

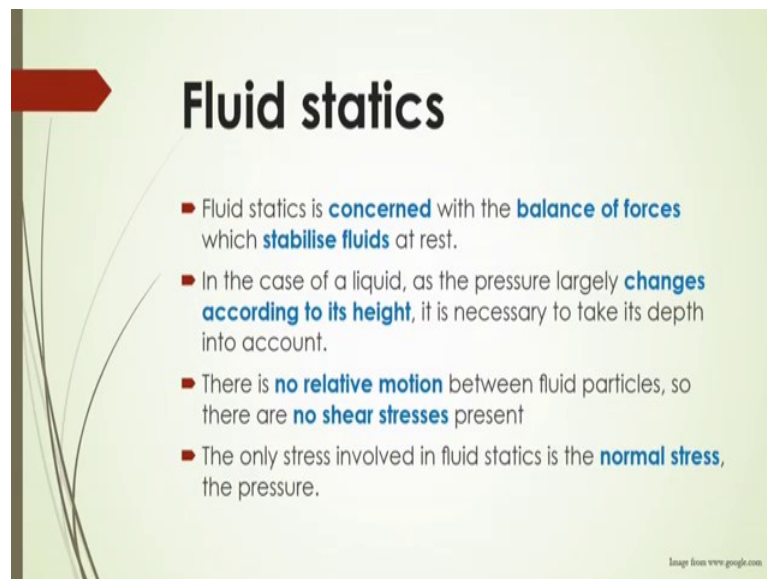
Fluid Flow Operations
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Lecture – 03
Fluid statics

Keywords: Fluid statics; Absolute and Gauge pressure; Hydraulic pressure; Characteristic of pressure; Measurement of pressure

Welcome, to massive open online course on Fluid Flow Operations. Hello, everybody. We have in the previous class discussed about the fluid properties and in this lecture we will discuss about the Fluid Statics; how the fluid will exert the pressure on the surface of the solid whenever it will be in rest even if will be accelerated with uniform of velocity and then how it will act or how it will give a force to the surface of the body.

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So, in this case this fluid statics is generally concerned with the balance of the forces which stabilise the fluid of whenever it will be in rest. In the course of liquid, you will see some pressure there will be change according to its height and it will be necessary to take its depth whenever it will be considered for measuring how much actually pressure will exert by this fluid on the surface of the fluid body which is immersed in the fluid for the force will be exerting on the bottom surface of a container in which it will be kept.

So, in this case whenever fluid will be statics in motion statics in condition; that means, that will be in rest in condition. So, in that case there will be no relative motion of course, between the fluid particles and that is why we can consider that the fluid will not have any shear stress in this condition and the only stress will be involved in the fluid statics is the normal stress and which is called pressure.

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Pressure in fluids at rest

- For uniform pressure:** If a uniform pressure acts on a flat plate of area A and a force F pushes the plate, then one can write

$$p = F/A$$
- In this case, p is the pressure and F is the pressure force.
- For non-uniform pressure:** If the pressure is not uniform, the pressure acting on the minute area ΔA is expressed by the following equation:

$$p = \lim_{\Delta A \rightarrow 0} \frac{\Delta F}{\Delta A} = \frac{dp}{dA}$$

The slide includes a diagram of a surface with a small area element ΔA and a normal force ΔF_n acting on it.

And you will see there are two types of pressure will be exerting on the surface of the solid and one is called uniform pressure another is called non-uniform pressure.

In case of a uniform pressure, you will see if a uniform pressure acts on a flat plate of area considering A here as per this diagram and this diagram which area is A and a force F pushes the plate then one can write that p will be is equal to F by A . What is p ? p is denoting denoted by pressure and or F is called force which is exerted on the plate surface and in case p is the pressure and F is the pressure force here. In this case p is the pressure and F is the pressure force here not capital p it is F , F is the pressure force.

And for non-uniform pressure if the pressure is not uniform you can say that pressure acting on the minute area which is represented by ΔA and it would be expressed by the following equation here now that is small p will be equals to limit of ΔA tends to 0 the Δp upon ΔA . Here, it will be actually a ΔF upon ΔA instead of Δp here F is a force and small p is called pressure.

So, it will be represented by $\int p \, dF$ by dA . So, we are having that uniform pressure if a uniform pressure acts on a flat plate of area A and a force F pushes the plate then one can write p is equal to F by A . In this case p is the pressure and F is the pressure force. For non-uniform pressure if the pressure is not uniform then the pressure acting on the small area ΔA which should be expressed by this dF by dA .

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Units of pressure

- Pascal (Pa) is the the unit of pressure,
- It is also expressed in bars or metres of water column (mmH₂O)
- in some cases atmospheric pressure is used
- 1 atm is standard 1 atmospheric pressure in meteorology and is called the standard atmospheric pressure.

1 atm = 760 mmHg (at 273.15 K, $g = 9.80665 \text{ m/s}^2$)
= 101325 Pa

Name of unit	Unit	Conversion
Pascal	Pa	1 Pa = 1 N/m ²
Bar	bar	1 bar = 0.1 MPa
Water column metre	MmH ₂ O	1 mmH ₂ O = 9806.65 Pa
Atmospheric pressure	atm	1 atm = 101325 Pa
Mercury column metre	mmHg	1 mmHg = 133.322 Pa
Torr	torr	1 torr = 1 mmHg

Now, what is the unit of pressure? Generally, Pascal is the unit of pressure which is widely considered and it is also expressed in bars or meters of water column that is in millimetre of water column. And in some cases atmospheric pressure is used that is called 1 atmosphere with reference pressure and 1 atmosphere is a standard 1 atmospheric pressure in meteorology and is called the standard atmospheric pressure. 1 atmosphere will be equals to 760 millimetre mercury at 273.15 Kelvin, whereas g will be is equal to 9.80665 meter per second square, g is called gravitational acceleration and this atmospheric pressure that is 1 atmospheric pressure will be is equal to 101325 Pascal.

So, in this table here see one relationship between the different form of that is different type of units or different forms of units to express the pressure is given here. Like Pascal is 1 Pascal is equal to 1 Newton per meter square and 1 bar will be equals to 0.1 mega Pascal and water column meter is 1 unit that is millimetre water is considered and then it will be is equal to 9806.65 Pascal 1 millimetre mercury equivalent to here and in this

way from this table you can calculate what should be the pressure even pressure also can be expressed in terms of unit torque that is torque, 1 torque equals to 1 millimetre mercury.

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Absolute pressure and gauge pressure

- Two methods used to express the pressure:
- Based on the perfect vacuum:** It is called absolute pressure
- Based on the atmospheric pressure:** It is called the gauge pressure
- Gauge pressure = absolute pressure - atmospheric pressure
- a pressure under 1 atmospheric pressure is expressed as a negative pressure or vacuum

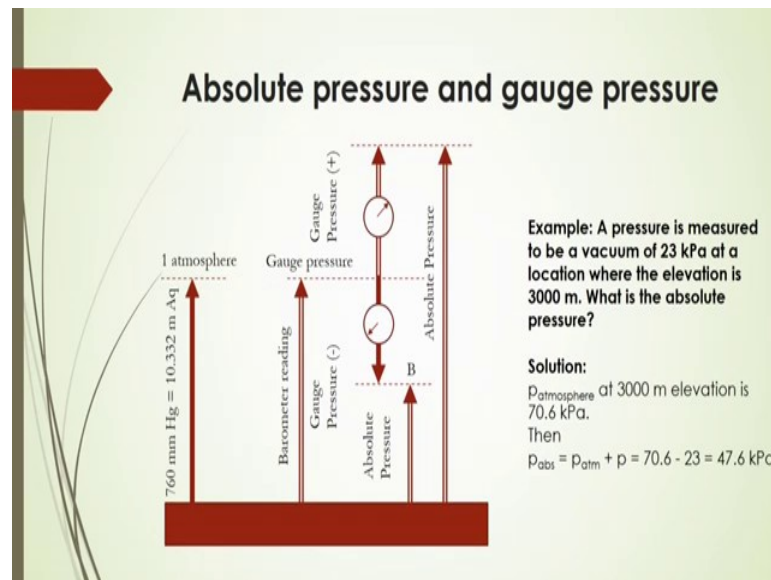
Standard atmosphere
101.3 kPa
14.7 psi
30 in Hg
760 mm Hg
1.013 bar
34 ft water

Standard atmosphere (at 40° latitude at sea level)

Now, absolute pressure and gauge pressure is the two another important form of pressures you see. Generally, two methods to express this pressure; one is based on the perfect vacuum and it is called the absolute pressure and based on the atmospheric pressure it is called the gauge pressure. Gauge pressure is equal to absolute pressure minus atmospheric pressure and a pressure under 1 atmospheric pressure is expressed as a negative pressure which is called vacuum. And 1 standard atmospheric pressure at 40 degree latitude at sea level it is considered for the standard atmospheric condition that is here absolute standard atmosphere, it is considered as at 40 degree latitude at sea level.

And that standard atmospheric pressure that is 101.3 kPa it is considered and one 14.7 psi, 30 units mercury, 760 millimetre mercury, 1.013 bar and 34 feet of water. This is the different of that is of unit or this is the different form of expressing the pressure. So, all are like this is the standard pressure or standard atmospheric pressure you can say. So, this is 101.3 kPa the standard atmospheric pressure and that will be of course, considered at the sea level of 40 degree latitude.

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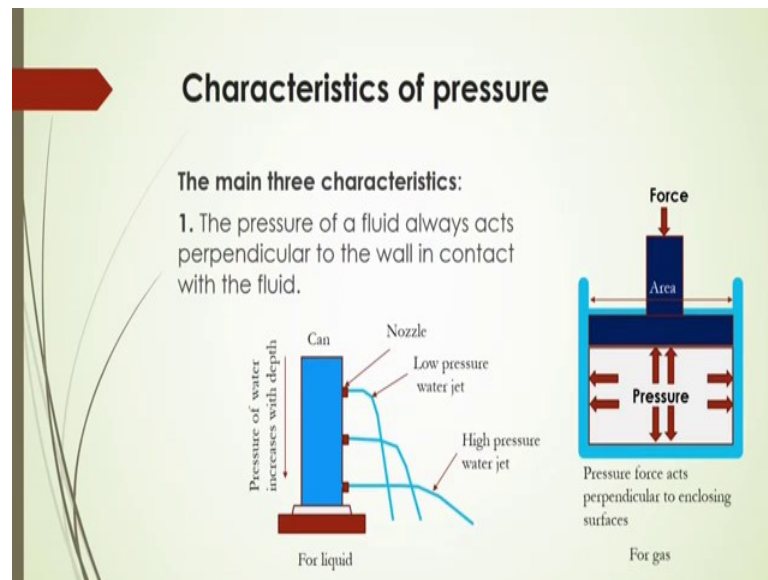


Here see one example of the pressure a pressure can be measured to be a vacuum of 23 is measured to be a vacuum of 23 kilo Pascal at a location where the elevation is 3000 meter then what should be the absolute pressure here.

So, here $p_{\text{atmosphere}}$ at 3000 meter elevation is 70.6 kPa. So, then we absolutely because to pre atmosphere plus that is p then it will be that is p is vacuum pressure is 23 kPa. So, it will be 70.6 minus 23 then it will because to 47.6 kPa. And here in this diagram see from the sea level here it will be 1 atmospheric pressure and here the barometer reading like this here this in this from this 1 atmospheric pressure above this atmospheric pressure this is called gauge pressure and below this atmospheric pressure it is called the vacuum pressure. And then this this up to this it will be called as a absolute pressure and then what should be that absolute pressure?

So, from the simple relationship of this gauge pressure will be equal to the absolute pressure minus atmospheric pressure or you can say that absolute pressure will be is equal to the gauge pressure plus atmospheric pressure. So, absolute pressure will be is equal to here, what should be the absolute pressure in this case from the sea level you have to add atmospheric 1 atmospheric pressure with the gauge pressure, then you can have the absolute pressure there.

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Now, let us consider that what will be the characteristics of the pressure? The main three characteristics of the pressure is first of all the pressure of a fluid will always acts perpendicular to the wall in contact with the fluid. Here see the diagram for the liquid whenever it will be kept in a what is that in a liquid column you will see from the bottom to top how pressure will be acting on this surface of the column of this container. And in this case at the bottom the pressure will be higher, so, you can get that high pressure water jet form from a bottom.

Whereas, in the middle section you will see there will be a relatively low pressure that is why low pressure relatively low pressure water jet and from the top here there will be a very less pressure, now that low pressure water jet you can obtain. The water is coming from the nozzle that is why you are getting the water from the nozzle as a jet. So, this jet the force will be more at the bottom whereas, at the top it will be relatively low. So, it depends on the height of the column of liquid in the container.

Now, pressure of water increases with the depth. So, we can say that the main one of the main important characteristics that the fluid always acts perpendicular to the wall in contact with the fluid and it will be related it will relate to the height of the liquid column. Here, again in case of gas you will see if you force on this container suppose this container contains the gas and if you apply the force from the top of this container and you will see since the gas is compressible fluid then it will go downward, but

pressure inside that container will be higher. So, in that case if you are at increasing the force there you can get more pressure.

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Characteristics of pressure

2. The values of the pressure acting at any point in a fluid at rest are equal regardless of its direction.

From geometry

$$dy = ds \sin \beta \quad (\text{Eq. 3})$$

$$dx = ds \cos \beta \quad (\text{Eq. 4})$$

The weight of the triangle pillar is doubly infinitesimal, so it is omitted. Therefore

$$p_x - p_y = \rho \frac{dx}{2} a_x \quad (\text{Eq. 5})$$

$$p_y - p_x = \rho \frac{dy}{2} (a_x + g) \quad (\text{Eq. 6})$$

So $p_x = p_y = p$ (Eq. 7)

Pressure acting on a minute triangular prism

$$\sum F_x = ma_x : p_x dy dz - p ds dz \sin \beta = \rho \frac{dx dy dz}{2} a_x \quad (\text{Eq. 1})$$

$$\sum F_y = ma_y : p_y dx dz - p ds dz \cos \beta - \rho g \frac{dx dy dz}{2} = \rho \frac{dx dy dz}{2} a_y \quad (\text{Eq. 2})$$

a is acceleration of fluid element

Now, another important characteristic is that the pressure value which will be acting at any point in a fluid at rest will be equal to will be equal and in all directions all directions whenever it will be exerting in a particular point. So, the values of the pressure acting at any point in a fluid at rest are equal regardless of its direction.

See the diagram here one suppose this is one container here this is called triangular prism here some liquids is that there. So, how pressure acting on a minute triangular prism there? So, in this case if we draw a free diagram of this that is triangular prism then we can say what are the force acting on this surfaces of this prism.

So, in this from this bottom there will be a force like this p_y into $dx dz$ and in this case p_x into $dy dz$ because p this is in x -direction what will be the pressure that is p_x , in the y -direction the pressure is p_y . So, in the x -direction what should be the force acting on this surface? In this surface, in the x -direction it will be p_x into $dy dz$, p_x into $dy dz$; whereas, in the y -direction the force would be p_y into $dx dz$ dx and dz is called the minute area and here dy and dz is also minute area in the x -direction.

So, and another force if it is acting in this surface of the fluid perpendicular to the surface of the fluid surface then you will see the pressure is p into force is to p into $dx ds$ into dx .

What is that ds ? ds is nothing, but the a small area of the surface of the liquid. So, in this case, but total force would be p into ds into dx .

So, if we consider the force balance in x and y direction, we can say that summation of F_x will be equals to m into a_x . Then if we substitute this force then p_x into $dy dz$ minus p into $ds dz$ into sine beta. What is that? beta is the inclination angle here beta of the surface of the prism with the horizontal surface. So, here beta. So, what should be the force acting in the x -direction for this? So, it will be p into $ds dz$ here if we consider that then it will be acting like this. It will be in the y -direction it will be in this direction. So, it will be minus because here from the bottom the force would be to go to p_y into $dx dz$.

So, it will be now in the x direct positive y -direction and the negative y -direction it will be into called $p ds dx$ into sine beta. So, that will be is equal to what should be the acceleration in the x direction. So, it will be ρ into $dx dy dz$ by 2 into a_x ; that means, a ρ into volume average volume or that is total the this volume of this prism then it would be ρ into volume; that means, we have to mass; mass into acceleration in the x -direction that it will be force. So, in the x -direction force balance will be this and in y -direction it would be again total sum of force will be equals to m into a_y ; a_y is the acceleration in the y direction. Then it will be what is that in the y -direction it will be what this $p_y dx dz$ minus p into $ds ds z dz$ at \cos beta minus ρg into $dx dy dz$ by 2 that will be equals to here ρ into $dx dy dz$ by 2.

So, here it will be in the y -direction here in the y direction, earlier that will be your x direction. So, x directional force balance it will be here in the x -direction $p_x dy dz$ minus here it will be acting in this direction in the x -direction it will be in the x -direction and whereas, in the y -direction this force balance this one and this one this and this whereas, in x -direction these and this direction. So, from this to force balance if we consider from this from the geometry also we can we can have dy is equal to ds into sine beta and dx will be equal to ds into \cos beta.

So, from this geometry and along with this of course, balance you can simply have this p_x minus p is equal to ρ into dx by 2 into a_x and in the y -direction it would be p_y minus p into ρ into dy by 2 into a_y plus g . So, from these two equations if we compare equation – 5 and 6 we can say that p_x will be equals to p_y and that would be equal to p

and p_x is equal to p_z that will be is equal to again p . So, overall we can say that in the a p_x will be equals to p_y equals to p_z that will be equals to p .

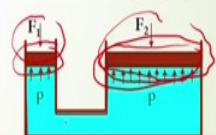
So, in all directions to the pressure acting will be equal to each other.

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Characteristics of pressure

3. The fluid pressure applied to a fluid in a closed vessel is transmitted to all parts at the same pressure value as that applied (Pascal's law).

- When the small piston of area A_1 is acted upon by the force F_1 , the liquid pressure $p = F_1/A_1$ is produced and the large piston is acted upon by the force $F_2 = pA_2$. Thus

$$F_2 = F_1 \frac{A_2}{A_1} \quad (\text{Eq. 8})$$


So this device can create the large force F_2 from the small force F_1 . This is the principle of the hydraulic press.

So, the third characteristics of the pressure is that the fluid pressure applied to a in a closed vessel is transmitted to all parts of the same pressure value as that applied in the fluid. So, when the small piston of area A_1 is acted upon by the force F_1 here F_1 here the liquid pressure p will be F_1/A_1 ; that means, force per unit area. Similarly, you can say that the same pressure will be acting on this cross sectional area of this piston and in this case it will be p is equal to again F_2/A_2 .

So, if we equate these two pressure then you can have this equation – 8. So, this is nothing, but that F_2 is equal to $F_1 \times A_2/A_1$. So, in this case what we can say that this device can create the large force F_2 from the small force F_1 this is the principle of the hydraulic press. So, here see if we are produce larger surface area and then accordingly you can say what should the force acting on this that will be based on this small what is that force F_1 here acting on this surface A_1 . So, this is called the principle of hydraulic press. So, this is also one important characteristics of the pressure.

So, one is first one is the called that that the pressure of a fluid always acts that would be perpendicular to the wall in contact with the fluid. Second, one is that in the all

directions, the pressure will be exerting with equal value and third one is that in this case if the fluid pressure applied to a fluid in a closed vessel and if it is transmitted to all parts of this container at the same pressure value as that applied there. So, in this case, if we apply the pressure F 1 in a small cross sectional area all with small force F 1 what should be the pressure is the exerting there that would be p and that p always will be the same in this large cross sectional area which will be resisting by this force F 2.

So, F 2 should be should be some multiplication factor of this F 1 which is applied in the small cross sectional area. So, this will be if suppose A 2 is the double of A 1; that means, here cross sectional area in this layer is double of this. So, we can say that the force will be required to resist this force then will be double up this earlier force.

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Pressure of fluid at rest

- In a fluid at rest the pressure varies according to the depth.

$$pdA - \left(p + \frac{dp}{dz} dz \right) dA - \rho g dA dz = 0 \quad (\text{Eq. 9})$$

$$\frac{dp}{dz} = -\rho g \quad (\text{Eq. 10})$$

$$p = -\rho g \int dz = -\rho g z + c \quad (\text{Eq. 11})$$

$$c = p_0 + \rho g z_0 \quad (\text{Eq. 12})$$

$$p = p_0 + (z_0 - z)\rho g = p_0 + \rho g h \quad (\text{Eq. 13})$$

base point is set at z_0

Another important point, that how we can estimate the pressure whenever any fluid is acting or any fluid is exerting any pressure on the surface of the solid and that of course, you have to quantitatively express what will be the pressure there.

Now, if we consider that a fluid is at rest and the pressure varies according to the depth and then we can say here if we consider this element of fluid here, this point suppose there is a fluid element like this of dz and cross sectional area is dA in the z-direction that is the vertical direction, then we can say that if we do the of force balance over this a fluid element we can have that what should be the p at this surface here then we can say p into dA that will be force acting vertically upward.

And it will be balanced by the force acting in this surface in the negative z-direction that will be equal to p plus dp by dz into dz ; that means, here since the pressure is related to the height of the liquid column if we consider that to would be uniform pressure division along with this z ; that means, the pressure gradient is dp by dz then at this dz at what should be the pressure here. So, total the pressure change what this height dz to be dp by dz into dz . So, p plus dp plus dp by dz into dz this will be your of pressure acting over this surface here in the vertical direction negative vertical direction.

Now, here what with the weight of this fluid that would be dW and it will be equal to ρg into dA into dz ; dz is the height of this liquid column and cross sectional area is equal to dA . So, dA into dz is the volume; volume into ρ it is mass into g that will be equal to force. So, this will be gravitational force.

So, if we vertically balance this force then we can say that p into dA minus p plus dp by dz into dz into dA it will be force and it is also force minus gravitational force which is acting downward vertically. So, ρ into g into dA dz that will be equal to 0. So, this is the force balance or you can say what will be the in the positive z-direction what is the force and negative force that force and the negative direction just that will be equal to equal when the fluid element will be in rest. So, in that case from this equation number 9 we can say that dp by dz will be equal to minus ρ into g .

So, by this equation number 10 we can have this for to be the pressure gradient along with the height of this liquid column. So, it will be equal to simply minus ρg minus ρg is called the specific gravity of the fluid. So, from this if we integrate this in differential equation of this dp by dz is equal to minus ρg with this boundary condition at z is equal to z_0 , p should be is equal to p_0 that means at the bottom, if I consider that the p is equal to p_0 then we can have after integration the constant of integration will be is equal to p_0 plus $\rho g z$. And, hence we can say that p will be equal to p will be equal to p_0 plus z_0 minus z into ρg that will be equal to p_0 plus ρg into h because z_0 minus z that would be equal to what h . So, this is your final equation that is called equation of statics of the fluid.

So, by this equation we will be able to find out what should be the pressure at a certain depth of the column. Suppose, this is the pond and here at a certain depth of h what should be the pressure here? Simply by equation number 13 you will be able to calculate

whereas, p_0 here it will be is equal to atmospheric pressure. So, p minus p_0 ; that means, the exact pressure minus atmospheric pressure that is total what atmospheric pressure plus the a height column pressure that will be is equal to $\rho g h$.

So, by this equation you will be able to find out what will be the pressure at a certain depth in the pond or any water column or in a container there you can have the pressure calculation for that.

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Pressure of gas with respect to height of atmosphere

- In this case, the density of gas changes with pressure. As the altitude increases, the temperature decreases

$$p v^n = \text{constant} \quad (\text{Eq. 14})$$

$$\text{At } z=0 \text{ (sea level), } p = p_0, \rho = \rho_0 \quad (\text{Eq. 15})$$

$$\frac{p}{\rho^n} = \frac{p_0}{\rho_0^n} \quad (\text{Eq. 16})$$

Eq. 10, $\frac{dp}{dz} = -\rho g$ yields $dz = -\frac{dp}{\rho g} = -\frac{1}{g} \frac{p_0^{1/n}}{\rho_0} p^{-1/n} dp = -\frac{1}{g} \frac{p_0}{\rho_0} \left(\frac{p_0}{p}\right)^{1/n} d\left(\frac{p}{p_0}\right)$ (Eq. 17)

Now, in case of gas what will happen the density of the gas changes with pressure? So, as the altitude increases the temperature also decreases. So, based on that accordingly the pressure of the gas will be changing. So, based on that $p v^n$ will be equals to constant. Here n is the poly tropic constant. So, in this case $p v^n$ is equal to constant if this is the non ideal gas. So, here n should be or for ideal gas it will be is equal to n is equal to equal to 1. So, $p v^n$ is equal to constant so, at z is equal to 0 at sea level you can say that p is equal to p_0 and ρ is equal to ρ_0 .

So, you can have that p by ρ to the power n that will be equal to p_0 by ρ_0 to the power n . Again from equation number 10, we can we know that dp by dz that will be is equal to minus ρg . So, if you substitute you these equation number 16, in this equation number 10 which will be yielding that dz would be equals to minus dp by ρg . So, that will because 2 minus 1 by g in to ρ a p_0 to the power 1 by n by ρ_0 into p to the power minus 1 by n so, into dp .

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Pressure of gas with respect to height of atmosphere

- Integrating from $z = z_0$

$$z = \int_0^z dz = \frac{1}{g} \frac{n}{n-1} \frac{p_0}{\rho_0} \left[1 - \left(\frac{p}{p_0} \right)^{(n-1)/n} \right] \quad (\text{Eq. 18})$$

By Eq. 18

$$\frac{p(z)}{p_0} = \left(1 - \frac{n-1}{n} \frac{\rho_0 g}{p_0} z \right)^{n/(n-1)} \quad (\text{Eq. 19})$$

From Eq. 16 & 19

$$\frac{\rho(z)}{\rho_0} = \left(1 - \frac{n-1}{n} \frac{\rho_0 g}{p_0} z \right)^{1/(n-1)} \quad (\text{Eq. 20})$$

So, finally, we can have after integration at z is equal to z_0 then z will be equals to this equation; equation number 18. Now, by this equation number 18, finally, we will be having the relationship of p at a certain height based on the height relative to the; that means, atmospheric pressure will be equals to that is $1 - \frac{n-1}{n} \frac{\rho_0 g}{p_0} z$ into ρ_0 to the power $\frac{n}{n-1}$. From equation 16 and 19, we can combined it represent this equation 19, in terms of density.

So, it will be density at particular height by a relative density based on that density at this sea level then it will be equals to ρ_0 equals to $1 - \frac{n-1}{n} \frac{\rho_0 g}{p_0} z$ into ρ_0 to the power $\frac{1}{n-1}$. So, by this equation number 19 and 20, you can calculate what should be the pressure at a certain height and what should be the density at a certain height by equation number 19 and 20 respectively.

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Pressure of gas with respect to height of atmosphere

- When the absolute temperatures at sea level and at the point of height z are T_0 and T respectively, from equation

$$pv = RT \quad (\text{Eq. 21})$$

$$\frac{p}{\rho T} = \frac{p_0}{\rho_0 T_0} = R \quad (\text{Eq. 22})$$

From Eqs. 19 to 22

$$\frac{T(z)}{T_0} = 1 - \frac{n-1}{n} \frac{\rho_0 g}{p_0} z \quad (\text{Eq. 23})$$

From Eq. 23

$$\frac{dT}{dz} = -\frac{n-1}{n} \frac{\rho_0 g}{p_0} = -\frac{n-1}{n} \frac{g}{R} \quad (\text{Eq. 24})$$

In aeronautics, it has been agreed to make the combined values of $p_0 = 101.325 \text{ kPa}$, $T_0 = 288.15 \text{ K}$ and $\rho_0 = 1.225 \text{ kg/m}^3$ the standard atmospheric condition at sea level.

The temp. decreases by $0.65 \text{ }^\circ\text{C}$ every 100 m of height in the troposphere up to 1 km high and $n = 1.235$.

And pressure of gas with respect to height of atmosphere. So, we know that when the absolute temperatures at sea level and at the point of high z are T_0 and T respectively, from equation pv is equal to a RT . Then we can have this p by ρRT equals to p_0 by $\rho_0 T_0$ in is equal to R and from equation this 19 and 22, you can express that the how temperature will be changing with respect to height also. So, based on this equation number 23, you will be able to calculate the temperature change along with the height if you know the atmospheric pressure and the poly tropic constant and also the density at sea level and what should be the height there.

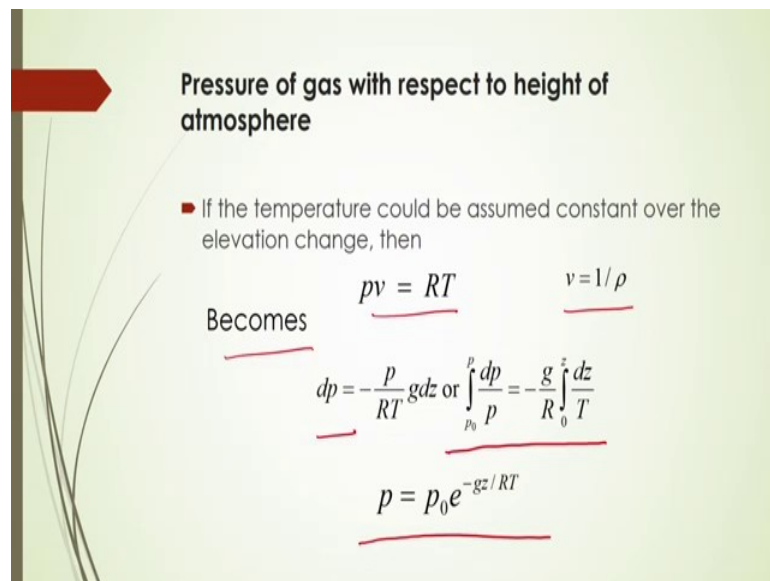
Again, from equation number 23 you can express this dT by dz at what would the temperature gradient this will be is equal to simply minus n minus 1 by n into g by R ; that means, here how temperature will be decreasing whenever you are go up up to a certain height from the atmosphere. The temperature decreases by 0.65 degree centigrade every 100 meter of height in the troposphere up to 1 -kilometre-high and n is equal to 1.235 .

So, in aeronautics it has been agreed to make the combined values of that is p_0 is equal to 101.325 kPa kilo per scale and T_0 is equal to 288.15 Kelvin and ρ_0 ; that means, the density at that sea level that will be $1.225 \text{ kg per meter cube}$, the standard atmospheric condition at the sea level then temperature will be decreased by 0.65 degree centigrade

every 100 meter of high in the trostosphere troposphere up to 1 kilo meter where n should be considered as 1.235.

So, these are the formula by which you will be able to calculate easily what will be the temperature there at a certain height, what will be the pressure at a certain height, what will be the density of the of fluid at a certain height from the sea level.

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Pressure of gas with respect to height of atmosphere

- If the temperature could be assumed constant over the elevation change, then

Becomes $p v = RT$ $v = 1/\rho$

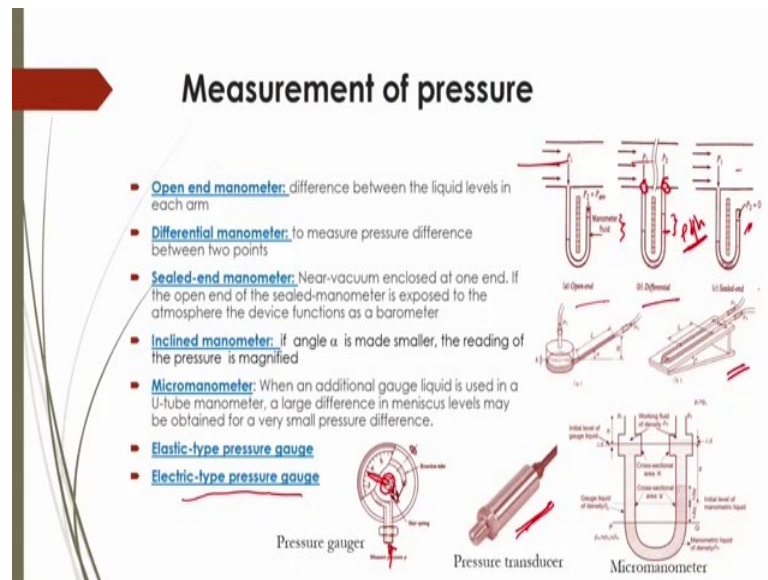
$$dp = -\frac{p}{RT} g dz \text{ or } \int_{p_0}^p \frac{dp}{p} = -\frac{g}{R} \int_0^z \frac{dz}{T}$$

$$p = p_0 e^{-gz/RT}$$

Again, pressure of gas with respect to height of atmosphere you can say that if the temperature could be assumed constant over the elevation change then we can say pv is equal to RT. So, in that case v is equal to 1 by rho that will becomes that dp will be equals to this one becomes dp be equals to minus p by RT gdz or if you integrate this from p 0 to p we can have the relationship of p is equal to p 0 into e to the power minus gz by RT.

So, in this case we are considering that ideal fluid, so and that if the temperature could be assumed constant over the elevation change.

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Again, how to measure the pressure whenever liquid is exerting on the fluid any continuous surface or any solid surface by its liquid column or fluid column then how actually you have to measure this pressure? Then there are various instruments or devices are being used to measure this pressure. Like manometer and also sometimes micro manometer and some pressure gauge and also different type of other electronic devices by which you can measure the pressure there.

In this case, manometer is one important of equipment which is laboratory which is in used in laboratory and you can easily measure what should be the hydrostatic pressure there. So, open end manometer is one type and there are other types like differential manometer, sealed-end manometer here in this diagram it is shown. This is open end, this is differential manometer and this is sealed manometer. In the open-end manometer, the difference between the fluid liquid level in each arm should be there. So, from which you will be able to calculate what should be the pressure exerted. Here in this case the difference so, from this difference will be able to calculate what will the pressure by ρg into h .

And, again here this differential manometer to measure the pressure difference between two points here suppose here one points and another here another point so, what would be the pressure difference between these two points you can calculate by using this manometer. In the manometer generally the mercury or carbon tetrachloride a fluid are

being used. So, based on that difference of this arm what is the difference of this liquid column or that is or mercury column or if you are using carbon tetrachloride and carbon tetrachloride column based on that you will be able to calculate ρg into h . So, ρ here it, will be what will be the fluid? If it is mercury, then you have to consider this density of the mercury. If it is suppose carbon tetrachloride you have to consider the density of the carbon tetrachloride here. So, by this you will be able to calculate what would be the pressure given by this through it whenever it will be flowing through pipe like this.

And, here it will be the sealed-end manometer in this case here it will be near vacuum that could be enclosed at one end and if the open end of the sealed manometer is exposed to the atmosphere the device functions as a barometer. Whereas, inclined manometer is also one important manometer by which you can measure the pressure. It is generally being used to modify the pressure reading that generally if at a certain angle the manometer is being used that the pressure the reading of that pressure will be magnified. So, this is the advantage of using this inclined manometer.

And micro meter also sometimes being used when an additional gauge liquid is used in a U-tube manometer, a large difference in meniscus levels may be obtained for a very small pressure difference. So, for a very small pressure difference sometimes it is very difficult to observe the liquid height change in the U-tube manometer. So, in that case the micro manometer is being used in that case the column; that means, here U-tube that is here tube size sometimes it will be very small. So, in that case for small pressure differences you can have a more that is differences of the height of the fluid layer and then you can calculate the additional against liquid pressure in that case.

And, other type of pressure gauge it is called elastic type pressure gauge; that means, here this elastic one we are this by this elastic behaviour of this that is here it is called one leg. So, by here spring the spring force it will be just moving if any pressure is exerted on it. And, another is electric type pressure gauge sometimes it is called pressure transducer. So, this is electrical that by what is that some voltage change when a bar voltage change whenever in the pressure exerted in the column. So, from this voltage change you will be able to find out how the pressure is by what is the pressure inside the column.

there is A point here like this A and here point 1, here point 2 and point 3 and what is that here point 4, point 4 and then B.

So, based on these first of all what you have to do you have to consider these two points A and 1. So, what should the $P_A - P_3$ sorry if you are not considering one directly if we consider that 3 then you can have $P_A - P_3$. You can divide also $P_A - P_1$ then $P_1 - P_2$ then $P_2 - P_3$. Since in this column only water is there so, we can directly take this $P_1 - P_A - P_3$ then $P_A - P_3$ based on this principle ΔP that will be equals to $-\rho g \Delta z$; that means, here we can say $P_1 - P_2$ that will be is equal to $-\rho g (z_1 - z_2)$.

So, this is your equation general equation. So, from this equation you can say if it is point 1 and 2 instead of 1 and 2 it will be coming A and 3, A and 3. So, here $P_A - P_3$ will be equals to minus 1000 that is ρ this since it is water in minus 1000 into 9.81 into what is the length here $z_1 - z_2$, if we consider that datum line is at the bottom of this U-tube manometer then it will be b like here the length is given here up to this is b. So, it would be $b + 0.1524 + A + 1.52$ into sine 30 degree since it is inclined at 30 degree and minus B.

Similarly, $P_3 - P_4$ based on this what will be the value? $P_4 - P_B$ what will be that $P_4 - P_B$ here it will it is $P_4 - P_b$. So, after I think if we add this if we add this we can have $P_A - P_B$; we can have $P_A - P_B$. So, $P_A - P_B$ will be equals to what $P_A - P_B$ will be equals to after summation of all those things and substituting the value of density of respective mercury and water then you can get this 11381.95 Newton per meter square. So, this is your final solution.

Then what will be the pressure difference between A and B? So, you can easily calculate. So, this is the principle general principle any in any I think by any U-tube manometer you can calculate the pressure difference by this principle.

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Try yourself:

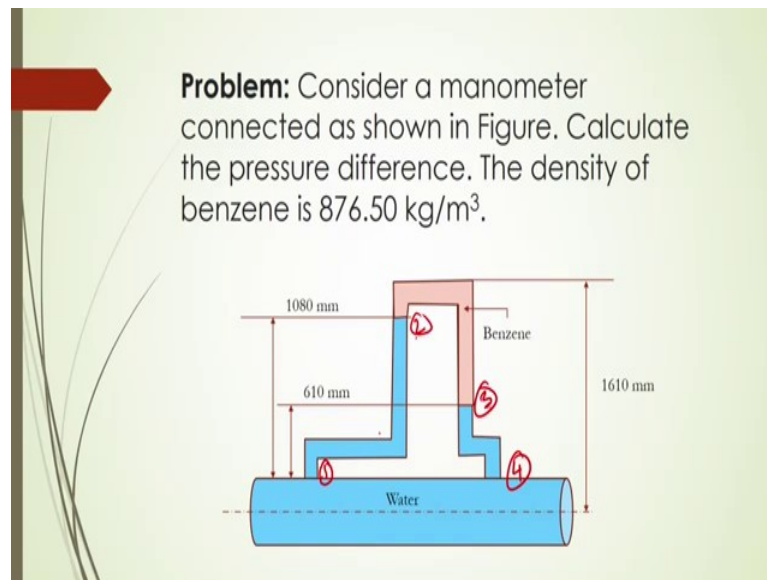
Find out: $p_{\text{water}} - p_{\text{oil}}$ as per Figure shown below
 Where specific gravity of oil is 0.86 (Ans. 10780 Pa)

You can try also this way like find out the pressure difference of water and oil at this point 1 and 6 here. So, what will be the P P what P 1 minus P I think 6 here. So, what will be the value here you can calculate.

So, similarly first of all you have to consider P 1 minus P 2, then it will be from this I think if you consider that this bottom line the what is that datum line. So, you just calculate here the what is that P 1 minus first of all P 1 minus here P 1 minus P 2 and then P 2 minus P 3, then P 3 then P 3 minus P 4 and then P 4 minus P 5 and then P 5 minus P 6, what will be the value? So, in this case if you add this finally, it will come P 1 minus P 6 is equal to what.

So, right hand side as per that rho into here z 1 minus z 2 into z or rho g into z 1 minus z 2. Similarly, here rho g rho here it will be water and it will be oil or mercury oil this is oil and into z into here it will be z 2 minus z 2 minus z 3 and this component will be 0 because z 2 and z 3 are same distance from the bottom. And P 3 minus P 4 it will be again P 3 minus P begin rho oil. So, rho oil into z into again here P z 3 minus z 4 similarly like this ok. Similarly, like this then finally, P 1 minus P 6 and after substitution of this you can get what should be the final value of this 107 10780 Pascal.

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Now, another example; suppose there are more than one water is used more than one liquid is used for I think for U-tube manometer. So, they are how to calculate that pressure difference, that is the same way you have to consider you have to just mark it out what should be the here this is one like here this will be 2, then 3, then here 4 what will be the pressure difference between P 1 and 4.

So, here P 1 first you have to consider this P 1 minus P 2 and then P 2 minus P 3 and then P 3 minus P 4 and after that if you sum it up you will get that P 1 minus P 4. And respective whenever you are considering that two point in that case in that length what is the liquid is then you have to consider the density of that particular liquid.

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Water pressure acting on a bank or a sluice gate

- the total force due to the water pressure acting on a bank built at an angle to the water surface
- The total pressure dP acting on a minute area dA .
- When the centroid of A is G , its y coordinate is y_G and the depth to G is h_G
- The total force F_t equals the product of the pressure at the centroid G and the underwater area of the bank wall.



Force acting on dam

$$dF = \rho g h dA = \rho g y \sin \theta$$

$$F_t = \int_A dF = \rho g \sin \theta \int_A y dA$$

$$F_t = \rho g \sin \theta y_G A = \rho g h_G A$$

Another important characteristics of the fluid is that is the force acting whenever there will be a dam or sluice gate is there. So, if what will be the water pressure whenever it will acting on a bank or a sluice gate. So, the total force in this diagram you will see this is one dam here this is the water body and this is the gate and see how force is applied here and we can see that you can see the total force due to the water pressure acting on a bank this is bank and then acting on a bank that is built at an angle to the water surface that is at theta.

The total pressure dP here if you consider P here total pressure dP acting on a minute area dA here if we consider that the total pressure acting on a minute area dA here dA minute area considered here so, what will be the pressure there? when the centroid of A is G centroid of A is G , its y coordinate is y_G and what is that and the depth to that is center of gravity is h_G . Then the total force F_t equals the product of the pressure at the centroid G and the underwater area of the bank wall.

So, this is the principle by which you can calculate the total pressure acting on the total area of the bank by this fluid. So, dF will be is equals to then $\rho g h$ into dA . This is the principle what are all be the pressure here then dF by dA that we $\rho g h$ at a certain height h here.

So, in is equal to ρg into $y \sin \theta$ as per θ you can calculate what will be the h value here. And, also this F_t will be is equal to then total dF then ρg into $\sin \theta$

into; what is that, y is nothing, but total cross sectional area will be is equal to in the y-direction it will be is equal to y into dA then total F t with your rho g sin theta y G into a that will be rho g into h G into A.

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The centre of pressure

The action point of P (i.e. the centre of pressure C) at which a single force P produces a moment equal to the total sum of the moments around the turning axis (x axis) of the sluice gate produced by the total water pressure acting on all points of the gate

$$y_c = y_G + \frac{I_G}{Ay_G} = y_G + \frac{h^2}{12y_G}$$

$$F_t y_c = \rho g I_x$$

I_x = the geometrical moment of inertia
 $I_x = I_G + Ay_G^2$
 I_G is the geometrical moment of inertia of area for the axis which is parallel to the x axis and passes through the centroid G

$I_G = \frac{1}{12}bh^3$ $I_G = \frac{\pi}{64}d^4$

Then, where is the actually centre of pressure? The centre of pressure it will be considered it will be denoted by y_c that is centre exactly in which location this pressure will be exerting. The action point of P that is pressure that is called centre of pressure C at which a single force P produces a moment equal to the total sum of the moments around the turning axis that is at x axis here in this case of the sluice gate produced by the total water pressure acting on all points of the gate here.

So, this y_c can be calculated from this y_G plus I_G by Ay_G that will be is equal to y_G plus h^2 by $12y_G$. Why? This is I_G is nothing, but if there is a rectangular shape then it would be I_G will be equals to $\frac{1}{12}bh^3$ and I_G will be equals to $\frac{\pi}{64}d^4$. So, in this case y_c if you substitute for this rectangular shape gate then you can calculate the y_c centre of the force.

So, in this case I_x would be equal to I_G plus A into y_G square where I_x is the where I_x is the geometrical moment of the inertia of the area for the axis which is parallel to the x axis and passes through the centroid G. And this I_x this is I_G , sorry. I_G is the I_G is the geometrical moment of inertia of area for the axis which is parallel to the x axis and

passes through the centroid G. So, from this force balance you would be able to calculate what will be the centre of the pressure acting by this fluid on the surface of the gate.

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What is the force to tear a cylinder?

- T is the force acting per unit length of wall which tears this cylinder longitudinally
- If the tensile stress due to T is lower than the allowable stress, safety is assured.
- By utilising this principle, a thin-walled pressure tank can be designed

$$2Tl = pdl$$

$$T = pd/2$$

Now, what is the force to tear a cylinder. This is also important pressure characteristics, that is exerted by the liquid ah. In that case a T if we consider that force acting per unit length of wall which tears this cylinder longitudinally. The if the tensile stress due to T is lower than the allowable stress that safety is assured there and by utilizing this principle a thin walled pressure tank can be designed here.

So, in this case if you consider that this force T is applying in this direction and pressure is acting in this direction then at which of force that you can tear this cylinder. So, in this case if we consider this T Te then 2 into T into l; l is the length of this pipe, this is then you can get 2Tl that would be equals to p into dl; p is the pressure and d into l is nothing, but the what is that area.

So, P into area; this is force. This force these will be balanced this force will be balanced this is the surface area, dl is the surface area, the surface area on which this pressure is acting to total surface is d into l. So, total force is this whereas, in this direction force is acting to tear this cylinder is T. So, T will be acting on this surface this surface is nothing, but here are 2 into l. So, here in this 2T into l; 2 means here T is the force per unit length then this in the two surface since this 2 over this length this T is acting. So, in the both sides it is acting. So, that is the 2 into Tl so, then T will be equal to pd a by 2.

So, by this equation you will be able to calculate what should be the force to be acting. So, that at that you will not be able to tilt this cylinder. So, the pressure should be; pressure should be p into d by 2 or force should be p into d by 2 and above this t you have then you will have that the cylinder is going to tear.

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Problem

- Consider the gate in Fig. below to be a quarter circle of radius 80 cm with the hinge 8 m below the water surface. If the gate is 1 m wide, what force P is needed to hold the gate in the position shown?
- Solution

$$F_t = \rho g \sin \theta y_G A = \rho g h_G A$$

$$= 1000 \times 9.81 \times (8-0.4) \times (0.8 \times 1) = 93200 \text{ N}$$

Another problem that if suppose we consider the gate in figure shown here in this case one gate like this below to be a quarter circle of radius 80 centimetre with the hinge 8 meter below the water surface. If the gate is 1-meter-wide; gate is 1-meter-wide what force P is needed to hold the gate in the position shown?

So, in this case total force will be is equal to $\rho g \sin \theta y_G A$. So, ρg into $h_G A$ as per formula discussed earlier. So, it will be total this force.

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■ **Example:** A 60-cm gate (square shape) has its top edge 12 m below the water surface. It is on 45° angle and its bottom edge is hinged as shown in Fig. What momentum force per unit length is needed to just open the gate?

The total force F is calculated to be

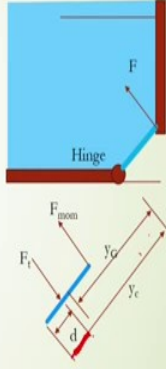
$$F_t = \rho g h_G A = 9810(12 + 0.3 \sin 45^\circ)(0.6 \times 0.6)$$

$$= 43130 \text{ N}$$

$$y_G = \frac{h_G}{\sin 45^\circ} = \frac{12 + 0.3 \sin 45^\circ}{\sin 45^\circ} = 17.27 \text{ m}$$

$$y_c = y_G + \frac{I_G}{A y_G} = 17.27 + \frac{0.6 \times 0.6^3 / 12}{(0.6 \times 0.6)(17.27)} = 17.272 \text{ m}$$

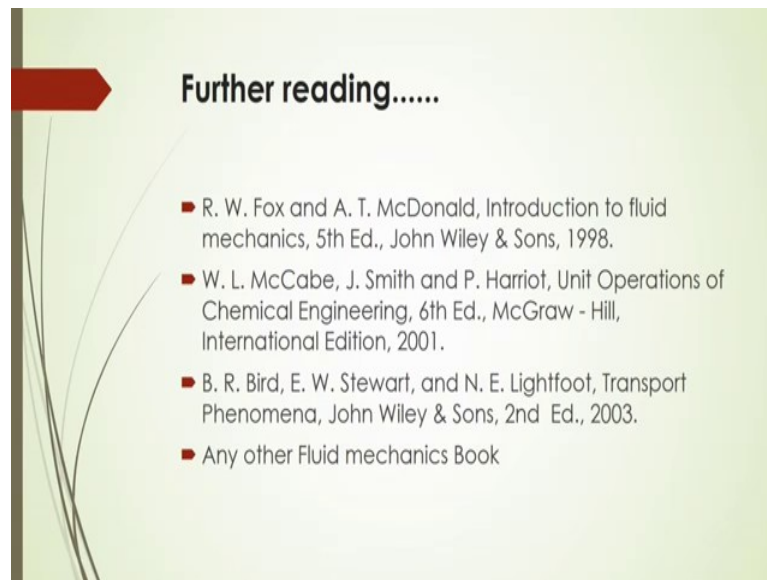
$$d = y_G + 0.3 - y_c \approx 0.3 \text{ m}$$

$$F_{mom} = \frac{F_t d}{L} = \frac{43130(0.3)}{0.6} = 21940 \text{ N}$$


Another example if suppose a 60 centimetre gate, that is square shape, has its top edge 12 meter below; this is 12 meter below, the water surface it is on 45 degree angle. It is on 45 degree angle and its bottom edge is hinged as shown in the figure what momentum force per unit length is needed to just open the gate here? So, again you have to calculate out of the total force acting here and what will be the y_G ? What is the y_G here, like this y_G ?

So, y_G will be equal to 17.27. What is the y_c ? y_c is a function of y_G then y_G plus I_G by $A y_G$. So, it will be coming like this and it will be 17.272 and then d what is the gap between this here from this y_c and this is y_c and what is the total length is like this. If you subtract this total y_c from this total length, then you will be able to calculate the d value here this gap of this at which this force is acting on the gate. So, here d will be is equal to then y_G plus 0.3 minus y_c it would be 0.3 meter and in that case momentum will be is equal to F_t into d by l . So, it will be coming 21940 Newton.

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And another important aspect that we can also calculate different aspect of that fluid characteristics of fluid motion. If suppose any fluid is moving with a certain acceleration, then what should be the pressure exerting on the surface by this fluid and when suppose any fluid is moving upward or in the downward direction or if it is flowing or if it is moving in a container at an acceleration which is equal to the gravity then whether is there any pressure will be exerted or not that will be discussed in the next lecture.

And also other if there is an horizontal oscillation uniform acceleration is there on the fluid, then how fluid surface will be just tilted from its original position or the original surface location and what will be the spill over of the liquid you can also calculate. So, that will be discussed in the next lecture that is module of fluid statics.

So, I think it will be helpful for you how actually pressure can be calculated. This is whenever there will be a certain liquid column at a depth then what will be the pressure and how the pressure force is exerted on the dam or bank of the river that you can calculate by this module of this lecture. So, I think I should also suggest for more reading about this fluid statics by this I think text book on or any other textbook you can follow or to read this content.

So, thank you [music].