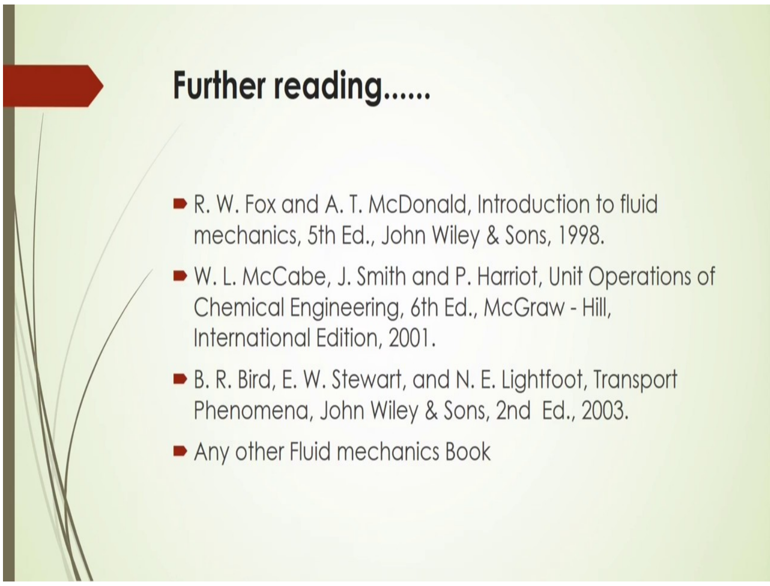

So, we have discussed a the continuity equation this is in one-dimension, two-dimension, three-dimensions and also how to derive those continuity equation in all directions also and what should be the like a fact and in that case how actually a we can calculate the velocity distribution in either directions with an example and I think you can go through this examples again. And, try to actually access yourself by considering different value the cross sectional area it will velocity distribution and following the other a text book also you can I think a we learn and the you can understand easily.

So, thanks for all today. Thank you for attention.