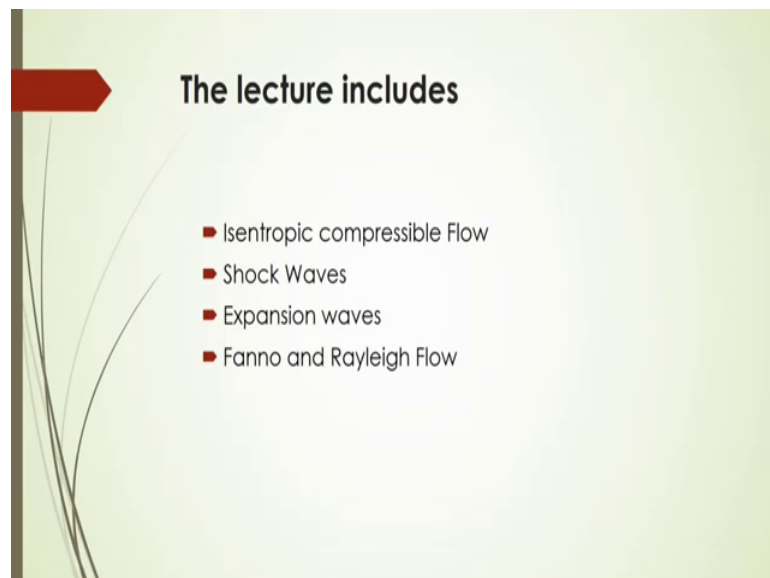


Fluid Flow Operations
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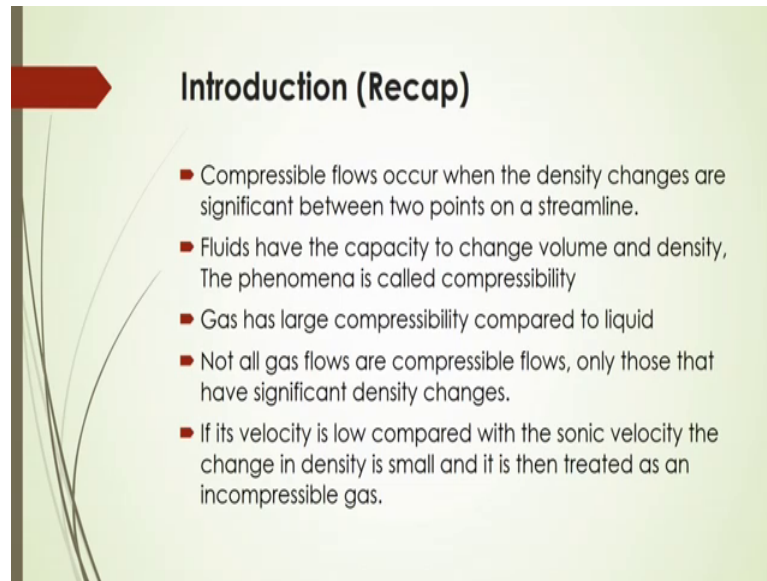
Module - 10
Lecture - 26
Compressible Flow: Part 2

Welcome to massive open online course on fluid flow operations. In this lecture we will continue the portion of compressible flow as a part 2. So, in this lecture we will discuss the isentropic compressible flow, shock waves, expansion waves and what is Fanno and Rayleigh flow there.

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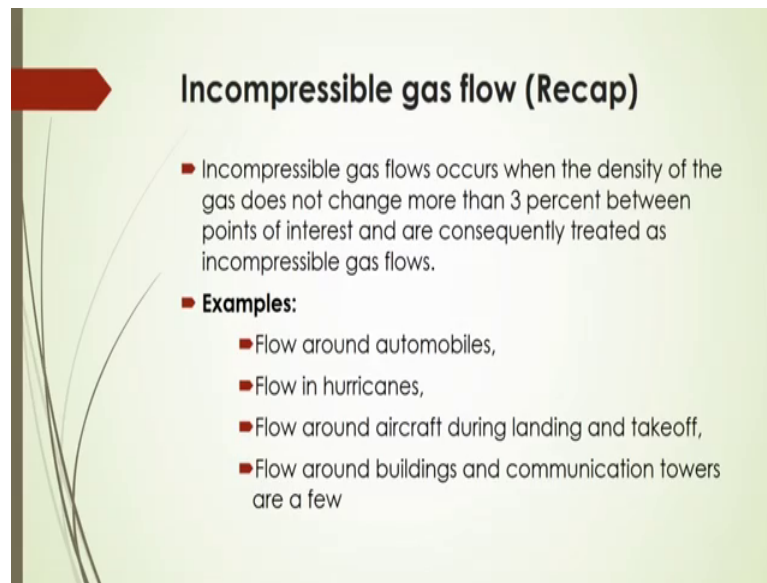
Introduction (Recap)

- Compressible flows occur when the density changes are significant between two points on a streamline.
- Fluids have the capacity to change volume and density, The phenomena is called compressibility
- Gas has large compressibility compared to liquid
- Not all gas flows are compressible flows, only those that have significant density changes.
- If its velocity is low compared with the sonic velocity the change in density is small and it is then treated as an incompressible gas.

So, as we have already discussed the definition of a compressible flows which occurs when the density changes are significant between two points on a stream line. Fluids have the capacity to change volume and density the phenomena is called compressibility and also we have discussed that the gas has large compressibility compared to liquid as per definition.

Not all the gas should be compressible there should be some criteria to be its compressibility; only those that have significant density changes. And if its velocity is low compared to the sonic velocity the change in density will be small and it is then treated as an incompressible gas.

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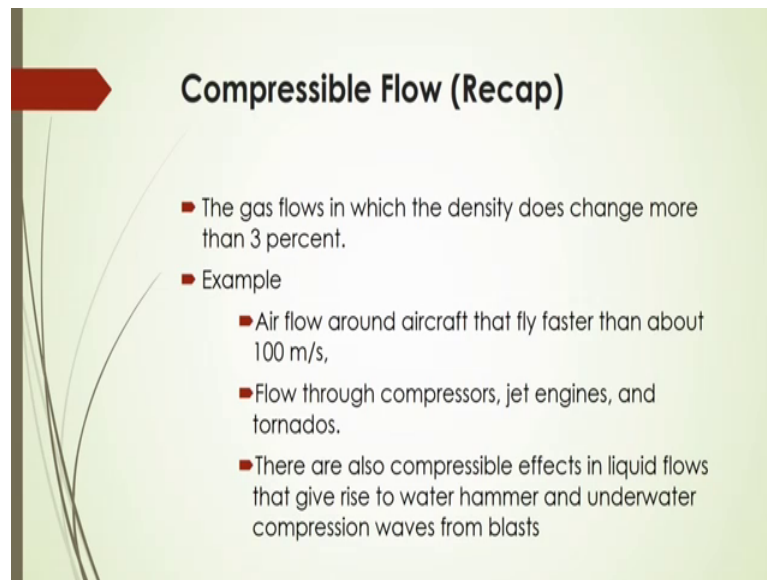
Incompressible gas flow (Recap)

- Incompressible gas flows occurs when the density of the gas does not change more than 3 percent between points of interest and are consequently treated as incompressible gas flows.
- **Examples:**
 - Flow around automobiles,
 - Flow in hurricanes,
 - Flow around aircraft during landing and takeoff,
 - Flow around buildings and communication towers are a few

Now, what are the conditions for the gas to be incompressible? In this case we have discussed that the previous lecture that incompressible gas flows occurs when the density of the gas does not change more than 3 percent between points of interest and are consequently is treated as a incompressible gas flows

In this case some example it is given that flow around automobiles, flow in a hurricanes, flow around aircraft during landing and takeoff and also flow around buildings and communication towers; in that case the gas is considered to be a incompressible. In that case there will be a certain change of density of the gas which will be less than 3 percent between two points of interest

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Compressible Flow (Recap)

- The gas flows in which the density does change more than 3 percent.
- Example
 - Air flow around aircraft that fly faster than about 100 m/s,
 - Flow through compressors, jet engines, and tornados.
 - There are also compressible effects in liquid flows that give rise to water hammer and underwater compression waves from blasts

So, we can then say that if the density change is more than 3 percent then it will be only compressible, otherwise if it is less than 3 percent it will be called as incompressible gas.

So, the gas flows in which the density does not change more than 3 percent; it will be considered as that is incompressible whereas, if its change is more than 3 percent then it will be compressible. Like air flow around aircraft that fly faster about 100 meter per second in that case this flow will be called as compressible flow; when flow through compressors jet engines and tornadoes; those cases the flows will be considered as compressible.

And there are also compressible effects in liquid flows in that case like water hammer and underwater compression waves from the blasts are considered to be a compressible liquid.

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Compressibility (Recap)

- Bulk modulus $K = \frac{\Delta p}{\Delta V/V} = -V \frac{dp}{dV}$ (1)
- Compressibility is defined by $\beta = \frac{1}{K}$ (2)

For water of normal temp./press., $K = 2.06 \times 10^9 \text{ Pa}$
For air of normal temp./press., $K = 1.40 \times 10^5 \text{ Pa}$
In the case of water, $\beta = 4.85 \times 10^{-10} \text{ 1/Pa}$,
Shrinks by 0.005% if press. increased 1 atm.

Fluid of volume V at pressure p
By increase Δp , decrease v by ΔV

Now, we have already given the definition of the compressibility; it depends on bulk modulus which is defined as that is the Δp by ΔV by V ; that means, here ratio of the change of pressure drop per unit volume change of a system. So, in that case this bulk modulus is defined as given in equation number 1 and then compressibility is defined by that is inverse of this bulk modulus, this will be here beta and it will be 1 by K and this is given in equation number 2.

And we know that what would be the compressibility for water air and in case of water shrinks by 0.005 percent if pressure increased to one atmospheres; in that case what should be the beta and also what should be the compressibility that is bulk modulus that is given here. And we have already given in the previous lectures also, so you have to remember this values for your further consideration and calculation also.

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Since, $\rho V = M = \text{constant}$,

$$K = \rho \frac{\Delta p}{\Delta \rho} = \rho \frac{dp}{d\rho} \quad (3)$$

■ The bulk modulus K is closely related to the **velocity c of a pressure wave propagating in a fluid**, which is given by the following equation

$$c = \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{K}{\rho}} \quad (4)$$

And also if you substitute this gas equation here ρb is equal to m then in that case K should be is equal to in terms of that is density change then it will be ρ into $d p$ by $d \rho$ that will be again that bulk modulus in terms of this density change how it will be happen.

And also the bulk modulus K is closely related to the velocity c of a pressure wave that is propagating in a fluid and this is given by the following equation; that is a c will be is equal root over of dp by $d \rho$ that will be is equal to root over K by ρ . So, if you know the bulk modulus of the fluid and density then you can easily calculate what should be the sonic velocity or what will be the velocity of a pressure wave that propagating in the fluid, you can calculate by this equation number 4.

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Isentropic Flow: Flow in a pipe-Effect of sectional change

- Assume the flow in a pipe with a gradual sectional change as shown in Figure
- Continuity Equation: $\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0$ (5)
- Momentum Equation: $-dpA = (\rho u) du$ (6)
- Isentropic relation: $p = m\rho^k$ (7)
- Sonic velocity: $c = \sqrt{dp/d\rho}$ (8)

Let us consider here that isentropic flow here isentropic flow; so flow in a pipe that effect of sectional change. Now let us assume the flow in a pipe of a gradual sectional change as shown in here. In this case what should be the continuity equation? That continuity equation it is given as that $d\rho/\rho + du/u + dA/A$ that should be is equal to 0 and momentum equation for this compressible flow it will be minus dp into A ; that will be A into ρu into du .

And then what should be the isentropic relation for the compressible gas? This is your isentropic relation that is p is equal to m into ρ to the power k , here m is constant and k ; k is called isentropic index and which is defined as the ratio of specific heat capacity of the fluid at constant pressure and constant volume respectively. And sonic velocity will be in terms of pressure change based on the density change.

So, in that case this sonic velocity is defined as c is equal to root over dp by $d\rho$. So, this five equation number 5, 6, 7, 8 will give you the respective continuity equation, momentum equation, isentropic relation and sonic velocity from which you will be able to calculate what will the flow characteristics even how pressure is developed and how temperature changing based on the sonic velocity and also what could be the equations in terms of MAC number that you can easily calculate.

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From Eqns (5), (6) and (8),

$$-c^2 d\rho = \rho u du = \rho u^2 \frac{du}{u} \quad (9)$$

$$M^2 \frac{du}{u} = -\frac{d\rho}{\rho} = \frac{du}{u} + \frac{dA}{A} \quad (10)$$

$$\therefore \frac{du}{dA} = \frac{1}{M^2 - 1} \frac{u}{A} \quad (11)$$

As per eqn (11),

- > If $M < 1$, $du/dA < 0$: the flow velocity decreases with increased sectional area,
- > If $M > 1$, $-d\rho/\rho > du/u$: the density decreases at a faster rate than the velocity increases for supersonic flow.

Consequently, for mass continuity, in order to increase the flow velocity the section area should increase rather than decrease, as for subsonic flow.

Now, from equation number 5, 6 and 8 after simplification you can write here, this minus c square into d rho that will be is equal to rho u d u is equal to rho u square into d u by u.

And then we can write here M square into du by u that will be is equal to minus d rho by rho and is equal to d u by u that will be plus dA by A and which implies that d u by dA that will be is equal to 1 by M square minus 1 into u by A; here m is called MAC number; MAC number is defined as what is that? We have already given earlier that will be is equal to u by c that is flow velocity by the sound velocity. So, in that way if we simplify after substitution of those and then we can get this simplified form of equation for the velocity change with respect to cross sectional area in terms of MAC number here.

Now, as for equation 11; we can say that if MAC number is less than 1; what does it mean that d u by d A should be less than 0? So, in that case we can say that the flow velocity decreases with increased sectional area. Again if MAC number is greater than 1 then you can say minus d rho by rho is greater than d u by d u as in given equation number 10. Then in that case the density decreases at a faster rate; then the velocity increases for the supersonic flow. That means, this supersonic which means m is greater than 1; that is why it is called the supersonic if it m less than 1 that it would be subsonic flow.

So, consequently for the mass continuity in order to increase the flow velocity; the

section area should increase the rather than decrease as for subsonic flow.

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Subsonic flow and supersonic flow in one-dimensional isentropic flow

Changing item	Flow state			
	Subsonic		Supersonic	
	Convergent	Divergent	Convergent	Divergent
Changing area	-	+	-	+
Changing velocity/ Mac number	+	-	-	+
Changing density/pressure/ temperature	-	+	+	-

Now, here some characteristics feature for the subsonic flow and supersonic flow in one dimensional isentropic flow. If we change the item like this changing area changing velocity or MAC number and changing density pressure or temperature; then what should be the flow state?.

Whether it would be subsonic or supersonic in a convergent divergent or convergent divergent sectional area? So, in that case if we say that if there is a change of changing area. So, in that case flow state should be negative for convergent 5 for the subsonic flow whereas, it should be positive in case of divergent flow for the subsonic flow. But for supersonic flow convergent to be again negative and divergent should be positive.

So, changing area will give you only divergent subsonic and divergent supersonic flow. And if you change the velocity or MAC number and keeping other constants in that case you will see that it will give you subsonic convergent flow and also it will give you the divergent supersonic flow.

But in that case subsonic divergent and supersonic convergent flow will not happen. And again if we change the density, pressure or temperature you can say that there will be existence of the subsonic divergent and supersonic convergent flow only. So, in that case there will be no flow state of subsonic convergent and supersonic divergent flow there.

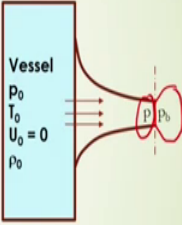
So, from this table you can assess in which cases of changing of these items you can expect that subsonic and supersonic flow state for the convergent and divergence section of a 5.

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Isentropic Flow: Case: Convergent nozzle

- Assume a gas of pressure p_0 , density ρ_0 and temperature T_0 flows from a large vessel through a convergent nozzle into the open air of back pressure p_b isentropically at velocity u , as shown in Figure.
- Putting p as the outer plane pressure, from eq. (38) (Lecture 25: Compressible flow part 1)

$$\frac{k}{k-1} \frac{p}{\rho} + \frac{1}{2} u^2 = \frac{k}{k-1} \frac{p_0}{\rho_0} \quad \checkmark \quad (12)$$



Now, let us discuss the isentropic flow in case of convergent nozzle. So, in that case if you assume a gas of pressure p_0 , a density ρ_0 and temperature T_0 flows from a large vessel here as shown here figure through a convergent nozzle into the open air of back pressure here created as p_b ; isentropically at velocity u . Then we can say that by substituting p as the outer plane pressure from equation number 38; even given in earlier lectures that is in the lecture 25 that for compressible flow part 1.

So, in that case if you substitute this you can have this equations of conservation of energy equation here that will be is equal to k by k minus 1 into p by ρ plus half of u square that will be is equal to k by k minus 1 into p_0 by ρ_0 . So, this equation will give you the what is that what should be the pressure p at this sections if you know the velocity of the flow there and also if you know the characteristics pressure of the compressible fluid there.

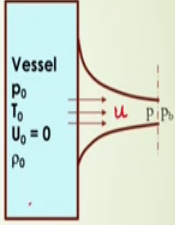
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Using $T = m\rho^{k-1} = mp^{(k-1)/k}$ [$m = \text{constant}$] (13)

Equation (12) becomes

$$u = \sqrt{2 \frac{k}{k-1} \frac{p_0}{\rho_0} \left[1 - \left(\frac{p}{p_0} \right)^{(k-1)/k} \right]} \quad (14)$$

Therefore the flowrate is

$$\dot{m} = \rho u A = A \sqrt{2 \frac{k}{k-1} p_0 \rho_0 \left(\frac{p_0}{p} \right)^{2/k} \left[1 - \left(\frac{p}{p_0} \right)^{(k-1)/k} \right]} \quad (15)$$


Now, using this isentropic flow condition of T is equal to m into ρ to the power k minus 1 that will be is equal to m into p to the power k minus 1 by k ; where m is constant in that case equation twelve becomes. So, after simplification it will be u will be equals to root over 2 k by k minus 1 into p_0 by ρ_0 into one minus p by p_0 to the power k minus 1 by k .

So, in this case, how to calculate this u ? Once you know the pressure at the last vessel and also density of the fluid at that particular section at a particular temperature here at temperature T_0 here in this case t p_0 T_0 u_0 and p_0 and ρ_0 ; if you know these then you will be able to calculate what will be the velocity at this sections here you can easily calculate.

And therefore, based on this u you can calculate the; what should be the velocity what should be the flow rate of the compressible fluids. So, m that mass flow rate will be is equal to ρu into A ; so after substitution of u you can have this simplified form of equation here A into root over 2 k ; k minus 1 into what is that p_0 into ρ_0 into p_0 by ρ_0 or to the power 2 by k and here ρ_0 by k to the power 2 to the power 2 to the power two by k into 1 minus p by p_0 to the power k minus 1 by k . So, this equation number fifteen will give you the volumetric flow rate or mass flow rate of the compressible fluid whenever it will be passing through the convergent section.

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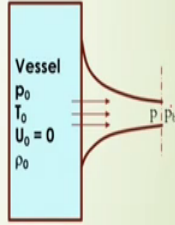
Critical pressure for maximum mass flowrate

$$m = \rho u A = A \sqrt{2 \frac{k}{k-1} p_0 \rho_0 \left(\frac{p}{p_0}\right)^{2/k} \left[1 - \left(\frac{p}{p_0}\right)^{(k-1)/k}\right]} \quad (12)$$

Differentiate the above equation (12) with p/p_0 , and equating zero for maximum mass flowrate we get

$$\frac{\partial m}{\partial (p/p_0)} = 0 \Rightarrow \frac{p}{p_0} = \left(\frac{2}{k+1}\right)^{k/(k-1)} \quad (13)$$

m is maximum for this p as in equation (13). The pressure is called critical pressure. For air flow, $p_{\text{critical}}/p_0 = 0.528$

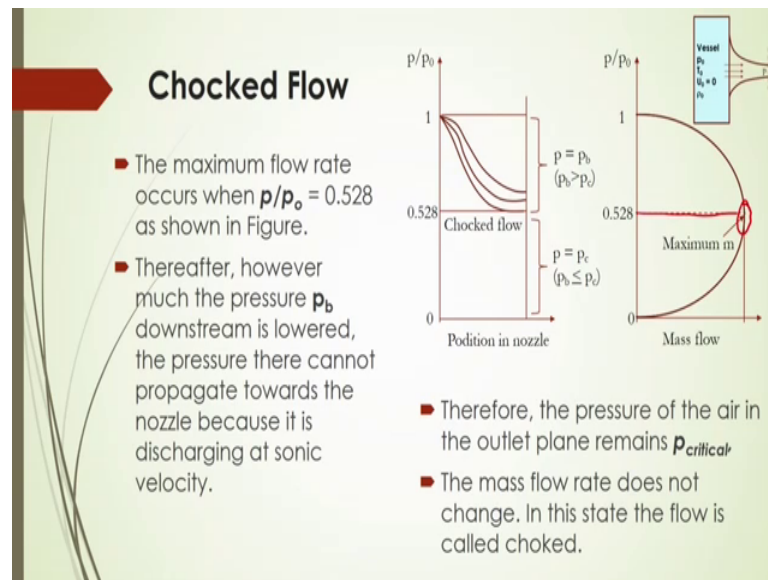


Now, what should be the critical pressure for the maximum mass flow rate? Here the mass flow rate is given in equation number 12. So, after differentiation of this equation twelve with p by p_0 p by p_0 in this case and equating 0 for maximum mass flow rate we get this here equation number 13.

So, we can get this p by p_0 will be is equal to what? 2 by k plus 1 to the power k by k minus 1 ; that will be your maximum; that means, at this condition then we have maximum mass flow rate. And at that maximum mass flow rate you can expect the pressure should be able to p ; that will be is equal to p_0 into 2 by k plus 1 to the power k by k minus 1 .

So, in this case m is the maximum for this pressure p as in equation number 13 and the pressure is called critical pressure at this particular condition. And for airflow this critical pressure to the total pressure at this vessel will be is equal to 0.528 .

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Now, if we consider that this maximum flow rate occurs when p by p_0 is equal to 0.528 as shown figure here. If we plot this p by p_0 with respect to the position in the nozzle of this convergence section here, then we can have at this value of p by p_0 is equal to 0.528; there will be a constant pressure; that means, the pressure which will be almost equals to the 52.8 percent of the total pressure of the vessel.

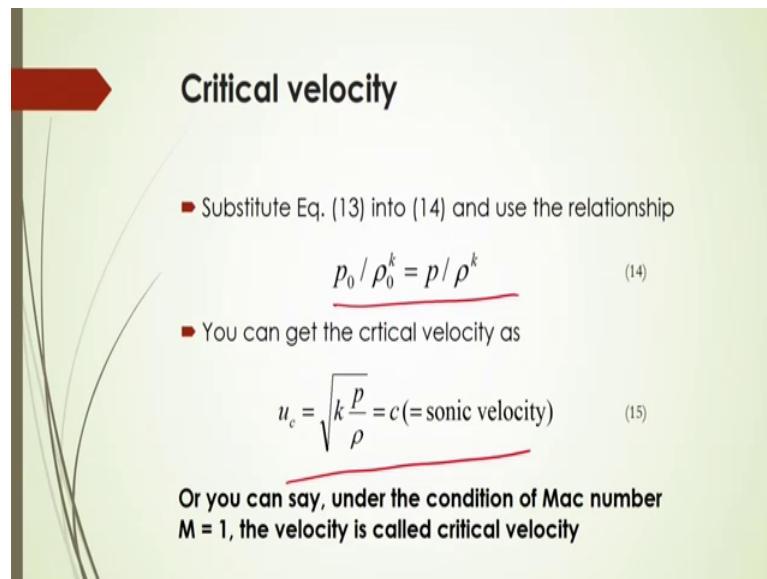
And at this condition you will see p should be is equal to p_b and this p_b should be is greater than p_c . Whereas, if are getting this p by p_0 what then 52 that is 0.528, then in the case in this case p should be is equal to critical pressure and then this what is that p_b should be less than equals to critical pressure here.

So, in this case we can say; however, mass of the pressure p_b ; downstream is lowered the pressure there cannot propagate flow rate the nozzle because it is discharging at that sonic velocity. Because there here sonic condition that is pressure and for this sonic velocity there you can say that the pressure cannot propagate towards the nozzle. Therefore, the pressure of the air in the outlet plane remains constant there and it would be equals to critical pressure. And the mass flow rate does not change in that case and it will be maximum there and in the in this state the flow is called choked flow.

So, here it will be maximum flow at this p by p_0 that will be is equal to 0.528 and at this condition of this p by p_0 is equal to 0.528, there you will see that outlet plane pressure will remains equal critical pressure. So, this condition is called choked condition and the

flow at this condition it is called choked flow.

(Refer Slide Time: 19:11)



Critical velocity

- Substitute Eq. (13) into (14) and use the relationship

$$p_0 / \rho_0^k = p / \rho^k \quad (14)$$

- You can get the critical velocity as

$$u_c = \sqrt{k \frac{p}{\rho}} = c (= \text{sonic velocity}) \quad (15)$$

Or you can say, under the condition of Mac number $M = 1$, the velocity is called critical velocity

Whereas substituting this equation number 13 that here given here at this condition p by p_0 is equal to 2 by $2k + 1$ to the power k by $k - 1$ in equation number 14 and use the relationship of this p_0 by ρ_0 to the power k is equal to p by ρ to the power k ; then you can get the critical velocity of u_c as root over k into p by ρ . So, that will be is equal sonic velocity.

So, in this case at choked condition the critical velocity should be is equal to root over k in root over k into p by ρ_0 which is equal to the sonic velocity. Or you can say that under the condition of MAC number is equal to 1 ; this is u_c if it is c then of course, that u by u_c that will be is equal to 1 . So, in this in this case that the MAC number will be is equal to 1 and under this condition the velocity will be called as critical velocity.

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Critical Density and Critical Temperature

- For the condition of $M = 1$, the critical density and critical temperature can be obtained at the critical velocity as

$$\frac{\rho_c}{\rho_0} = \left(\frac{2}{k+1} \right)^{1/(k-1)} = 0.634 \quad (16)$$
$$\frac{T_c}{T_0} = \frac{2}{k+1} = 0.833 \quad (17)$$

At the critical outlet state of $M = 1$, the critical pressure falls to 52.8% of the pressure in the vessel, while the critical density and the critical temperature respectively decrease by 37% and 17% from those of the vessel.

Now, again based on this critical velocity or this condition of this critical pressure at the condition of MAC number is equal to 1, you can calculate the; what should be the critical density there. So, the critical density and the critical temperature can be obtained at the critical velocity as by this equation ρ_c by ρ_0 that will be is equal to 2 by k plus 1 to the power 1 by k minus 1 from which you can calculate what is should be the ρ_c .

So, here it will be coming as 0.634 at this critical velocity and then from which you can say that critical density is equal to 63.4 percent of this density of the fluid. And again that critical temperature will be is equal to as per this; you can say that it will be 2 by k plus 1 that will be is equal to 83.3 percent of by initial temperature of the fluid there.

So, why in this case, but this will this 83 percent or 63 percent is these values are of course, will be changing based on the compressible fluid type. There in that case if specific speed capacity ratio will be changing; accordingly you can have these values here because k will change as for that fluid properties. So, at the critical outlet state of MAC number is equal to 1, you have to remember that the critical pressure falls to 52.8 percent of the pressure in the vessel. While the critical density and the critical temperature decrease by 37 and 17 percent from those of the vessel respectively.

So, this is the for the case for you can say that here convergent section convergent nozzle and what should be that convergent divergent nozzle.

(Refer Slide Time: 22:27)

Case: Convergent – Divergent nozzle (Called as de Laval nozzle)

- As shown in Figure, a convergent nozzle followed by a divergent length.
- When back pressure p_b outside the nozzle is reduced below p_0 , flow is established

If the fluid flows out through the throat section without reaching the critical pressure the general behaviour is the same as for incompressible fluid

And in that case sometimes this condition is called that the de Laval nozzle. So, as shown in figure in the slide a convergent nozzle followed by a divergent length. So, this geometry it is called as de Laval nozzle geometry. And when that back pressure p_b here outside the nozzle is reduced below p_0 flow will be established.

So, in that case if the fluid flows out through the throat section without reaching the critical pressure; the general behaviour is the same as for incompressible fluid.

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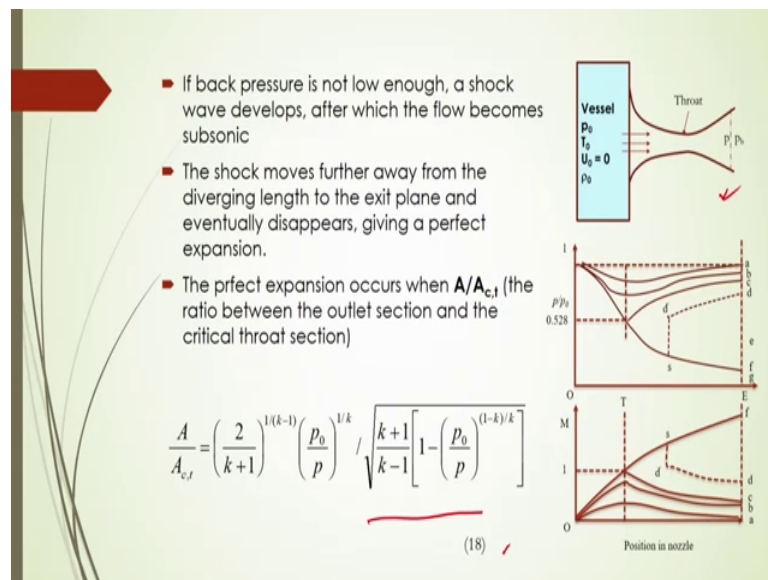
- When the back pressure decreases further, the pressure at the throat section reaches the critical pressure and $M = 1$ and supersonic flow starts to happen
- Unless the back pressure is low enough, supersonic velocity cannot be maintained

Now, when the back pressure decreases further; the pressure at the throat section reaches

the critical pressure. And in that case MAC number should be is equal to 1 and you can say that supersonic flow starts to happen at that particular condition.

And if you are not getting that the back pressure is low enough then of course, the supersonic velocity cannot be maintained at that particular condition. So, it is to be remembered that the back pressure if it is decreased further, then the pressure at the throat section will reach to the critical pressure at MAC number is equal to 1 and in that case supersonic flow you can expect.

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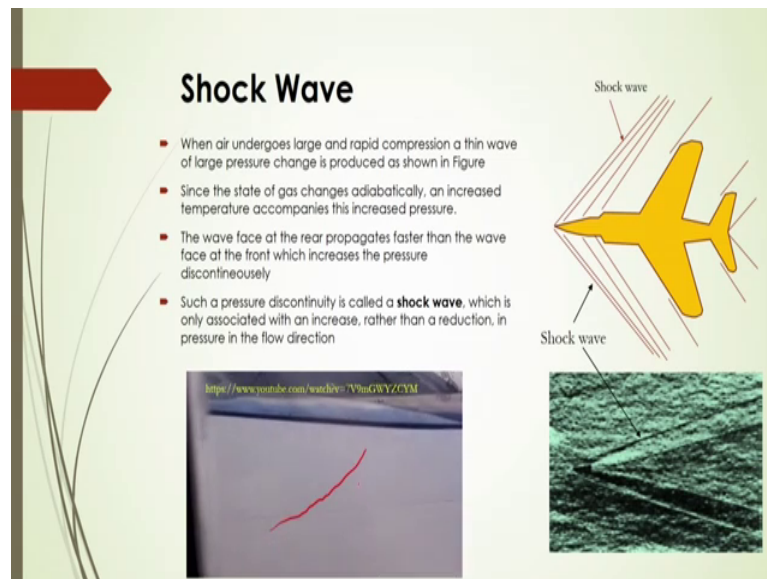
Also if back pressure is not low enough a shock wave develops in there and after which the flow becomes subsonic. And the shock waves further away from the diverging length to be exist and in that case the exit plane and eventuality disappears and giving a perfect expansion there as shown in figure here.

And the perfect expansion occurs when A by A c that is the ratio of this cross sectional area of the critical cross sectional area the ratio between; the outlet section and the critical throat section in that case you will have this ratio as shown in equation number 18 as 2 by k plus 1 to the power 1 by k minus 1 into p 0 by p divided by this portions of this k plus 1 by k minus 1 into 1 minus p 0 by p to the power 1 minus k by k whole to the power 1 by 2.

So, in that case the perfect expansion will happen only for those ratio of this cross

sections from a section a to critical cross section here at the throat.

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Now, what is the shock wave? We can say that if any compressible gas that undergoes large and rapid compression; you will see a thin wave of large pressure will be changed and it will be produced as shown in figure here. And since the state of gas changes adiabatically an increased temperature will happen in this particular cases of pressure increment. And in that case the wave pressure the near propagates faster than the wave face at the front, which increases the pressure discontinuously.

Now, if this pressure occurs discontinuously; you will see there will be a propagation in the rare part that will be faster than the wave. And also in the front side you will see that it will increase the and this phenomena will be called as shock wave phenomena. And in this case this shock wave phenomena will only be associated with an increase rather than the deduction in the pressure in the flow direction.

So, here in this you will see one video; you will see there as obtained from this source here see how the shockwave wave is form, whenever this flight is moving at a certain velocity that this is MAC number is greater than 1 and there we will observe that the there will be a certain line of this your absorption observation of this; that is the compressible compressibility of the gas at the wing side at that particular flow velocity.

Here this is the mark here obtained this mark is obtained here this white line this is

shown that this is called shockwave here. So, this phenomena is happened if this aero plane is moving at the velocity higher than the sonic velocity; that means, that the sound velocity. And this critical pressure will observed there and from that critical pressure that the what is that; the increment of this pressure will happen discontinuously and due that discontinuous pressure, you will get this type of wave formation at the front side of this wing there.

(Refer Slide Time: 27:51)

Normal Shock Wave

- Shock waves are of two types: **Normal** and **oblique** both with respect to the direction of flow
- If a long cylinder is partitioned with Cellophane film or aluminium foil to give a pressure difference between the two sections, and then the partition is ruptured, a shock wave develops.
- The shock wave in this case is at right angles to the flow, and is called a **normal shock wave**. The device itself is called a **shock tube**.

And then you will have these two types of shocks in this flow condition; one is called normal another is called oblique type of that is shockwave. And if you are having that long cylinder in a partitioned with the cellphone film or aluminium foil to give a pressure difference between the two sections and then the partition will be called as a shockwave. In that case the shockwave will be at the right angles to the flow and it will be called as normal shockwave.

So, whenever you will see interesting that the shockwave should be always at 90 degree angles to the flow of the fluid. Whereas, this normal what is that called oblique that is the shockwave; it will be developed only at the tangential direction of the flow or tangential direction of the shockwave there. So, it will be called as the oblique shockwave

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$A_1 = A_2$

P_1, ρ_1, T_1	P_2, ρ_2, T_2
$c_1, u_1, M_1 > 1$	$c_2, u_2, M_2 < 1$

Δx

Continuity	$\rho_1 u_1 A_1 = \rho_2 u_2 A_2$	(19)
Momentum	$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$	(20)
Energy Equation	$\frac{u_1^2}{2} + \frac{k}{k-1} \frac{p_1}{\rho_1} = \frac{u_2^2}{2} + \frac{k}{k-1} \frac{p_2}{\rho_2}$	(21)

Now, if we consider the continuity equation for shockwave; whether it is the normal or tangential you have to follow that continuity equation that we have already discussed earlier; that how to absorb the velocity change as per a pressure change there. And based on this continuity equation and for that you have to have the momentum equation and it will be represented by this equation number 20 and it is as $p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$ is equal to $p_2 + \rho_2 u_2^2$. And if you substitute the velocity at the sections one from this continuity equation then you will have the expression for this velocity u_2 based on this momentum equation.

Again if you have this energy equation it is called that Bernoulli's equation for the compressible fluid; then in that case you can write this equation as $\frac{u_1^2}{2} + \frac{k}{k-1} \frac{p_1}{\rho_1}$ that will be is equal to in the sections it will be $\frac{u_2^2}{2} + \frac{k}{k-1} \frac{p_2}{\rho_2}$. So, based on this equation number 2 again you can express what should be the pressure at the section 2 and what should be the velocity at the section 2 based on the pressure and the compressibility index or you can say sometimes the it is called that isentropic indexes.

So, based on that you can have the change of velocity pressure even what should be the density other variables if you are keeping constants there. So, at this sections A_1 and A_2 ; if you are keeping the sectional area is constant then only you can apply this equations and you can have the simplified form without this what is that the velocity u_1 and u_2 . In

that case the velocity will be almost equals to same because this cross sectional area does not change; if this cross sectional area change of course, accordingly you will have the velocity u_1 and u_2 .

But in this case if it is a same cross sectional area only u_1 should be considered here. So, in that case ρu into a it will be is equal to constant that will be your continuity equation. Whereas, momentum equation it will come only p plus ρu square that will be is equal to constant. And similarly energy equation it will come u square by 2 plus k by k minus 1 into p_1 by ρ_1 as per equation number 21; it will be is equal to constant.

(Refer Slide Time: 31:37)

From Eqns (19) and (20)

$$u_1^2 = \frac{p_2 - p_1}{\rho_2 - \rho_1} \frac{\rho_2}{\rho_1} \quad (22)$$

$$u_2^2 = \frac{p_2 - p_1}{\rho_2 - \rho_1} \frac{\rho_1}{\rho_2} \quad (23)$$

Substituting Eqns (22) and (23) into (21)

$$\frac{u_1}{u_2} = \frac{\rho_2}{\rho_1} = \frac{[(k+1)/(k-1)](p_2/p_1) + 1}{[(k+1)/(k-1)] + p_2/p_1} \quad (24)$$

Or

$$\frac{T_2}{T_1} = \frac{\rho_2}{\rho_1} = \frac{[(k+1)/(k-1)] + p_2/p_1}{[(k+1)/(k-1)] + p_1/p_2} \quad (25)$$

since $p = \rho RT$

Rankine-Hugoniot Equations

Show the relationships between the pressure, density and temperature ahead of and behind a shock wave.

Now, from these equations 19 and 20 here it is given and based on these equations; you can calculate what should be the velocity at the sections 1. So, u_1 square will be is equal to that is after simplification it will become 2 minus p_1 by p_1 by ρ_2 minus ρ_1 into ρ_2 by ρ_1 . And similarly u_2 will be again as per this equation number 23.

Now, after substitution of this equation number 22 and 23 again in equation number 21 here at this energy equation; then we can have after simplification this equation or you can express this equation by this by equation number 25; just after substitution of this equation of state that is p is equal to $\rho r t$. So, this is coming what will be the velocity ratio of this two sections or density ratio of this two sections. Once you know the pressure at two sections as well as the isentropic index for this compressible fluid.

So, in this case this equation number 25 it is sometimes referred as Rankine Hugoniot equations. And in this case you have to remember that this relationship between the pressure density and temperature ahead and behind of a shockwave. So, for this shockwave you have to calculate or how to calculate this temperature ratio and velocity ratio you can have it from this equation number 25 and 24 respectively.

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From Equations (19) and (20)

$$\frac{p_2}{p_1} = 1 + \frac{2k}{k+1}(M_1^2 - 1) \quad (26)$$

$$\frac{p_1}{p_2} = 1 + \frac{2k}{k+1}(M_2^2 - 1) \quad (27)$$

Therefore

$$M_2^2 = \frac{2 + (k-1)M_1^2}{2kM_1^2 - (k-1)} \quad (28)$$

A shock wave develops only when the upstream flow is supersonic

Similarly, from equation number 19 and 20 we can express this pressure ratio in terms of MAC number. So, here you see if we consider that MAC number at section 1 based on the velocity at section 1 with considering or with comparing with the sonic velocity; then you can get this MAC number at that section 1 as that is u_1 by c . So, based on this MAC number you can calculate this pressure at this section 2; if you know the pressure at section 1 and also if you know the MAC number at section 1; of course, the you have to provide the value for this compressibility index

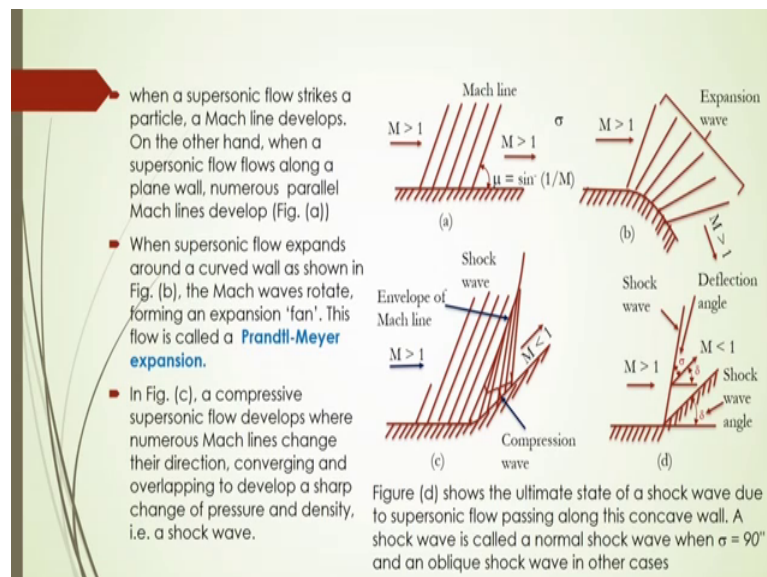
Similarly, this pressure issue also you can calculate based on the MAC number at section 2 also. So, in that case what should be the pressure at section 1 that you can calculate by knowing the pressure at section 2; by equation number 27. So, based on this equation number 26 and 27 if you can be simplify these two equations by just dividing, then you can have the simplified form of equations which relates the MAC number of 1 and 2; only in terms of that the isentropic indexes. So, this equation number 28 is the basic equations for MAC number or velocity change of that shockwave only based on the

isentropic indexes.

So, in this case you have to remember that a shockwave will develop only when the upstream flow is supersonic. So, upstream flow is supersonic; whether it will be supersonic or subsonic that you have to calculate for that you have to get the M_2 value. M_2 means that is the Mach number at that section 2, that is the upstream flow upstream and it is the downstream flow, but in upstream flow is M_1 .

So, in that case the what should be the M_1 value; in terms of M_2 that you can calculate. So, once you know this M_2 value at the downstream flow then you will be able to calculate M_1 from this equation number 28. So, after that you can assess whether this shockwave will develop or not.

(Refer Slide Time: 35:47)



Now, another important that the how this Prandtl Meyer expansion can be formed based on this subsonic and supersonic criteria of this compressible flow. So, in that case if you have a supersonic flow that will strikes a particle a Mach number line develops on the wave.

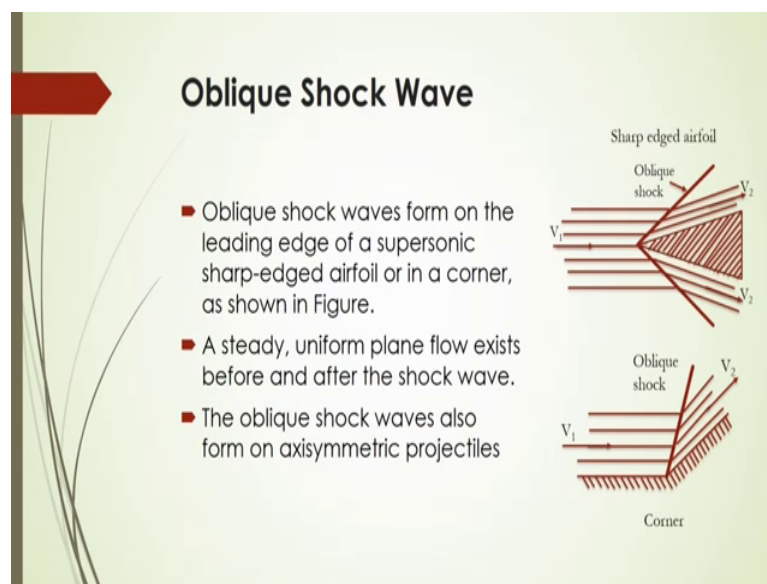
And on the other hand you see when a supersonic flow flows along a plane wall then in that case numerous parallel Mach lines can be developed there. And in that case the supersonic flow expands around a curved wall as shown in figure b and in this case the Mach waves rotate and forming an expansion fan and this type of phenomena will be

then called as the Prandtl Meyer expansion phenomena.

In figure c, it is shown that the compressible supersonic flow which will develop where the numerous MAC lines will change their direction. And also it will be converging and overlapping to develop a sharp change of pressure and density and this type of phenomena will happen only in the shockwave case. And in the figure d it will be seen that the ultimate state of a shockwave will be due to the supersonic flow that will pass along this concave wall.

And a shockwave is called a normal shockwave when this sigma value as shown in figure it will be 90 degree and also for an oblique shockwave in other cases it will happen if sigma is greater than 90 degree or less than 90 degree there.

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An oblique shockwave form on the leading is of a supersonic sharp edged airfoil or in a corner also it is in the case of that what is that other than the normal shockwave. And in that case you have to see that whether this shockwave is formed from the sharp edged airfoil or not. So, this is important because this oblique shockwave only it will come only the tangential flow condition; this is not the normal it will form. So, oblique shockwave will only form if you are having that sharp edged at the airfoil that is the geometry.

A steady uniform plane flow that will also exist before and after the shockwave in this case and it will be seen that in the oblique shockwave waves also from an axis symmetric

projectile as shown in here figure.

(Refer Slide Time: 38:25)

$$u_{1n} = u_1 \sin \sigma \quad u_{1t} = u_1 \cos \sigma$$

$$u_{2n} = u_2 \sin(\sigma - \delta) \quad u_{2t} = u_2 \cos(\sigma - \delta) \quad (29)$$

From the momentum equation in the tangential direction, since there is no pressure gradient,

$$u_{1t} = u_{2t} \quad (30)$$

From the momentum equation in the normal direction,

$$u_{1n}^2 - u_{2n}^2 = \frac{2k}{k-1} \left(\frac{p_2}{\rho_2} - \frac{p_1}{\rho_1} \right) \quad (31)$$

$\sigma = 90^\circ < \delta_{\max} < \sigma = \sin^{-1}(1/M_1)$

u_n = normal velocity
 u_t = tangential velocity

And also for this what should be the tangential flow and also the normal flow and how to calculate that type you know that the angle of this tangential and the normal velocity happened at this direction here are shown in figure. So, you can calculate what will be the velocity at this normal flow condition and at the tangential flow condition this shown in equation number 29.

And from the momentum equation in the tangential direction; since there is no pressure gradient you can have this equation number 30. At this condition you can then have this relationship for this normal shockwave flow between these two sections and it will be as a function of pressure and density as shown in equation number 31.

So, in this case this phenomena will be only applicable if you are getting that delta max within a shockwave angle of 90 degree to that is the sigma at any angle and this will be calculated based on this MAC number sine inverse into 1 by M 1.

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When combined with Eq, (29), we get

$$\cos \delta = \left(\frac{k+1}{2} \frac{M_1^2}{M_1^2 \sin^2 \sigma - 1} - 1 \right) \tan \sigma \quad (32)$$

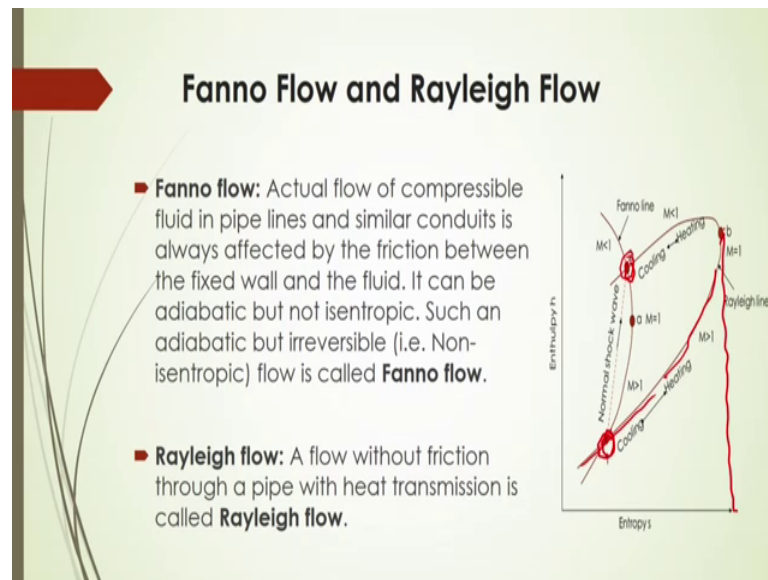
$\sigma = 90^\circ < \delta_{\max} < \sigma = \sin^{-1}(1/M_1)$

If $\delta < \delta_{\max}$, The shock wave is attached to the sharp nose
 If $\delta > \delta_{\max}$, the shock wave detaches and stands off from nose

Similarly, if we combine with that equation number 29 we get that $\cos \delta$ will be equal to this as equation number 32 it is shown. So, based on which you can have an assessment for the shockwave whether if it will be sharp nose or it will be stands off from the nose or not.

If δ is less than δ_{\max} ; then the shockwave will be attached to the sharp nose and if δ is greater than δ_{\max} the shockwave detaches and stands off from the nose. So, here in this figure it is shown that there will be a attachment of the shockwave to the wedge and the detachment of the shockwave to the nose; how it will be occurred based on this δ angle.

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Now, finally, we will discuss that what should be the Fanno flow and the Rayleigh flow. So, whenever you are getting that the shockwave there at a certain flow rate then of course, you will have the compressibility friction in the flow lines.

So, in that case actual flow of the compressible fluid in pipelines and the similar conduits will be always affected by the friction between the fixed wall and the fluid and it can be adiabatic, but not isentropic in that case. So, such an adiabatic but irreversible or it is called non isentropic flow will be called as Fanno flow. And also if the flow without friction through a pipe with heat transmission occurs then this phenomena will be called as Rayleigh flow.

So, in this figure it is shown that how the enthalpy will change based on this entropy and based on piece at that MAC number if it is less than 1 that is subsonic flow then how Fanno line will form. And also if it is greater than 1 that is MAC number is greater than 1 then how Rayleigh line will form.

And at that Rayleigh line you will see at a certain value of entropy change; we will see there will be maximum value of this enthalpy. And also how this enthalpy will be changing with this Rayleigh line based on this friction then you can have from this profile here. And also this Fanno line also will be occurred at this particular non isentropic flow condition based on the MAC number and also based on the enthalpy entropy relationships in that case.

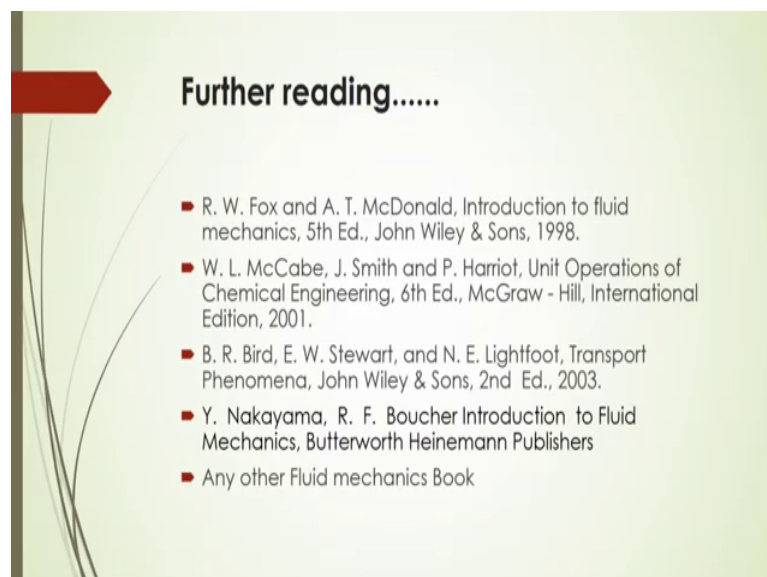
So, we will see there will be intersection between this Fanno line and the Rayleigh line. So, if we have these two intersections of this Fanno line and this Rayleigh line and if you add these two sections to points there; you can have this normal shockwave line there. So, this dotted line will be represented as the normal shockwave line.

So, we have I think learned something about that the compressible flow phenomena for convergent divergent sections; when it will be forming whether a at a differential geometrical area change or velocity change or other temperature pressure change, how you can assess the convergent and divergent flow based on this compressible fluid. And also how the shockwave is formed and normal and oblique shockwaves are how it will be from that we have discussed.

So, I think you will get some idea about this you can learn more about this shockwave phenomena because the details more details of the shockwave phenomena is not the scope of this lecture here. So, basic things are here how to calculate that velocity pressure density and temperature change based on this normal shockwave and the oblique shockwave.

And how this enthalpy and entropy change based on this MAC number and how this Fanno line or Rayleigh line are formed. And from the Fanno line and Rayleigh line profile you can have the normal shockwave phenomena. So, I will suggest you to read more about this from the text books suggested here.

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And so in this case; I would stop now.

Thank you for your attention.