

**Fluid Flow Operations**  
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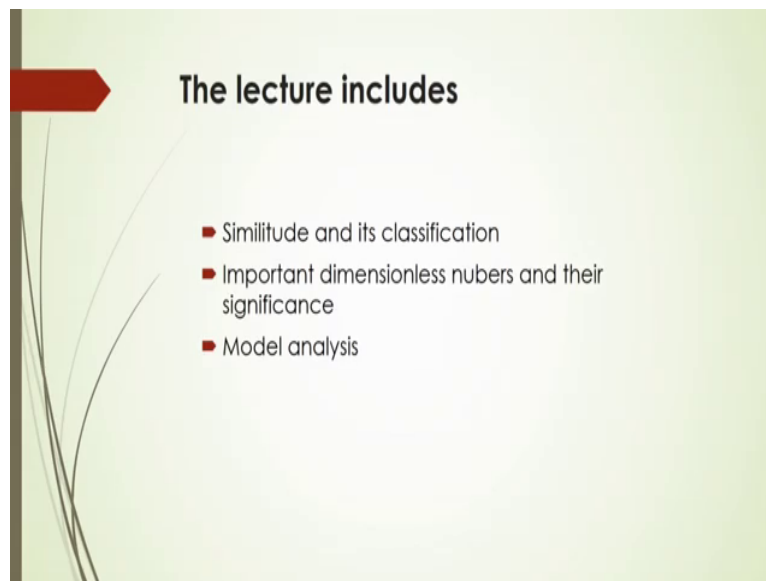
**Module - 09**

**Lecture - 24**

**Dimensional Analysis and Law of Similarity: Part 3- Law of Similarity and Significant Dimensionless Number**

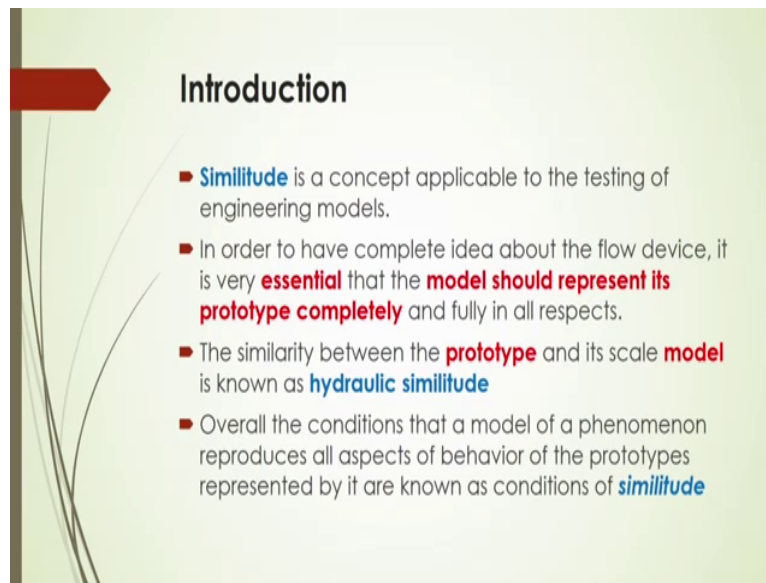
Welcome to this massive open line online course on Fluid Flow Operations. Here in this lecture we will discuss the Law of Similarity and Significant Dimensionless Number. We have already discussed the method how to analyse this dimensional group and also how to form the dimensionless groups by Buckingham pi theorem and as well as Rayleigh's methods.

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In this lecture we will discuss the similitude and its classification and also important dimensionless numbers and their significance and also model analysis.

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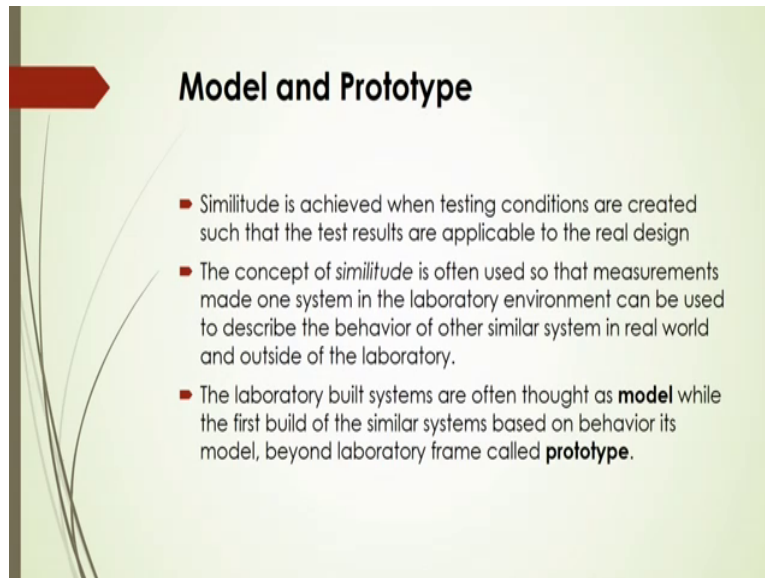
**Introduction**

- **Similitude** is a concept applicable to the testing of engineering models.
- In order to have complete idea about the flow device, it is very **essential** that the **model should represent its prototype completely** and fully in all respects.
- The similarity between the **prototype** and its scale **model** is known as **hydraulic similitude**
- Overall the conditions that a model of a phenomenon reproduces all aspects of behavior of the prototypes represented by it are known as conditions of **similitude**

Now, in this case we have to know what is that similitude? It is a tool of concept which is applicable to the testing of engineering models. And in order to have complete idea about the flow device it is very essential that the model should represent its prototype completely and fully in all respects. And also whenever you are the going to analyse the experimental processes and design from small scale to the large scale, you have to analyze that particular that is prototype based on the similarity law.

And the similarity between the prototype and its scale model is known as the hydraulic similitude. So, in that case overall the conditions that a model of the phenomena reproduces all aspects of behaviour of the prototypes that will be represented by it and which are called as conditions of a similitude.

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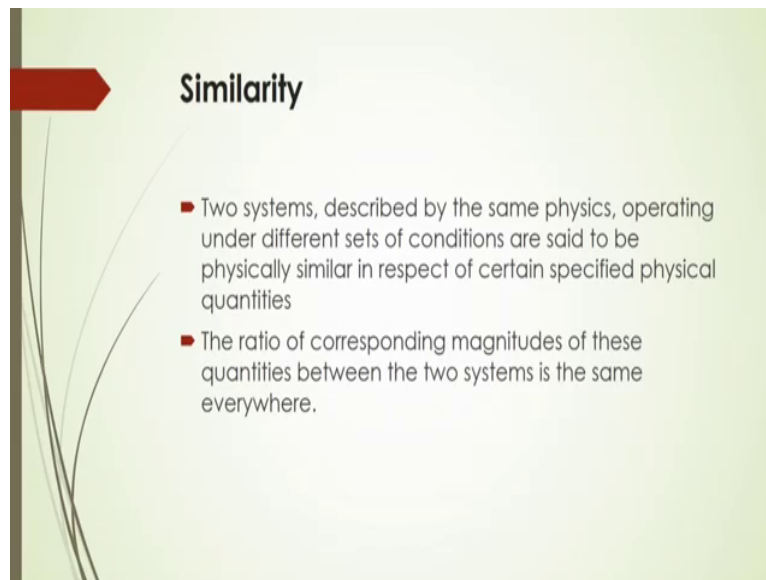
**Model and Prototype**

- Similitude is achieved when testing conditions are created such that the test results are applicable to the real design
- The concept of *similitude* is often used so that measurements made one system in the laboratory environment can be used to describe the behavior of other similar system in real world and outside of the laboratory.
- The laboratory built systems are often thought as **model** while the first build of the similar systems based on behavior its model, beyond laboratory frame called **prototype**.

And then what is that actually that we have told that there will be a model and the prototype. Of course, the similitude is achieved when the testing conditions are actually created such that the test results are applicable to the real design. And the concept of that similitude is often used to measure one system in a laboratory environment and which can be used to describe the behaviour of other similar system in real world and outside the laboratory. So, whenever you are doing any experiments to design a suppose palm then you have to use those concepts or what is that results of that small scale system in laboratory to actually develop the large scale pump there.

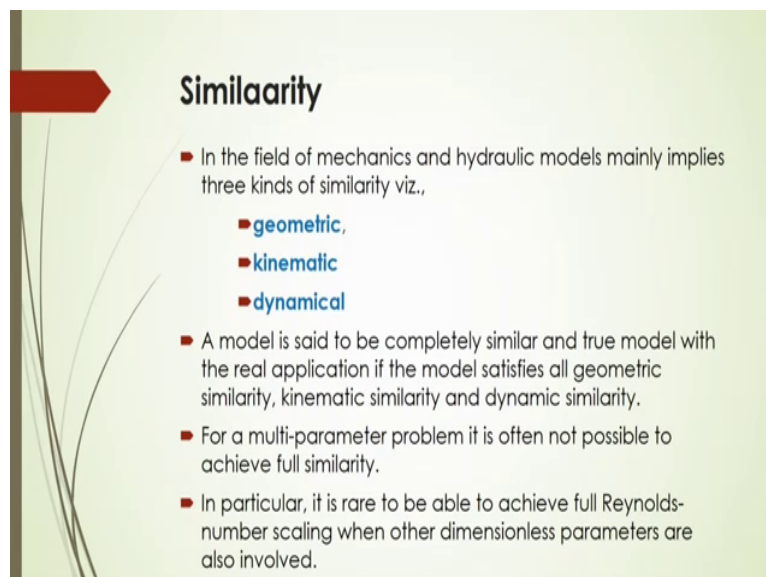
So, there will be a certain what is that length of that this small scale to large scale by which you can actually designed the equipment there. So, those that tool or you can say that length is called the similitude. So, the laboratory build system are often thought as model while, the first build of the similar systems based on the behaviour of its model beyond the laboratory frame it will be called as prototype there. So, here small scale it is that is called model and then what is that large scale it will be converted to a prototype things.

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So, in that case so, similarity the tool that describe the two systems by the same physics operating under different sets of conditions that are said to be physically similar in respect of certain specified physical quantities. And the ratio of the corresponding magnitudes of these quantities between the two systems will be the same everywhere based on the similarity law.

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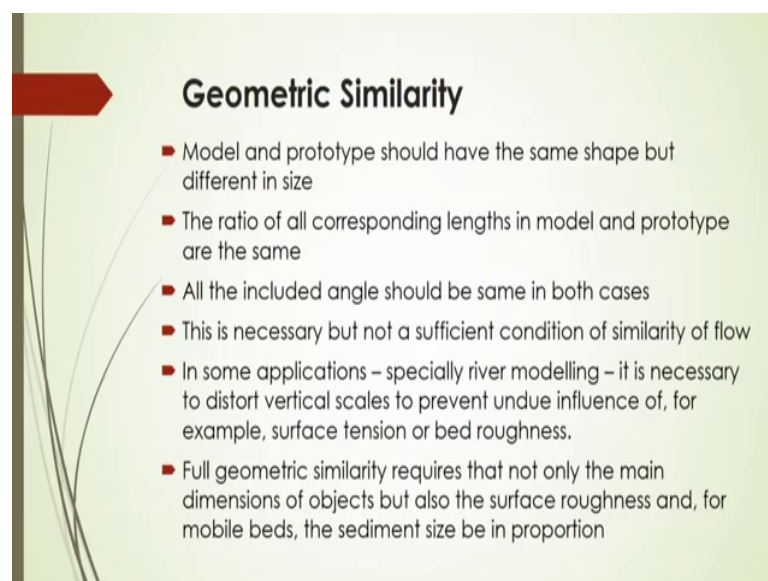
Now, you will see in the field of mechanics and hydraulic models mainly implies three kinds of similarity such as geometric kinematic and dynamical. A model is said to be completely similar and true model with the real application, if the model satisfied all geometric similarity

even kinematics similarity and also you can say dynamic similarity. So, for multi parameter problem it is often sometimes not possible to achieve full similarity. In that case you have to carefully use the particular portion of that similarity like here sometimes you will not get that other than dynamic similarity; it will not be the same as that what is that kinematics similarity.

Or, geometric similarity it will be same, but other similarity will not be same. So, in that case you have to wisely use those models to scale up from the small scale to the large scale. In particular it is rare to be able to achieve full Reynolds number sometimes scaling when other dimensionless parameters are also involved there. Because, some correlations will be developed based on the experimental data in that case that correlations will involve more than one dimensionless groups like Reynolds number, Weber number, even Moulton number, even you will see sometimes capillary number it will come.

So, all those dimensionless groups there are those are very significant whenever you are going to analyse the fluid flow phenomena. So, whenever those groups will be coming and expressed as that is related to each other; in that case some dimensionless groups will have the full scale whereas, other may not be have the same full scale range. So, that case you have to wisely use that model up to which extent you can use these models to actually used for make a prototype for that case. So, in that case you have to consider that certain range of operating conditions based on which you have to device, you have to make a device within a particular that operating condition.

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**Geometric Similarity**

- Model and prototype should have the same shape but different in size
- The ratio of all corresponding lengths in model and prototype are the same
- All the included angle should be same in both cases
- This is necessary but not a sufficient condition of similarity of flow
- In some applications – specially river modelling – it is necessary to distort vertical scales to prevent undue influence of, for example, surface tension or bed roughness.
- Full geometric similarity requires that not only the main dimensions of objects but also the surface roughness and, for mobile beds, the sediment size be in proportion

So, the prototype model will be based on that operating conditions. Now, let us discuss about that different similarity. Here what is that geometric similarity? The model and prototype should have the same shape, but different in size in this type of geometric similarity. And the ratio of all corresponding lengths in model and prototype should be same. And also if there is an angle involves in that case all the included angle should be same in both cases and this is necessary, but a not sufficient condition of similarity of flow.

So, in that case in some applications specially river modelling it is necessary to distort vertical scales to prevent undue influence of for example, of surface tension and bed roughness. So, in that case you have to carefully consider that in which directions the length to be considered. So, for the river you cannot actually consider that vertical directions of that flow there. So, it is sometimes it will be neglected based on that is horizontal movement of the fluid. And flow geometric similarity requires that not only the main dimensions of objects but, also the surface roughness and for mobile beds or the sediment size be in proportion.

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■ If  $l_1$  and  $l_2$  are the two characteristic physical dimensions of any object, then the requirement of geometrical similarity is

$$\text{Model Ratio} = \frac{l_{1m}}{l_{1p}} = \frac{l_{2m}}{l_{2p}} = l_r$$

■ where  $l_r$  is the scale factor or sometimes known as the model ratio. The suffices  $m$  and  $p$  refer to model and prototype respectively

■ Two triangles have *geometric similitude* if the triangles have equal angles or equal length ratios.

Right circular cylinders: Diagram showing a large cylinder with diameter  $D$  and height  $h$ , and a smaller cylinder with diameter  $D/2$  and height  $h/2$ .

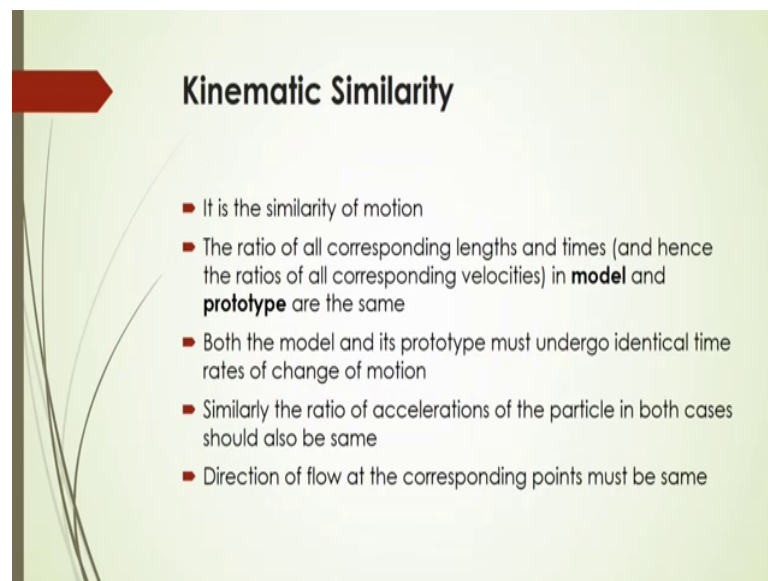
Parallelepipeds: Diagram showing a large rectangular prism with dimensions  $a$ ,  $b$ , and  $c$ , and a smaller one with dimensions  $a/2$ ,  $b/2$ , and  $c/2$ .

Triangular prisms: Diagram showing a large triangular prism with base  $a$  and height  $h$ , and a smaller one with base  $a/2$  and height  $h/2$ .

In this case you see some figure here it is given that some similarities like right circular cylinders, parallel what is that oboids shape, even sometimes triangular prisms. In that case if suppose that if  $l_1$  and  $l_2$  are the two characteristics physical dimensions of any object then the requirement of geometrical similarity will be model ratio as  $l_{1m} / l_{1p} = l_{2m} / l_{2p} = l_r$  for model  $p$  for prototype. Similarly, these ratio will be that is  $l_{2m} / l_{2p}$  and that should be is equal to  $l_r$ ;  $l_r$  is the ratio of this dimensions.

So, when  $l_r$  is the scale factor in that case sometimes when as the model ratio is considered for the scale up of the processes you have to fix it off this ratio constant. So, that by changing this  $l_1$  or  $l_2$  in such a way that the ratio should be constant there. So, accordingly you can scale up the system you can scale up the unit just by keeping the ratio constant there. And if two triangles have the geometric similitude if the triangles have equal angles or equal lengths ratio. So, this is the main important laws of this geometric similarities.

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**Kinematic Similarity**

- It is the similarity of motion
- The ratio of all corresponding lengths and times (and hence the ratios of all corresponding velocities) in **model** and **prototype** are the same
- Both the model and its prototype must undergo identical time rates of change of motion
- Similarly the ratio of accelerations of the particle in both cases should also be same
- Direction of flow at the corresponding points must be same

Now, let us consider this kinematic similarity. So, in that case you will see that motion you have to consider that motion. So, in this case the similarity of the motion to be considered for this kinematic similarity. And the ratio of all corresponding lengths and times also you can say that the ratio of the corresponding velocities are supposed to be here it will come because their lengths and times you are considering. So, accordingly what should be the velocity.

In the model and the prototype in both cases you have to consider and it should be same and both the model and its prototype of course, will undergo the identical time rates of change of motion there. And you will see that here the similarity acceleration of the particle in both cases should be maintained. And in that case the ratio should be constant for that motion and also you have to consider the direction of the flow and the corresponding points there. Because, you cannot actually compare that the direction of the vertical and horizontal at the same way.

So, in that case if you are considering the horizontal directions then all the corresponding dynamic even what is that geometric similarity should be considered in that direction. So, it will more easier way to represent, but you cannot mix up it with the horizontal and vertical directions for that. So, direction of the flow at the corresponding points must be same on that case and the paths of homologues moving particles are geometrically similar there. And also the ratio of the velocities of the homologues particles are equal useful ratios between model and particles for the kinematics are given here as well like this  $V_m$  by  $V_p$  that is velocity ratio for this model to prototype.

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- The paths of homologous moving particles are geometrically similar.
- The ratios of the velocities of homologous particles are equal.
- Useful ratios between model and prototype, for Kinematic similarity as well

Velocity :  $\frac{V_m}{V_p} = \frac{L_m/T_m}{L_p/T_p} = \frac{L_m}{L_p} \cdot \frac{T_m}{T_p} = \frac{L_r}{T_r}$

Acceleration :  $\frac{a_m}{a_p} = \frac{L_m/T_m^2}{L_p/T_p^2} = \frac{L_m}{L_p} \cdot \frac{T_m^2}{T_p^2} = \frac{L_r}{T_r^2}$

Discharge :  $\frac{Q_m}{Q_p} = \frac{L_m^3/T_m}{L_p^3/T_p} = \frac{L_m^3}{L_p^3} \cdot \frac{T_m}{T_p} = \frac{L_r^3}{T_r}$

And this will it will come  $L_m$  by  $T_m$  that  $L_m$  is the length of the length in case of model and time in case of model. So,  $L_m$  by  $T_m$  it should be as what is that velocity and similarly for prototype it should be  $L_p$  by  $T_p$ . So, in that case you will see time maybe in the model maybe higher than there what is the prototype, but this  $L_m$  by  $T_m$  that is velocity should be in such a way that the ratio should be constant whenever you are going to scale up this.

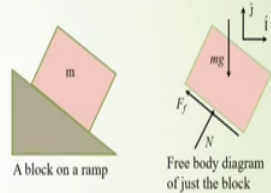
So, in that case  $L_m$  by  $L_p$  divided by  $L_m$   $T_m$  by  $T_m$  it will be so, simply  $L_r$  by  $T_r$  you have to consider here. Similarly, for acceleration it will be  $L_p$  by  $T_p$  square as for this equation and similarly for discharge you can consider this  $Q_m$  by  $Q_p$ . And it will be ultimately as per dimensions it will come  $L_r$  cube by  $T_r$  here. So, based on these ratio for this model and prototype this kinematics similarity to be considered for your device.



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**Example:** Consider a submarine modeled at 1/40th scale. The application operates in sea water at 0.5 °C, moving at 5 m/s. The model will be tested in fresh water at 20 °C. Find the power required for the submarine to operate at the stated speed.

■ The variables, which describe the system, are:



Variable	Application	Scaled mode	Units
$L$ (diameter of submarine)	1	1/40	(m)
$V$ (speed)	5	Calculate ✓	(m/s)
$\rho$ (density)	1028	998 ✓	(kg/m <sup>3</sup> )
$\mu$ (dynamic viscosity)	$1.88 \times 10^{-3}$	$1.00 \times 10^{-3}$ ✓	Pa.s (N s/m <sup>2</sup> )
$F$ (force)	Calculate	To be measured	N (kg m/s <sup>2</sup> )

Let us do an example here: consider a submarine that model that at 1 by 40th scale and the application operates in seawater at 0.5 degree centigrade. And in that case submarine is moving at 5 metre per second. In this case the model will be tested in fresh water at 20 degree centigrade. Find the power required for the submarine to operate at the stated speed. Now, in this case the variables which describe the systems are, what are the variables here? Diameter of the submarine you have to consider, velocity of the submarine, even density, even viscosity and the force.

Now, in that case what will be the applied that is the scale? Here in this case which are given respectively and in this case you have to calculate the force. And a scale mode in this case 1 by 40th you have to that is scale it up and you have to calculate then velocity you have to what is that, water condition you have to test it. So, the density of the water will be 998 here and also the viscosity it is as per this and then what should be the force that should be measured and respective units are given here.

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**Solution**

- Dimensional analysis is used to re-arrange the units to form the Reynolds number (Re) and pressure coefficient ( $C_p$ ). These dimensionless numbers account for all the variables listed above except F, which will be the test measurement. Since the dimensionless parameters will stay constant for both the test and the real application, they will be used to formulate scaling laws for the test.

$$Re = \frac{\rho V L}{\mu}$$

$$C_p = \frac{2\Delta P}{\rho V^2}, F = \Delta P L^2$$

$$V_{model} = V_{application} \times \left(\frac{\rho_a}{\rho_m}\right) \times \left(\frac{L_m}{L_a}\right)$$

$$F_{application} = F_{model} \times \left(\frac{\rho_a}{\rho_m}\right) \times \left(\frac{V_a}{V_m}\right)^2 \times \left(\frac{L_a}{L_m}\right)^2$$

This gives a required test velocity of:

$$V_{model} = V_{application} \times 2.19$$

The force measured from the model at that velocity is then scaled to find the force that can be expected for the real application:

$$F_{application} = F_{model} \times 21.9$$

The power P in watts required by the submarine is then:

$$P[W] = F_{application} \times V_{application} = F_{model} [N] \times 17.2 \text{ m/s}$$

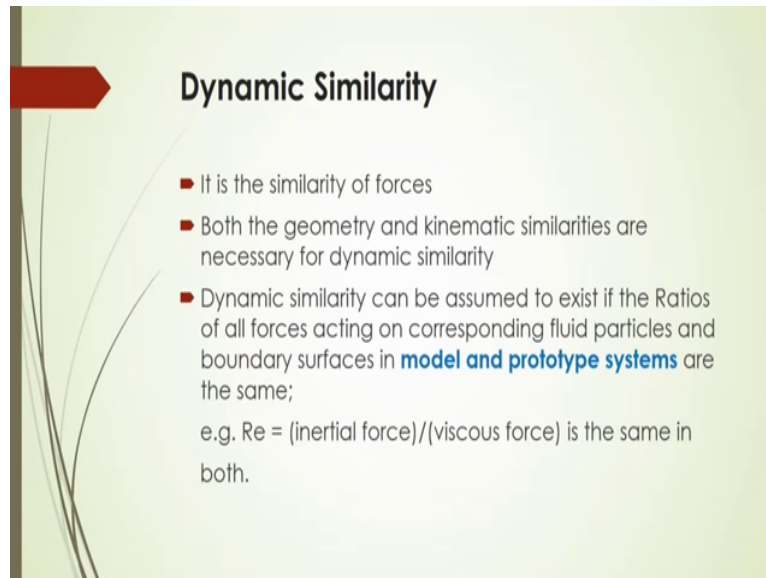
So, how can actually do this analysis here? So, first of all based on the dimensional analysis you can rearrange the units to form the Reynolds number first and the pressure coefficient there. So, these dimensionless number occur for all the variables listed above except F and which will be the test measurements. And since the dimensionless parameters will take constant for both the test and the real application there, they will be used to formulate scaling laws of the test in this case. So, as per dimension analysis based on the variables we are getting this what is that Reynolds number as one dimension this groups.

And also  $C_p$  that is what is called the pressure coefficients here and you have to then use you that similarity laws in this case, what will be the velocity model and velocity in the application in the real life. So, based on which in this case we are just rearranging this based on this similarity of this Reynolds number here, based on this Reynolds number. Then we are getting this velocity for the model that will be the velocity the application into rho a by rho m into mu m by rho mu a; a is for application m for model here. In this case the application may be it is prototype application in this case it is prototype. And also in that case then how to calculate that force that is applied here in the application part.

So, it will be simply what is that based on the definition of the force here, it will come here this F model into this one. Now, this gives the required tests velocity as V model should be equal to this as per V application into 2.19 as per this scale. And then what is that the force measured from the model at that velocities then scaled to find the force. And that can be

expected for the real application as the sake F application that will be equal to F model into 21.9. And the power P in watts required by the submarine is then it will be as per calculation here it is given. Now, F model 21.9 that is as per scaled up will be having. So, here we are getting this because this F application we are calculating from this part and then what should be the model, then you can calculate as per this.

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**Dynamic Similarity**

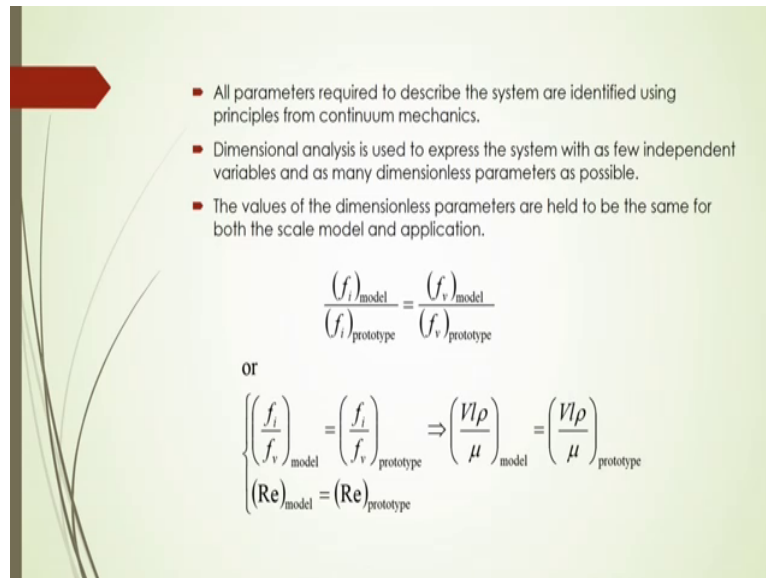
- It is the similarity of forces
- Both the geometry and kinematic similarities are necessary for dynamic similarity
- Dynamic similarity can be assumed to exist if the Ratios of all forces acting on corresponding fluid particles and boundary surfaces in **model and prototype systems** are the same;

e.g.  $Re = (\text{inertial force})/(\text{viscous force})$  is the same in both.

Now, let us consider that dynamic similarity, now it is the case for forces. So, in that case you have to consider whether this forces will be in the similar for the model and prototype case or not. Because, that you have to again fix it up the ratio of the forces in such a way that you have to make the dimensionless those in terms of forces there.

Both the geometry and the kinematics similarities are required for the analysis of this dynamic similarity. And in that case this dynamic similarity to be assumed to exist if the ratio of all forces acting on the corresponding fluid particles and the boundary surfaces in model and prototype systems. And as an example if you consider that Reynolds number this is basically the ratio of the two forces; one is inertia force and other is viscous force. So, this is ratio of inertia force to the viscous force and it should be the same in both and the analysis for dynamic similarity.

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- All parameters required to describe the system are identified using principles from continuum mechanics.
- Dimensional analysis is used to express the system with as few independent variables and as many dimensionless parameters as possible.
- The values of the dimensionless parameters are held to be the same for both the scale model and application.

$$\frac{(f_i)_{\text{model}}}{(f_i)_{\text{prototype}}} = \frac{(f_v)_{\text{model}}}{(f_v)_{\text{prototype}}}$$

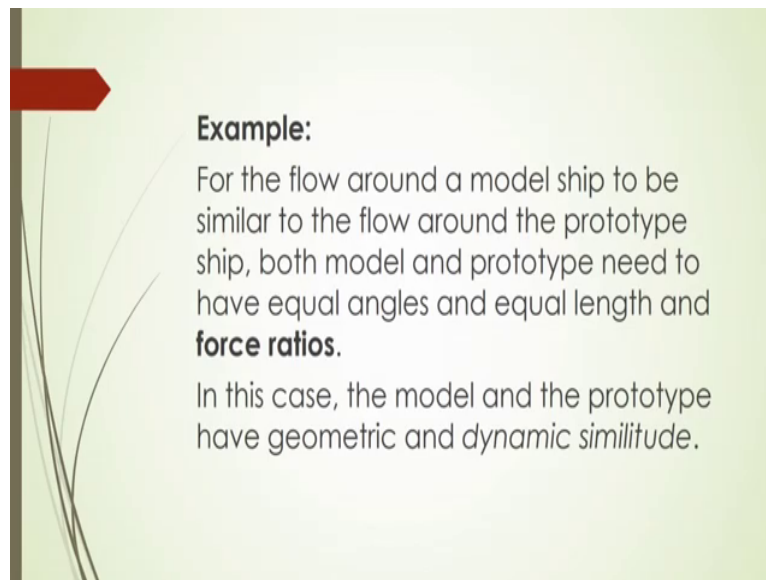
or

$$\left\{ \begin{array}{l} \left( \frac{f_i}{f_v} \right)_{\text{model}} = \left( \frac{f_i}{f_v} \right)_{\text{prototype}} \Rightarrow \left( \frac{Vl\rho}{\mu} \right)_{\text{model}} = \left( \frac{Vl\rho}{\mu} \right)_{\text{prototype}} \\ (\text{Re})_{\text{model}} = (\text{Re})_{\text{prototype}} \end{array} \right.$$

And all parameters required to describe the system are identified using principles from continuum mechanism. In that case dimensional analysis to be used to express the system with as few independent variables there and as many dimensionless parameters as possible ok. So, on that case the values of the dimensionless parameters are held to be the same for both the scale model and its application that is called prototype.

So, here you can say that  $f_i$  model and  $f_i$  prototype that will be  $f_v$  model and  $f_v$  prototype. So, in this case you can say that Reynolds number as a model and Reynolds number as prototype, in that case it is coming here  $V l \rho$  by  $\mu$  and model and  $V l \rho$  by  $\mu$  as prototype here. So, these are the ratios of these two forces here inertia forces to the viscous force and it should be remain same for the analysis.

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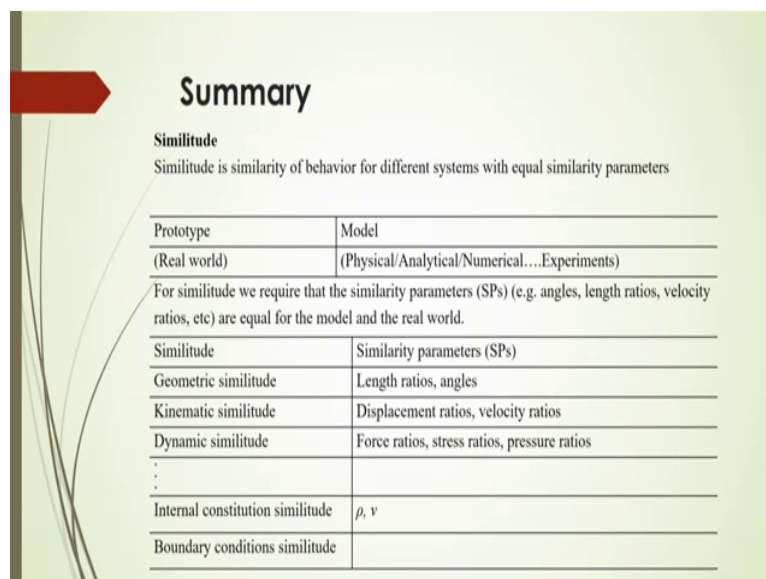
**Example:**

For the flow around a model ship to be similar to the flow around the prototype ship, both model and prototype need to have equal angles and equal length and **force ratios**.

In this case, the model and the prototype have geometric and *dynamic similitude*.

Now, for the flow around a model ship to be similar to the flow around the prototype ship, both model and prototype needs to have equal angles and equal length and force ratios; then only you can say that the model and the prototype have geometric and dynamic similarity.

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**Summary**

**Similitude**  
Similitude is similarity of behavior for different systems with equal similarity parameters

Prototype	Model
(Real world)	(Physical/Analytical/Numerical...Experiments)

For similitude we require that the similarity parameters (SPs) (e.g. angles, length ratios, velocity ratios, etc) are equal for the model and the real world.

Similitude	Similarity parameters (SPs)
Geometric similitude	Length ratios, angles
Kinematic similitude	Displacement ratios, velocity ratios
Dynamic similitude	Force ratios, stress ratios, pressure ratios
.	
Internal constitution similitude	$\rho, \nu$
Boundary conditions similitude	

Like in this case if I go to the summary of the similitudeness, in this case similarity of the behaviour for different systems with equal similarity parameters like this here prototype and model. Now, real world that is like model here physical analytical numerical experiments that. So, model to be developed based on the experimental study in the laboratory and based

on that what is that mechanistic. Or, you can say the basic theory from the basic theory like physical law and from the analytical way or numerical way and or by experiments that you have developed the model.

And based on that model by keeping the ratios of the force or keeping the similarity of that what is that forces motion even geometry then in that case you have to develop the prototype based on those experimental observations. And for similitude we required that the similarity parameters in that case like angles, lengths, ratios, velocity etcetera. And all are should be equal for the model and the real worlds scenario and similarity parameters it is called similitude. So, all the parameters will be considered as a common name as similitude. And the geometric similitudes means here length ratios angles to be considered.

Kinematics similitude it should be displacements ratio, velocity ratios even you know that flow rate ratios ah. And also you will see some other similarities like a dynamic similitude; in that case force ratios, stress ratios, pressure ratios are used to be considered here. And some other things like internal constitution of similitude in that case sometimes density, velocity should be considered.

And boundary condition similitude also to be considered because, the boundary layer whenever fluid will be flowing through the flat surface or over the surface of the any equipment there some boundary layer theory to be considers. In that case some what is that flow patterns of the boundary when turbulent or laminar conditions to be sometimes considered there in the case of model as well as prototype.

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**Forces acting on the prototype and the model**

- In order to make the flow conditions similar, the respective ratios of the corresponding forces acting on the prototype and the model should be equal.
- The forces acting on the flow element are:

Elasticity force:  $F_E = KL^2$

Gravity force:  $F_G = mg = \rho L^3 g$

Inertial force:  $F_I = ma = \rho L^3 \frac{L}{T^2} = \rho \left(\frac{L}{T}\right)^2 L^2 = \rho u^2 L^2$

Pressure force:  $F_P = \Delta p A = \Delta p L^2$

Surface tension force:  $F_S = \sigma L$

Viscous force:  $F_V = \mu \left(\frac{du}{dy}\right) A = \mu \left(\frac{u}{L}\right) L^2 = \mu u L$

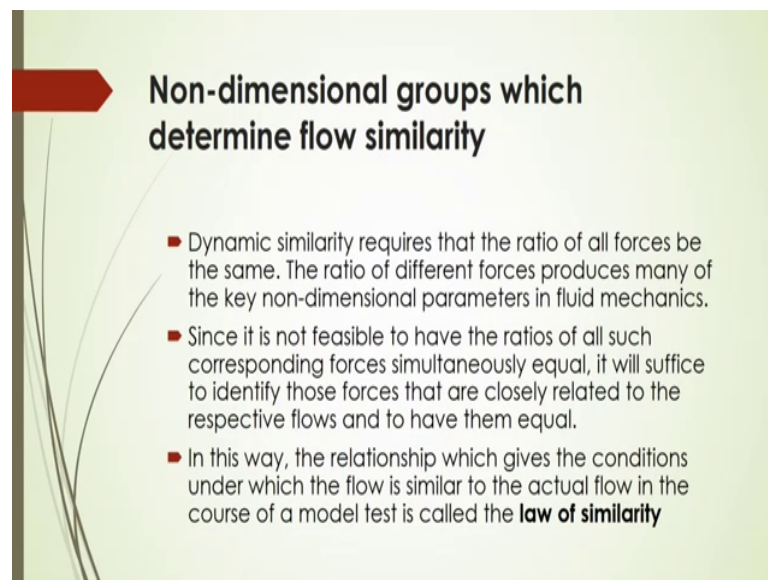
Now, we will discuss what should be the different forces that acting on the prototype and the model. Because, whenever you are going to analyse the model and prototype based on the forces then you have to know how what will be the general mode general forces in the fluid related processes are generally taking part, that those courses to be actually no in this case. So, in order to make the flow condition similar the respective ratios of the corresponding forces acting on the prototype and the model that should be equal.

And those forces acting on the flow element are like elasticity force that is defined as  $F_E$  is equal to  $KL^2$  and gravity force like your  $F_G$  that will be  $m$  into  $g$ . That means, as a dimensions  $\rho L^3 g$ , inertial force like  $F_I$  that will be defined as mass into acceleration that will be  $\rho L^3$  into  $L$  by  $T^2$ . And finally, it is coming  $\rho u^2 L^2$ ; similarly pressure force is very important. In that case it will be defined as  $\Delta p$  into cross sectional area that will be called as pressure force and it is defined as  $\Delta p$  into  $L^2$  that is the dimensions for the cross sectional area.

The surface tension force it is simple, that is surface tension into length because the surface tension is nothing, but the tension acting on the surface per unit length. So, that is why  $\sigma$  into  $L$ ; that means, surface tension that is force per unit length into length that will be your total force of the surface tension. And the viscous force is simply as per Newton's law that  $\mu$  is into  $\frac{du}{dy}$  this  $\frac{du}{dy}$  is called the shear rate and then the proportionality constant it is called as  $\mu$  that  $\mu$  is called viscosity.

So, viscous force it will be  $\mu \frac{du}{dy}$  then into cross sectional area because this  $\mu \frac{du}{dy}$  it is called stress. Stress is nothing, but force per area then if you multiply by area then it will come as viscous force here. So finally, it will be coming as  $\mu u$  by  $L$  into  $L^2$  that will be is equal to  $\mu u$  into  $L$ .

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**Non-dimensional groups which determine flow similarity**

- Dynamic similarity requires that the ratio of all forces be the same. The ratio of different forces produces many of the key non-dimensional parameters in fluid mechanics.
- Since it is not feasible to have the ratios of all such corresponding forces simultaneously equal, it will suffice to identify those forces that are closely related to the respective flows and to have them equal.
- In this way, the relationship which gives the conditions under which the flow is similar to the actual flow in the course of a model test is called the **law of similarity**

Now, known dimensional groups which determined this close similarities are like this here in that case dynamic similarities very important because, those dynamic similarity actually based on the force. So, dynamic similarity requires that the ratio of all forces to be the same, the ratio of different forces produces many of the key non-dimensional parameters in the fluid mechanics. So, all those dimensionless parameters to be actually analysed based on the method that is given in your dimensional analysis like Buckingham pi theorem and also Rayleigh's method.

Either one have to use to get this dimensionless parameters based on the forces acting on the particular process systems. And to design those process systems or process unit then you have to scale it up from the small scale or model to the prototype based on this force. And then you have to before going to that similarity laws analysis that you have to make the source in terms of dimensionless groups. Now, in this case if it is not feasible to have the ratios of all such corresponding forces; in that case you have to consider that or you have to identify some forces that are closely related to the respective flows and to have them equal. And in this way



the relationship which gives the conditions under which the flow is similar to the actual flow; in the course of a model test that will be called as law of similarity.

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**Reynolds number**

- only the viscous force and inertia force are of importance where the compressibility of the fluid may be neglected and in the absence of a free surface, e.g. where fluid is flowing in a pipe, an airship is flying in the air or a submarine is navigating under water.

Reynolds number (Re) =  $\frac{\text{Inertia force}}{\text{Viscous force}} = \frac{F_I}{F_V} = \frac{(\rho L^2 u)u}{\mu(u/L)L^2} = \frac{\rho u L}{\mu}$

Osborne Reynolds (1842/1912), British Scientist and Mathematician

**Consequently, when the Reynolds numbers of the prototype and the model are equal the flow conditions are similar.**

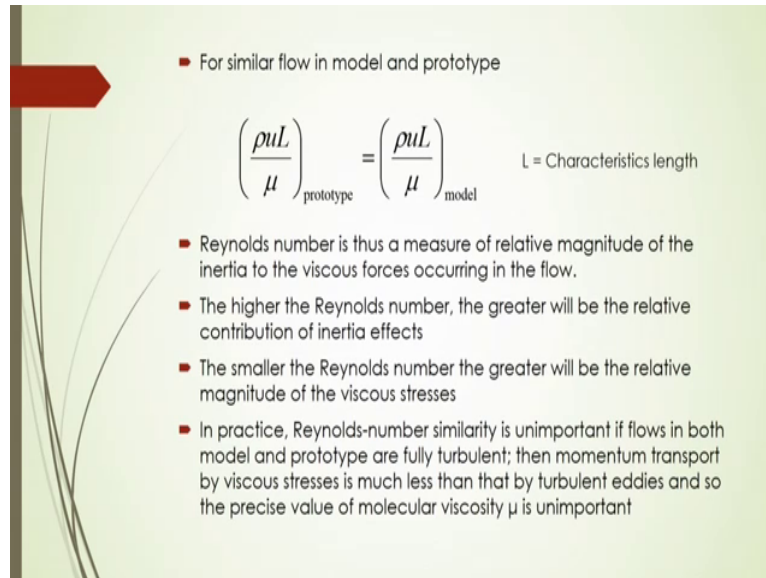
So, in that case you have to consider that different forces. Now, based on the different forces some significant groups are coming or significant numbers it is called since it does not have any unit so, we are considering as a number. So, Reynolds number like what is that Weber number, even you know that it is called what is that Reynolds number, Weber number, Capillary number, even Froude number. So, all those numbers are coming into account for the analysis of the flow; only the viscous force and the inertia force are of importance for the compressibility of the fluid may be neglected and in the absence of a free surface.

So, in that case this two forces are very important on the viscous force and inertia force. Example, where the fluid is flowing in a pipe even when flow they are in air ship whenever it will be flying in the air or submarine is navigating under water in that case there the flow of that is submarine. And because of which there will be some drag force will be acting because of that fluid is just making a friction over the surface of the submarine. So, in that case what should be the force is important for this type of examples that is called Reynolds number. So, Reynolds number is defined as the inertia force to the viscous force.

So, this is actually F I by F v is denoted. So, we can write this what is that inertia force as rho L square u into u by here viscous force is mu u L mu into u by L into L square as it is given in the earlier slides. Then accordingly you just substitute and have the simplification of this

form of this  $\rho u L$  by  $\mu$ . So, it is called Reynolds number. So, in that case when the Reynolds number of the prototype and the model are equal then you can say that flow conditions will be similar.

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- For similar flow in model and prototype

$$\left( \frac{\rho u L}{\mu} \right)_{\text{prototype}} = \left( \frac{\rho u L}{\mu} \right)_{\text{model}} \quad L = \text{Characteristics length}$$

- Reynolds number is thus a measure of relative magnitude of the inertia to the viscous forces occurring in the flow.
- The higher the Reynolds number, the greater will be the relative contribution of inertia effects
- The smaller the Reynolds number the greater will be the relative magnitude of the viscous stresses
- In practice, Reynolds-number similarity is unimportant if flows in both model and prototype are fully turbulent; then momentum transport by viscous stresses is much less than that by turbulent eddies and so the precise value of molecular viscosity  $\mu$  is unimportant

For similar flow in that case in model and prototype we can write here this Reynolds number the prototype and the Reynolds number the model. So,  $L$  here is the characteristic length it may be based on that design of the equipment, sometimes some equipment will be considered for this characteristic length as diameter. And some equipments will consider the characteristic length as the what is length there.

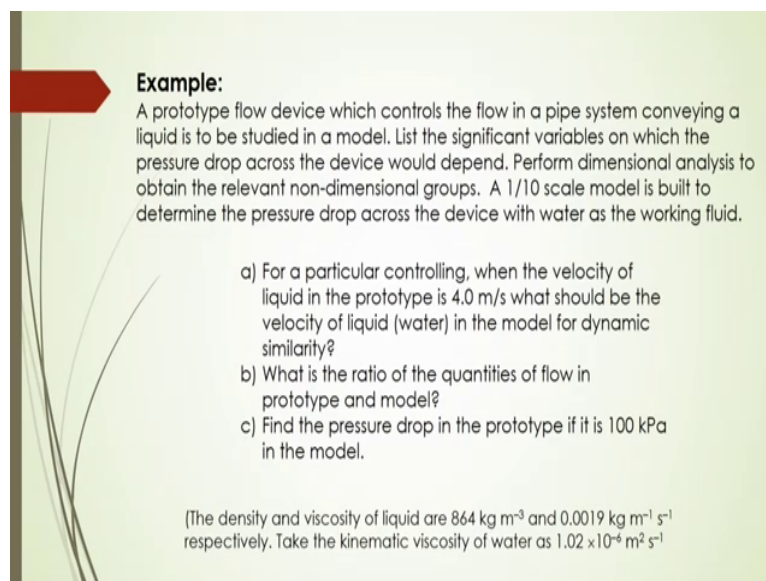
So, Reynolds number in this case will be a measure of relative magnitude of the inertia to the viscous force that will account the flow characteristics in the pipe. And based on which you are just applying the similarities law and the higher the Reynolds number you will see that greater will be the relative contribution of the initial effects. But, you can do either way either see if you want to get the higher Reynolds number sometimes you can use that very low viscous fluids in that case you can get more velocity, when other force inertia force will be higher. And also if you are using more viscous fluids in that case inertia force will be lower to get this constant Reynolds number there.

So, to get the higher Reynolds number you have you can increase the velocity, you can increase the density, you can increase the length of the or characteristic length of the system or you can lower the or you can use the fluid which have the lower viscosity. So, in that case

the higher Reynolds number the greater will be the relative contribution of the initial effects. And, the smaller the Reynolds number in that case you can use higher viscosity and also you can use lower density denser liquid low velocity, low and small characteristic lengths. So, in that case smaller the Reynolds number the greater will be the relative magnitude of the viscous stresses.

So, either way you can keep this ratio constant by increasing viscous force or decreasing what is that inertia force there. So, in practice Reynolds number similarity is unimportant if the flow is both flow in both the model and prototype are fully turbulent. Sometimes in turbulent condition you will see will not have that what is that viscous effect there. So, the momentum transport by viscous stresses is much less than that by the turbulent eddies and so, the precise value of molecular viscosity  $\mu$  is unimportant in that case. So, it will not be a good way to design the equipment at that condition of that higher Reynolds number that is turbulent condition there.

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**Example:**  
A prototype flow device which controls the flow in a pipe system conveying a liquid is to be studied in a model. List the significant variables on which the pressure drop across the device would depend. Perform dimensional analysis to obtain the relevant non-dimensional groups. A 1/10 scale model is built to determine the pressure drop across the device with water as the working fluid.

- For a particular controlling, when the velocity of liquid in the prototype is 4.0 m/s what should be the velocity of liquid (water) in the model for dynamic similarity?
- What is the ratio of the quantities of flow in prototype and model?
- Find the pressure drop in the prototype if it is 100 kPa in the model.

(The density and viscosity of liquid are  $864 \text{ kg m}^{-3}$  and  $0.0019 \text{ kg m}^{-1} \text{ s}^{-1}$  respectively. Take the kinematic viscosity of water as  $1.02 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$ )

Let us do an example in this case for analysis this: a prototype flow device which controls the flow in a pipe system that conveying a liquid is to be studied in a model. And list the significant variables on which the pressure drop across the device which will depend and perform dimensional analysis to obtain the relevant non-dimensional groups. And in that case a 1 by 10's scale model to be built to determine the pressure drop across the device with water as the working fluid.

So, in this case for a particular controlling in that case when the velocity of the liquid in the prototype will be 4.0 metre per second, then what should be the velocity of the liquid in the model for dynamic similarity. And what is the ratio of the quantities of the flow in prototype and model? And also find the pressure drop in the prototype if it is 100 kilo Pascal in the model. The density and the viscosity of the liquid to be considered as 864 kg per meter cube and 0.0019 kg per meter per second respectively. And kinetic viscosity of the water to be considered  $1.02 \times 10^{-6}$  metre square per second. So, based on this problem you have to analyze and we have to find out the velocity of the liquid.

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**Solution**

- The pressure drop  $\Delta p$  is expected to depend upon the gate opening  $h$ , the overall depth  $d$ , the velocity  $V$ , density  $\rho$  and viscosity  $\mu$ .
- Relevant variables:  $\Delta p, h, d, v, \rho, \mu$
- Dimensions are:
- Number of variables:  $n = 6$
- Number of independent dimensions:  $j = 3$  (M, L and T)
- Number of non-dimensional groups:  $n - j = 3$
- Choose  $j (= 3)$  repeating variables:  $d, v, \rho$

$\Delta p$	$ML^{-2}T^{-2}$
$h$	$L$
$d$	$L$
$v$	$LT^{-1}$
$\rho$	$ML^{-3}$
$\mu$	$ML^{-1}T^{-1}$

Now, let us have the solution for this the pressure drop  $\Delta p$  is expected to depend upon the what is that gate opening  $h$ , the overall depth  $d$ , the velocity, density  $\rho$  and the viscosity  $\mu$  as per this what is that model here is an example that based on this example. So, in that case relevant variable should be  $\Delta p, h, v, \rho$  and  $\mu$  and dimensions are what is that given here in the box that for this  $\Delta p, h, d, v, \rho, \mu$  this dimensions are given. So, number of independent dimensions are 3, that is fundamental dimensions that is mass length and time number of non-dimensional group it is  $n - j$  is equal to 3. So, as per that Buckingham pi theorem you have to identify the repeating variables such as this  $d, v$  and  $\rho$ .

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Form dimensionless groups by Buckingham method by non-dimensionalising the remaining variables:  $\Delta p$ ,  $h$  and  $\mu$ .

$$\pi_1 = \frac{\Delta p}{\rho v^2}$$

$$\pi_2 = \frac{h}{d}$$

$$\pi_3 = \frac{\mu}{\rho v d}$$

$$\Rightarrow \pi_3' = (\pi_3)^{-1} = \frac{\rho v d}{\mu}$$

$$\pi_1 = f(\pi_3, \pi_3')$$

i.e.,  $\frac{\Delta p}{\rho v^2} = f\left(\frac{h}{d}, \frac{\rho v d}{\mu}\right)$

And then do the dimensional analysis from the dimensionless groups by Buckingham method by non-dimensionalising the remaining variables  $\Delta p$ ,  $h$  and  $\mu$ . So, in that case we are getting this dimensionless group sets  $\pi_1$ ,  $\pi_2$  and  $\pi_3$  based on this dimensional analysis of Buckingham pi theorem. And then we can have this  $\pi_3$  that is as per convenient for this  $\pi_3$  as inverse of this  $\pi_3$  here as  $\pi_3'$  it will be simply the significant group as what is that Reynolds number  $\rho v d$  by  $\mu$ .

So, if you make a what is that relationship among the groups here like  $\pi_1$  should be as a function of  $\pi_3$  and  $\pi_3'$ . So, it will come Reynolds number sorry this is  $\Delta p$  by  $\rho v^2$  it is Euler number is a function of this  $h$  by  $d$  ratio and the Reynolds number there.

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Dimensional similarity requires that all non-dimensional groups should be the same in model and prototype;

i.e.,

$$\pi_1 = \left( \frac{\Delta p}{\rho v^2} \right)_{\text{prototype}} = \left( \frac{\Delta p}{\rho v^2} \right)_{\text{model}}$$

$$\pi_2 = \left( \frac{h}{d} \right)_{\text{prototype}} = \left( \frac{h}{d} \right)_{\text{model}}$$

$$\pi_3 = \left( \frac{\rho v d}{\mu} \right)_{\text{prototype}} = \left( \frac{\rho v d}{\mu} \right)_{\text{model}}$$

From the last dimensionless group

(a)

$$\frac{v_p}{v_m} = \frac{(\mu/\rho)_p}{(\mu/\rho)_m} = \frac{0.0019/864}{1.02 \times 10^{-6}} \times \frac{1}{10} = 0.215$$

Therefore

$$v_m = \frac{v_p}{0.215} = \frac{4.0}{0.215} = 18.60 \text{ m/s}$$

Subscripts m for model; p for prototype

Now, if we are going to analyze that the plan and model and prototype, in that case dimensional similarity will be required and also all non-dimensional groups should be the same in this model and prototype. So, in this case we can say that for pi 1 group that is rho delta p by rho v square this is Euler number. So, in that case it will be in the prototype and the model what should be that. Similarly, pi 2 this ratio prototype and model what should be that, even for Reynolds number what should be the prototype and what should be the model that should be the same in model and prototype.

So, based on these similarity laws we can say that based on this here then we can say that v p by v m ah. So, we can have this mu by rho by mu by what is that rho here in this case then we can say that it will be coming as 0.215. Therefore, what should be the v m as we know that the prototype is given as 4.0 and then after substitution of this value and simplification it is coming 18.60. So, if we know that prototype value what should be the model actually value or once you know the model value what should be the prototype value based on this equation you can calculate. So, in this case this m and p are the model and prototype as for this.

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(b) Flow quantity ratio is

$$\frac{Q_p}{Q_m} = \frac{v_p}{v_m} \left( \frac{d_p}{d_m} \right)^2 = 0.215 \times 10^2 = 21.5$$

(c) Pressure drop ratio is

$$\pi_1 = \left( \frac{\Delta p}{\rho v^2} \right)_{\text{prototype}} = \left( \frac{\Delta p}{\rho v^2} \right)_{\text{model}}$$
$$\Rightarrow \frac{(\Delta p)_p}{(\Delta p)_m} = \frac{\rho_p}{\rho_m} \left( \frac{v_p}{v_m} \right)^2 = \frac{864}{1000} \times 0.215^2 = 0.04$$
$$\therefore (\Delta p)_p = 0.04 \times (\Delta p)_m = 0.04 \times 100 = 4.0 \text{ kPa}$$

And then flow quantity ratio should be also same that case  $Q_p$  by  $Q_m$  should be is equal to  $v_p$  by  $v_m$  to  $d_p$  by  $d_m$  square. So, based on this model and prototype value we can have this  $Q_p$  by  $Q_m$  should equal to 21.5. And pressure drop ratio as per this Euler number equality for prototype and model we are having this  $\Delta p_p$  for the prototype; it would be is equal to 0.04 into  $\Delta p_m$ .

As per this here this from this  $\pi_1$  dimensionless groups, then we are we are getting this ratio of this pressure drop here as 0.04. So, from this ratio we can calculate this prototype pressure drop that will be is equal to 4.0 kPa. So, we can use this actually examples to analyze that how to actually get the value for a prototype based on the model parameters even sometimes from the prototype model how to get the model parameters to their.



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**Froude number**

■ When the resistance due to the waves produced by motion of a boat (gravity wave) is studied, the ratio of inertia force to gravity force is important

$$\frac{\text{Inertia force}}{\text{Gravity force}} = \frac{F_I}{F_G} = \frac{\rho^2 u^2 L^2}{\rho L^3 g} = \frac{u^2}{gL}$$

■ In general, in order to change  $v^2$  above to  $v$  as in the case for **Re**, the square root of  $u^2/gL$  is used. This square root is defined as the Froude number **Fr**,

$$Fr = \frac{u}{\sqrt{gL}}$$

William Froude, 1810-1879, British Scientist

Let us do another dimensionless number it is called Froude number, it is very important in this case here resistance due to the waves produced by the motion of a boat that is a gravity wave. And the ratio of inertia force to the gravity force are very important in this case. And this ratio that is inertia force to gravity force it is called the Froude number. So,  $F_I$  by  $F_G$  that will be is equal to  $u$  square by  $gL$ . So, it is called Froude number. So, in order to change the velocity about to be as in the case of Reynolds number the square root of the  $u$  square by  $gL$  is used. So, this square root is defined as the actually Froude number which is mostly used as a definition of this Froude number here.

So, this Froude number will be used to again for the similarity analysis as per earlier example here. Now, if we consider that Froude's law of similarity in that case again then  $u$  by root of a  $gL$  for the model it should be same for the prototype also. The equality of the Froude's number for two kinds of flow can ensure that dynamic similarity only if gravitational force are the prime importance there. And the case where the free surface of the fluid plays a major role in that case this gravitational force is important; flows in an what is that open channel with wave that is gravitational force is important.



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### Froude's law of similarity

$$Fr = \left( \frac{u}{\sqrt{gL}} \right)_{\text{model}} = \left( \frac{u}{\sqrt{gL}} \right)_{\text{prototype}}$$

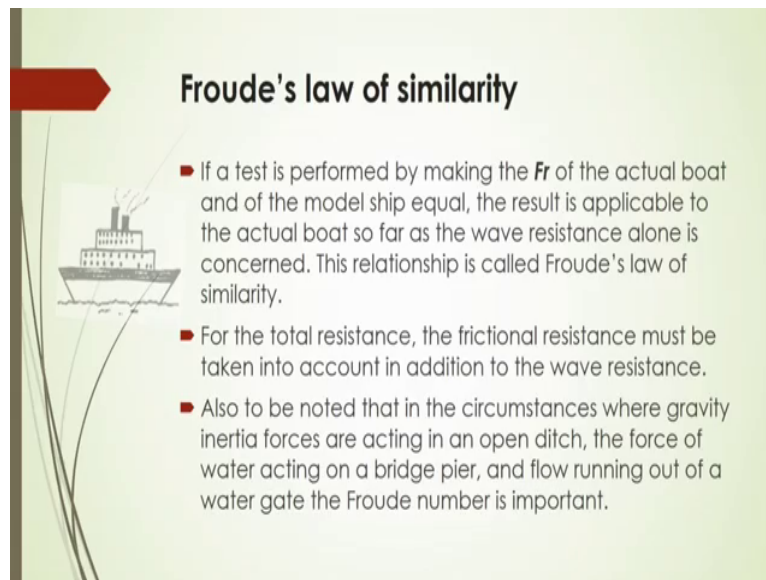
- The equality of Froude's number for two kinds of flow can ensure dynamical similarity only if gravitational force are of prime importance
  - The cases where the free surface of the fluid plays a major role as
    - Flow in an open channel with wave
    - Flow of jet from an orifice
    - Flow over the spillway of a dam
    - Wave motion on the surface of water by ship

**Note that:** Both conditions (geometric and kinematic) of dynamic similarity may not be satisfied at the same time. The Re and the Fr cannot be same simultaneously for two flows because the variation of force coefficient with Reynolds number represent the effect of skin friction while the variation with Froude's number represents the contribution of wave making resistance

Flow of jet from an orifice, flow over the spillway of a dam, wave motion on the surface of the water by ship. Those are very important feature to considered for that gravitational force is a prime factor. In this case you have to remember that both conditions geometric and kinematic of dynamic similarity may not be satisfied at the same time.

The Reynolds number and the Froude number cannot be same simultaneously for the two flows because, the variation of force coefficient with Reynolds number represents the effect of skin while, the variation with Froude's number represents the contribution of the wave making resistance there. So, that is why these are geometric and kinematics similarity may not same in all cases whenever you are going to analyze the similarity based on the Froude law.

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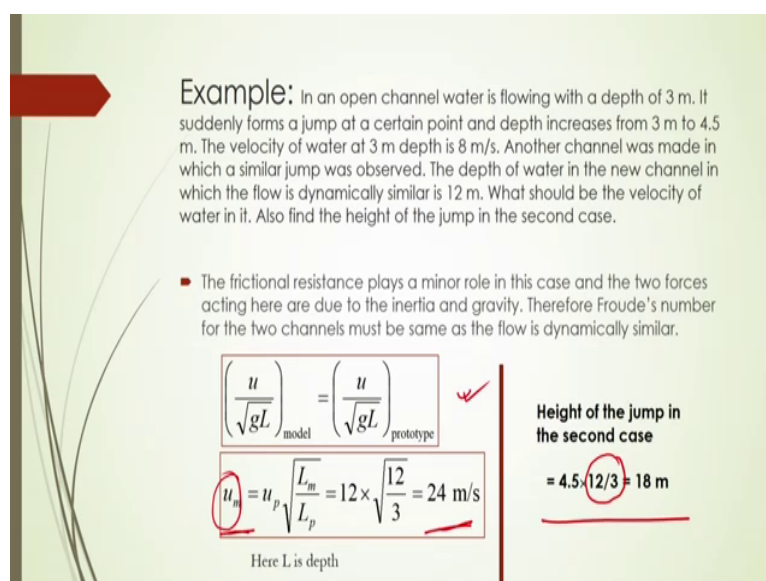


### Froude's law of similarity

- If a test is performed by making the  $Fr$  of the actual boat and of the model ship equal, the result is applicable to the actual boat so far as the wave resistance alone is concerned. This relationship is called Froude's law of similarity.
- For the total resistance, the frictional resistance must be taken into account in addition to the wave resistance.
- Also to be noted that in the circumstances where gravity inertia forces are acting in an open ditch, the force of water acting on a bridge pier, and flow running out of a water gate the Froude number is important.

Now, if a test if you are performing by making the Froude number of the actual boat and the model ship equal. So, in that case the results is applicable to actual boat. So, far as the wave resistance alone the alone will be considered there. And this relationship is called the Froude's law similarity. For the total resistance the frictional resistance must be taken into account in addition to the wave resistance there. Also to be noted that in the circumstances where, that gravity inertia forces are acting in an open ditch, the force of water acting on the bridge pier and flow running out of a water gate the Froude number should be important there.

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**Example:** In an open channel water is flowing with a depth of 3 m. It suddenly forms a jump at a certain point and depth increases from 3 m to 4.5 m. The velocity of water at 3 m depth is 8 m/s. Another channel was made in which a similar jump was observed. The depth of water in the new channel in which the flow is dynamically similar is 12 m. What should be the velocity of water in it. Also find the height of the jump in the second case.

- The frictional resistance plays a minor role in this case and the two forces acting here are due to the inertia and gravity. Therefore Froude's number for the two channels must be same as the flow is dynamically similar.

$$\left(\frac{u}{\sqrt{gL}}\right)_{\text{model}} = \left(\frac{u}{\sqrt{gL}}\right)_{\text{prototype}}$$
$$u_m = u_p \sqrt{\frac{L_m}{L_p}} = 12 \times \sqrt{\frac{12}{3}} = 24 \text{ m/s}$$

Here L is depth

**Height of the jump in the second case**  
 $= 4.5 \times (12/3) = 18 \text{ m}$

Let us do an example here: in an open channel water is following with a depth of 3 metre. It suddenly forms jump at a certain point and depth increases from 3 metre to 4.5 metre. And the velocity of the water at 3 metre depth is 8 metre per second, another channel whenever you considering that should be made in which a similar jump was observed. And the depth of water in the new channel in which the flow is dynamically similar and it will be 12 metre in depth. And also then in that case particular system what should be the velocity of the water in it and also find out the height of the jump in the second case.

So, in this case you have to analyze the frictional resistance. This frictional resistance would be the minor role in this case and the two forces acting here are due to the inertia and the gravity should be the measure force. And also a Froude's number for the two channels must be same as dynamic similarity. So, we can have this relationship here  $u$  by root over  $gL$  by model that will be is equal to  $u$  by root over  $gL$  prototype. So, we can have this based on this similarity. So, you can say that  $u_m$  should be is equal to  $u_p$  into root over  $L_m$  by  $L_p$  and after substitution of the value then we are getting 24 metre per second here, this  $u_m$  that is model velocity here as are this.


And based on these then you can calculate what is the height of the jump in the second case then 4.5 into here again 12 by 3 here. So, it is coming as a what is that 18 metre. So, this the Froude's number you can easily actually used for this type of examples for this considering this by considering this Froude number based on this dynamic similarity.

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### Weber number

- When a moving liquid has its face in contact with another fluid or a solid, the inertia and surface tension forces are important:
 
$$\frac{\text{Inertia force}}{\text{Surface tension}} = \frac{F_i}{F_s} = \frac{\rho v^2 L^2}{TL} = \frac{\rho v^2 L}{T}$$
- In this case, also, the square root is selected to be defined as the Weber number **We**

$$We = v\sqrt{\rho L/T}$$
- **We** is applicable to the development of surface tension waves and to a poured liquid.



Moritz Weber  
(1871–1951),  
German  
Professor

And another important number is called Weber number, whenever you are considering any suppose droplet or bubble or disperse phase in the continuous medium like gaseous dispersed in the continuous medium of liquid in that case how this bubble or drop is formed. What in the maximum size of the bubble or droplet that is that will be stabilized in the liquid medium. So, in that case you have to consider that Weber number, that Weber number actually basically based on inertia force and the surface tension force. Because, whenever you are producing in a drop or a bubble that it should be stabilized based on the surface tension force.

So, to get the stabilized form of this droplet or particles in a medium then you have to consider this two forces there. And based on these two forces Weber number is defined and there will be a certain value of critical Weber number based on which you can say that whether this bubble or a droplet fluid is stabilized there or not. And the size would be stabilized or not and whether it should be spherical or not.

So, when a moving liquid has its face in contact with another fluid also or a solid there also it is important Weber number because, their surface tension effect is coming as well as inertia force. So, inertia and surface tension forces are defined here as that inertia force as  $\rho v^2 L$  and the surface tension force is as  $\sigma$  or sometimes  $T$  is noted down.

So, if we consider these two ratios we are getting this value as  $v$  into root over  $\rho L$  by  $T$ . So, in this case square root is selected to be defined as the Weber number. So, it will be coming like this and also Weber number is applicable to the development of the surface tension waves. And also to a poured liquid where droplet and bubbles flow phenomena are important. Some examples like I have told that there will be suppose absorption of gases in a liquid medium or suppose when the gas liquid reactions where gas is dispersed in the liquid medium as a disperse flow is a bubbles. So, you have to know how this bubbles are forming whether the bubbles are uniform in shape or not when this bubbles will come in maximum size and stabilized form.

What should be the Weber number based on which you can say, if critical Weber number it is coming 1.2 you see that in air water system that bubbles will be forming stabilized. So, there so, in that case Weber number is very important.

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## Mach number

- When a fluid flows at high velocity, or when a solid moves at high velocity in a fluid at rest, the compressibility of the fluid can dominate so that the ratio of the inertia force to the elasticity force is then important


$$\frac{\text{Inertia force}}{\text{Elastic force}} = \frac{F_i}{F_e} = \frac{\rho v^2 L^2}{KL} = \frac{v^2}{K/\rho} = \frac{v^2}{a^2}$$

- Again, in this case, the square root is selected to be defined as the Mach number  $M$ ,

$$M = v/a$$

$M < 1$  Subsonic flow  
 $M = 1$  Sonic Flow and  
 $M > 1$  Supersonic flow

When  $M = 1$  and  $M < 1$  and  $M > 1$  zones are coexistent, the flow is called transonic flow.

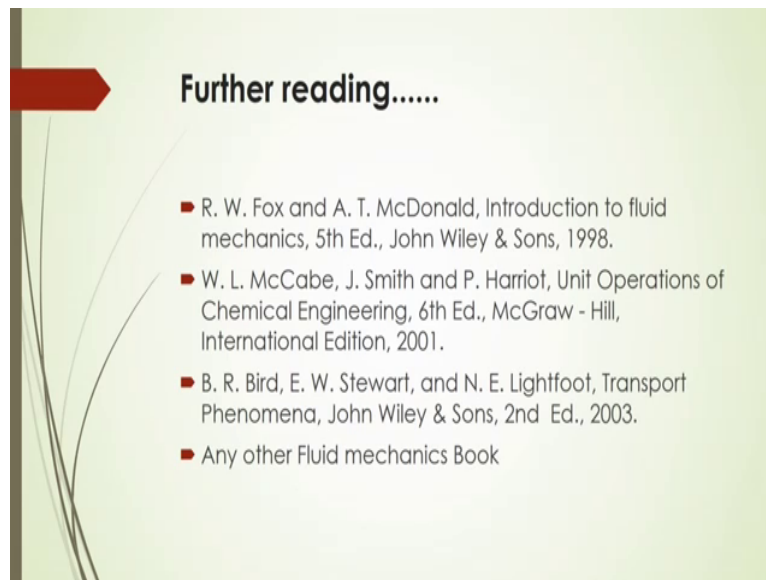


Ernst Mach (1838-1916)  
Austrian physicist

Another important number is Mach number, when a fluid flows at high velocity or when a solid moves at high velocity in a fluid at rest the compressibility of the fluid can dominate. So, that the ratio of the inertia force to the elasticity force then it will be important there. And in that case inertia force to the elastic force would be coming as  $v$  square by a square and again in this case the square root is been selected to be defined as the Mach number here. So, number will be is equal to  $v$  by  $a$ .

If Mach number is less than 1 it will be called as the subsonic flow and if  $M$  is equal to 1 to be called sonic floor. And if it is greater than 1 then it will be called supersonic flow. When  $M$  is equal to 1 and  $M$  less than 1 and  $M$  is greater than 1 zones are coexistent, the flow is called transonic flow. So, in the zone where this three ah; that means,  $M$  is equal to 1,  $M$  less than equals to 1 and  $M$  is greater than 1 zones are coexistent in that case it will be called as transonic flow.

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So, I will suggest you to read further regarding this what is that similarity law you can how to apply this similarity law from this text books. So, again some other books reference books also you can follow.

So, in this lecture we have actually learnt how to apply the different similarity law like geometric similarity law, what is that dynamic similarity, kinematics similarity. And what are the different forces those forces are important and how the ratios are coming and how to define those ratios. And where particularly those forces are applicable based on the similarity law and also how to define design the systems based on the similarity law from scale to lab scale or model scale to the prototype scale. So, I think this will be helpful for better understanding of this fluid flow phenomena and also design of the equipment based on the fluid flow phenomena and you can apply the similarity law.

Of course, before going to that applying similarity law you have to know the dimensional analysis. Because, the this dimensional analysis will give you the formation of dimensionless groups. And the dimensionless groups will be the important actually one important measure, that is the consideration for analysis of that model and prototype based on the similarity law. So, without that dimensionless groups it will not be actually acceptable for this analysis of this similarity laws to design the equipment.

So, thank you for this lecture today. Next class, next lecture we will be discuss something else that will be I think in different model. We will discuss like compressible flow and how it will be a flowing and how to be different from the other flow.

So thank you.