

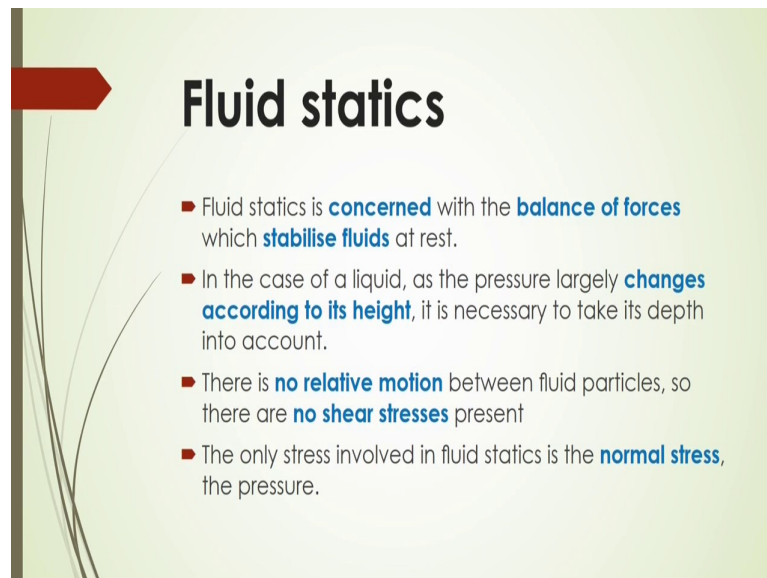
Fluid Flow Operations
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Lecture - 04
Fluid Statics-continued

Keywords: Floating body; Buoyancy force; Resulting force; Archimedes principle; Restoring force; Metacentric height; Equi-accelerated motion; Rotational motion

Welcome to massive open online course on Fluid Flow Operations. So, we have discussed earlier about the fluid statics in the previous class or previous lecture. Then in this lecture, we will discuss more about this fluid statics. In this case, we will discuss about the statics of the fluid when it will be in a uniform acceleration. So, how that pressure will be exerting on the bottom of a container of this fluid whenever this fluid will be moving with the certain acceleration. And if we go back what is that fluid statics first of all we know that the fluid static is concerned with the balance of forces that stabilize the fluid at rest.

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Fluid statics

- Fluid statics is **concerned** with the **balance of forces** which **stabilise fluids** at rest.
- In the case of a liquid, as the pressure largely **changes according to its height**, it is necessary to take its depth into account.
- There is **no relative motion** between fluid particles, so there are **no shear stresses** present
- The only stress involved in fluid statics is the **normal stress**, the pressure.

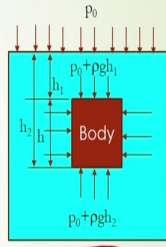
So, here also we will actually I used that concept that whenever fluid will be rest what should be the balance on the forces for stabilizing that force liquid. And based on that concept, we will be analysing what should be the force whenever it will be in moving condition. And in case of a liquids as the pressure largely changes according to its height, of course, you have to know to take its depth into account. And there is no relative

motion whenever we are considering that the fluid statics whenever fluid will be in rest, and of course, there will be no shear stress are present in that particular statics of fluid. And the only stress involved in the fluid statics is the normal stress and this will be called as the pressure that we have already discussed in the previous lectures.

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Pressure on floating body

- Consider a cube is placed in a liquid. The pressure acting on the cube due to the liquid in the horizontal direction is balanced right and left.
- For the vertical direction, the force F_1 acting on the upper surface
- The force F_2 acting on the lower surface
- **From this equation, the body in the liquid experiences a buoyancy equal to the weight of the liquid displaced by the body.**



$$F_1 = (p_0 + \rho g h_1) A \quad (\text{Eq. 1})$$

$$F_2 = (p_0 + \rho g h_2) A \quad (\text{Eq. 2})$$

Resultant Force: $F = F_2 - F_1 = \rho g (h_2 - h_1) A = \rho g h A = \rho g V$

volume of the body in the liquid is V (Eq. 3)

Now, in this case, if we consider that what should be the pressure if there is body is floating on a liquid. So, if you consider a suppose a cube, it is shown in the picture that a body like this your brown colour. This body is dip into a liquid. And you are seeing that there are several forces are acting on this body. Now, this shape of the body is considered here as a cube. So, by displaced in the liquid and the pressure acting on this cube will be due to the liquid that is horizontal direction whenever it will be flowing in a certain acceleration. But if there is no acceleration, then of course it will be the principle of the fluid statics at rest.

So, if you consider this cube that the pressure acting on the cube will be due to the liquid in the horizontal direction which will be balanced right and left here. So, in the horizontal direction, you will see in the this the arrow sign, it is denoted by the horizontal direction what will be the force even in the vertical direction, what will be the force. But for the vertical direction, the force F_1 and if you consider that F_2 that is acting on the lower surface again it on the vertical direction, then if we balance on these forces then what should be the resultant force that will be here.

So, if you consider that F_1 is acting downward on the cube that is it will be is equal to of course, the mass into that is what is that not mass if F force will be equals to this one F_1 will be is equal to p_0 plus ρg into h_1 into A . What is this A , A is the cross sectional area. So, force is equal to p_0 , p_0 is the atmospheric pressure that is acting upon the in the vertical direction from the top on the surface of the liquid.

So, again if we consider that F_2 acting upon the lower surface, then it will be p_0 plus ρg into h_2 ; h_2 is the depth of the fluid up to this surface of the bottom of this cube. And from this equation, the body in the liquid experiences a buoyancy which will be equal to the weight of the liquid that will be displaced by the body. So, if you know this F_1 and F_2 , then what should be the resultant force. So, resultant force will be is equal to resultant force will be is equal to F_2 minus F_1 and is equal to ρg into h_2 minus h_1 into A that will be equal to $\rho g h A$ that will be ρg into V .

So, what will be the pressure on the floating body that can be calculated from the resultant force here. So, resultant force F will be is equal to F_2 minus F_1 . What is F_1 ? F_1 is the p_0 plus ρg into h_1 into area; that means, pressure into cross sectional area. And F_2 is the pressure at point 2 where at depth h_2 that will be is equal to p_0 plus $\rho g h_2$ into cross sectional area. So, resultant force will be the just F_2 minus F_1 . And if you substitute this value F_1 and F_2 here, then we can get after simplification, this resultant force will be equal to ρg into V .

So, what is V ? V is nothing but the volume of the body in the liquid. So, if we have the volume of the liquid that is immersed in the liquid then what should be the force applied on this body, it will be is equal to ρg into V . And the same applies to the case where a cube is floating. In this case, the cube is floating and the some part of this body will be above the surface of the liquid. So, in this case, again we can get from that equation the body and the liquid which will be experiencing a buoyancy equal to the weight of the liquid which will be displaced by the body and this is called the Archimedes principle. So, based on this Archimedes principle, you can have what should be the force acting on the body.

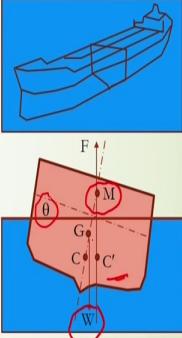
So, the centre of the gravity whenever it will be displaced liquid by the liquid then you can have in which location this centre which location this buoyancy will be acting that is

called centre of buoyancy. So, this centre of the gravity of the displaced body is called the centre of the buoyancy, and is the point of action of the buoyancy force.

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The stability of a floating body

- A body of weight W floating in the water with an inclination of small angle.
- The location of the centroid G does not change with the inclination of the body.
- But since the centre of buoyancy C moves to the new point C' , a couple of forces $W_s = F_s$ is produced and this couple restores the body's position to stability.
- The forces of the couple W_s are called restoring forces.
- The point M on the vertical line passing through the centre of buoyancy C' (action line of the buoyancy F) and the centre line of the body is called **metacentre** and GM is called the **metacentric height**.



If M is located higher than G , the restoring force acts to stabilise the body, but if M is located lower than G , the couple of forces acts to increase the roll of the ship and so make the ship unstable.

Now, what should be the stability of a floating body, you have to know how much it will be stabilised and when what will be the force acting on the body that can be remain stable. Now, a body of weight W if you consider that is floating in the water, you can inclined of a small angle like θ here this body shown in the brown colour in this picture. And you see that this body is floating some part of this body is outside this liquid, and some part inside the liquid.

So, in this case, if it is inclined to small angle, you will see the centre of gravity, it will not change to the inclination of that body. But since the centre of the buoyancy C moves to the new point C' here, C' dash, you will see a couple of forces that is W_s and is equal to F_s , W is the weight of the solid body and F is the force applied on the solid body whether it is in vertically or upwardly. So, this will be produced and this couple of forces restores on the body's position to stabilize.

Now, the forces of the couple W_s are called restoring forces, this restoring forces W is called restoring forces. Now, the point M here the it is called metacentric point on the vertical line that will be passing through the centre of the buoyancy here, which will be acting in the line of the buoyancy. And the centre line of the body which will be called

this is called the metacentre and the G M, G M that is distance between centre of gravity and this metacentric point will be called the metacentric height.

So, this case point M from the vertical line passing through the centre of the buoyancy C dashed and the centre line of the body will be called metacentre. And G M will be called the metacentric height. If M is located higher, then this G the restoring force acts to stabilize to the body acts to stabilize the body and you will see there will be a there will be a further action of force if you apply, then M will not be located exactly wherever it is now. But if M is located lower than G; lower than the G, the couple of the forces that will act to increase the increase the role of the body. Or if you suppose this is a ship then you can say that that this force acts to the increase the role of the ship and so make the ship unstable here.

So, when the body of a body will be stable or unstable that depends on this metacentre as well as centre of gravity. Now, here very important point that the M should be located higher than the G, then only the body should be stabilized in the liquid. But if M is located lower than the G, the centre of gravity, then the couple of forces that acts to increase the to increase the unstable condition of the body. And the make the body totally unstable and there will be some other forces acting on the force that will be totally unstable condition.

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The Metacentric Height

- The metacentric height is recommended as:
 - For Merchant Ships Particularly Liners it is: 0.3 to 1 m
 - Sailing Ships it is: 0.45 to 1.25 m
 - Battle ships: 1 to 1.5 m
 - River craft: upto 3.6 m
- Metacentric height

$$GM = CM \pm CG = \frac{I}{V} \pm CG \quad (\text{Eq. 5})$$

I = moment of inertia, V = Volume of liquid displaced. Negative sign is used if G is above C and positive if G is below C
- Further righting movement of the body

$$= F.MG \sin \theta = \rho g V M G. d \theta \quad (\text{Eq. 6})$$

Now, how to actually that calculate that metacentric height where it is very important to know because the stabilization of this body in the liquid that depends on this metacentric height. The meta centric height generally is recommended as for merchant ships particularly for liners if it is 0.3 to 1 metre. And for sailing ships this metacentric height should be 0.45 to 1.25 metre. And for battle ships generally it is considered as 1 to 1.5 metre. And the river craft whenever it will be designed that metacentric height should be up to 3.6 metre.

So, metacentric height is equals to that is $G M$ it will be $C M$ that is centre of buoyancy centre of buoyancy to the centre of metacentric height that is $C M$ plus minus $C G$ that is centre of buoyancy to the centre of gravity. Then it that is I by V plus minus $C G$. What is I here, this I is called moment of inertia by which this the body should be rotate in a certain angle. And V is the volume of the liquid that is displaced by the body and negative sign is used in this case if C is above as G is above C , that means, this is the centre of gravity if it is above the centre of buoyancy. And it will be considered as positive, if the centre of gravity is below the centre of buoyancy.

Further, you can say for righting movement of the body you can say this will be is equal to F into $M G$ into $G \sin \theta$ that will be equal to $\rho G V M G$ into $d \theta$. As per geometry you can calculate what will be the $M G$, $M G$ generally $M G$, $M G$ is in this angle θ . So, in this what will be the vertical direction, what will be the value, then you can easily calculate or in the horizontal direction, what will be the value that you can calculate as per the geometry.

So, F into $M G$, that means, in the horizontal direction what will be the value could be here that will be $M G$ into $\sin d \theta$. And here this ρG into v into $M G$ into $d \theta$, because this $\sin d \theta$ is almost very that means, very small. So, you can directly take it as $d \theta$ here. So, from this equation, you will be able to calculate when actually this the body will be just right to movement or to get the movement here. And metacentric height what should be the height of the metacentric that you can calculate, what will be the distance between the centre of gravity and the metacentric height. And also from the centre of buoyancy to the centre of gravity by equation number 5 is given here.

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Example: A rectangular pantoon is 7 m long and 3 m wide. The Mass of the pantoon is 30 tonne (see Figure below). Determine the position of the centre of the gravity above the base of the pantoon such that it does not overturn in still water. Take the specific volume of the water as 0.98 m³/tonne

▪ **Solution:**

Mass of pantoon = 30 tonne.

Volume of water displaced = $30 \times 0.98 = 29.4 \text{ m}^3$.

Depth of immersion of ship $h = \text{Volume/area} = \frac{29.4}{7 \times 3} = 1.4 \text{ m}$

Distance of the buoyancy **C** above the base = $h/2 = 1.4/2 = 0.7 \text{ m}$

Now $CM = I/V = \frac{(l^3/12)}{(l \times b \times h)} = 0.536 \text{ m}$

OC = 0.7 m; CM = 0.536 m and

OM = OC + CM = 0.7 + 0.536 = 1.236 m

For stability, M should be above G or M coincides with G

Therefore Maximum height of centre of gravity above the base = OG = OM = 1.236 m

Now, let us do an example for this. Let us see suppose a rectangular pantoon is a 7 metre long and 7 metre long and 3 metre wide. And the mass of the pantoon is given as 30 tonne see in the figure here. Determine the position of the centre of the gravity above the base of the pantoon such that it does not overturn in still water that means here what should be the position of the centre of the gravity that it will not make any turn to unstable. Now, take the specific volume of the water as 0.98 here that will be metre cube per tonne.

So, in this case, if I know that mass of the pantoon here as per problem, it is 30 tonne and volume of water displaced it will be equals to 30 into 0.98, because the specific volume is 0.98 metre cube per tonne. So, for 30 tonne what should be the volume, so volume of the water that is displaced that will be equals to 29.4 metre cube. Similarly, for depth of immersion of the ship that is h you can calculate from the relationship volume by area. Now, what should be the volume, volume is already calculated 29.4 and divided by area. Area is it is given because length is given 7 and width is given 3. So, the surface what should be the area that will be 7 into 3, so it will be 1.4 metre.

Now, what should be the distance of the buoyancy C from the base that is here 0.0 here it will be considered that 0. So, from 0 to centre of gravity what should be the distance here. So, it will be is equal to total h by 2. What will be the height that means, what will

be the height of that body that is immersed in the liquid, so that height you have to consider to get the centre of buoyancy.

So, this will be half of this height. This height is I think 1.4 metre that we have calculated here because depth of immersion of ship will be is equal to 1.4 this is ship or pantoon you can say now in this case h by 2 that will be equals to 0.7 metre. Now, what will be the C M, C M will be is equal to I by V . What is I ? I as the inertia those these inertia will be is equal to I will be is equal to length into b cube divided by 12 from the geometry this. So, l is the length, l is equal to length and b is the width. So, you can easily calculate what will be the inertia.

So, here it is calculated l b cube by 12 into that is I by V , I is equal to this you can calculate you know that l , you know the b . And V , V is the volume, volume is nothing but l into b into h . So, from this relationship, you can calculate this what will be the CM. So, this CM is calculated as 0.536 metre. Whereas, OC you know that OC that means depth of immersion it is given depth of immersion divided by 2 that is OC it is 0.7 metre. And what is that C M is nothing but the distance between this centre of buoyancy to the metacentric height this.

So, what should be the volume, it will be is equal to 0.7 it will be is equal to CM is equal to 0.536 already we have calculated here. And so OM will be is equal then total OM, OM will be is equal to OC plus CM, OC is 0.7. And OC M is equal to 0.536. So, total is equal to 1.236 metre. For stability you have to know what should be the value of G and G or M and accordingly whether this G or M is coincide to each other or it may be M will be higher than the G or not. So, for stability M should be above the G or M coincides with the G.

So, therefore, maximum height of the centre of the gravity above the base should be OG that will be is equal to OM, OG will be is equal to OM. So, it will be is equal to 1.236 metre. So, if your body is immersed up to this 1.236 metre in into the liquid, then it will be stabilised. So, this is the position of the centre of the gravity above the base of the pantoon such that it does not overturn in the still water. So, this is way this is the way how you can calculate what should be the metacentric height from the bottom of the subject, and whether it will be stabilized or not.

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Fluid Statics when containers moves

- When a vessel containing a liquid moves in a straight line or rotates, if there is no relative flow of the liquid while the vessel and liquid move as a body
- It can be treated as the mechanics of a stationary state

Now, important another point that here to calculate the fluid statics when the containers of the liquid will be moving at a certain acceleration. So, when a vessel that will contain a liquid moves in a straight line or if it is rotates, then what should be the fluid statics that we will calculate here. And also it can also be treated as the mechanics of the stationary state to calculate the pressure whenever it will be moving at certain acceleration.

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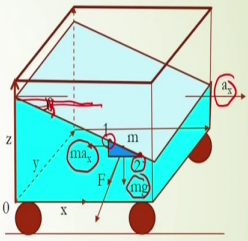
In case of equi-accelerated straight-line motion

- Consider that a vessel filled with liquid is moving in a straight line at a constant acceleration on the horizontal level
- Further consider a minute element of mass m on the liquid surface, where its acceleration is \mathbf{a} , the forces acting on m are gravity in a vertical downward direction = $m\mathbf{g}$, and the inertial force in the reverse direction to the direction of acceleration - $m\mathbf{a}$
- If θ is the angle formed by the free surface and the x direction, the following relation can be easily obtained:

$$p_2 - p_1 = -\rho a_x(x_2 - x_1) - \rho(a_z + g)(z_2 - z_1) \quad (\text{Eq. 7})$$

$$0 = -\rho a_x(x_2 - x_1) - \rho(a_z + g)(z_2 - z_1) \quad (\text{Eq. 8})$$

$$\tan \theta = (z_2 - z_1) / (x_2 - x_1) = a_x / g \quad (\text{Eq. 9})$$



For constant pressure line (free surface), $p_1 = p_2$

The acceleration $a = F / m$ (Eq. 10)

Therefore $p = \rho ah$ (Eq. 11)

Now, in case of equi-accelerated straight-line motion, suppose the liquid is moving horizontally in a container at a certain acceleration, then what will be the force acting on the force exerted by the liquid. And how the liquid surface will be just moving or it will be changes from its original surface rotation. Now, if we consider that a vessel that is filled with liquid is moving in a straight line at a constant acceleration of a x here that is certain acceleration this fluid container is moving. And if we further consider a minute element of mass m on the liquid surface, where its acceleration will be a the force is acting on m or gravity in a vertical downward direction that will be is equal to mg , mg .

And the inertia force, in the reverse direction that will act and in the direction of acceleration and that will be is equal to ma that will be is equal to ma . So, if θ is the angle that formed by the free surface, whenever it will be moving and in the x direction to in the x direction, then the following relation can be easily obtained. So, in this case, whenever it will be moving with an acceleration, so there will be a tilting of this surface, and this will be making θ angle with the horizontal axis. So, in this condition, we can say that p_2 minus p_1 , this is 2 and 2 this two point.

So, p_2 minus p_1 what should be the force per unit area in this case it will be balanced after balancing these forces we can directly obtain this equation number 7 for pressure difference at this two points. So, it will be minus ρ into a x into x_2 minus x_1 minus ρ into a z plus g here into z_2 minus z_1 ; z_2 minus z_1 in the vertical direction and x in the horizontal direction.

Now, at point 1 and 2, you will see the pressure will be same. There will change in pressure here in the surface. So, you can directly say that p_2 and p_1 will be same. So, p_2 minus p_1 will be equals to 0. So, from this equation number 8, from equation number 7 by substituting p_2 minus p_1 is equal to 0, we can get $\tan \theta$ will be equals to z_2 minus z_1 divided by x_2 minus x_1 . So, it will be is equal to nothing but a x by g just comparing with this equation number 8 and 9.

So, you can say that $\tan \theta$ will be is equal to a x by g . So, this is your $\tan \theta$. So, this is your inclination angle by which you can say what should be the acceleration if you know the centre of acceleration gravitational acceleration, then you can easily calculate what will be the angle at which this liquid will be tilted from its horizontal position

during its acceleration. So, the acceleration will be is equal to F by m here and p will be is equal to ρ into h , ρ into a into h .

So, by this equation, you can easily calculate if suppose a liquid container suppose the oil container in a oil track, it will be moving in a horizontal in on the road at a certain acceleration, then what will be the surface of the oil that will be tilted from its horizontal location. You can easily calculate by this equation 9. So, in this case, what will be the angle that will be calculated in this case you have to know what will be the acceleration of the track at which it will be moving forward.

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Example

- A 120-cm-long tank contains 80 cm of water and 20 cm of air maintained at 60 kPa above the water. The 60-cm-wide tank is accelerated at 10 m/s^2 . After equilibrium is established, find the force acting on the bottom of the tank.

Solution

The distance x can be related to y

$$\tan \alpha = \frac{a_x}{g} = \frac{10}{9.81} = \frac{y}{x} \quad \therefore y = 1.019x$$

Area $120 \times 20 = \frac{1}{2}xy = \frac{1.019}{2}x^2$ Therefore $x = 68.63; y = 69.94 \text{ cm}$

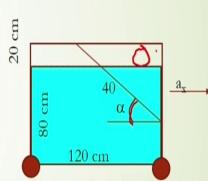
The pressure will remain unchanged in the air above the water since the air volume does not change.

From Eq. (7)

$$p_A = 60\,000 + 1000 \times 10 \times (1.20 - 0.6863) + 9810 \times 1.0 \text{ m} = 74\,900 \text{ Pa}$$

$$p_B = 60\,000 + 9810 \times (1.00 - 0.6994) = 62\,900 \text{ Pa}$$

The average force on the bottom

$$F = \frac{p_A + p_B}{2} A = \frac{74\,900 + 62\,900}{2} (1.2 \times 0.6) = 49\,610 \text{ N}$$


And then another example, you can say if suppose a 120 centimetre long tank that contains 80 centimetre of water, in it and 20 centimetre of air that is maintained at 60 kilo Pascal above the water, then you have to calculate what will be the force acting on the bottom of the tank. So, the 60 centimetre wide tank is accelerated at 10 metre per second square. So, in this case, the distance x can be related by y here, so that $\tan \alpha$ if it is making angle of α , this liquid surface during acceleration then it will be what is that a_x by g . So, a_x is given to you that will be is equal to 10 and g equal to 9.18 and that will be y by x , then y will be is equal to 1.019 after substitution of this value of a_x and gravitational acceleration. And what should be the area, area will be 120 into 20, then it will be is equal to simply half into x into y .

So, the relationship it will get you can get here 120 into 20 that will be is equal to 1.019 divided by 2 into x square, because there is a relationship y and x is here. And therefore, after solution of these two equation, we can get this x will be equals to 68.63 and y will be equals to 69.94 centimetre. Now, the pressure will remain unchanged in the air above the water, since the air volume does not change here.

So, from the equation number 7, then we can calculate p A will be is equal to this here the atmospheric pressure is given that is 60000 and plus here other part of this fluid pressure and then finally, you can get 74900 Pascal. And p B will be is equal to similarly here. So, we can have that total force that is average force will be is equal to p A plus p B by 2 into A that will be is equal to 49610 Newton. So, from this equation, you know what will be the total force acting on the bottom of the tank during its acceleration with 10 metre per second square.

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Example

Problem: An open rectangular tank 2 m wide, 4 m long and 1.5 m deep contains water upto a depth of 1 m. The tank is accelerated horizontally parallel to its length at 3.27 m/s^2 . Calculate the volume of water spilled during its motion

Solution:

Volume of water in the tank before it is accelerated = $1 \times 4 \times 2 = 8 \text{ m}^3$

Slope of the free liquid surface after it is given constant acceleration
 $\tan \theta = a/g = 1/3$

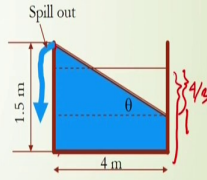
Therefore rise in water level on rear end of tank
 $= \text{slope} \times \frac{1}{2} \times \text{length} = 2/3 \text{ m}$

Water level on rear end of tank = initial level + rise
 $= 1 + 2/3 = 1.667 \text{ m} > 1.5 \text{ m}$ [so water spilled out]

The depth of water at front end = $1.5 - 4/3 = 0.167 \text{ m}$

Volume of water remained in the tank
 $= ((1.5 + 0.167)/2) \times 4 \times 2 = 6.667 \text{ m}^3$

Therefore volume of water spilled out during the motion
 $= 8 - 6.667 = 1.333 \text{ m}^3$



Another example, suppose if we move out some water at a certain acceleration, then you will see the tilting of the water surface will be there. And if there will be more that means, if there is a acceleration at a certain acceleration that the liquid will be spilled over from this container. So, what should be that I think volume in the container or what will be the volume of the water that is spilled over, whenever it will be moving at a certain velocity.

So, if we consider that an open rectangular tank of 2 metre height and 4 metre long and 1.5 metre deep that contains of up to a depth of 1 metre here up to 1 metre of depth. And then the tank is accelerated horizontally parallel to its length at 3.27 that is metre per second square with an acceleration. And this case they calculate then calculate the water which is spilled over during its motion.

So, first of all you have to calculate the volume of water in the tank before it is accelerated. So, it will be is equal to 1 into 4 into 2 that will be is equal to 8 as per geometry as per geometry and what will be the volume of water in the tank here. And the slope of the free liquid surface after it is given constant acceleration, it will be is equal to $\tan \theta$ as per we have earlier discussed, then $\tan \theta$ will be is equal to ax by g , it will be is equal to 1 by 3 . So, therefore, rise in the water level on the rear end of the tank will be equals to this slope into half of length, this slope into half of length, then it will be is equal to 2 by 3 metre.

Now, water level on the rear end of the tank it will be equals to initial level plus rise that will be is equal to 1 plus 2 by 3 that will be 1.1667 metre of course, it will be greater than 1.5 metre, because above this 1.5 metre the water will be spilled over. So, the depth of water at front end will be is equal to here, this is 1.5 metre, this 1.5 metre minus this is what is that up to this 4.4 by 3 . So, it will be is equal to 0.167 metre.

So, volume of water that remained the volume of water that remained in the tank, it will be is equal to 1.5 plus 0.167 divided by 2 into area that will be is equal to what 4 into 2 . So, it will be total 6.667 metre cube. So, earlier that the volume of the liquid was that since what the it is I think and the horizontal position 8 , whereas in this case the volume of water spilled over, then can be calculated this 8 minus 6.667 , it will be is equal to 1.33 metre cube. So, this much amount of liquid will be spilled over whenever it will be moving at a acceleration of this 3.27 metre per second square.

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In case of equi-accelerated vertical motion

- Consider an open container filled with liquid moves **vertically upward** with a uniform linear acceleration, a_z .
- A small column of liquid is acted upon by F, W and acceleration force

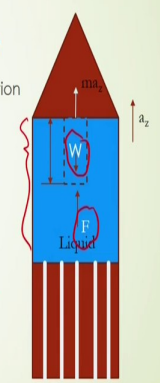
$F = \text{pressure force} = p \cdot dA$
 $W = \text{weight of the liquid column} = \rho h \cdot dA \cdot g$
 $m = \text{Mass} = (\rho h \cdot dA \cdot g) / g$

Force balance: $F - W = ma_z$

- i.e., $p \cdot dA - \rho h \cdot dA \cdot g = [(\rho h \cdot dA \cdot g) / g] a_z$

So,

$p = \rho gh + \rho g h a_z / g = \rho gh(1 + a_z / g)$
For vertically downward force
 $p = \rho gh - \rho g h a_z / g = \rho gh(1 - a_z / g)$



Rocket flows up with liquid

Now, in case of equi-accelerated vertical motion what will be the pressure. Now, consider an open container that is filled with liquid that will be moved upward vertically with a uniform linear acceleration of a_z , this is in to the vertical direction and a small column of liquid is acted in this case upon by F, F W and acceleration force. Now, F is pressure force, this will be calculated as p into dA , where W is equal to the weight of the liquid column here, what will be the weight of the liquid column that will be ρg into h into dA . And then m is equal to mass, mass will be is equal to what is that total weight divided by gravitational acceleration.

So, weight of the liquid is here ρ into h into dA into g and mass is then mass will be is equal to this ρg into dA into g divided by gravitational acceleration g . So, force balance if we do, then we can get F minus W that will be is equal to $m a_z$ at which it this liquid will be moving upward that acceleration a_z . Now, if we substitute this F here that is p into dA and W here ρh into dA into g , then it will be equals to m into a_z , m is already calculated here, and a_z is equal to here.

And so finally, after simplification, we can get p will be is equal to $\rho g h$ plus $\rho g h a_z$ divided by g that will be $\rho g h$ into $1 + a_z$ divided by g . Similarly, if we consider it the acceleration will be vertically downward, then we can say that p will be is equal to ρg into h minus here in this case g of course, you have to consider this vertically downward condition here, in this case acceleration will be in the negative direction. So, a

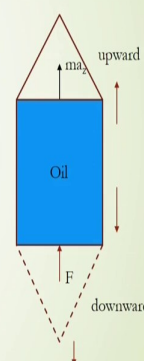
z will be is equal to minus a z. So, here rho g h into finally it will come 1 into 1 minus a z into g. So, this is your formula of the pressure drop whenever it will be going downward at a certain acceleration. So, from this whenever suppose any liquid container is moving up in a rocket at a certain acceleration. So, what you can say what should be the pressure exerted by this liquid on the bottom of this container that you can easily calculate by this formula.

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Example: An open rectangular tank 2.5 m long and 2 m wide (normal to the plane of paper) contains oil of specific gravity 0.85 upto a depth of 1.5 m. Calculate the force on the bottom of the tank, when it is being moved with an acceleration of $g/2$ m/s^2 in vertically upward and downward directions.

Solution:
 $h = 1.5 \text{ m}$, $a_z = g/2 \text{ m/s}^2$; density = $0.85 \times 1000 = 850 \text{ kg/m}^3$

- When tank is moving upward:
 $p = \rho gh + \rho gha_z/g = \rho gh(1+a_z/g) = 1912.5 \text{ N/m}^2$
 Area of the bottom tank = $2.5 \times 2 = 5 \text{ m}^2$
 So pressure force on the bottom = $1912.5 \times 5 = 9562.5 \text{ N}$
- When tank is moving downward:
 $p = \rho gh - \rho gha_z/g = \rho gh(1-a_z/g) = 637.5 \text{ N/m}^2$
 So pressure force on the bottom = $637.5 \times 5 = 3187.5 \text{ N}$



Suppose an open rectangular tank 2.5-metre-long and 2-metre-wide contains oil of specific gravity of 0.85 up to depth of 1.5 metre, then calculate the force on the bottom of the tank when it is being moved to tank acceleration of g by 2 metre per second square in vertically upward and downward direction. So, we just know that h will be equals to 1, and a z is equal to what and density is equal to what.

So, if you know these parameter then you just apply this formula and substitute this value, you can easily calculate what will be the pressure there in the vertical direction. And area of the bottom tank is simply you can calculate here this is 2.5 into 2 that is 5 metre square. And so the so the pressure force on the bottom will be is equal to 9562.5 Newton. Similarly, for the downward condition you can just have this pressure by considering the acceleration in the negative of that given acceleration here. So, here directly you just substitute then you can get 3187.5 metre Newton. So, you will see that

there will be a almost three times greater than pressure greater than pressure, then the what is that downward movement of the rocket or container here.

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In case of equi-accelerated inclined motion

- Force Balance: for upward

$$F \cdot \sin\theta = m a_x$$

$$F \cdot \cos\theta - mg = m \cdot a_z$$
- Dividing

$$\tan\theta = \frac{a_x}{(g + a_z)} = \text{slope of free surface}$$
- Similarly for downward, the slope of free surface:

$$\tan\theta = \frac{-a_x}{(g + (-a_z))}$$

In case of equi-accelerated inclined motion what will be that pressure here. So, in this case if you see this diagram, so inclined the container is moving at an angle phi here, the positive the direction of the this here in this direction, with a with an acceleration a then what should be the pressure acting on the fluid surface there so and bottom of the surface there.

So, in this case force balance, if we do then for upward, you can say F into sine theta into will be is equal to m a x in the x direction and F cos theta minus mg that will be is equal to m a z. So, here we can have this force balance in the horizontal and upward directions. And in this case, if we divide these two forces, then you can get the tan theta tan theta here this is the theta that theta is actually the surface of liquid which may after getting acceleration at a certain acceleration quantity. And then this theta this will be making this angle with the horizontal surface of the liquid there.

So, similarly for downward the slope of free surface will be tan theta is equal to if we consider ax negative of ax here, so minus ax divided by g plus minus of a z. So, you can calculate from this relationship what should be the tan theta here what will be the angle whenever it will be flowing through the inclined angle at a at an acceleration of a.

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Example: An open rectangular tank 2.5 m long and 2 m wide contains water upto a depth of 1 m. Calculate the slope of the free surface of the liquid when it is accelerated at 2 m/s². 30 degree inclined up and downward

Solution

- For upward:
 $a_x = a \cos\phi = 2 \cos 30^\circ = 1.732 \text{ m/s}^2$; $a_z = a \sin\phi = 2 \sin 30^\circ = 1.0 \text{ m/s}^2$
 $\tan\theta = a_x / (g + a_z) = 1.732 / (9.81 + 1.0) = 0.1602$ implies, $\theta = 9^\circ$
- For downward
 $\tan\theta = a_x / (g - a_z) = 1.732 / (9.81 - 1.0) = 0.1968$ implies, $\theta = 11^\circ$

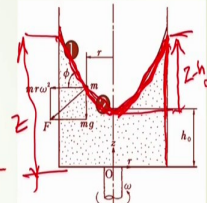
Now, suppose this is an open rectangular tank 2.5-metre-long and 2-metre-wide that contains water up to a depth of 1 metre. In this case, what should be the slope of free surface of the liquid when it is accelerated at 2 metre per second square. And in this case 30 degree inclined up and downward angle to be considered. So, in this case, again you have to calculate in the x direction what will be the a_x that is acceleration in the x direction that will be is equal to $a \cos \phi$, so it will be 1 metre per second square.

Whereas, the $\tan \theta$ it will be then here a_x by $g + a_z$, then it will be 0.1602; and after solution of θ will be almost equals to 9 degree centigrade 9 degree. And for downward then θ will be is equal to around 11 degree, minus 11 degree, because here we are considering the downward direction movement of the liquid. So, by this formula we will be able to calculate this what should be the fluid statics in the upward and downward direction when it will be moving at an inclined surface.

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In case of rotational motion

- A cylindrical vessel filled with liquid is rotating at constant angular velocity ω .
- If ϕ is the angle formed by the free surface and the horizontal direction,



(Eq. 12)

$$\tan \phi = \frac{mr\omega^2}{mg} = \frac{r\omega^2}{g}$$

But also

$$\tan \phi = \frac{dz}{dr} \quad (\text{Eq. 13})$$

Therefore

$$\frac{dz}{dr} = \frac{r\omega^2}{g} \quad (\text{Eq. 14})$$

(Eq. 15)

After integration

$$z = \frac{r^2\omega^2}{2g} + c \quad (\text{Eq. 15})$$

(Eq. 16)

$$z - h_0 = \frac{r^2\omega^2}{2g}$$

Centrifugal head on free surface

The free surface is therefore a rotating parabolic surface.

$$p_2 - p_1 = \frac{\rho\omega^2}{2} (r_2^2 - r_1^2) \quad (\text{Eq. 17})$$

Now, in case of rotational motion what should be the pressure in this case. If suppose a cylindrical vessel that is filled with a liquid that rotating at a constant angular velocity ω , in this case you will see there will be a formation of vorticity. So, as per this diagram, here this is the surface of the liquid. And this if ϕ is the angle formed by the free surface and the horizontal direction, then you can calculate $\tan \phi$ will be is equal to $m r \omega^2$ divided by $m g$. So, from this you can calculate r into ω^2 by g , but also $\tan \phi$ that will be is equal to dz by dr from this at any point if you are considering that it will be dz by dr .

Here at a r distance if you are considering this point, then it will be simply dz by dr what should be the $\tan \phi$ here So, this $\tan \phi$ that will be is equal to dz by dr , it will be is equal to $r \omega^2$ by g . Now, you just consider here that at z is equal to h_0 and at r is equal to 0 , c should be is equal to 0 if we if we integrate this equation here constant of integration. So, after integration you can get this z will be equals to here $r^2 \omega^2$ by $2g$ plus here c , c is the constant of integration and this constant of integration will be is equal to h_0 and if we consider this boundary condition here.

So, finally, you can get this $z - h_0$ that will be is equal to $r^2 \omega^2$ by $2g$, h_0 is the what is that initial liquid height in the container. And after rotation this liquid height will be up to a certain height to be moved up and to be forming a mod x

like this. So, you will see this curved surface, and how and what will be the height of this curved surface will be moving up that you can calculate from this relationship of equation 16.

So, this is your I think $z - z_0$, then it will be what is that this is $z - h_0$, this if we are considering z here. So, here $z - h_0$, if we if we consider up to this, this is your z , and this is your z . If you consider this is your z then it will be $z - h_0$ is equal to this 1. So, from the initial surface how long this liquid will be moving up at a certain rotation. And now what should be the pressure at two points this 1 and 2, if we consider then $p_2 - p_1$ is equal to $\rho \omega^2$ by 2 into r_2 square minus r_1 square this is at your at this point 1 what will be the r_1 at this point what should be the r_2 for this curved surface.

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Example

Problem: The cylinder as shown in Fig. is rotated about the center axis as shown. What rotational speed is required so that the water just touches point A. Also, find the force on the bottom of the cylinder.

Solution:

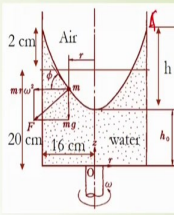
The volume of the air before and after must be the same. Recognizing that the volume of a paraboloid of revolution is half of the volume of a circular cylinder of the same radius and height, the height of the paraboloid of revolution is found $\pi \times 0.16^2 \times 0.02 = (1/2) \pi \times 0.16^2 \times h$; therefore, $h = 0.04$ m

and

$0.04 = (\omega^2 \times 0.16^2) / (2 \times 9.81)$ as per Eq. 16

The pressure on the bottom:
where $p_0 = 9810 \times 0.20 - 0.04 = 1570$ Pa. So,

$p = \frac{1000 \times 5.54^2}{2} r^2 + 1570 = 15346r^2 + 1570$



The pressure is integrated over the area

$\int_0^{0.16} (15346r^2 + 1570) 2\pi r dr = 142.1 \text{ N}$

And let us do an example here. If suppose a cylinder as shown in the figure is rotated about the centre axis as shown here and what rotational speed to be maintained, so that the water just passes the point A and also find the force on the bottom of the cylinder ok. So, here again we have to apply the things what will be the pressure exerted by this p minus p_0 here, it will be this ρ into ω square by 2 into r square minus r_1 r 1 square.

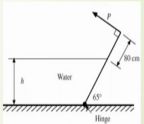
And here ω can be calculated from this equation number here as per equation 16, then what will be the ω square. And from this also then what will be the p here, p

should be is equal to as per this equation number. And then finally, if you integrate this total pressure will be is equal to this 142.1 n and then omega will be is equal to by this equation what will be the omega is coming. So, based on this, we will be able to calculate what will be the rotational speed to require the water to pull up to a certain point here.

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Try yourself

- Show that the rise of liquid along the wall of a cylinder above the initial level is the same as the depression of the liquid at the axis of rotation
- A cylinder having oil of sp. Gravity 0.8 is revolved above its vertical axis. Find the speed in rpm if pressure at a point A taken 1 m radially from the axis is same as that of another point B taken at a distance of 2 m from the axis. The point B is 1 m above the point A. **Ans: 24.4 rpm**
- Find the force P needed to hold the 2-m-wide gate in Fig. in the position shown if $h = 1.2$ m. **Ans: 4860 N**



Now, other also you can try also in this case some problems are given, you can try yourself based on this lecture.

(Refer Slide Time: 45:28)

Further reading.....

- R. W. Fox and A. T. McDonald, Introduction to fluid mechanics, 5th Ed., John Wiley & Sons, 1998.
- W. L. McCabe, J. Smith and P. Harriot, Unit Operations of Chemical Engineering, 6th Ed., McGraw - Hill, International Edition, 2001.
- B. R. Bird, E. W. Stewart, and N. E. Lightfoot, Transport Phenomena, John Wiley & Sons, 2nd Ed., 2003.
- Any other Fluid mechanics Book

And I think I should suggest more reading for this for this further understanding of this fluid statics like here references are given. So, if you follow this, you can have more illustration based on this that is fluid statics in the moving condition.

So, thank you.