Fluid Flow Operations

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Lecture – 06
Fundamentals of Flow Part 2

Keywords: One, two and three dimensional flows; Convective acceleration; Local acceleration; Substantial derivatives; Rotational and Irrotational flow; Angular velocity; Vorticity; Stokes' Theorem

So, welcome to massive open online course on Fluid Flow Operations. In this lecture, we will discuss about the Fundamentals of Flow as a Part 2. In part 1, we have discussed about the mathematical representation of fluid flow how it can be represented by Eulerian method or by Lagrangian method and also we have discussed about the streamline, streak line, path line, timeline and also stream function and velocity potential, how this velocity potential is related to the stream function and also what are the different types of fluid flow like whether it is a steady state or unsteady state, if it is viscous or inviscid that has already been discussed.

And, weather the flow the flow is uniform or non uniform, how the flow is fully developed and also whether it is laminar flow or turbulent flow, how this laminar and turbulent flow be calculated or can be identified and also we have discussed something about what will be the basis of the two-dimensional, three-dimensional and also two-dimensional flow.

Now, in this lecture we will discuss about the acceleration will be represented in three-dimensions and also what should be the circulation and how it can be mathematically represented, what would to be the vorticity and how it can be mathematically represented in two and three-dimensions and also what will be the angular velocity, how it can be represented, how the pressures are acceleration of the fluid flow can be represented in three-dimensions even in a single directions also.

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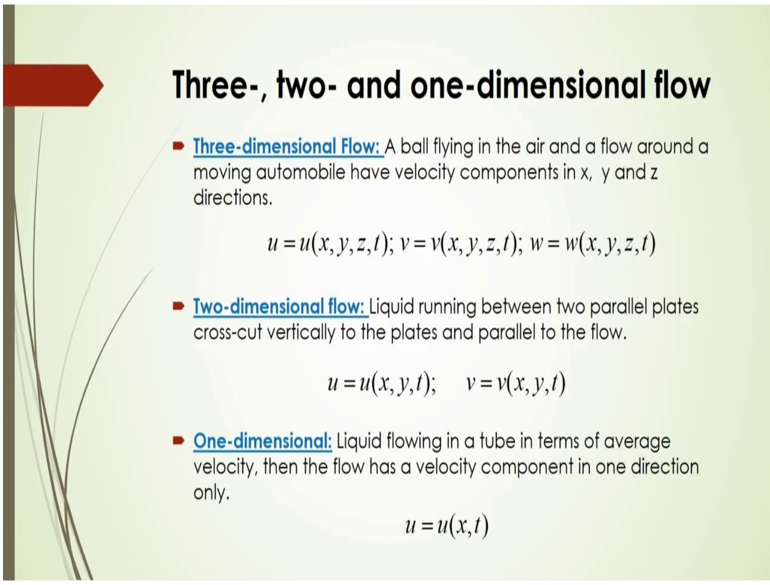

Now, as a two, three and one-dimension case, we can say that this three-dimensional flow is a represented by as a function of x, y, z that is in the spatial coordinates in x, y and z and also as a function of time. So, if suppose the velocity of the fluid is represented in the x, y and z directions, then how it can be represented? In three-dimension, it will u will be as a function of x, y, z and t. Similarly, in the b; that means, the y direction again it would be a function of in x, y and z direction and also time. If w is the velocity component in the z direction then also this w can be represented as a function of x, y, z and t.

In two-dimensions also we can represent the fluid parameters like if liquid is running between two parallel plates and cross cut vertically to the plates and parallel to the flow then we can represent it as u, that is u is the velocity of the fluid as a function of only x and y direction and also as a function of time. Similarly, v will be represented by that in the x and y directions then it will be x, y and t.

For one-dimensional case, if suppose the liquid is flowing through a narrow tube and its velocity will be average velocity, you will see that there will be no variation of the velocity in the y and z directions except x direction. So, in that case the flow as a velocity component in one direction should be represented only in the x, y or z directions either one. So, u will be a function of only in the x direction or it is in the y direction or in the z direction that is you will be do x, t; the t is the time here or you can say v will be is equal to y, t or z will be w will be is equals to only that is z, t.

So, in this way you can represent the one-dimensional, two-dimensional and three-dimensional flow.

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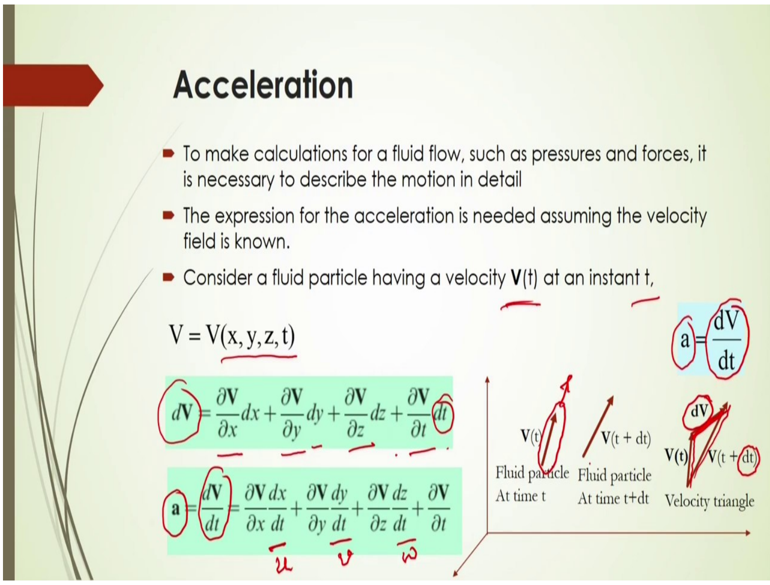

Now, it is considered the acceleration of the fluid flow and to make these calculations for this fluid flow such as spacious and forces, it is necessary to describe this motion of the fluid in details x as a as in terms of what is that acceleration.

Now, the expression for the acceleration is needed as you in the velocity field is known to you. Now in that case if you consider a fluid particle that have a velocity V (t) here velocity V (t)at an instant t then we can say that V should be as a function of x, y, z and t. And, if we differentiate t to with respect to that special coordinates and the time in stand then we can say that dV will be is equal to dau V dau x into dx plus dau V dau y into y plus dau V dau z into dz plus dau V dau t into dt.

In this case this dau V by dau t if we divide this dau dV by dt; that means, on both sides if we divide then we can get dV by dt that is actually the acceleration of the fluid. Now, this will then come it will be dau V dau x into dx by dt plus dau V dau y into dy by dt plus dau V dau z into dz by dt plus dau V like dau t. Now, this one will be just dx by dt will be represented by u and dy by dt it is V and dz by dt it is w in the x, y and z direction respectively.

Now, see this velocity vector if suppose the fluid particle is moving in the in this direction at a velocity V (t)and after a time t plus delta t; that means, here within a small period of time if the fluid particles just moving ahead then the velocity will be V as a function of t plus delta t. And, in this case if we represent this as a velocity triangle here, this is the V at t and dt time interval. So, here in this case this is V (t)then what will be this is one this will be your just increment of this velocity with respect to dt that will be represented by dV.

So, here if you know this dV with respect to this small time dt, then you can easily calculate or represent this acceleration as dV by dt which is denoted by a here.

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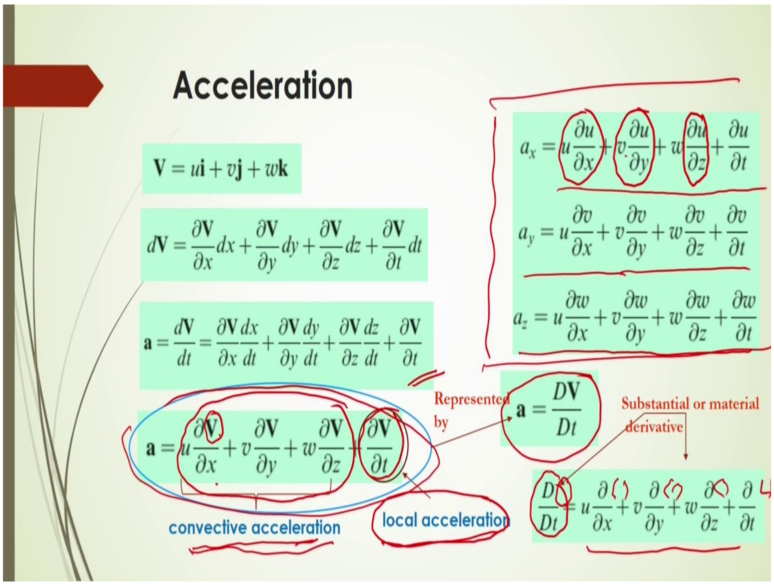

Now, if we consider this velocity to be represented by a vector notation, then you can represent this as V will be is equal to ui plus vj plus wk. Here u, v and w are the velocity components in the x, y and z direction and these are scalar quantities. And, then dV already we have discussed the dV will be is equal to dau V dau x into dx plus dau V dau y into dy plus dau V dau z into dz plus dau V dau t into dt and then yes, acceleration will be by this equation and finally, this.

Now, you will see if we can see here the two some terms here. Now, up to this terms of this acceleration; that means, here the spatial terms here u into dau V dau x plus v dau V dau y plus w dau V dau z this three terms will be represented or will be called as the convective acceleration; that means, here only due to the fluid flow or at a particular time instants only in the x, y and z directions, but there will be time will be fixed.

But, if we consider again this terms here only at any particular directions that is any x or any that is there is no actually effect of that special locations, then only time then it will be called as only local acceleration. So, total acceleration will be convective acceleration and the local acceleration.

Now, if we consider each component of this acceleration in x, y and z direction what should be that? Now, in this case V should be in the x direction if I considered that in the x direction then this V should be u and then acceleration in the x direction will be is equal to u into dau u by dau x like this here and in the y direction to be dau u dau y and in the z direction it will be dau u dau z. So, finally, it will be u into dau u dau z dau x v into dau u dau y plus w into dau u dau z plus dau u dau t.

So, here in this these are the components. So, you are observing here very interesting point that u; that means, in the x directional velocity scalar quantities also varies with x, y and z directions and also with time t. So, a x acceleration then it will be as a function of this. Now, only if you are considering the x direction, there will be no variation of the velocity of u with y direction then dau u dau y will be 0, and in the z direction if it is not the effective then dau u by dau z will be equals to 0. So, in that case only in the x direction the a x will be is equal to u into dau u dau x and plus dau u by dau t.

Now, very interesting that if you are not having any component of the velocity in the x, y and z direction only u will be there, then at a particular time then only dau u dau t will be is equal to acceleration. Similarly, in the y direction the acceleration components will be is equal to u dau v dau x plus v dau v dau y plus w dau v dau z plus dau v dau t. Similarly, in the yes z direction it would be represented by this equation. So, we are getting the acceleration in the x, y and z direction and what should be its components in x, y and z direction, how it can be represented? You can represented by this equations.

Now, the general equation should be is equal to this general equation should be is equal to this, and from the general equations we are getting two components here. One is that convective acceleration and another is local acceleration. So, from this general terms you can easily calculate what should be the component of the acceleration or any other variables there.

Now, this things can be represented by other important notations it is called substantial or material derivatives. So, dau capital DV by Dt this is called the substantial or material derivative. So, this substantial or material derivative is defined as u dau dau x plus v dau dau y plus w dau dau z plus dau dau t.

So, in this case this capital D by Dt here any parameter if you substitute respective parameter it will come here, here and here and also here. So, if you are considering will v then it will come v v v like this. So, the total acceleration will be represented by DV by Dt by this substantial derivatives.

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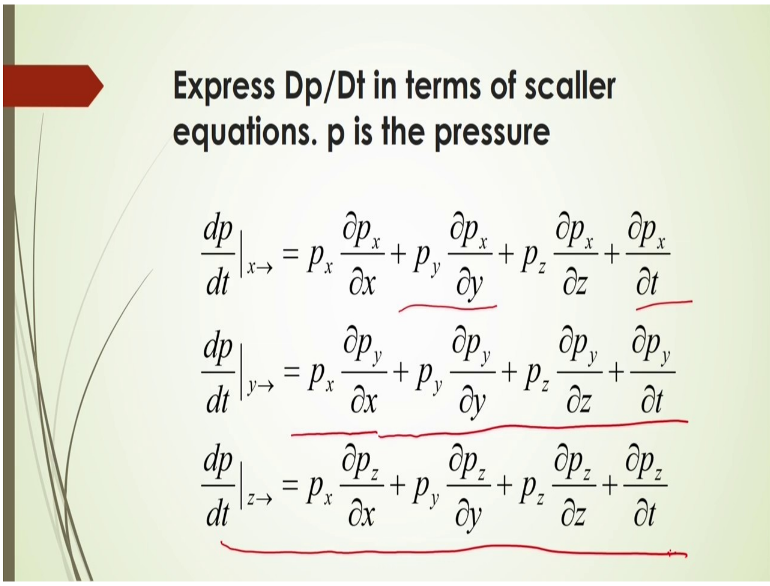

Next if we apply this substantial derivative to the pressure. So, what will be that? So, in the x direction the component should be that dp by dt that would be is equal to here p x dau p dau x plus p y dau p dau y dau p x dau y plus p z dau p x dau z t plus dau p x dau t. So, here this dau p x dau x, here the pressure gradient or pressure per unit length in the x direction how to be changing. If there is no change of pressure in the y direction or z direction, then you can simply omit these those terms and you can consider it.

And, also for steady state operation, there will be no change of pressure then you can that the dau p x by dau t that will be equals to 0. Similarly, in the y direction you can represent this dp by dt; that means, variation of the operation the with respect to time in the y direction it will be like this. Similarly, in the z direction the velocity pressure variation with respect to time it will be represented by this.

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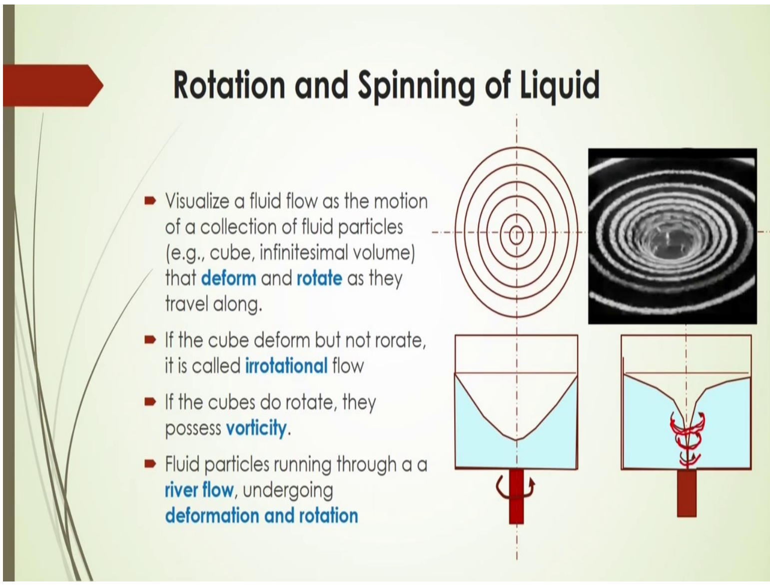

Now, another important characteristics of the fluid flow is called rotation and the spinning of liquid, what is that? We have see the diagram one video is there. So, from this video you will see how liquid element will be rotated and how it will make a vortex what the surface and how depth of this vortex will be there by this video you can see, so, fluid element how it will be rotated. If you inject some dye in a rotating surface, you will see these dye particles how it will be forming a streamline a rotational streamline here.

So, and then this is streamline will be represented by a mathematical expression that will be discussed here how this rotation will be there and what will be the vortex out of the vorticity the terms or to be the intensity of the vorticity and how this vorticity can we calculated from the rotation of this fluid and it will be represented.

So, if we visualize this fluid flow as the motion of collection of fluid particles that is if you are considering cube or infinite symbol volume that deform and rotate as they travel along. Now, if the cube deform, but not rotate, if any cube suppose is deform, but not rotate it will be called irrotational flow.

Now, if the cubes do rotate then they possess some vorticity. Here you see that vorticity, from this video also you can observed that vorticity. You see how depth is liquid particles is going downward and the formation of fluid particles as a cube as cone and from the bottom in the bottom you will see the fluid particles will be coming and form a cone shaped there and at the end point there will be a one single particles there and ah. So, this way it will be a vortex here this is the vortex and that in this particular direction it will be rotating and this vortex how to deforming here.

And, if the cubes to rotate then they will form vorticity and the fluid particles running through a suppose in a river uses sometimes there will a formation of vorticity that will underway undergo a rotation and the deformation there.

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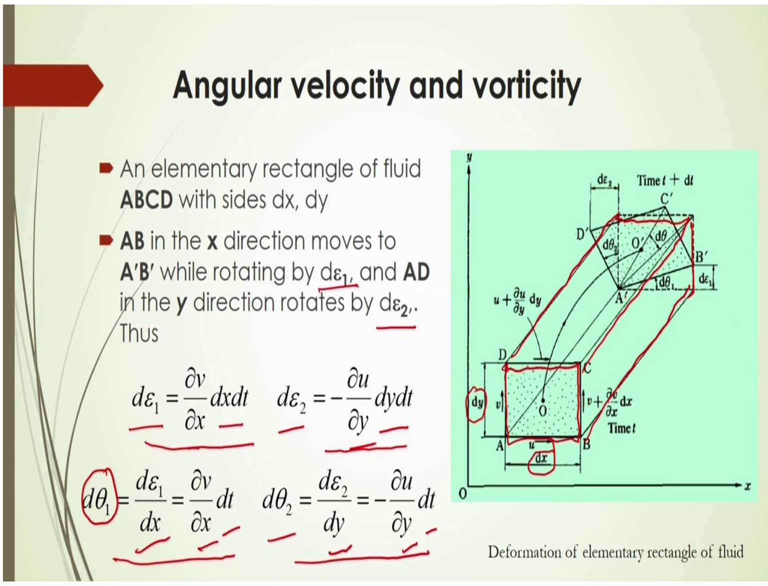

Now, angular velocity and vorticity, what will that velocity of that rotation? You have to find out that will be called as angular velocity. Now, if an elementary rectangular of the fluid here in the diagram is considered as ABCD with the sides small sides dx and dy this is the what is that elementary rectangle here. This is one face of this rectangle and this one like this is your one rectangle. Now, in this case this is one face now ABCD with sides here dx and dy and AB in the x direction right AB in the x direction moves to A dash B dashed moves to dash B dashed. Here moves AB is move moving to A dashed and B dashed here and also you will see that other parts also DC will be moving to D dash C dash.

Now, very interesting that if we twist this if we rotate this fluid element with a certain angle d epsilon 1 and AD in the y direction rotates by d epsilon 2; one is d epsilon 1 at an angle another is d epsilon 2 then in this case we can write d epsilon 1 will be is equal to dau v dau x into dx dt and d epsilon 2 will be is equal to dau u dau y into dy dt. So, in that case what should be the theta of deformation that will be represented by and it will be defined by the d epsilon 1 by dx is equal to dau v dau x into dt.

Now, d theta 2 similarly from the geometry d epsilon 2 by dy and it will be minus dau u dau y into dt. So, in this way we can calculate what should the d epsilon 1 and d epsilon 2. And d theta 1 how it will be related to that then from this two deformations of this epsilon 1 and epsilon 2 we can easily calculate this d theta 1 what is the angular deformation of that.

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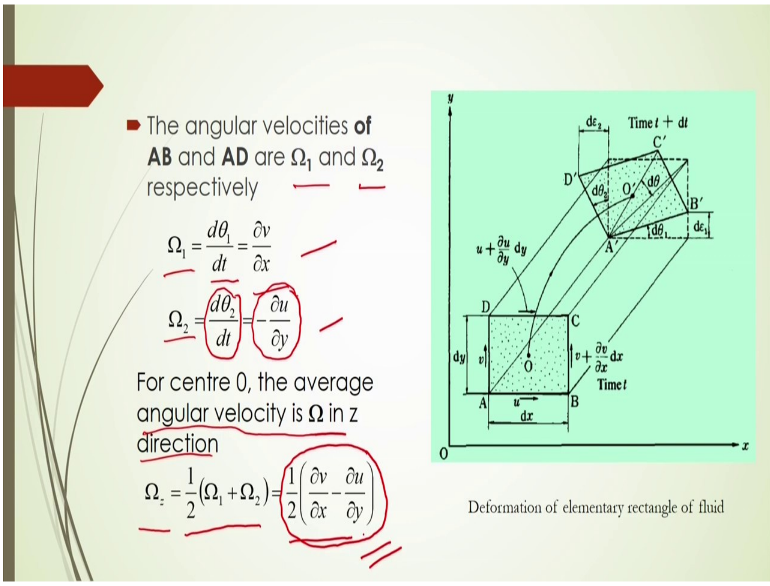

So, from that angular deformation we can calculate the angular velocity of AB and AD which will be denoted by omega 1 and omega 2 respectively for omega 1 will be defined as d theta 1 by dt and which will be dau v dau x. Similarly, omega 2 will be dau theta by dau dt; that means, per unit time how this deformation is happened. So, it will be minus d u dau u dau y here since the counter clockwise this movement is there will be a negative.

And, also for centre O, if we consider the average angular velocity is omega in the a z direction, then in the z direction angular velocity to be represented by omega z which will be average of this deformation in angular deformation of omega 1 and omega 2. So, average of this half of omega 1 plus omega 2. Now, if you substitute this omega 1 and omega 2 from this and this equation respectively. Then we can simplify and get this equation of omega in the z direction that is angular velocity in the z direction it will be is equal to what is that half of dau V dau x minus dau u dau y. So, this is your angular velocity.

So, you have to remember this angular velocity how to calculate if you are deforming this any face of this element fluid with a with an deformative angle of d theta 1 and d theta 2 and the which that angular velocities of omega 1 and omega 2 then you can represent this of the average angular velocity by this averaging of these two angular velocity of this.

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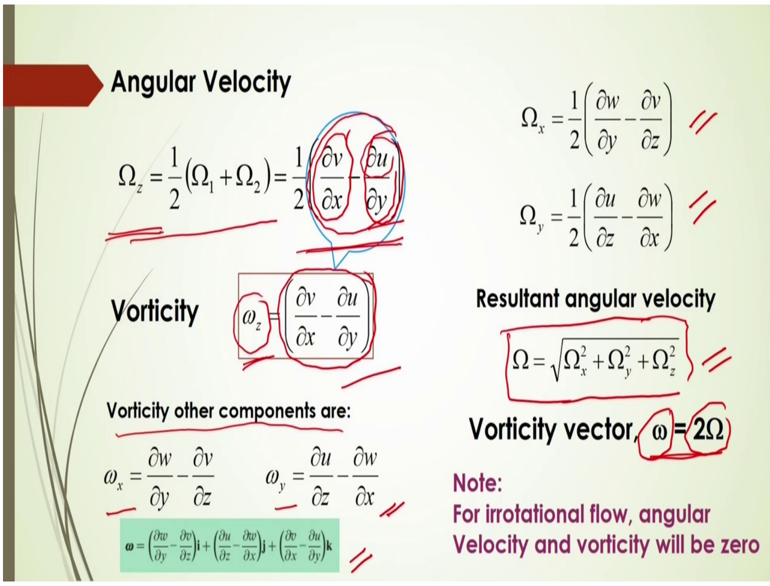

So, in that case finally, we can say that this angular velocity will be is equal to half average of this or two angular velocity and it will be represented by the Cartesian coordinates of this dau v dau x; that means, what will be the velocity in the y direction is with respect to the distance in the x direction and also the velocity change of y of u x direction with respect to the distance of the y direction. That means it is called this terms it is called velocity gradient. So, this velocity gradient in the x and y direction.

Now, summation of this velocity gradient of this y direction and in the x direction that will be represented by this angular velocity here. Now, from this part of this angular velocity only the relative gradient of this angular relative gradient of this velocity of this y and x direction it will be represented as the vorticity here. So, vorticity is defined as dau v dau x minus dau y dau y the z directions.

So, here we can say that this is only that what is the relationship in the z direction by angular velocity and the vorticity. So, omega z will be equals to what 2 times of angular velocity. So, omega z is the vorticity. So, very simple vorticity vector will be or vorticity will be 2 times of angular velocity. Now, other components of the vorticity can be represented by this omega x that will be the x direction to be dau omega dau w dau y minus dau v dau z and also in the y direction this vorticity will be dau u dau z minus dau w dau x. So, this will be your vorticity in the other x and y direction respectively.

And, also by vector components you can say by vector notations you also represent by this equations and also angular velocity in the x direction it will be like this and in the y direction it will be like this. So, resultant angular velocity will be root over of that square of this angular velocity if the summation of angular velocity in the x, y and z directions. So, here this represented by this equation. So, it will be square root of angular velocity in the x, y and z direction and its summation of the square of the angular velocity.

Now, vorticity vector if we have considering the in the vector; that means, in the general one notation in the x, y and z action then omega vorticity will be is equal to 2 times of angular velocity. So, in this case very interesting that you have to remember also for e rotational flow angular velocity and the vorticity will be 0. So, if you are getting e rotational flow; that means, here there will be no angular velocity there will be no verticity there.

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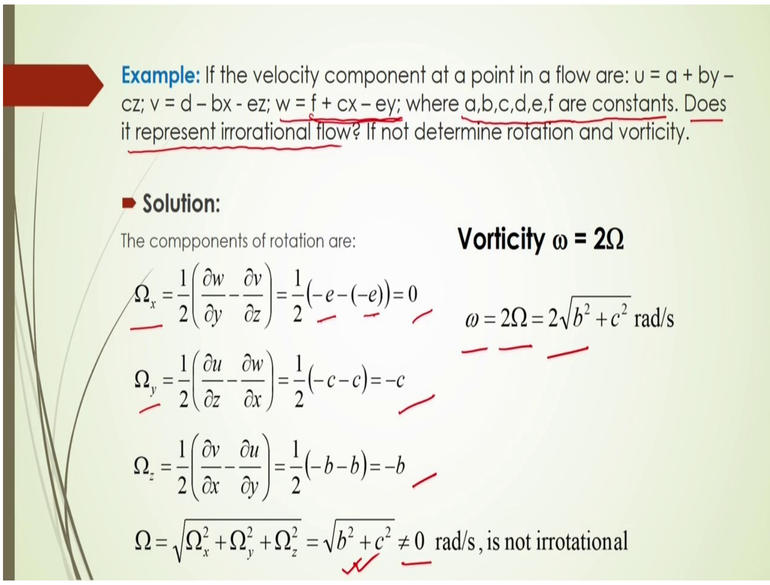

As an example if we consider this if the velocity component point at a point in a flow rate u that will be is equal to a plus by minus cz and b will be is equal to d minus ez and w is equal to f plus cx minus ey here; where a, b, c, d, e, f are constants. Now, in this case you have to tell whether the fluid is moving as an irrotational flow or rotational flow. If it is not irrotational flow, then calculate what should be the rotation and vorticity of this fluid flow.

Now, first you have to calculate the component of the rotation. What are the components? This omega x that is in the x direction it will be half of dau w dau y minus dau v dau z. So, it will be half of now dau w by dau y, you can you can calculate it from this equation w is equal to f plus cx minus ey by derivative by its derivation. So, here dau w dau y it will be minus e and dau v dau z to be minus e. So, here minus c minus of this minus e that will come 0. Similarly, in the y direction omega will be is equal to minus c in the z directions omega will be is equal to minus b.

Now, finally, the resultant this angular velocity will be is equal to root over of omega x square plus omega square plus omega z square that will be is equal to root over or after substitution of this omega x omega y and omega z, it will come root over b square plus c square this will not be equals to 0. What does it mean? The flow is not irrotational, it is rotational flow.

So, in this case what should be that rotation and vorticity? Now, rotation this is the rotation root over b square plus c square. Now, what should be the vorticity? Vorticity will be is equal to 2 times of rotation; that means, omega will be is equal to 2 of omega; this is simple 2 into root over b square plus c square. This is in terms of gradient per second.

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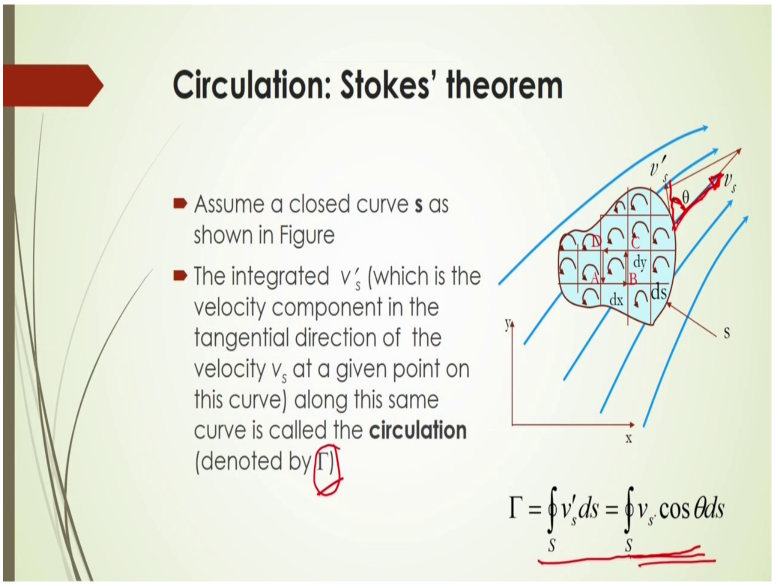

Now, another important characteristics it is called circulation that is Stokes 'theorem as per Stokes' theorem. We can consider here that if you consider a closed curve here in a streamline of streamline of fluid flow as shown in figure here the integrated v s dashed; this integrated v s dash v s dash is nothing, but the velocity component in the tangential direction of this velocity v s. This velocity is v s, now, its tangential will be v s dashed now, at a given point in the curve.

Now, the integrated v s dash at given point on this curve along this same curve is called the circulation. This circulation will be denoted by omega sorry; it is represented by gamma. So, this gamma will be represented by this integration of this v s dashed ds. And, this v s dash is nothing, but the if it is a making an angle theta then it v s cos theta. So, after substitution of this v s cos theta, it will come here like integration of v s cos theta v s.

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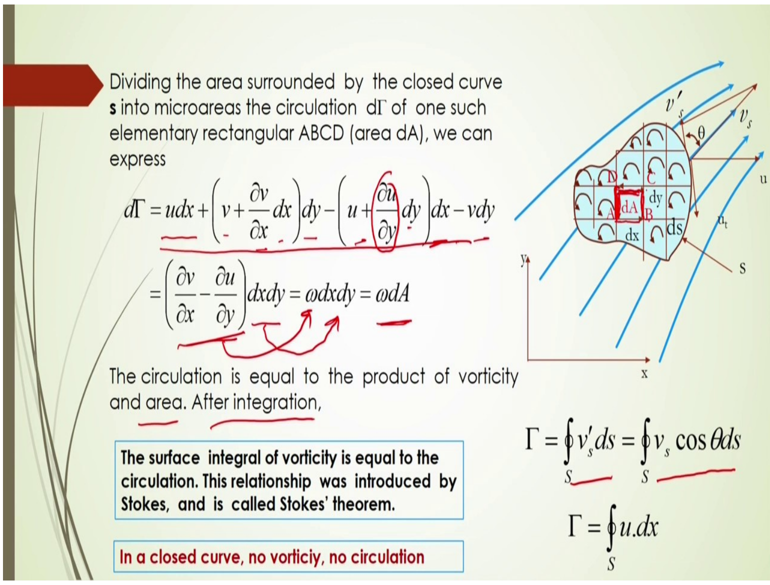

Now, if we divide the area surrounded by this closed curve s into micro areas the circulation gamma of one such elementary rectangular ABCD; ABCD here in the diagram area dA we can express d gamma will be equals to u into dx plus v plus dau v dau x into dx into dy minus u plus dau u dau y into dy into dx minus vdy. That means, here in this direction what will be that u into dx in this direction here it will be v plus dau v dx because this velocity will be changing with respect to x here and into dy and minus here x direction the x directional a velocity in this case here u plus dau u dau y because in the y direction this velocity will be changing that is why the gradient will be dau u by dau y into dy and then you have to multiply by dx here because here some distance will be there area then after that here it will be what is that minus vdy, this is vdy.

So, this is the resultant what is that circulation in this case. So, finally, after simplification you can say dau v dau x minus dau u dau y into dx dy; that means, nothing, but this portion is omega and this portion is nothing, but this area. So, this omega into area will be represented for the circulation of this fluid element here. Now, the circulation is equal to the product of this then vorticity and area. After integration you can have this gamma will be is equal to v s dash ds than v s into ds by integrating whole surface of this element.

Now, this surface integral of vorticity is equal to the circulation; this is the statement. This relationship was introduced by Stokes' and is called Stokes' theorem. In a closed curve there will be no vorticity, no circulation.

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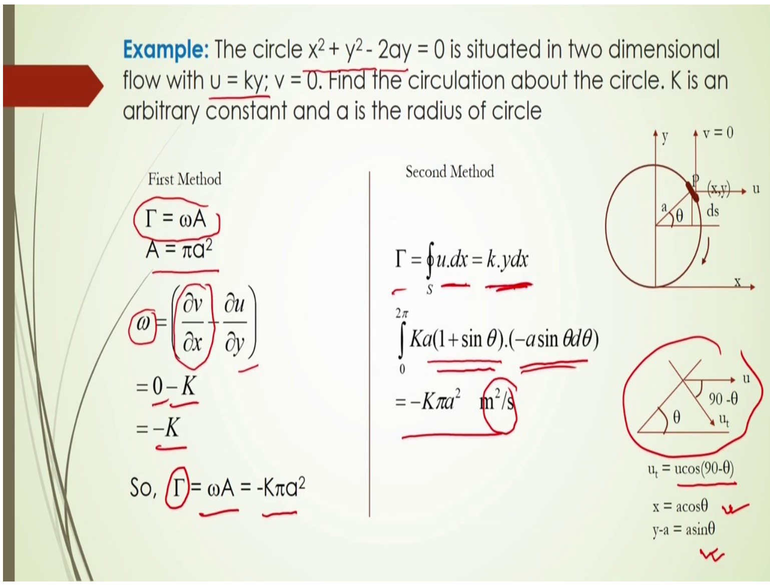

Let us do an example for this like the circulation if we represent by like if suppose a circulation is happening to the surface of the circle or circulation happens in such a way that we can representing that a circle and what will be the equation of the circle that will be x square plus y square minus 2 ay equals to 0, where a is the radius of the circle and is situated in two dimensional flow with u is equal to ky and v is equal to 0.

Now, in this case what should be the circulation about the circle and also k is an arbitrary constant and a is the radius of the circle by considering it what should be the circulation of velocity there? So, in this case if we considered that gamma; gamma is nothing, but as per theory it would be a w into A; that means, vorticity that means, vorticity into what is that into area and area is nothing, but pi a square since it is a circle and omega is the vorticity it is the dau v dau x minus dau u dau y.

So, here dau v dau x is nothing, but the 0, because we are getting from this u is equal to ky. So, we are there is no change of velocity in the x direction. So, here dau v dau x will equals to 0; whereas dau u dau y that will be is equal to minus that will be is equal to K because this u will changing with respect to y that is in the y direction then it will be minus K after derivative of this equation of u is equal to ky. Then finally, it will be coming minus K. So, gamma will be is equal to; that means, circulation gamma will be is equal to omega into A that will be the minus K into pi a square.

Also, the second method you can calculate in other way like that gamma will be is equal to integration of this u into dx because in the x direction you are getting that circulation, so, in this case, the y how it will be changing u into dx that will be k into y into dx. So, overall based on the circle if you substitute this k and y here y; y is nothing, but here a into 1 plus sin theta into minus a into sin theta d theta. You can have it from the geometry like this u t if it is a into cos theta 90 minus theta here as per diagram here as per diagram. Then x will be is equal to a cos theta and y minus a is equal to a sin theta.

Here based on this you can say that what should be the value of y? y would be is equal to a into 1 plus sin theta and dx will be is equal to minus what is that a sin theta d theta. So, after substitution and getting the integration with within the limit of 0 to 2 pi we can get minus K into pi a square that will be as bit as square per second it is.

So, we can get this then circulation based on this vorticity and also the rotational if there is a there is a to represent the circulation in terms of angular velocity that also can be possible because the angular velocity and the a vorticity is directly related. Then angular velocity from the angular velocity or you can calculate the vorticity; from the vorticity you can calculate the circulation.

So, those are related to each other very interesting that here you have already discussed that how this circulation or angular velocity is related. Angular velocity is the that is half of vorticity and also vorticity is related to that circulation as vorticity we multiply with this vorticity by cross sectional area then you can get the what is that here the circulation.

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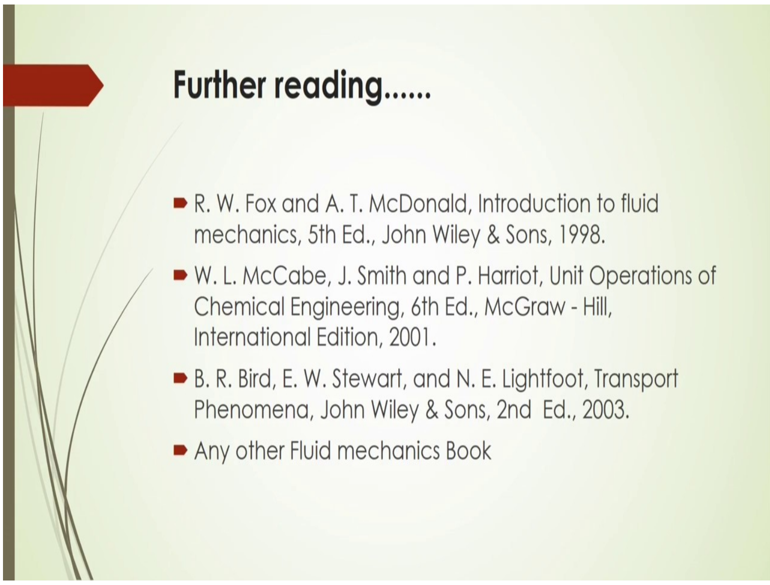

So, I am just suggesting to, but the reading here from the Fox, McDonald textbook for Fluid Mechanics and also McCabe, Smith you can follow that is Unit Operations of Chemical Engineering, 6th edition or higher edition also you can follow, there also some this fluid characteristics are described. And, mostly for this other part of the circulation even transport different processes you can get in details also from this textbook of Bird, Stewart and Lightfoot that is Transport Phenomena and any other text book you can follow for more in details ah, but this is the basis for this circulation vorticity and the rotational velocity.

Thank you.