

**Fluid Flow Operations**  
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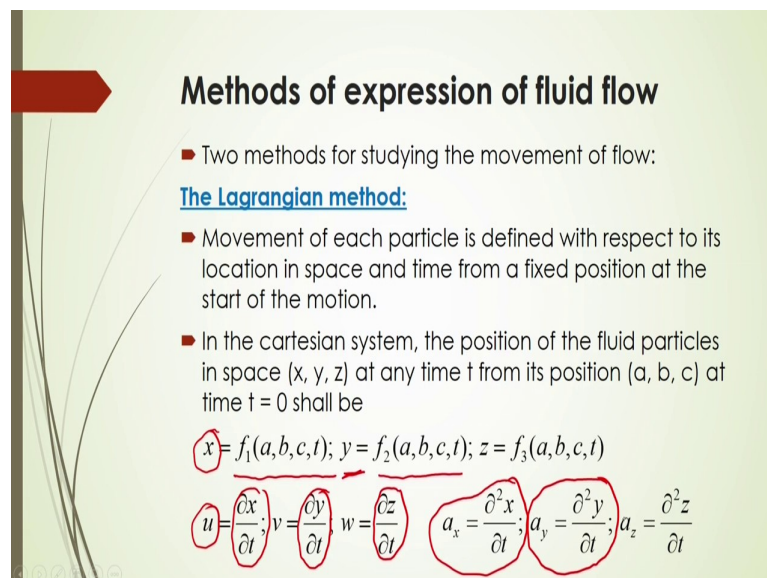
**Lecture - 05**  
**Fundamentals of Flow**

**Keywords:** Fundamentals of flow; Lagrangian method; Eulerans method; Lines of flow; Fluid particles; Types of fluid

Welcome to massive open online course on Fluid Flow Operations. In this lecture we will discuss about the Fundamentals of Flow, here how the flow will be flowing and what will be the mathematical expression for the flow. And how this mathematical expression can be x, y represented. Generally, I am showing that there are two methods; one is the Lagrangian methods and another is Eulerians method. We will be discussing here this two methods of this fundamental flow. And based on this two methods are some characteristics features of this a flow will come here.

Like a how whenever fluid is flowing through the pipe. And then what will be the streamline of a flow? And what would be the stream function for that? How stream function is related to a streamlined function? And also how the potential line of the fluid flow can be expressed is will be discussed here.

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**Methods of expression of fluid flow**

- Two methods for studying the movement of flow:  
The Lagrangian method:
  - Movement of each particle is defined with respect to its location in space and time from a fixed position at the start of the motion.
  - In the cartesian system, the position of the fluid particles in space (x, y, z) at any time t from its position (a, b, c) at time t = 0 shall be

$$x = f_1(a, b, c, t); y = f_2(a, b, c, t); z = f_3(a, b, c, t)$$
$$u = \frac{\partial x}{\partial t}; v = \frac{\partial y}{\partial t}; w = \frac{\partial z}{\partial t} \quad a_x = \frac{\partial^2 x}{\partial t^2}; a_y = \frac{\partial^2 y}{\partial t^2}; a_z = \frac{\partial^2 z}{\partial t^2}$$

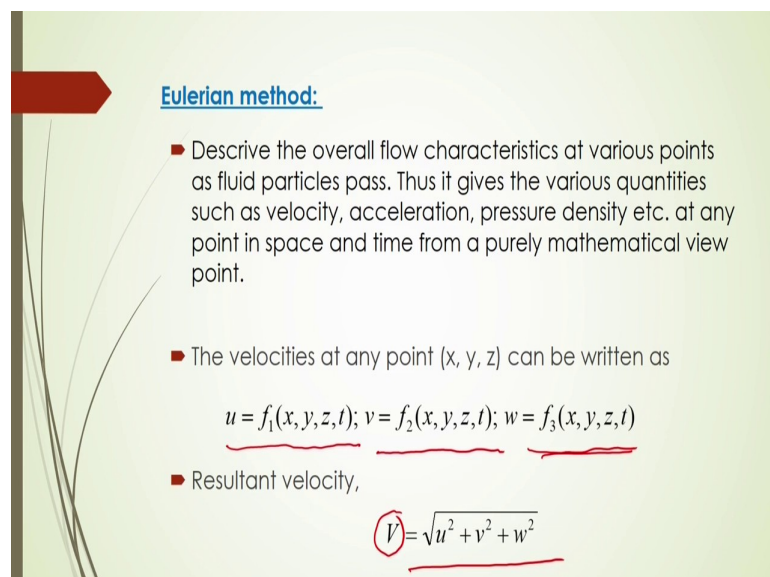
So you know that this mathematical method to express this a fluid flow there are two methods. one is called the Lagrangian method and in this case the movement of each particle is defined with respect to its location in space and a time from a fixed position at a start of the motion.

And in this case generally Cartesian system if we represent this mathematical expression for the fluid flow the position of the fluid particles in space will be represented as a function of a some space and the and time. So, in that case if we say that at a particular position the particle is there then you can expressed this a position of the particle in terms of x.

So, it will be is equal to function of some location a b c and with respect to time t. Similarly, at a position y then how this are the position of the fluid particles in the y direction then how it will be represented? Again it would be a function of a some location a some parameters and then with respect to time. And in this case, what will be the velocity of that particular solid fluid particles they are.

Then it will be represented by u in the x direction and this will be defined by  $\frac{dx}{dt}$ . And v in the y direction it will be  $\frac{dy}{dt}$  and also and the z direction it will be  $\frac{dz}{dt}$ . Similarly, the isolation of the particle in the x direction and y direction and also other direction like z direction can be represented by this expression here ax and ay and az.

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**Eulerian method:**

- Describe the overall flow characteristics at various points as fluid particles pass. Thus it gives the various quantities such as velocity, acceleration, pressure density etc. at any point in space and time from a purely mathematical view point.
- The velocities at any point (x, y, z) can be written as
 
$$u = f_1(x, y, z, t); v = f_2(x, y, z, t); w = f_3(x, y, z, t)$$
- Resultant velocity,
 
$$V = \sqrt{u^2 + v^2 + w^2}$$

So, this is the method to express this a mathematical express the fluid flow as for this Lagrangian method. Similarly, Euler methods also one important description to express the fluid flow characteristics here.

So, in this case it a generally describes the overall flow characteristics at a various points as the fluid particles whenever it will be passed through. And in this case it gives to the various quantities as has velocity of acceleration, pressure, density, etc at any point in space and time from a purely mathematical view point here.

So, the velocities at any point x y and z in x y and z direction respectively can be written as like this u will be as a function of here of f 1 x y and z t. And v will be again function of this space and time as similarly for of w in the z direction it will be represented by a function of your f x y z and t.

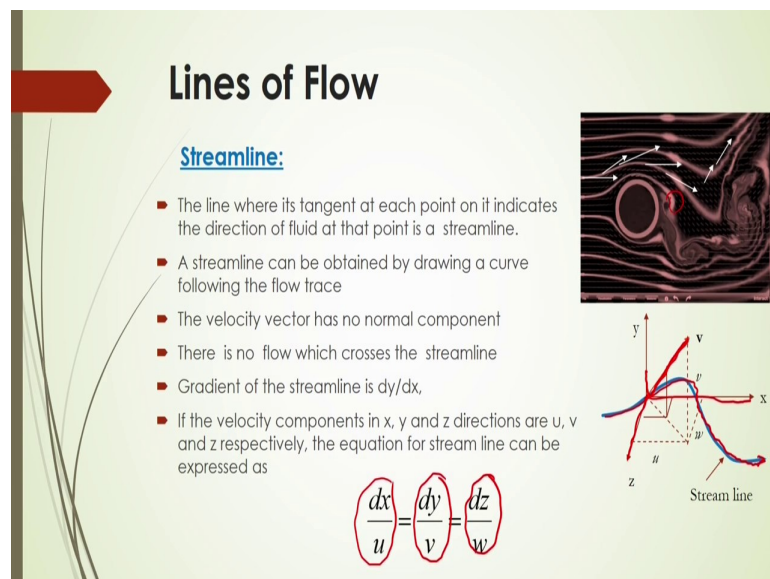
And finally, the resultant velocity based on this x y z directional velocity will be represented by this v, v will be is equal to root over then u square plus v square plus w square where; u, v and w are the velocity components in x y and z direction respectively.

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## Lines of Flow

**Streamline:**

- The line where its tangent at each point on it indicates the direction of fluid at that point is a streamline.
- A streamline can be obtained by drawing a curve following the flow trace
- The velocity vector has no normal component
- There is no flow which crosses the streamline
- Gradient of the streamline is  $dy/dx$ ,
- If the velocity components in x, y and z directions are u, v and z respectively, the equation for stream line can be expressed as

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$$


Now, whenever fluid flow flowing through a particular direction, so you will see in a pipe there will be a; there will be a some particles that to be flowing in a particular of path. So, that whenever particles will be moving in his particular path what will be the line of that path; it will be called as path line.

Similarly, if you join in those particles and it will be a some line a of that fluid particles it will be sometime are represented by this a streamline. But this streamlined is defined as a the line where its a tangent at its point on it indicates the direction of fluid at that point in a streamlined.

So, in that case you see here the video one videos that here how this fluid particle in a particular line it will be flowing. And whenever it will pass through an object it will a get an obstruction and it will spread over the surface of this a solid particles and it will get the disturbance and still it will be from falling at a certain a streamline there.

But a at that particular location you will see there will be a because of that is streamline, but beside this points you will see some other lines there will be falling a certain fashion. So, certain path the particles will be followed in that case the line this case where it is tangent other particles tangent that is point on it is a will be indicating the direction of the fluid flow at that particular point that will be called as streamline here.

So, here it is shown in the a picture and also a streamline can be obtained by drawing a curve following the flow trace here shown in the video. And also here for if we represent this a trace by this a blue line here in this case suppose this is the streamline. So, at particular point for suppose this is in y direction, this is in x, this is in z direction a this is in z direction and this is in x direction.

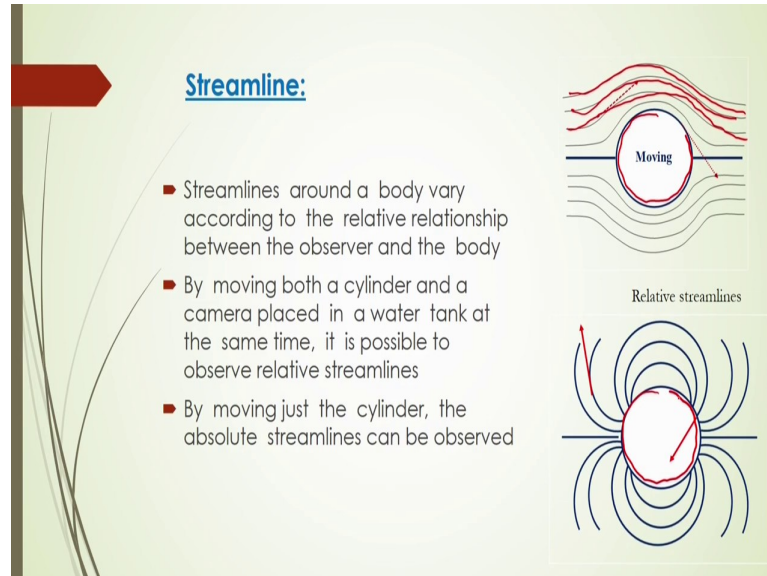
So, if there is a result in velocity suppose in the v then what will be the velocity component in the x and y direction. And from which you will be a getting what should be the slope of this line there. So, a streamline can be obtained by drawing a curve here just by tracing the this point of tangent a on the line.

So, the velocity vector has no normal component in this case of streamline. There will be no flow who is crosses the streamline also. And a gradient of the streamlined will be represented by  $d u$ ;  $d u$  by  $d y$  or  $d u$   $d y$  by  $d x$  here. So, if the velocity component in x y and z direction are u v and z respectively. Then the equation for streamline can be expressed by this  $d x$  by u that will be is equal to  $d y$  by v and equals to  $d z$  by w.

So, main a important point here that the velocity vector will not have any normal component and there will be no flow which crosses this streamline. So, because that to

the direction of this fluid will be as for the tangent of the particle flowing at this point there.

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Also the streamlines around a body vary according to the relative relationship between the observer and the body. If the body here if the body will be moving here this line this is one body the white face here. So, this will be one body and around this body the fluid will be flowing and the streamline of the fluid flow who is represented by this curve here.

So, in this case the movement will be related to each other. So, in this case if moving of this of a solid particles is observed by the observer then you will see the a streamline surround a body that will vary this according to the relative relationship between the observer and the body. And by moving both a cylinder and a camera, suppose if you are observing by camera if the camera is I think if it is moving a with the observer and also observer and the cylinder also moving. So, parallelly if both are moving and then the there will be a relative movement of both the observer and the cylinder.

So, in that case a by moving both the cylinder and a camera if replaced in a water tank at the same time it will be possible to observe the relative a streamlines there. So, by moving just to the cylinder the absolute streamlines can be observed here. So, if it is so, suppose cylinder it is moving at a certain directions and also streamline is also as per this

flow. So, you will see there will be a relative movement there will be relative relativity of that movement of the streamlines and the solid particles there.

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**Example:** In a two-dimensional flow, the components of velocity are given by:  $u = ax$ ;  $v = -ay$ . Find the type of streamlines and draw them.

**Solution:**  
 $u = ax$ ,  $du/dx = a$ ;  $v = -ay$ ,  $dv/dy = -a$   
 $dy/dx = v/u = -y/x$   
 $dy/y = -dx/x$   
 $\ln(y) = -\ln(x) + \text{constant}$   
 $\ln(xy) = \text{constant}$   
 (this is the equation of streamline)

$\ln(xy) = 2$   
 $y = \frac{\text{Exp}(2)}{x}$

Now we have a discussed about the streamline what will be the streamline function is there. So, and what will be the velocity component in the u v and w direction and based on this we can express the equation for streamline by this equation here.

So, based on this equation let us do an example here. In a two dimensional flow if the components of the velocity are given by suppose u is equal to ax here and v is equal to minus ay. So, find the type of streamlines and draw them here. So, very interesting that here it is given the velocity component u will be is equal to ax.

And d then d u by d x will be is equal to what a and also v is equal to minus ay. So, d v by d y will be is equal to minus a. Now, d y by d x from this a relationships we can express that it will be v by u. So, it will be minus y by ax after substitution and then you can express these a d u by d y that will be the minus d x by d x.

So, these things are coming from this first d y by d x is equal to minus y by x. So, and after this equation; after this equation if we take the logarithm on both sides they are a integrate on both sides then you can get that  $\ln y$  will be is equal to minus  $\ln x$  plus constant here. So, finally, you can say that  $\ln xy$  would be is equal to constant.

So, this is the equation of the streamline. So, this  $\ln xy$  that will be is equal to constant will give you this type of a streamline here this is hyperbola type streamline. So, if your velocity component is following suppose some  $u$  is equal to a  $x$ . Suppose here  $u$  will be is equal to  $2x$  or something some a straight lined is form that is the equation or velocity is expressing by this and  $v$  is equal to minus  $ay$ .

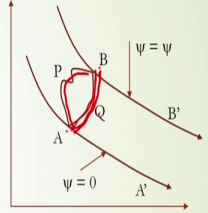
Then from this equation and based on the equation of streamline you can have this a final form of a streamlined equation. So, based on this what type of streamline equation you can easily calculate. Now, suppose this constant is coming any value suppose if it is  $\ln xy$  is equal to 2, then a what should be the  $y$  value that will be is equal to. If a suppose  $\ln xy$  is equal to 2, then  $x$  then  $y$  will be is equal to 2 by sorry  $e$  to the power or exponent of 2 divided by  $x$ . So, if you change the  $x$  value; if you change the  $x$  value then respective  $y$  value what should be that if you plot it then you can get this type of hyperbola type of streamline. So, this is a one example how to calculate the streamline.

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### Stream Function

- If we consider a two-dimensional steady, incompressible flow where  $AA'$  and  $BB'$  are the stream line representing the flow.
- The flow per unit time across points A and B of the two stream lines is called stream function  $\psi$
- The line joining AB may be APB or AQB But the flow per unit time will remain same.
- Let  $\psi$  at point A is zero, then  $\psi$  at point B will be flow per unit time.i.e.,  $\psi$ .
- The function  $\psi(x, y, t)$  can be defined by amount of discharge across the coprdinates as: →

**N.B: The discharge between two streamlines will be the difference in the values of the stream functions**



Discharge across  $dy$   
 $\partial\psi = u\partial y \Rightarrow u = \frac{\partial\psi}{\partial y}$

Discharge across  $dx$   
 $\partial\psi = -v\partial x \Rightarrow v = -\frac{\partial\psi}{\partial x}$

Now, what will be the stream function? Another important that if you consider a two dimensional steady incompressible flow where  $AA$  dashed and  $BB$  dashed as shown in figure here are the streamline this will be representing the flow. Then the flow per unit time that across the points A and B of the two streamlines here then it will be called as a stream function.

So, the flow per unit time across the points A and B; across the point A and B, so, it will be represented by a stream function that will be denoted by  $\psi$  here. So, the line joining AB maybe here C, this line joining AB maybe APB or AQB, but the flow per unit time will remain same here. So, this is the main important point that to be express the stream function you will see that the flow per unit time across points A and B will remain same.

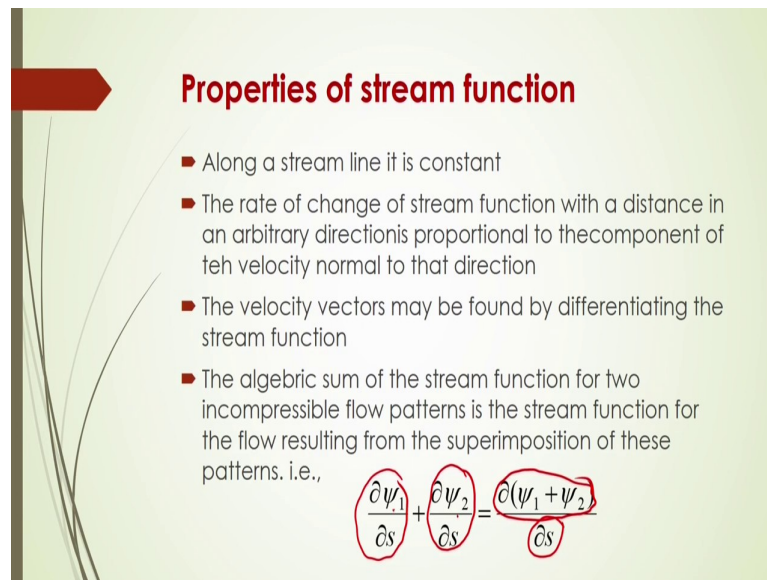
Does not matter whether it will go this different way like APB or AQB, but the flow per unit time will remain same. So, based on this concept if we considered that  $\psi$  at point A is 0 then  $\psi$  at point B will be flow per unit time that is  $\psi$ . The then the function  $\psi$  which will be a function of  $x$   $y$  and  $t$  this will be defined by the amount of discharge across the coordinate.

As here  $\frac{d\psi}{dy}$  that will be is equal to  $u$  into  $\frac{d\psi}{dy}$   $\frac{d\psi}{dx}$  that will implies the  $u$  will be equals to  $\frac{d\psi}{dy}$ . So, this will be discharge across  $s$   $x$ . So,  $\frac{d\psi}{dy}$  will be is equal to  $-v$   $\frac{d\psi}{dx}$  which will imply the  $v$  is equal to  $-\frac{d\psi}{dx}$ . So,  $\frac{d\psi}{dy}$  by  $dy$  it will be that is the discharge across  $dy$  whereas,  $\frac{d\psi}{dx}$  will be discharge across  $dx$ .

So, this is basically the how much water will be flowing a per unit time across the point A and B of two streamlines there. So, the distance between two streamlines will be the difference in the values of the stream functions. So, based on this you can calculate what would be the stream what would be the discharge per unit time based on this concept.



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**Properties of stream function**

- Along a stream line it is constant
- The rate of change of stream function with a distance in an arbitrary direction is proportional to the component of the velocity normal to that direction
- The velocity vectors may be found by differentiating the stream function
- The algebraic sum of the stream function for two incompressible flow patterns is the stream function for the flow resulting from the superimposition of these patterns. i.e.,

$$\frac{\partial \psi_1}{\partial s} + \frac{\partial \psi_2}{\partial s} = \frac{\partial (\psi_1 + \psi_2)}{\partial s}$$

Now, what will be the properties of stream function? Along a streamline of course, it will be constant and the rate of discharge of a stream function with a distance in an arbitrary direction is proportional to the component of the velocity normal to that direction. And the velocity vectors may be found by differentiating the stream function here.

And the algebraic sum of the stream function for two incompressible flow pattern is the stream function for the flow that will result from the superimposition of this patterns. So, here in this case  $\frac{d\psi_1}{ds} + \frac{d\psi_2}{ds}$  that will be is equal to  $\frac{d(\psi_1 + \psi_2)}{ds}$  here. So, this is very important that this  $\psi_1$  and  $\psi_2$  are superimposition of these patterns here.

So, whenever you are considering that a streamline of course, that should be that is (Refer Time: 19:46) must be constant and the rate of charge of stream function, rate of change of that stream function with respect to distance in any arbitrary direction that will give you the; that will give you the change of rate of change of stream function based on the velocity normal to that direction.

And the velocity vectors also can be found by differentiating this system function. And of course, the summation of two stream function will be superimposition and just by summing up those based on a certain distance.

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**Example:** The stream function of a flow is given by the expression,  $\psi = 2x^2 - y^2$ . Find out the components of the velocity and the resultant velocity at a point denoted by  $x = 2, y = 3$

$$u \Big|_{at\ y=3} = \frac{\partial \psi}{\partial y} \Big|_{at\ y=3} = -2y \Big|_{at\ y=3} = -2(3) = -6$$
$$v \Big|_{at\ x=2} = -\frac{\partial \psi}{\partial x} \Big|_{at\ x=2} = -4(x) \Big|_{at\ x=2} = -4(2) = -8$$
$$V = \sqrt{u^2 + v^2} = \sqrt{(-6)^2 + (-8)^2} = 10$$

The stream function of a flow if it is given by the expression psi is equal to 2 x square minus y square as an example. So, in this case what should be the components of the velocity and the resultant velocity at a point that will be denoted by x is equal to 2 and y is equal to 2. That means, at point 2 and 3 what should be the velocity component and the resultant velocity if you know the stream function there.

So, stream function is y is equal to 2 x square minus y square then if you differentiate it with respect to y at point 3 then you can get minus 2 y; that means, minus 2 into y is equal to 3 here so it will be minus 6. Similarly, if you differentiate it that dou psi dou x at point 2 it will be coming as a 8 since it will be negative for a velocity in the y direction it will be represented by minus 8.

And so resultant velocity will be is equal to here v will be is equal to root over u square plus v square, so, this will be coming as 10. So, what will be the resultant velocity if the stream function will be 2 x square minus y square at any point then you can easily calculate by this stream function.

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**Example:** What is the stream function if (i) uniform indefinite flow parallel to x axis, (ii) flow parallel to y axis, (iii) inclined stream

- (i)  $u = u_0$ ,  $v = 0$ , therefore,  $-\frac{\partial \psi}{\partial x} = 0$  Hence  $\psi = f(y)$  only
- Or  $u = \frac{\partial \psi}{\partial y} = u_0 \Rightarrow \psi = u_0 y + \text{constant}$
- (ii) Similarly flow parallel to y axis,  $\psi_1 = v_0 x + \text{constant}$
- (iii) To get stream function for an inclined stream, the above mentions are superimposed

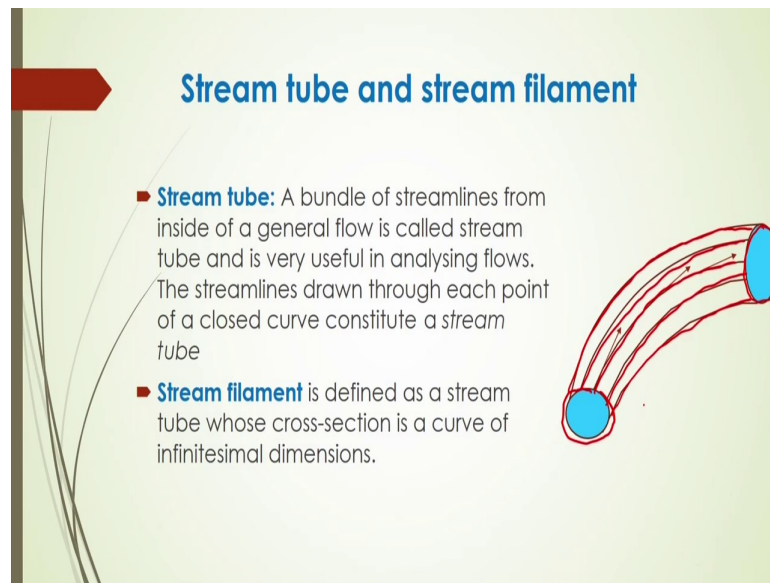
$$\psi_2 = \psi + \psi_1 = u_0 y - v_0 x$$

And also another example like if there will be an uniform indefinite flow parallel to x axis. And also if flow parallel to y axis if it is inclined stream then what should be the stream function. So, if is there uniform indefinite flow parallel to x axis; that means, if u is equal to u 0 and then v will be equals to 0.

Therefore, we can say minus dou psi dou x that will be equals to 0. Hence psi will be equals to a function of y only or u will be is equal to dou psi dou y that will be is equal to u 0 from which after integration you can calculate what will be the psi. So, psi will be is equal to here u 0 into y plus constant.

Similarly, flow parallel to y axis it will be psi 1 will be equals to v 0 x plus constant. And to get the stream function for an inclined stream the above mentions are of course, superimposed like this here. So, it will be psi 2 will be equals to psi this one in the here psi and then plus psi 1 this is psi 1 then stream function for an inclined stream it will be is equal to u 0 y minus v 0 x.

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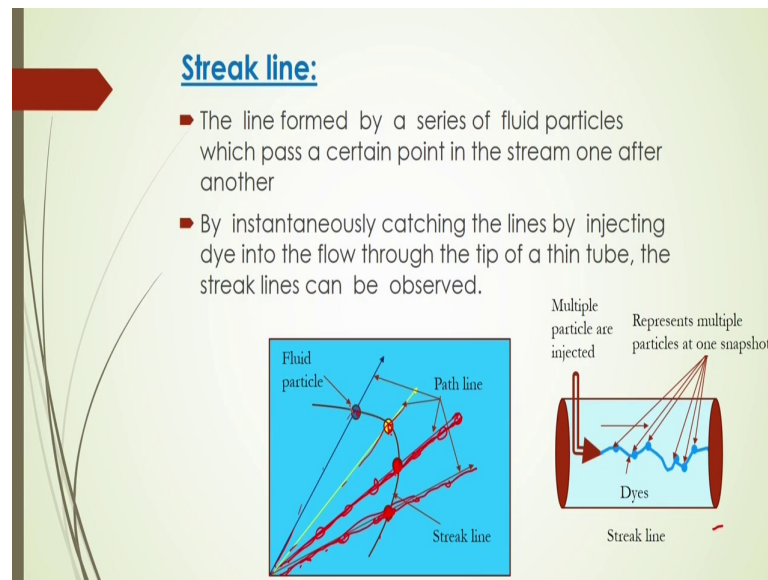
Now, another important characteristics of the fluid flow that it is represented by the stream tube and the stream filament. Here what is stream tube? A stream tube is nothing, but the bundle of a streamlines from inside of a general flow which is called stream tube as shown in the figure here.

We are just bundle of the stream function a streamline bundle of the streamline here that will make this stream tube and here. So, in this case why this stream tube should be required to know. This is very useful in analyzing flows basically for just considering a certain what is that control volume to analyze the fluid flow.

The streamlines drawn through each point of a closed curve that will constitute a stream tube, so, this is basically one control volume you can say. So, stream filament is also one other important of; that means, characteristics to represent the fluid flow. In this case the stream filament is the nothing, but a stream tube whose cross section is a curve of infinitesimal a dimensions.

So, in this case this cross section should be very a small. So, if you consider that cross section is very small are as a point one here you will say that you can represent the fluid flow in a refine manner.

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Another important one is called streak line. In this case you will see the figure here if we suppose allow some dye in a fluid flow you will see how the dyes particles will be following a path as particles will following a path here in the flow.

So, the path that given by this are produced or generated by the fluid particles are here in this case by dye particles it will be called as a streak line here. The line formed by a series of fluid particles in this case which pass a certain point here in the stream one after another. So, this is the definition of the streak line.

So, the it will be a line that will be formed by a series of fluid particles. And of course, it will pass through a certain point in the stream one after another. And also that us instantaneously if you catch the lines by injecting dye into the flow through that tip of a thin tube the streak lines can be easily observed.

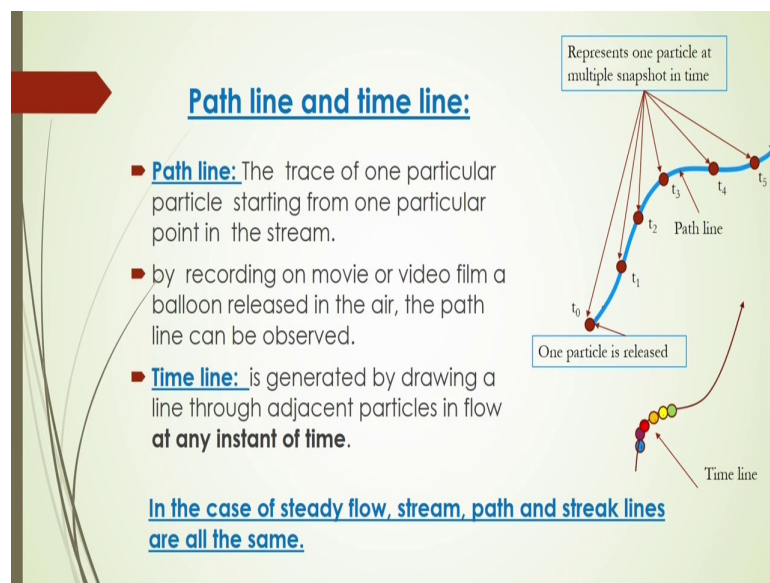
So, how here see this is one the streak line see here if you represent these this is one path line and this is one path line that this another simple this particles will follow these this point, this point, this point with respect to time. And here these particles will follow this time, this time here this position, this position, this position.

So, the joining of this position changing is called the path line here. This is also another path line by this particle this is also another a path line by this particles. Now at a certain point in a stream will see in this case if you consider that only this particles will

whenever at a certain point it will follow; then this particle this particle by taking a camera or by snapshot.

You will see at that particular location. these particles will be there you need on their particular path line. So, if you joining this; if you join this lines then you can get what should be the streak line of these fluid particles there.

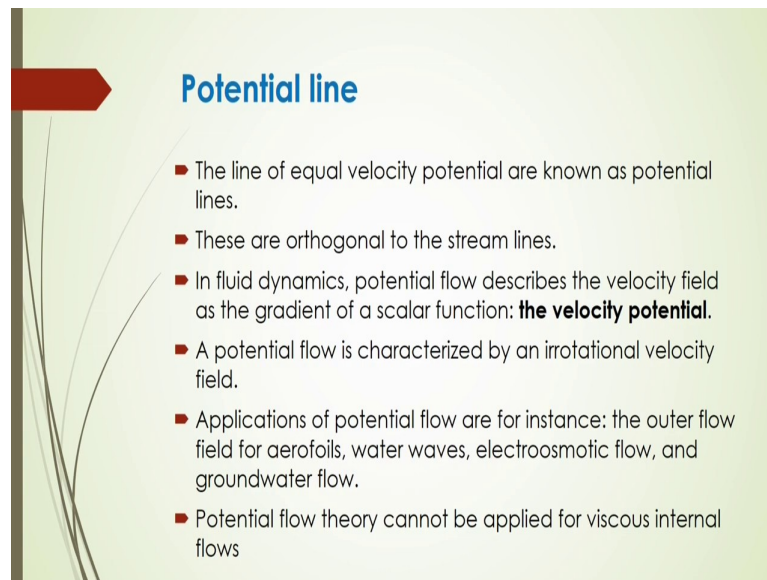
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And path line is nothing, but the trace of one particle starting from one particular point in the stream. And by recording one movie or video film a balloon released in the air you can observe that the path line of that particular balloon that is released in the air. And also another important line that represent by, represent that, represent the fluid flow characteristics that is generated by drawing a line through adjacent particles in the flow at any instant of time.

So, this is so, the here you can say this path line the particles will follow the at a different time interval are at different time where this particles will be there at the particular locations there. And timeline is there that here drawing if you draw the line through the adjacent particles in flow at any instant of time then you can say this will be your timeline they are. Now in the case of steady flow remember that the streamline, path line and streak lines all will be same.

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### Potential line

- The line of equal velocity potential are known as potential lines.
- These are orthogonal to the stream lines.
- In fluid dynamics, potential flow describes the velocity field as the gradient of a scalar function: **the velocity potential**.
- A potential flow is characterized by an irrotational velocity field.
- Applications of potential flow are for instance: the outer flow field for aerofoils, water waves, electroosmotic flow, and groundwater flow.
- Potential flow theory cannot be applied for viscous internal flows

Another important characteristics called potential line the line of equal velocity potential are known as potential lines. There are orthogonal to the streamlines, so, this potential line will be the orthogonal to the streamlines. In fluid dynamics, you will see the potential flow that will describes the velocity field as the gradient of a scalar function which will be called as a velocity potential. That is characterized by the irrotational velocity field.

And these are applied for the outer flow description of the field for aerofoil's, water waves and electro osmotic flow and also you can say groundwater flow. Also potential flow theory cannot be applied for a viscous internal flow. So, potential flow that limitation is that you cannot apply this potential flow theory for the viscous internal flows.

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**Velocity potential**

- Consider a flow takes place in a pipeline with a pressure difference. The direction of flow will take place from higher to the lower pressure.
- Thus the velocity of flow in a certain direction will depend upon the potential difference which is known as velocity potential and denoted by  $\phi$

$$\partial\phi = u\partial x \Rightarrow u = \frac{\partial\phi}{\partial x}$$
$$\partial\phi = v\partial y \Rightarrow v = \frac{\partial\phi}{\partial y}$$
$$\partial\phi = w\partial z \Rightarrow w = \frac{\partial\phi}{\partial z}$$

The slide features a green background with a red arrow pointing right. The equations are handwritten in red ink, with the variables u, v, w and the partial derivatives of phi circled in red.

Now, how actually one can represent this velocity potential mathematically? If we consider a flow that takes place in a pipeline with a pressure difference, then in that case the direction of flow that will take place from higher or to the lower pressure. So, this is basically the potential difference. Now does the velocity of the flow in a certain direction generally depend upon the potential difference.

So, then it will be represented by the velocity potential and denoted by a phi here. So,  $\partial\phi$  will be is equal to  $u \partial x$ . So, from which you can say that what will be the  $u$  then it will be  $\partial\phi / \partial x$ . Similarly, in the  $y$  direction  $v$  will be represented by  $\partial\phi / \partial y$ . So,  $\phi$  is the velocity potential and  $v$  is the velocity and how then it will be related so how velocity at a particular direction.

How it will be related to the velocity potential you can easily expressed by this;  $u$  will be equals to  $\partial\phi / \partial x$  and  $v$  will be is equals to  $\partial\phi / \partial y$  and also  $w$  will be is equal to  $\partial\phi / \partial z$  in  $x$ ,  $y$  and  $z$  directions respectively. So, based on these functions you can define for the velocity a potential. And also how the velocity at a particular directions can be obtained from this velocity potential you can also have.



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**Relation of Velocity potential and stream function**

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$$

$$v = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

These Equations are called **Cauchy-Riemann Equation**

$$d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy$$

$d\phi = 0$  along potential line, therefore

$$\frac{dy}{dx} = -\frac{u}{v}$$

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy$$

$d\psi = 0$  along stream line, therefore

$$\frac{dy}{dx} = \frac{v}{u}$$

Thus the slope of potential line  
× slope of stream line = -1

Now, what should be the relation between velocity potential and stream function. Stream function we know that psi whereas, this velocity potential sorry velocity potential is phi. So, what will be the relation between phi and psi, so, if we considered that u in the x directional velocity u.

So, in that case as for velocity potential this will be  $\frac{du}{dx} = \frac{d\psi}{dy}$  where  $\frac{du}{dx}$  and as per that stream function it will be  $\frac{d\psi}{dy} = \frac{d\phi}{dx}$ . And in the y direction velocity will be equal to  $\frac{dv}{dy} = -\frac{d\psi}{dx}$  that will be is equal to minus  $\frac{d\psi}{dx}$ . And so these equations are called actually Cauchy's Riemann equation.

And this is represented by the what is that the velocity potential and stream function relationship. And this has given by Cauchy's Riemann and from this equation you will be able to calculate once you know the psi and phi and what should be the velocity there. And also if you know the velocity what should be the psi and phi also you can get it.

Now if you represent that we get this  $d\phi$  will be is equal to  $\frac{d\psi}{dy} dx + \frac{d\phi}{dy} dy$  into  $dx$  plus  $\frac{d\psi}{dy} dy + \frac{d\phi}{dy} dx$  into  $dy$ . And a along the potential line there will be no I think what is that velocity potential then  $d\phi$  will be equals to 0. So, based on these you can express what should be the; what is what should be the velocity?

What should be the I think tangent of that velocity potential line that will be  $\frac{dy}{dx}$ , that will be is equal to minus  $\frac{u}{v}$ . So, and also if we express that  $d\psi$  will be equals to

$\frac{d\psi}{dx} = u$  and  $\frac{d\psi}{dy} = v$ . And along the streamline then  $d\psi = 0$  based on which what will be the tangent for in the  $y$  and  $x$ .

Other than  $\frac{dy}{dx}$  will be equals to  $\frac{v}{u}$ . So, thus in this case so we can get the slope of this stream function or streamline and the potential line. So, for the potential and the slope will be is equal to  $\frac{dy}{dx}$  that will be minus  $\frac{u}{v}$ . Whereas, for streamline the what is the slope will be is equal to  $\frac{v}{u}$ . So, if we relate with that then you can say that slope of potential line will be is equal to negative of inverse of slope of streamline.

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**Example:** A stream function is given by the expression  $\psi = 3x^2 - 3y^2$ . What is the velocity potential function?

**Solution:**

$$u = \frac{\partial \psi}{\partial y} = -6y \qquad v = -\frac{\partial \psi}{\partial x} = -6x$$

$$\frac{\partial \phi}{\partial x} = u = -6y \Rightarrow \phi = -6xy + f_1(y) \quad (i)$$

$$\frac{\partial \phi}{\partial y} = v = -6x \Rightarrow \phi = -6xy + f_2(x) \quad (ii)$$

From (i) and (ii)

$$\phi = -6xy + C$$

C is a constant

Let us to an example here; suppose a stream function is given by the expression  $\psi$  that will be equals to  $3x^2 - 3y^2$ . Then what should be the velocity potential function based on this stream function? So, we know that  $u$  will be is equal to  $\frac{d\psi}{dy}$  if we in differentiate it we can get minus  $6y$ . Similarly,  $v$  will be is equal to minus  $\frac{d\psi}{dx}$  as per definition then it will be is equal to minus  $6x$ .

So,  $\frac{d\phi}{dx}$  as per definition of the velocity potential line. So, it will be is equal to  $u$  which is nothing, but minus of  $6y$  from which if you integrate it then you can get  $\phi$  will be equals to minus  $6xy$  plus some constant which will be a function of  $y$ . Again  $\frac{d\phi}{dy}$  in the  $y$  direction in that will be equal to  $v$  as per definition then  $v$  is equal to what?  $v$  is nothing, but  $\frac{d\psi}{dx}$  negative of this then it will be minus  $6x$  then from which after integration you can calculate what will be the  $\phi$  here.

So, phi will be is equal to minus 6 xy plus again constant that will be a function of x. So, from this equation i and ii you can easily say that this phi will be is equal to minus 6 x y plus some constant C here; C is the constant. So, you can easily calculate what should be the potential line a based on the stream function.

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**Flow Net**

- It is a graphical representation of streamlines and perpendicular equipotential lines.
- The condition of the orthogonality of the streamlines and potential lines is:
 
$$\frac{\partial \phi}{\partial x} \cdot \frac{\partial \psi}{\partial x} + \frac{\partial \phi}{\partial y} \cdot \frac{\partial \psi}{\partial y} = 0$$
- It is important to analyse the flow problem in which boundary configuration and flow conditions are such that the mathematical analysis is difficult.
- Example: Flow through a **Francis turbine runner**

The slide includes a diagram of a flow net with streamlines and potential lines, and a photograph of a Francis turbine runner.

Now, another important you can graphical important characteristics of the fluid flow which will be represented graphically so it is called flow net. This flow net it is a graphical representation of streamlines and perpendicular equipotential lines. Here see the diagram here see this as per what is that arrow that this lines are if we consider that streamlines.

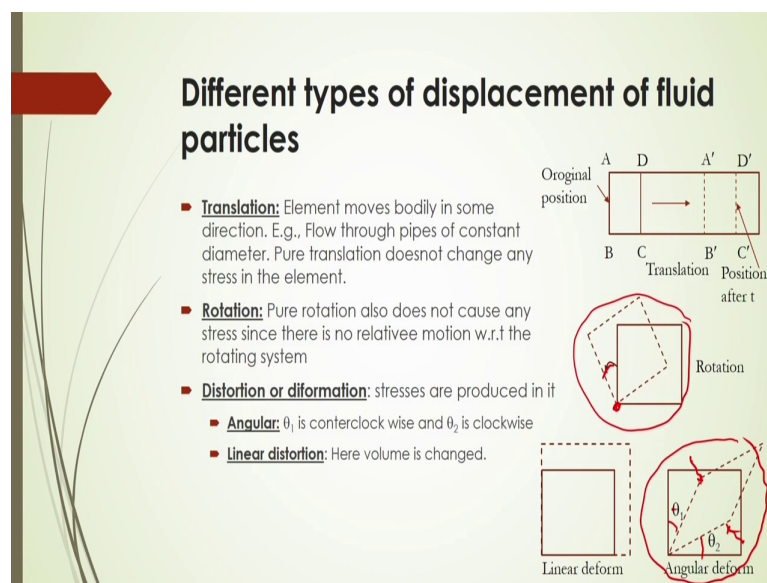
Then it will be the orthogonal to this streamline will be here it will be as potential the line here. So, potential line and streamline will be perpendicular to each other. The condition of the orthogonal orthogonality of the streamlines and the potential line is that dou phi dou x into dou psi dou x plus dou phi dou y into dou psi dou y that will be is equal to 0.

So, this condition should be full filled then only you can get this type of flow net there. So, where actually this flow net is being used or why it is so important? It is important to analyze the flow problem in which boundary configuration and flow conditions are such that the mathematical analysis is very difficult.

Like Francis turbine runner; in that case you will see flow through this runner there will be a complex flow pattern. So, to represent this flow pattern the flow net will be a tool to be used to characterize this flow. Now another important things that what we have to know that how different types of or how this fluid particles will be displaced.

Now different types of displacement of the fluid particles occur whenever fluid is flowing through a particular conduit. Like translation, like rotation, like distortion or deformation. What is that translation? As per figure here, see here.

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This element here moves this element moves bodily in some direction. Here suppose A is moving to CD, CD is moving to C dashed D dashed AB is moving to a dashed B dashed like this. So, flow through pipes of a constant diameter in that case pure translation does not change any stress in the element.

So, translation means; here the fluid particles will be displaced from one location to the another location here. So, without having any stress there so that is ideal translation. Whereas, rotation is the pure rotation it is called in that case when there will be a no cause of any stress. That means, there is no relative motion with respect to the rotating system.

So, in this case you will see if we considered this a rotation here the fluid particle just is pivoting now from one point to in a certain direction here it is just a rotating this here.

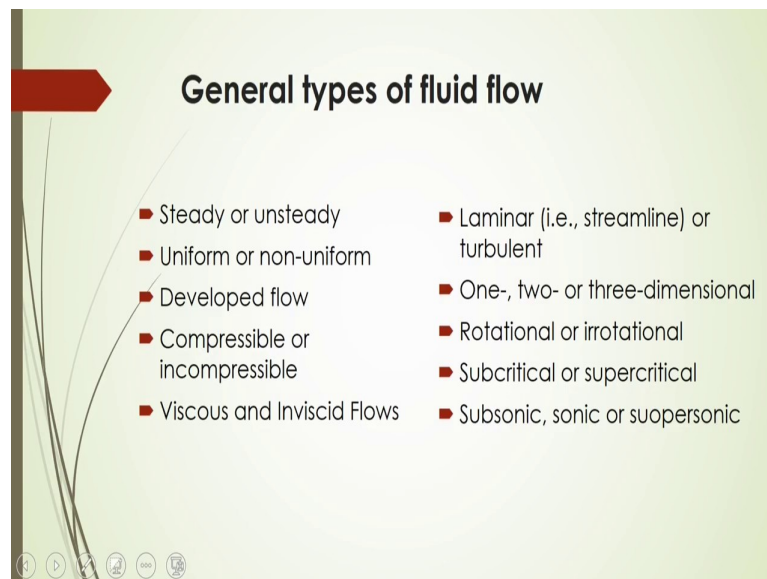
So, important that here there will be also know cause of this stress here for this ideal rotation.

There is no relative motion with respect to the a rotating system here. So, it will be just from this just simple just moving just pivoting at a particular point. Distortion or deformation in this case a stresses of course, will be produced on it and in this case two types of distortion we can get.

One is angular which is represented by maybe theta, how much theta angle is I think that is made based on the stresses. Now if we consider here see angular deformation here just simply the spacing out or by stressing on this two point in this direction you will see, there will be a deformation of this object or fluid particles there with an angle theta 1 and theta 2.

Now theta 1 is a counterclockwise angular distortion and theta 2 is called the clockwise angular distortion. Whereas, linear distortion in this case here volume of course, will be changed totally it will be stressed in such a way that the volume will be changed their. Like if, you increase the temperature the volume of the Cauchy's will be increases.

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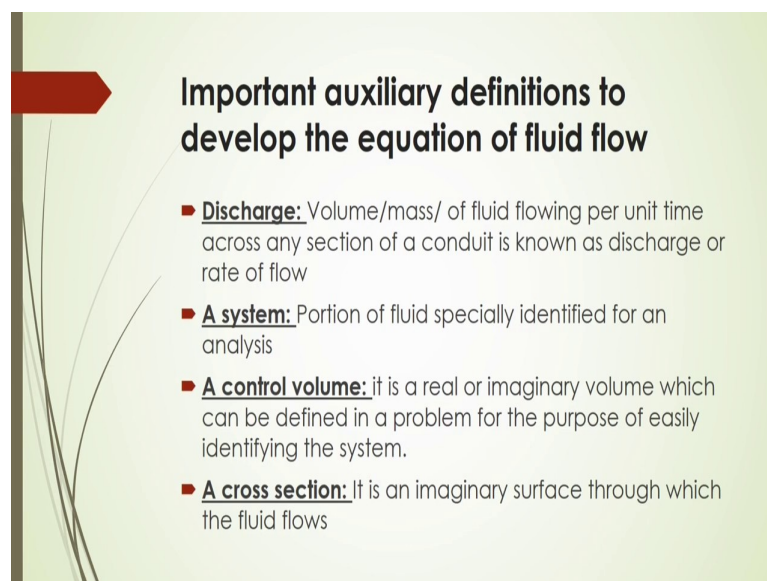
Now, general types of fluid flow. We will discuss what are the different types of fluid flow can represent that. One is like steady state and unsteady state; if it is flow is with respect to time then it will be unsteady state. If the flow does not change with time then it

will be steady state. Uniform and non-uniform sometimes the flow will be uniform and non-uniform.

Sometimes the velocity of the fluid or streamlines will be changing on its different location. Also developed flow or undeveloped flow; sometimes the flow will be initially it will be disturbing and that is after a certain time the flow will be getting uniform flow throughout the cross section of the fluid and so also you will see cross section of the pipe.

Also, you will see that the average velocity will be same average velocity of the same in each location of the pipe. And compressible or incompressible flow viscous or inviscid flows. There will be some laminar or turbulent flow one or two three-dimensional flow, rotational, irrotational, supercritical and super official, and subcritical and also subsonic, sonic and supersonic. These are the various types of fluid flow. We will be discussing one by one what is that.

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**Important auxiliary definitions to develop the equation of fluid flow**

- **Discharge:** Volume/mass/ of fluid flowing per unit time across any section of a conduit is known as discharge or rate of flow
- **A system:** Portion of fluid specially identified for an analysis
- **A control volume:** it is a real or imaginary volume which can be defined in a problem for the purpose of easily identifying the system.
- **A cross section:** It is an imaginary surface through which the fluid flows

Let us see other; what is the important auxiliary definitions to develop the equation of a fluid flow? Like; you have to know the discharge that is volume per mass of fluid that is flowing per unit time across any section of a conduit, which is known as discharge, or rate of flow. Before analyzing the equation of fluid flow, you have to define a system.

Sometimes you have to consider a portion of the fluid that specially identified for an analysis. And also you have to consider a control volume it is a rare it is a real or imaginary volume which can be defined in a problem or the purpose of easily identifying the system. Also cross section it is an imaginary surface through which this fluid will be flowing, so, that also should be defined in your case.

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**Steady and unsteady Flow**

- Steady Flow:** A flow whose flow state expressed by velocity, pressure, density, etc., at any position, **does not change with time**

$$\frac{dp}{dt} = 0 \quad \frac{dV}{dt} = 0$$
- Unsteady Flow:** a flow whose flow state **does change with time** is called an unsteady flow.
 
$$\frac{dp}{dt} \neq 0 \quad \frac{dV}{dt} \neq 0$$

Now, analysis of fluid flow now considered the steady and unsteady flow. The steady what is that steady flow? A flow whose flow state expressed by the velocity pressure density or other variables at any position that will not change with respect to time. So, you can express these steady flow as suppose this  $\frac{dp}{dt}$  will be equals to 0.

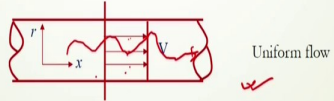
That means, pressure does not change with time also  $\frac{dV}{dt}$  will be equals to 0. What does it mean? Here there will be no change of velocity with respect to time; unsteady flow it is defined as the flow whose flow state will change with respect to time, so, that will be called an unsteady flow.

So, in this case you will see that  $\frac{dp}{dt}$  not will be equals to 0 there will be a certain value there at a certain rate that pressure will change with respect to time. And also the velocity will respect to time will change, so, that will be called acceleration so in that case it will be unsteady flow.

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### Uniform and non-uniform flow

- **Uniform flow:** The fluid or flow parameters at any given instant remains same at every point in space
- **Non-uniform flow:** On the other hand, if the fluid or flow parameters at any given instant rchange with distance in the flow.

$$\left. \frac{\partial V}{\partial s} \right|_{at t_0} = 0$$
$$\left. \frac{\partial V}{\partial s} \right|_{at t_0} \neq 0$$


Uniform flow

What is that uniform and non-uniform flow? How it will be defined? The fluid or flow parameters at any given instant that will remains same at every point in the space it is called then uniform flow. That means, here  $\frac{\partial V}{\partial s}$  at time  $t$  that will be equals to 0. That means, at any position if you go in the five when you pipe will see there will be no change of velocity so it is called uniform flow.

Non-uniform flow on the other hand if the fluid or flow parameters at any given instant will change with distance in the flow. So; that means,  $\frac{\partial V}{\partial s}$  at  $t_0$  that will not be 0 that will have some value, so, that the velocity will be a function of distance there. Like here in this case this is uniform flow at any point here their velocity will be same so that is why it is called uniform flow.

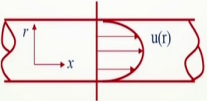
Whereas, you will see if the velocity pattern like this here; so, you will see the velocity at any point it will be changed with respect to distance or any location then it will be called as non-uniform flow.



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### Developed flow

- The velocity profiles do not change with respect to the downstream coordinate.
- The momentum also does not change in the flow direction.
- The pressure in the flow direction will balance the shear stress near the wall.



The developed flow may be unsteady, i.e., it may depend on time, such as when a valve is being opened or closed.

What is developed flow? In this case, you can get the velocity profiles will observe we will discuss later on. So, the velocity profiles whenever fluid is flowing at a certain velocity it will change with respect to I think excel position and time. So, if suppose this velocity profile do not change with respect to downstream any coordinates like x y z then you can say it will be developed flow.

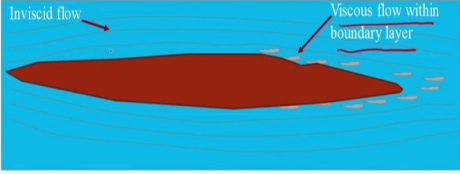
The momentum also does not change in the flow direction in this case. So, the pressure in the flow direction will balance the shear stress near the wall. So, this type of condition will be represented by the developed flow. So, when the velocity profiles does not change with respect to any coordinates in the downstream that is in the direction of this flow then you can say it will be a developed flow.

The developed flow may be unsteady that is it may depend on time such as when a valve is being opened or closed. In that case we will see there will be a velocity be changed or may not change that based on the operating condition. If it is a not changing then it will be a developed flow at unsteady condition.

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## Viscous and Inviscid Flows

- **Viscous flow:** All fluids have viscosity and if the viscous effects cannot be neglected, it is a viscous flow.
- **Inviscid flow:** the effects of viscosity can be completely neglected with no significant effects on the solution to a problem involving the flow.
- **Inviscid flows are easier to solve than viscous flows**



Now, what is the viscous and inviscid flows? Viscous flows that case all fluids that have viscosity and if the viscous effects cannot be neglected during the flow and also representation of the flow then it will be considered as a viscous flow. Whereas, the effects of viscosity can be completely neglected with no significant effects on the solution to a problem involving the flow.

So, in that case it will be called as inviscid flow. Inviscid flows are easier to solve then viscous flows. Viscous flow some extent it is complicated to represent here. So, in this case you will see here as per diagram here viscous flow within boundary layer here.

Whereas, inviscid flow beyond this objects surface, some away of this object there will be a no viscosity effect so that will inviscid flow. Whereas, adjacent to this solid surface the flow will be get disturbance and there will be a some change of this velocity pattern here. So, due to these it will be represented as a viscous flow within the boundary layer.

Now next one is a very important that whenever you are going to represent any velocity pattern or acceleration or density change, pressure change, of the fluid flow you have to consider it; whether it is in one dimensional or in two dimensional or three dimensional flow.

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**Three-, two- and one-dimensional flow**

- **Three-dimensional Flow:** A ball flying in the air and a flow around a moving automobile have velocity components in x, y and z directions.  
$$u = u(x, y, z, t); v = v(x, y, z, t); w = w(x, y, z, t)$$
- **Two-dimensional flow:** Liquid running between two parallel plates cross-cut vertically to the plates and parallel to the flow.  
$$u = u(x, y, t); v = v(x, y, t)$$
- **One-dimensional:** Liquid flowing in a tube in terms of average velocity, then the flow has a velocity component in one direction only.  
$$u = u(x, t)$$

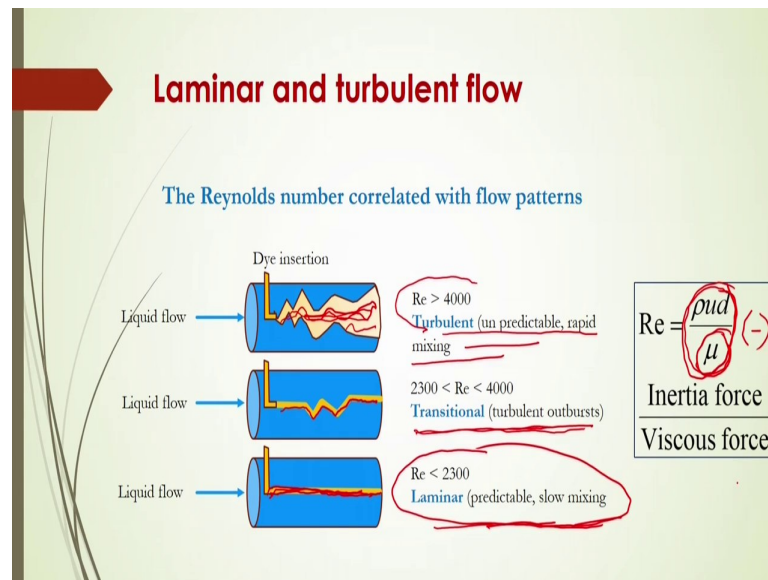
If you are considering that three-dimensional flow, suppose a ball whenever it will be flying in the air. And a flow around a moving automobile that will have velocity components in x y and z directions, so, it will be called as three dimensional flow. And the flow velocity will be represented by this a special coordinates like u x y z and the time.

And for two dimensional flow in this case the liquid running between the two parallel plates crosscut velocity to the plates and parallel to the flow. So in that case two dimensional flow like it will be only x and y direction and with respect to time; only two velocity will be considered here u and v.

Whereas, one dimensional flow in the suppose a narrow tube the fluid is flowing through that narrow tube you will see the flow will be in one dimension, two dimension there will be no variation of the velocity or acceleration. So, it will be called as one dimensional flow.

So, liquid flowing in a tube in terms of average velocity then the flow has a velocity component in one direction only. So, it will be called as one dimensional flow and it will be represented by u will be equals to u x t. So, u the velocity of the liquid will be represented only the special coordinate x and time.

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What are the laminar and turbulent flow? In this case, you will see one important criteria to consider whether this fluid will be laminar or turbulent. Whether it will be very streamline very slow in motion the fluid flow or the fluid particles will be moving haphazardly by just changing its location with respect to time.

Now if you consider that there will be a flow fluid particles will be moving in a certain fashion that the fluid particles will follow a straight line path. So, in that case it will be a laminar flow. Whereas, if the fluid particle should not follow that straight line; that means, here at any position it will change its what is that straight line path then it will be called as turbulent flow.

Here if we considered that by this diagram just see this diagram that; if we inject some dye the dye will be represented by this yellow colour you will see at a certain time if we inject it whenever it will be flowing through this flow stream you will see the solid particles or that is dye particles will be moving haphazardly like this. So, this will be what is that turbulent flow.

And whenever liquid flow in a certain fashion that the dye particles will follow some extent linear sometimes very just small a change of its location with respect to liquid flow; then it will be either considering laminar or turbulent. So, it will be called as transitional flow. And if that dye particles just follow the straight line there will be no zigzag path of the moving of these particles in this liquid flow.

There will be no change of its location from its one direction. So, then it will be called as laminar flow. Like here so one important criteria to represent this laminar or turbulent flow or transition flow it is called by it is called that Reynolds number. What is that Reynolds number?

The Reynolds number is a dimensionless group number, which will be defined as that the ratio of two forces that is called inertia force and the viscous force. The ratio of inertia force to the viscous force it will be called as Reynolds number. Like here, the mathematical representation of this Reynolds number is  $\rho u d$  by  $\mu$ . What is  $\rho$ ?

$\rho$  is the density of the liquid,  $u$  is the velocity at which the liquid is flowing through the pipe  $D$  is the diameter of the pipe. So, in this case and the viscosity is the viscosity is represented by  $\mu$ . So,  $\rho u d$  by  $\mu$  if we see that the what should be the dimension of this Reynolds number, it will be it will have it will not have any dimensions here.

So, in this case  $\rho u d$  by  $\mu$  you will see what will be the dimensions here it will be dimensionless. So, in this case if this Reynolds number suppose liquid is flowing with a certain velocity  $u$ , the density of the liquid  $\rho$  diameter of this pipe is given to you and you know the viscosity of the fluid.

And if after calculation about then what should be the Reynolds number? If Reynolds number is coming greater than 4000 then you can say it will be turbulent flow. Of course, it is applicable only for pipe flow for other channel flow this Reynolds number may change and other things that will be discussed later on.

So, in this case for pipe flow if Reynolds number is greater than 4000 then you can say it will be turbulent flow. It is totally unpredictable rapid mixing will be happening inside the tube or pipe. And if this Reynolds number within the range of 2300 to 4000 then you can say this will be a transitional flow; either laminar or turbulent mixed there.

So, you cannot consider that purely it will be laminar you cannot consider the purely turbulent so it will be transitional. So, like turbulent outburst is one type of example there. Like and also here if Reynolds number if it is less than 2300 then you can say it will be laminar flow.

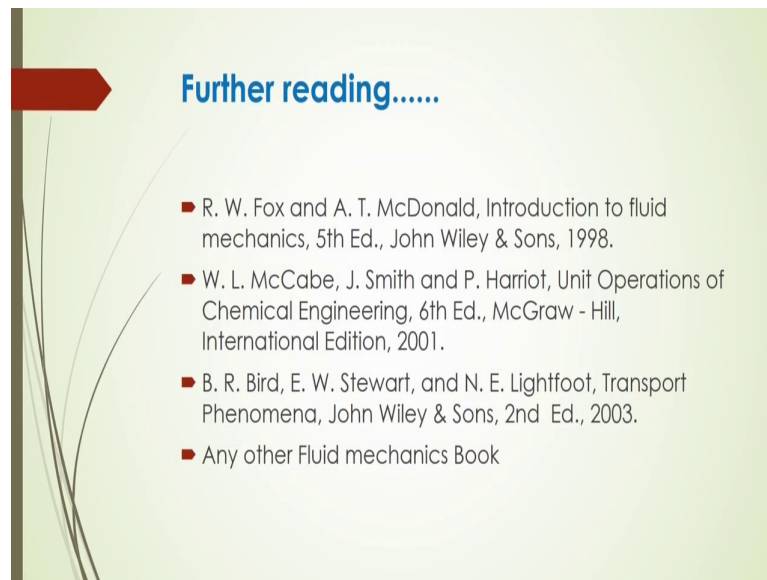
It is very obvious then you can see easily and also it will be predictable and slow mixing will be there. So, this is the flow different types of flow. And so we can have that different type of flow whether it will be the steady state, unsteady state, uniform, non uniform, viscous or non viscous or it is laminar or turbulent.

So, before going to a steady all other; that means, flow characteristics of the fluid you have to know all this things. A whenever you are going to represent the of fluid flow operations. Whether that operations is in a laminar condition or turbulent condition whether it is in whenever there will be a reaction is going on suppose if the what will be the flow pattern will there if the laminar or turbulent if you are supply if you are.

Suppose if you are controlling the temperature whether the laminar flow or turbulent flow. If you are considering any a chemical engineering operation or mechanical engineering operation heat transfer device there also you have to design all those things you have to consider that whether this laminar flow turbulent flow or that is inviscid flow or non inviscid flow.

Even some other steady or unsteady state there for further calculation you have to do. To know the velocity profile also we have to consider whether laminar or turbulent flow all this things you have to calculate their. So, before going to details of the fluid flow operations for mathematical representation of velocity profile or one dimensional flow two dimensional flow even some other device design, so, you have to know this flow characteristics that is fundamentals of the fluid flow there.

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The slide features a light green background with a dark green vertical bar on the left side. A red arrow points to the right from the top of this bar. The title 'Further reading.....' is written in blue text. Below the title, there is a list of four references, each preceded by a red square bullet point.

**Further reading.....**

- R. W. Fox and A. T. McDonald, Introduction to fluid mechanics, 5th Ed., John Wiley & Sons, 1998.
- W. L. McCabe, J. Smith and P. Harriot, Unit Operations of Chemical Engineering, 6th Ed., McGraw - Hill, International Edition, 2001.
- B. R. Bird, E. W. Stewart, and N. E. Lightfoot, Transport Phenomena, John Wiley & Sons, 2nd Ed., 2003.
- Any other Fluid mechanics Book

So, for further reading you please follow this introduction to fluid mechanics by McDonald Fox. And also other suitable fluid mechanics any book you can follow their. Mainly I can suggest that even Bird, Stewart and Lightfoot, is one important book a very very nice books for getting the velocity profile necessary all this things they are discussed. And also the McCabe, Smith also you can follow for the definition of the different types of a fluid flow.

So, thank you for this attention.