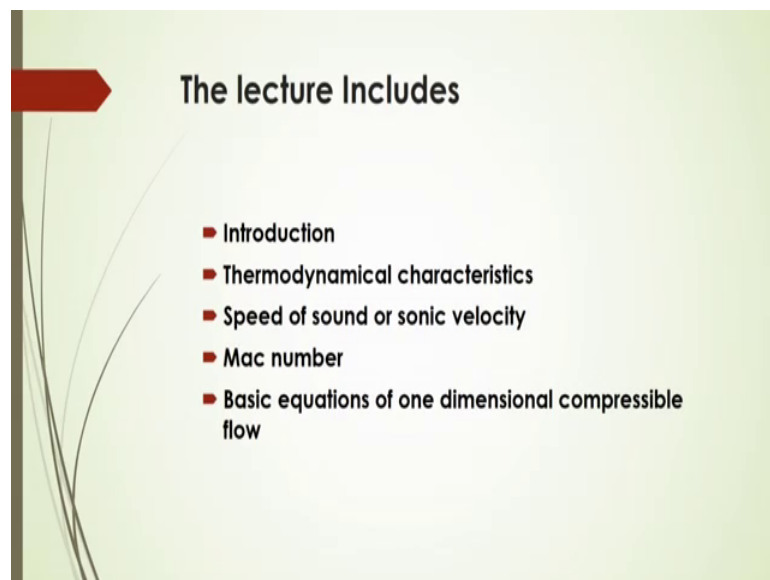


Fluid Flow Operations
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Module - 10
Lecture - 25
Compressible Flow: Part 1

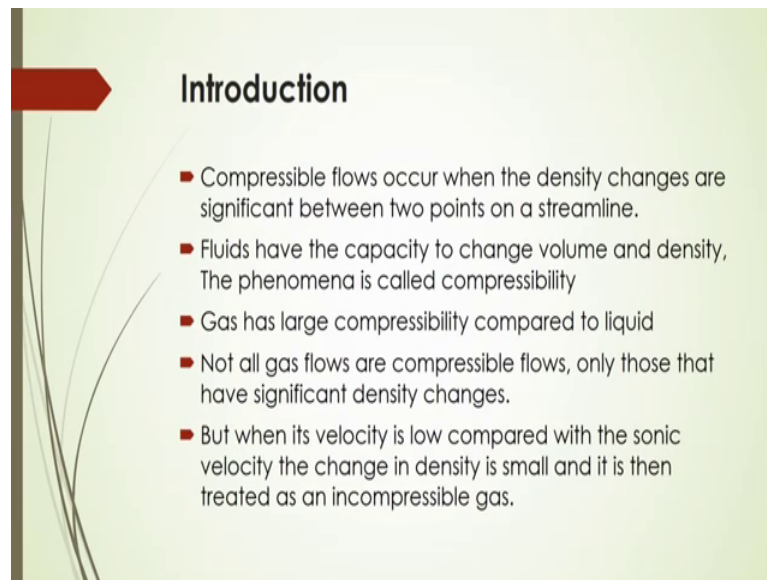
So, welcome to Massive Open Online Course on Fluid Flow Operations. In this lecture we will discuss about the compressible flow, till now we have discussed the flow phenomena based on the incompressible flow.

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So, in this lecture we will discuss how the compressible flow will be actually that keeping the characteristics of different aspects they are in flow condition. So, in this case we will discuss the some thermodynamical characteristics of the compressible flow, speed of sound or sonic velocity and Mac number which will be characteristics characteristic that is number or which will keep the flow behavior of the compressible flow or compressible fluid and also what are the basic equations of that compressible flow we will discuss in this lecture.

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Introduction

- Compressible flows occur when the density changes are significant between two points on a streamline.
- Fluids have the capacity to change volume and density, The phenomena is called compressibility
- Gas has large compressibility compared to liquid
- Not all gas flows are compressible flows, only those that have significant density changes.
- But when its velocity is low compared with the sonic velocity the change in density is small and it is then treated as an incompressible gas.

Now, as we know that compressible flow it will occur when the density changes are significant between two points on a stream line. In the case of incompressible flow will not be having that the density changes or we have not considered the density changes there, because this is if we consider some liquid then at a certain condition or atmospheric condition we are having that that liquid does not change liquid density does not change and also pressure or other things are changing though we are considering that unchanged density there.

So, in this case we are considering that the compressible flow based on this density changes. So, compressible flow will be happen to occur when we consider that there will be a change of density and that density should be a significant between the two points on a stream line and also fluids have the capacity to change the volume and density and the phenomena is called that compressibility.

So, we have already discussed that what is the definition of compressibility and what will be the compressibility factor for the different fluids. So, in that case there will be certain range of compressibility factor from which we can say that whether it will be compressible flow or incompressible flow or the fluid of that flow it is called the compressible fluid or incompressible fluid.

So, gas has large compressibility compared to the liquid. So, that is why gas is considered as a compressible fluid and also not all gas flows are compressible in the

flows, because the only those will have the significant change of density for them only it will be called as compressible fluid.

But when its velocity is low compared to the sonic velocity (sonic means sound velocity) the change in density will be very small and it is then treated as an incompressible gas. So, sometimes so the gas also will be considered as incompressible gas. So, in that case velocity should be actually compared to the sonic velocity and in that case if it is coming that the change in density is very small then only it will be considered as incompressible gas.

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Incompressible gas flow

- Incompressible gas flows occur when the density of the air does not change more than 3 percent between points of interest and are consequently treated as incompressible gas flows.
- Examples:
 - Flow around automobiles,
 - Flow in hurricanes,
 - Flow around aircraft during landing and takeoff,
 - Flow around buildings and communication towers etc.




Image: <http://ceaselesswind.com/tag/hurricane-outflow/>

So, let us see that incompressible gas flow here, this incompressible gas flow occurs on the density of the air, if I consider that air is incompressible gas then in this case we have to observe the density of the gas how it will be changing. So, if the density of the air or gas that does not change more than 3 percent between points of interest and are consequently treated as then incompressible gas flows.

Like here if there is a flow around automobiles you will see there will be a hardly change of that density of the gas and also flow in a hurricane and flow around aircraft during landing and takeoff. In that case the flow of air that will be treated as incompressible air and also flow around buildings and communication towers at a certain velocity it will be considered as an incompressible gas or incompressible airflow.

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Compressible Flow

- The gas flows in which the density does change more than 3 percent.
- Example
 - Airflow around aircraft that fly faster than about 100 m/s,
 - Flow through compressors, jet engines, and tornados.
 - There are also compressible effects in liquid flows that give rise to water hammer and underwater compression waves from blasts




Image: <https://www.youtube.com/watch?v=no-WTGgJ740>

Whereas the compressible mostly in general cases we are considering the gas is compressible flow only for those conditions when that the density does not change much more. So, in that case it will be that the incompressible, but there is a limit, if the gas flows in which the density does change more than 3 percent.

If you are observing the then density will be changing 3 percent, more than 3 percent then it would be called as compressible, otherwise all the gases will be incompressible see we are not getting that significant amount of change of density there. Like airflow around aircraft that fly faster than about 100 meter per second in that case the density will change more than 3 percent.

And also flow through compressors jet engines and tornados in that case this will be compressible because in that case density will change more than 3 percent. There are also compressible effects in liquid flows that gives rise to water hammer and also we have discussed that they are the underwater compression waves from blasts there. So, in that case those liquid also will be considered as an compressible flow.

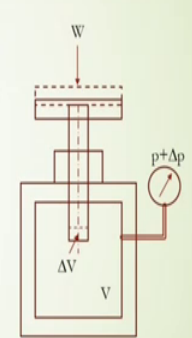
So, in general we are considering the liquid will be incompressible and gas is compressible, but this is a certain range of limit up where you can say that there will be exceptional all that sometimes water also will be considered as an as a compressible fluid.

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Compressibility

- Bulk modulus
$$K = \frac{\Delta p}{\Delta V/V} = -V \frac{dp}{dV} \quad (1)$$
- Compressibility is defined by
$$\beta = \frac{1}{K}$$

For water of normal temp./press., $K = 2.06 \times 10^9 \text{ Pa}$
For air of normal temp./press., $K = 1.40 \times 10^5 \text{ Pa}$
In the case of water, $\beta = 4.85 \times 10^{-10} \text{ 1/Pa}$,
Shrinks by 0.005% if press. increased 1 atm.



Fluid of volume V at pressure p
By increase Δp , decrease v by ΔV

And before going to the description of the fluid flow that is compressible flow, we have to know little bit about that characteristics of that fluid that whether it is compressible or on non compressible and then how it is defined actually. Already in the beginning of this course we have discussed in the flow characteristics module what will be the compressibility and how it is defined.

Here also we are again that just remembering those things that the compressibility how it will be defined here, compressibility is defined as that here by bulk modulus in this case. So, what is that bulk modulus? So, a bulk modulus is represented by K this is generally the change of pressure per unit volume change of that is cash. So, in that case this bulk modulus will be is equal to minus p into dp by dV as shown in here in equation 1. So, compressibility will be defined as and it will denoted by this beta and it will be inverse of thus that bulk modulus. So, this is simple as minus V into dV by dp it will be coming.

So, compressibility is the inverse of the bulk modulus. So, it is defined as like this whereas, bulk modulus is defined as how the pressure is changing whenever unit volume of or that is the volume change certain change of volume or per unit volume change of that pressure. So, based on which we are calculating this bulk modulus here by equation 1 and for water for normal temperature or pressure you can say that this bulk modulus will be as 2.06 into 10 to the power 9 Pascal and for air of normal temperature and pressure this bulk modulus will be 1.40 into 10 to the power 5 Pascal.

So, in the case of water beta will be is equal to what is the compressibility it will be 4.85 into 10 to the power minus 10 1 by Pascal. And similarly in this case you can say that this water is sinks by 0.005 percent if pressure increased in one atmosphere. So, in this case we are having that compressibility how this it is defined and what is the bulk modulus and it will be actually used to define that compressibility factor and also whether the fluid will be compressible or incompressible that by this definition we can assess it.

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Since, $\rho V = M = \text{constant}$,

$$K = \rho \frac{\Delta p}{\Delta \rho} = \rho \frac{dp}{d\rho} \quad (3)$$

■ The bulk modulus K is closely related to the **velocity c of a pressure wave propagating in a fluid**, which is given by the following equation

$$c = \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{K}{\rho}} \quad (4)$$

And here since we know that that rho V is equal to M that will be equals to constant as per gas law then we can have this bulk modulus as after substitution of this rho V is equal to M that will be is equal to then rho into delta p by delta rho that will be equal to rho into dp by d rho.

So, by this equation we can again have the bulk modulus based on the density instead of velocity instead of volume there. So, since we know that as per gas law in this case rho V is equal to M. So, after substitution we are getting this equation number 3 for the bulk modulus in terms of density.

Now, the bulk modulus K is closely related to the velocity of a velocity here let be here instead of a it will be c actual velocity c of a pressure wave that is propagating in a fluid or you can say a over the weather whatever notation you can use you can use so, we will be using here c instead of a throughout this that is lecture.

So, here velocity c of a pressure wave or propagating in a fluid which is given by the following equation that c will be is equal to what is that, c will be is equal to root over dp by $d\rho$ that will be is equal to root over K by ρ after substitution of this value of dp by $d\rho$ from equation number 3. Then we are getting this velocity c of a pressure wave that is propagating in a liquid or fluid or any other air also air or liquid whatever it is.

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Thermodynamic Characteristics

- A gas having the following relationship between absolute temperature T and pressure p

$$pv = RT \quad R = \frac{R_0}{M} \quad (5)$$
- R , is the universal gas constant ($R_0 = 8314 \text{ J/(kgK)}$) and M is the molecular weight.
- For example, for air, assuming $M = 28.96$, the gas constant is

$$R = \frac{8314}{28.96} = 287 \text{ J/(kgK)} = 287 \text{ m}^2/(\text{s}^2\text{K}) \quad (6)$$

And then, thermodynamic characteristics in that case gas having the following relationship between absolute temperature T and the pressure. So, pv will be is equal to RT and then R will be is equal to R_0 by M this will be R_0 by M . So, R_0 by M then here in this case R will be is equal to universal gas constant and R_0 will be is equal to 8314 joule per kg K and M is the molecular weight here. And for example, for air assuming M if it is the 28.96 the gas constant will be is equal to R is equal to 8314 by 2896 then it will be 287 joule per kg K that will be 287 meter square per second square K.

So, in this case we are having that what should be the universal gas constant. So, this is one of the thermodynamic characteristics that you have to consider for analyzing the flow of compressible fluid through the conduit.

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- Then, assuming internal energy and enthalpy per unit mass e and h respectively,
- The specific heat at constant volume is:
$$C_v = \left(\frac{\partial e}{\partial T}\right)_v \quad de = C_v dT \quad (7)$$
- The specific heat at constant pressure:
$$C_p = \left(\frac{\partial h}{\partial T}\right)_p \quad dh = C_p dT \quad (8)$$
$$h = e + pv \quad (9)$$

Then if we assume that there will be certain internal energy and enthalpy per unit mass that is denoted by e and h respectively. So, in that case how the specific heat of that compressible fluid can be expressed. So, the specific heat at constant volume can be expressed by that C_v is equal to de/dT at constant volume whereas, so this de ; that means, so change of enthalpy that will be is equal to then C_v into dT .

And the specific heat at constant pressure that will be is equal to C_p that will be equal to dh/dT at constant pressure and then dh should be is equal to C_p into dT . So, here h will be is equal to then $e + pv$ here. So, as per this h is enthalpy; enthalpy is nothing, but the summation of this internal energy and also the pressure and that is work done by that pressure for a particular volume change that will be pv . So, then enthalpy will be is equal to here h is equal to $e + pv$. So, this 7, 8 and 9 equation numbers will give you the definition for the specific heat at constant volume and specific heat at constant pressure there.

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According to the first law of thermodynamics, when a quantity of heat dq is supplied to a system, the internal energy of the system increases by de , and work $p dv$ is done by the system. Therefore,

$$de = C_v dT \quad dq = de + p dv \quad (10)$$

From the equation of state we can write

$$p dv + v dp = R dT \quad (11)$$

From Eq. (9)

$$dh = de + p dv + v dp \quad (12)$$

So, according to the first law of thermodynamics when we consider a quantity of heat dq is supplied to a system that compressible gas then the internal energy of the system increases by if it is de and then work done by that fluid system then it will be denoted by $p dv$. So, in that case the internal energy change will be is equal to $C_v dT$ and then dq that is the heat that amount supply to the system it will be related to that internal energy change and that will be equal to plus internal energy change and along with that p to dv .

So, in this case you can say that if we supply the small quantity of dq and the system then there will be change of internal energy that is denoted by de and also the work that is $p v$ is done by the system in that case. So, this energy change and also work done by the system will give you the total amount of that whatever supplied of heat is to be in the system and from the equation of state we can write in that case $p dv$ plus $v dp$ that will be is equal to $R dT$ because we know that $p v$ is equal to $R T$. So, from which after differentiation we can get this $p dv$ plus $v dp$ that will be is equal to R into dT that is denoted by this equation number here 11.

Now, from the equation number 9, this is equation number 9 h is equal to e plus $p v$ and from this equation number 9 we can write after again differentiation we can get this dh would be is equal to de plus $p dv$ plus $v dp$.

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■ In the case of constant pressure change, $dp = 0$, so Eqs. (11 & 12) becomes

$$p dv = R dT \quad (13)$$
$$dh = de + p dv = dq \quad (14)$$

■ Substituting Eqs (7,8,11,12) in Eq. 10, we can get

$$C_p - C_v = R \quad (15)$$

■ Now, $c_p/c_v = k$ (k : ratio of specific heats (isentropic index)), so

$$C_p = \frac{k}{k-1} R \quad (16) \quad C_v = \frac{1}{k-1} R \quad (17)$$

Now, in the case of constant pressure change where dp should be is equal to 0. So, in that case equation number 11 and 12 this equation 11 number and this equation number 12 from this we can write that $p dv$ will be is equal to $R dT$ and dh will be is equal to de plus $p dv$ that should be is equal to dq as per definition given in equation number 10 here.

Now, after substitution of the equation 7, 8, 11 and 12 into equation number 10 then we can get that C_p minus C_v that will be is equal to R ; that means, the difference in specific heat at constant volume and constant pressure what will be that differences that will come as only universal gas constant. So, we can get this relationship to calculate what should be the C_p in terms of R and also C_p C_v in terms of R there.

Once we know that the ratio of that C_p and C_v ; that means, the ratio of the specific heat of the fluid at certain pressure and volume there. So, if we consider, if we denote this ratio of the C_p by C_v as k , this k will be called as specific heats ratio or it is called sometimes isentropic index. So, this index will actually give you that what should be the factor 2 multiply with the universal gas constant to get this specific heat capacity. So, from this equation number 16 and 17 you can get the C_p and C_v value if you know that the isentropic index that is C_p by C_v is equal k .

So, these are the that is characteristics equation of this compressible fluid that you have to know. So, this isentropic index is very important because all those isentropic index

will be required to analyze the flow of that compressible fluid through the pipe or conduit or any other devices.

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- We may also determine the entropy change or assume an isentropic process ($ds = 0$).
- Then, one of the following equations may be used:

$$\Delta s = C_p \ln \frac{T_2}{T_1} - R \ln \frac{p_2}{p_1}$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{(k-1)/k}$$

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1} \right)^k$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{(k-1)/k} \quad (18)$$

So, in this case if we consider the entropy change another characteristics factor that is thermodynamic characteristic factor so, how to how to actually determine the entropy change. So, in that case if we consider that isotropic flow then ds should be is equal to 0, then one of the following equations may be used to calculate that what is that entropy change for the compressible fluid.

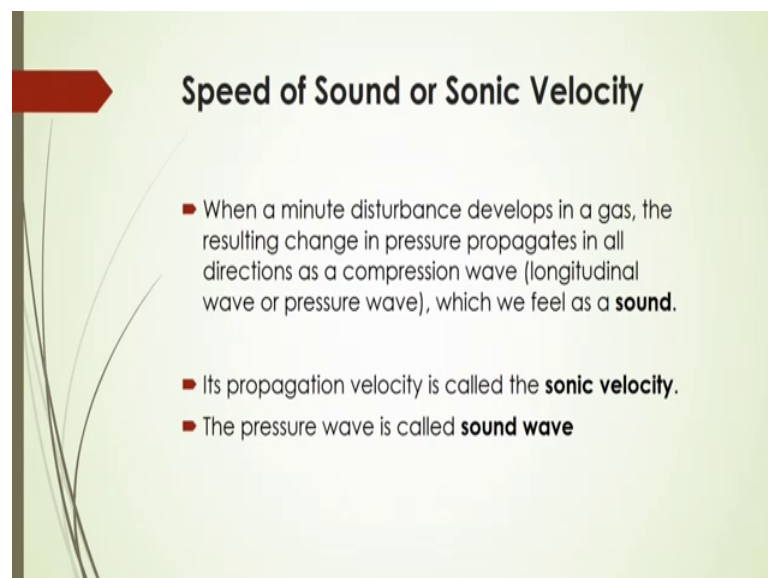
So, in that case Δs , s is entropy so, Δs that is entropy change. So, it would be C_p into $\ln T_2$ by T_1 minus R into $\ln p_2$ by p_1 . So, T_1 and T_2 are the temperature at 2 states or 2 sections you can say here for the flow. So, and p_1 and p_2 are the pressures at the 2 states or 2 sections also. So, generally if we consider that temperature change from T_1 to T_2 and pressure accordingly will be changing from p_1 to p_2 , then if we have the systems behavior by the changing of entropy then we can calculate that entropy change as by this equation number here by equation 18.

And in this case T_2 by T_1 to be related with the pressure change as p_2 by p_1 to the power k minus 1 by k and p_2 by p_1 that will be is equal to ρ_2 by ρ_1 to the power k and T_2 by T_1 is equal to p_2 by p_1 to the power k minus 1. So, in this way we can actually relate what should be the entropy change if we know that temperature ratio as

per the pressure change then what should be the entropy change by this equation number 18 here.

So, this one I am just telling, this one is actually by mistake it is written. So, please do not consider this equation here. So, only these 2 portions in terms of temperature change based on pressure change and pressure change based on density change you can calculate those portion from this equation number 18 and then substitute this corresponding value of this T_2 and by T_1 and the equation here then you will be able to calculate what to the entropy.

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Now, let us consider the speed of sound or it is sometimes called the sonic velocity when a minute disturbance if you observes when if you see that there is a gas flowing and it will results a certain change of pressure. So, in that case due to that disturbance, small disturbance there will be a certain change of what is that pressure and that change of pressure sometimes will provoke propagates in a liquid in all directions as a compression wave.

So, in that case the compression wave is actually we can feel as a sound there. So, we can say that sound how it will be actually produced in the fluid flow system that when a minute disturbance develops in a gas you can see it will results the certain change in pressure which will propagates in all directions as a compression wave and it will be

actually coming as a sound and its propagation velocity will be called as sonic velocity and the pressure wave will be called as sound wave here.

So, that you have to then have this concept that how actually sound is coming only thing is that you have to make a disturbance in the gas and that disturbance will change the pressure and that pressure change will propagate in all directions as a compression wave and those compression waves will be actually exposed as a sound. So, its propagation velocity will be denoted by or will be called as a sonic velocity and the pressure wave is called as a sound wave. Now, in this case if we consider the small amplitude wave that is shown in here in the figure below.

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- Consider the small-amplitude wave shown in Figure below traveling through a conduit at unsteady state condition.
- In Fig. (a) it is moving so that a stationary observer sees an unsteady motion
- In Fig. (b) the observer moves with the wave so that the wave is stationary and a steady flow is observed, and
- In Fig. (c) shows the control volume surrounding the wave. The wave is assumed to create a small differential change in the pressure p , temperature T , density ρ , and velocity V in the gas.

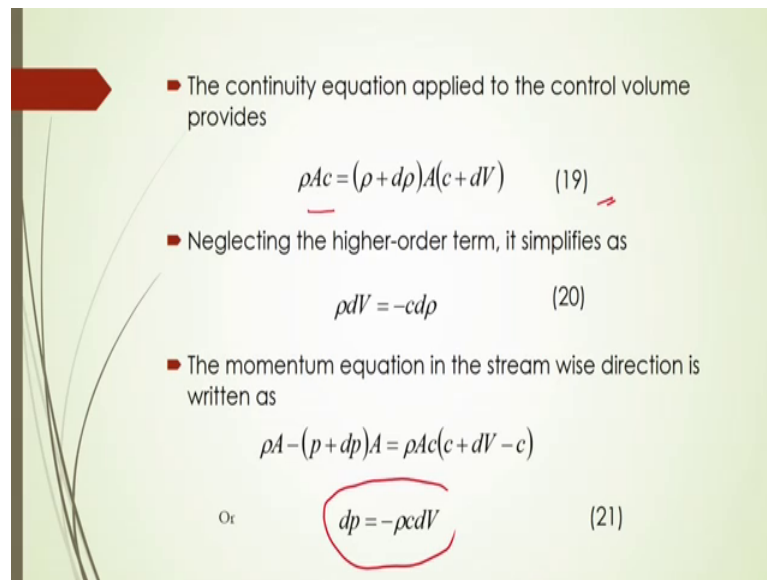
The diagrams are as follows:

- (a) Moving wave: A horizontal line with a red arrow pointing right labeled c . Below the line, on the left, are the values $p+d p$, $T+d T$, and $\rho+d \rho$. On the right are p , T , and ρ . A red arrow labeled dV' points right between the two sets of values. Below the line, it says $V'=0$.
- (b) Stationary wave: A horizontal line with a red arrow pointing right labeled c . Below the line, on the left, are the values $p+d p$, $T+d T$, and $\rho+d \rho$. On the right are p , T , and ρ . A red arrow labeled $c+dV'$ points right between the two sets of values. Below the line, it says $V'=c$.
- (c) Control volume: A dashed rectangular box. On the left side, a red arrow labeled $c+dV'$ points right. On the right side, a red arrow labeled c points right. Below the box, it says $\rho+d\rho/\Delta$ on the left and ρ/Δ on the right.

And that small amplitude sound is traveling through a conduit at a unsteady state condition. Then as per figure a you can say that it is moving so, that a stationary observer sees an unsteady motion in that case and as per figure b the observer moves with the waves so the wave is stationary and steady is observed in that case. And in figure c if we see that the control volume surrounding this propagating wave.

So, in that case the wave is assumed to create a small differential change in the pressure p and also you can say there will change in temperature and density and velocity V in the gas. So, if we have this actually change this pressure, temperature and density also the velocity of the gas.

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- The continuity equation applied to the control volume provides
$$\rho A c = (\rho + d\rho) A (c + dV) \quad (19)$$
- Neglecting the higher-order term, it simplifies as
$$\rho dV = -c d\rho \quad (20)$$
- The momentum equation in the stream wise direction is written as
$$\rho A - (p + dp) A = \rho A c (c + dV - c)$$

Or
$$dp = -\rho c dV \quad (21)$$

In that case we can then write one continuity equation by applying to the control volume and which will give you the equation number here as $\rho A c$ that will be equals to $\rho + d\rho$ into A into $c + dV$ as given in equation number 19. So, in this case neglecting the higher order term in this equation then you can after simplification have ρdV that would be equal to minus $c d\rho$.

And the momentum equation in that case you can write in the stream wise direction as $\rho A - (p + dp) A$ that will be is equal to $\rho A c$ into $c + dV$ minus c , where c is called that what is that sound velocity and in this case after simplification we can write this dp will be is equal to minus the ρc into dV .

So, based on those change of pressure temperature and velocity and density of the gas we can write the continuity equation through a whenever that compressible gas is flowing through a conduit at a steady state condition that equation can be represented by this equation number 19. And final it is 20 and for momentum equation will be represented by this here equation as dp will be is equals to minus ρc into dV .

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■ Combining the continuity and momentum equations results in

$$c = \sqrt{\frac{dp}{d\rho}} \quad (22)$$

(for small scale sound wave)

■ The lower-frequency (less than 18 000 Hz) sound waves travel isentropically so that

$$\frac{p}{\rho^k} = \text{const} \quad (23)$$

■ Therefore after differentiation

$$\frac{dp}{d\rho} = k \frac{p}{\rho} \quad (24)$$

Now, combining these continuity and this momentum equations that will results as c will be is equal to root over dp by d rho. So, this sound velocity will be is equal to square root of the ratio of the pressure change to the density change or you can say the square root of the pressure change with respect to density change there for a small scale sound wave this equation is applicable.

The lower frequency if it is coming less than 18000 hertz sound waves will be travelled isentropically and in that case you can say that this p by rho to the power k should be constant and therefore, after a differentiation we can get this equation number 23 as dp by d rho will be is equals to k into p by rho. Now, you have to substitute this or after substitution of this dp by d rho from equation number 24 in to equation number 22.

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■ The speed of sound or sonic velocity for such waves is then

$$c = \sqrt{\frac{kp}{\rho}} = \sqrt{kRT} \quad (25)$$

■ High-frequency waves travel isothermally resulting in a speed of sound of

$$c = \sqrt{RT} \quad (26)$$

■ In other words, the sonic velocity is proportional to the square root of absolute temperature. For example, for $k = 1.4$ and $R = 287 \text{ m}^2/(\text{s}^2 \text{ K})$, $c = 340 \text{ m/s}$ at 16°C (289 K)

Then we can get c will be is equal to what root over kp by ρ that will be is equal to simply root over kRT because p by ρ that will be is equal to here RT . So, we can get this sound velocity as what is that root over kRT . What is k ? k is nothing, but the isentropic index or it is called the ratio of the specific heat capacity of this compressible fluid at constant pressure and constant volume.

Now, high frequency waves sometimes travel isothermally and which will result in a speed of sound as here c will be is equal to root over RT , because in that case for isothermal cases here C_p by C_v will be is equal to 1. So, in that case this sound velocity should be only root over RT and in other words you can say these things that the sonic velocity is proportional to the square root of the absolute temperature.

For example, for k is equal to 1.4 and R is equal to 287 meter square per second square k , then in that case this c will be is equal to 340 meter per second at 16 degree centigrade. And so here interesting that if you know that the isentropic index then what should be the sound velocity at a particular temperature you can easily calculate from this equation number 25 and for isothermal case you can calculate it from equation number 26.

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■ Next, if the bulk modulus of fluid is K,

$$dp = -K \frac{dv}{v} = K \frac{d\rho}{\rho} \quad (27)$$

■ And

$$\frac{d\rho}{d\rho} = \frac{K}{\rho} \quad (28)$$

■ Therefore

$$c = \sqrt{\frac{dp}{d\rho}} = \sqrt{\frac{K}{\rho}} \quad (29)$$

Next if you consider that bulk modulus of the fluid as capital K. So, in that case this dp should be is equal to minus K into dv by v then it will be is equal to K into d rho by rho and then dp by d rho will be is equal to what is that K by rho. So, in that case c will be is equal to root over dp by d rho ultimate it is coming as root over K by rho.

So, sound velocity also you can calculate from the density, if you know that coefficient of bulk modulus of the fluid and also density of the fluid. So, by equation number 29 you can calculate the sound velocity provided the bulk modulus coefficient of bulk modulus you know and also density of the fluid.

Now, let us consider the definition for Mac number this is one of the important dimensionless number based on which this compressible Fluid is actually being characterized and in that case whether the fluid compressible fluid have the supersonic or what is the subsonic that depends on this range of this Mac number.

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Mac Number

- At the stagnation point of a body placed in a uniform flow, the pressure increases

$$\Delta p = \rho U^2 / 2 \quad (30)$$

- This increased pressure brings about an increased density

$$\Delta \rho = \Delta p / c^2 \quad (31)$$

- The ratio of flow velocity u to sonic velocity c , i.e. $M = u/c$, is called Mach number

$$M = \frac{U}{c} = \frac{1}{c} \sqrt{\frac{2\Delta p}{\rho}} = \sqrt{\frac{2\Delta p}{\rho}} \quad (32)$$

- From this equation, the Mach number M corresponding to a density change of 5% is approx. 0.3.
- For this reason steady flow can be treated as incompressible flow up to around Mach number 0.3.

Now, at the stagnation point of a body if it is placed in a uniform flow then the pressure increases as you know that what should be the kinetic energies there. So, it will be rho U square by 2 in that case this rho that is rho is the density of the gas and U is the velocity of the gas here. So, pressure change that pressure increase will be is equal to rho a square by 2 as given in equation number 30 here.

This increased pressure will bring about an increase the density, then in that case this delta rho that is increase in density due to this increased pressure it will be as delta p by c square, because we know that here in this equation number this equation number 29 this keeps that what is that here the sound velocity. So, this from that equation number 29 we can write this equation number 31 as delta a row that will be is equal to delta p by c square.

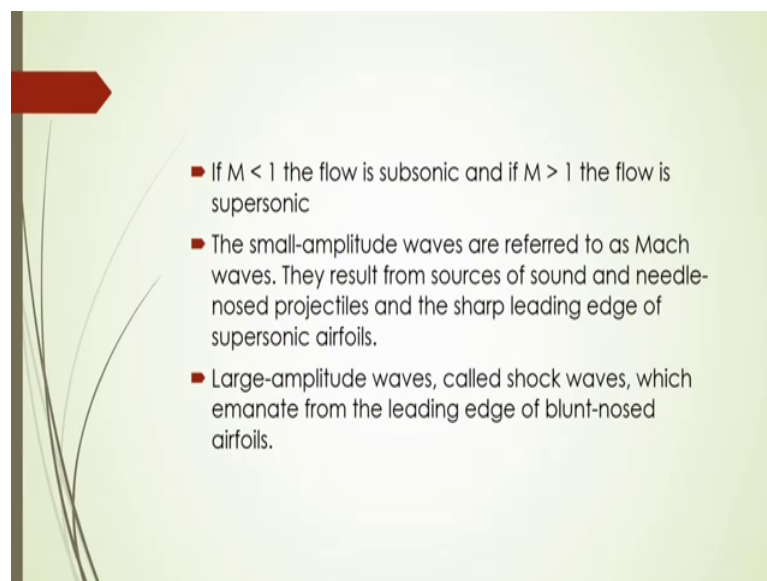
Now, this ratio of flow velocity U or that is that is velocity of the gas and also what is the sonic velocity c then we can defined as this Mac number as the ratio of this flow velocity of the fluid to the sonic velocity. So, it would be called as Mac number so as per that will be Mac number is denoted by M sometimes some text you will get that it will be denoted by Ma instead of only M sometimes it will be denoted by only M. So, Mac number is as M, M is equal to U by c.

So, if you substitute the value of U from equation number 30 then we can have this 1 by c into root over 2 delta p by rho and that will be is equal to root over 2 delta rho by rho

finally, it will come after substitution of this Δp value there. So, this equation number 32 is denoted or that is referred to as a Mac number and this Mac number in terms of that flow velocity of the fluid and sound velocity or this Mac number can be actually referred to as the change of density per unit density of the fluid into the square as per this equation number 32 as given here. Root over 2 into density change out of that certain velocity of that particular denser fluid.

And from this equation the Mac number 3 corresponding to a density change of 5 percent is approximately 0.3 here. So, if we are having the 5 percent change of this density then Mac number should be equal to 0.3 and for this reason you can say that a steady flow can be treated as incompressible flow up to around Mac number of 0.3. So, steady flow can be treated as incompressible flow up to around Mach number 0.3 beyond that it will be compressible and before 0.3 it will be treated as incompressible flow.

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If Mac number is less than 1 the flow is called subsonic flow and if M is greater than 1 that is Mac number is greater than 1 the flow is called supersonic flow. The small amplitude waves are referred to as Mach waves, they result from sources of sound and also needle nosed projectiles and the sharp leading edges of supersonic airfoils so, in that case Mach waves to be defined for those cases for a certain velocity of the fluid.

Large amplitude waves called shockwaves who is emanate from the leading edge of blunt nosed airfoils and in that case you will see that Mach number will be more than 1.

So, there should be a waves formation it will be very large the amplitude waves and these type of waves will be called as shock waves and we will discuss that shock waves later on also.

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Basic Equation for 1-Dimensional Compressible Flow

- Continuity Equation:
- For a constant mass flow m of fluid density ρ flowing at velocity u through section area A , the continuity equation is

$$m = \rho u A = \text{constant} \quad (33)$$

- By logarithmic differentiation

$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0 \quad (34)$$

Now, let us have the basic equation for one dimensional compressible flow. So, for that you have to know the continuity equation, now if we consider a constant mass flow of m of fluid density ρ that is flowing at velocity u through a sectional area A then the continuity equation can be written as m will be is equal to $\rho u A$ that should be constant as per that continuity equation earlier we have defined that the total mass should be constant.

So, in this case ρ is called density and uA is called volume that is the per unit time. So, in that case this mass flow rate would be remains constant there then if we take the logarithm of this equation number 33 on it is both sides then we are getting here $d\rho$ by ρ plus du by u plus dA by A that will be is equals to 0.

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■ Euler's equation of motion in the steady state along a streamline is

$$\int \frac{dp}{\rho} + \frac{1}{2}u^2 = \text{Constant} \quad (35)$$

■ For adiabatic conditions from Eq. (23)

$$p = m\rho^k \quad m = \text{constant} \quad (36)$$

■ So after derivative and integration of Eq. (36)

$$\int \frac{dp}{\rho} = \int mk\rho^{k-2}d\rho = \frac{k}{k-1} \frac{p}{\rho} + \text{Constant} \quad (37)$$

And then we can have after integration of this equation then we can represent the Euler's equation of motion in the steady state along a streamline as what is that root over integration of dp by rho plus half of u square that will be is equals to a constant here.

For adiabatic conditions from the equation number 23, that earlier we have shown that p would be is equal to m into rho to the power k and then m is equal to constant then. So, after derivative and integration of equation number 36 we can write here this integration of dp by rho that will be is equal to integration of mk rho to the power k minus 2 into d rho. And finally, it will come as k by k minus 1 into p by rho plus constant that is constant of integration.

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Bernoulli's Equation

- Substituting the Eq. (37) into Eq (35),
$$\frac{k}{k-1} \frac{p}{\rho} + \frac{1}{2} u^2 = \text{Constant} \quad (38)$$
- Or
$$\frac{k}{k-1} RT + \frac{1}{2} u^2 = \text{Constant} \quad (39)$$

Equations (38) and (39) correspond to **Bernoulli's equation for an compressible fluid.**

And after that we are going to substitute this equation number 37 in to equation number 35 here. Then what we can get this equation number after simplification that is k by k minus 1 to p by rho plus half of u square will be equals to constant or k by k minus 1 into RT because p by rho is equal to RT here plus half of u square is equal to constants.

So, this equation number 38 or 39 corresponds to the Bernoulli's equation for compressible for an incompressible that is this one is the I think for compressible fluid it will be compressible fluid not is incompressible so, it will be compressible fluid. So, this equation number 38 and 39 if you know that isentropic index that is k value then at a particular temperature what should be the velocity of the fluid and how the energy is conserved that you can assess by this equation number 38 or 39. So, these equations are called that Bernoulli's equation for the compressible fluid.

Now, what will be the total temperature the static temperature and the dynamic temperature as earlier for what we discussed in the case of incompressible fluid. So, in the same way you can have what is that static temperature dynamic temperature here in the case of compressible fluid.

Now, if fluid discharges from a very large vessel in that case u should be is equal to 0. So, using subscript 0 for the state variables in the vessel here, then equation number 39 yields here T 0 by T that will be represented by this equation number here 40 as 1 plus half of 1 by RT into k minus 1 by k into u square.

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Total temperature, the static temperature and the dynamic temperature

- If fluid discharges from a very large vessel, $u = u_0 = 0$ (using subscript 0 for the state variables in the vessel), eqn (39) yields

$$\frac{T_0}{T} = 1 + \frac{1}{2} \frac{1}{RT} \frac{k-1}{k} u^2 = 1 + \frac{k-1}{2} M^2 \quad \text{where } M = \frac{u}{c} = \frac{u}{\sqrt{kRT}} \quad (40)$$

Total temperature = T_0 = Mac number
 The static temperature = T
 The dynamic temperature = $\frac{1}{2} \frac{1}{R} \frac{k-1}{k} u^2$

And finally, it is coming as in terms of Mac number as 1 plus k minus 1 by 2 into M square because you know that M will be equals to u by c that is nothing, but u by root over kRT because c is equal to root over kRT here. So, the ratio of temperature to the actually stream temperature final temperature to the stream temperature this ratio will be represented by this equation number 40 in terms of Mac number as well as the temperature and velocity of the fluid, if provided that isentropic index of the compressible fluid is known to you.

So, in this case total temperature if you know that this T_0 , then the static pressure is T then the dynamic pressure will be represented by these terms here in this case this one is called what is that dynamic temperature. So, dynamic temperature will be is equals to half of 1 by R into k minus 1 by k into u square.

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Stagnation and Static Pressure

By Eq (40) we can write

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{k/(k-1)} = \left(1 + \frac{k-1}{2} M^2\right)^{k/(k-1)} \quad (41)$$

Stagnation Pressure = P_0
The static pressure = P

where $M = \frac{u}{c} = \frac{u}{\sqrt{kRT}}$ = Mac number

This is applicable to a body placed in the flow, e.g. between the stagnation point of a Pitot tube and the main flow.

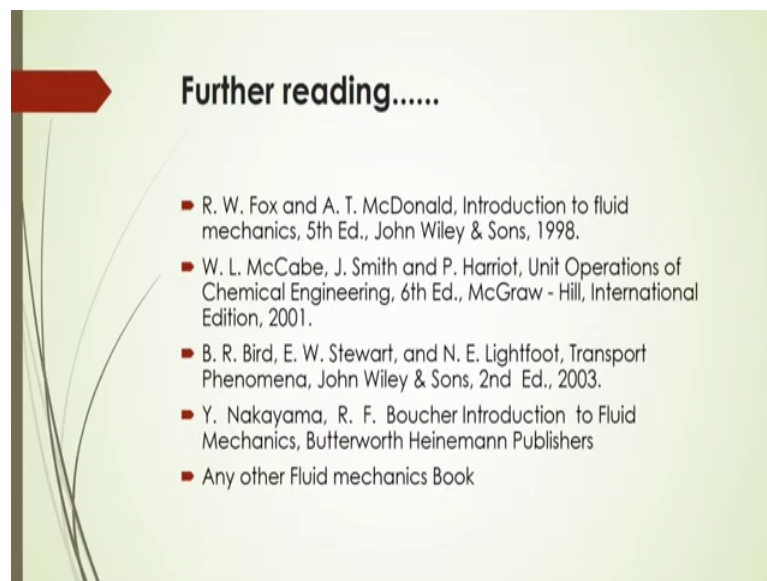
And then what is the stagnation and static pressure? By equation number 40 here this you can write that p_0 by p that will be is equal to T_0 by T to the power k by k minus 1 is equal to $1 + \frac{k-1}{2} M^2$ in terms of Mac number. So, here from this equation number 41 you can have the stagnation pressure, what is the stagnation and the static pressure as the static pressure as the p . So, in that case this p_0 by p as a function of temperature as well as the function of M it can be obtained. So, here this Mac number will be defined by this equation number this.

So, I think we have learned something about that what is that compressible fluid and how it is defined and how it is deviated from the incompressible fluid and generally based on the Mac number even you know that if it Mac number is 0 point, above 0.3 to would be compressible if it is less than 0.3 then it will be incompressible and also if density changed not less than 3 percent of the fluid then it will not be considered as a what is that compressible fluid. If it is the then density change greater than 3 percent then you have to consider that the fluid will be as a compressible.

So, these are the criteria and what will be the definition of that compressibility and how this compressibility can be calculated. And also what will be the basic equation of what is the Bernoulli's equation to express the conservation of energy for this compressible fluid you can have and you can get from this lecture and you can calculate based on the change of pressure based on the temperature, based on the other parameters even in

terms of Mac number, how it can be calculated this energy change and what would be the velocity of the compressible fluid for a particular temperature and even if you know the pressure and if you know the isentropic index then how to calculate this Bernoulli's equation and from that Bernoulli's equation how to calculate the velocity of the compressible fluid. So, I think it would be very useful for you for this compressible flow next lecture we will discuss again further about this compressible fluid.

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So, thank you for this lecture today.