Fluid Flow Operations
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Module - 09
Lecture - 23
Dimensional Analysis and Law of Similarity: Part 2-Dimensional Analysis: Buckingham's π Theorem

Welcome to a massive open online course on Fluid Flow Operations. We are discussing the module 9 of Dimensional Analysis and Law of Similarity. In this lecture as a part 2 we will discuss on the Dimensional Analysis, how it can be done by Buckingham's pi theorem. So, these lectures includes introduction Buckingham's pi method of dimensional analysis and some examples.

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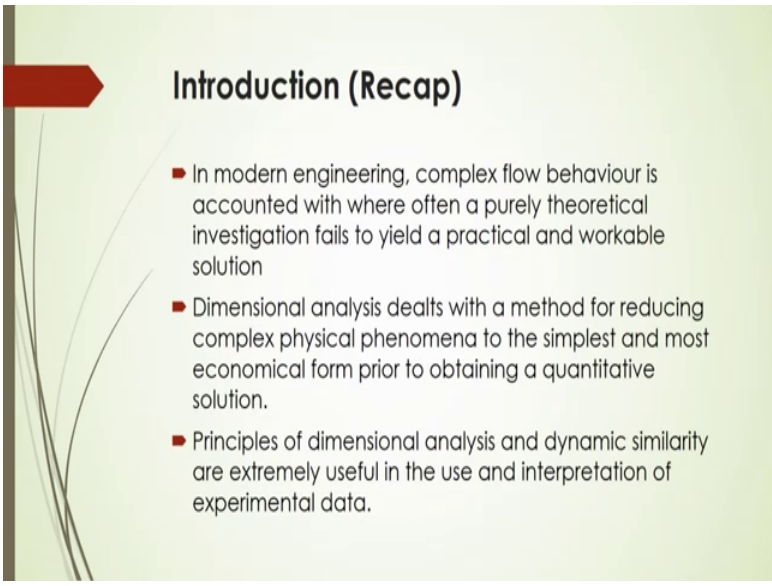

Now, we have already discussed in the previous lecture that what is that dimensional analysis and we know that in the modern engineering there are so, many complex flow behaviour involved about the process. So, in that case that complex flow behaviour accounted with the often sometimes purely theoretical investigation which sometimes fails to yield a practical and also the workable solution there.

And in this case this dimensional analysis is very important and deals with a method of reducing complex physical phenomena to the simplest and most economical form, that will give you the quantitative solution by this dimensional analysis. And, also if I talk about the principle of dimension analysis you have to consider the different variables and based on the variables you have to obtain a dimensionless groups. And, from those dimensionless groups sometimes you have to compare it from the some smaller scale to the larger scale and based on which we can define this phenomena as a similarity law.

And this then dimensional analysis and the similarity laws are interrelated and those are extremely useful in the use and interpretation of experimental data.

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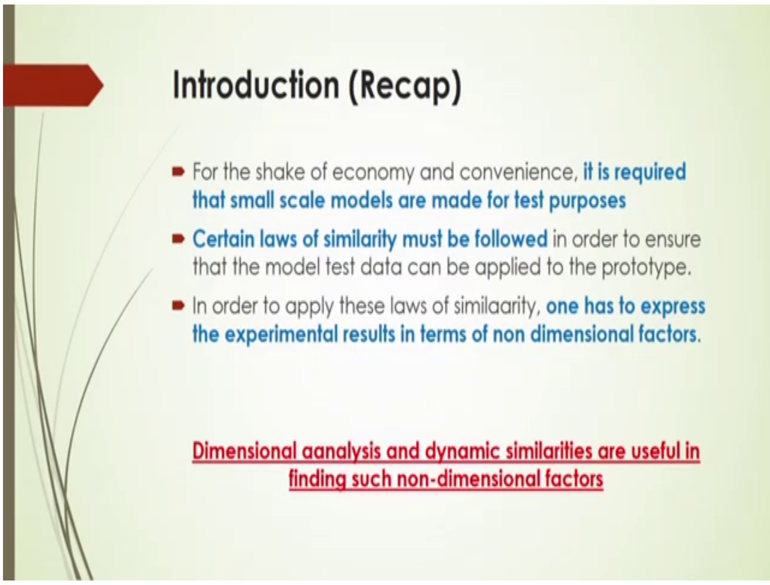

Now, for the sake of economy and convenience it is required to analyze that a smaller scale models with the prototype models for just designing of the certain flow, best equipments. And, in that case there you have to consider that symmetry variables flow variables and also what is that the physical properties as if as if variables. Now, either one of the variables if you change then accordingly for the smaller scale to larger scale how it can be varied that to be considered here.

And in that case you have to of course, follow certain laws of similarity in order to ensure that the model test data can be apply to the prototype. And of course, in that case I have to express the experimental results in terms of non-dimensional factors. So, in this figure you have to remember that this dimensional analysis and the dynamic similarities are useful in finding such non-dimensional factors by which you can scale up the process from small to large there.

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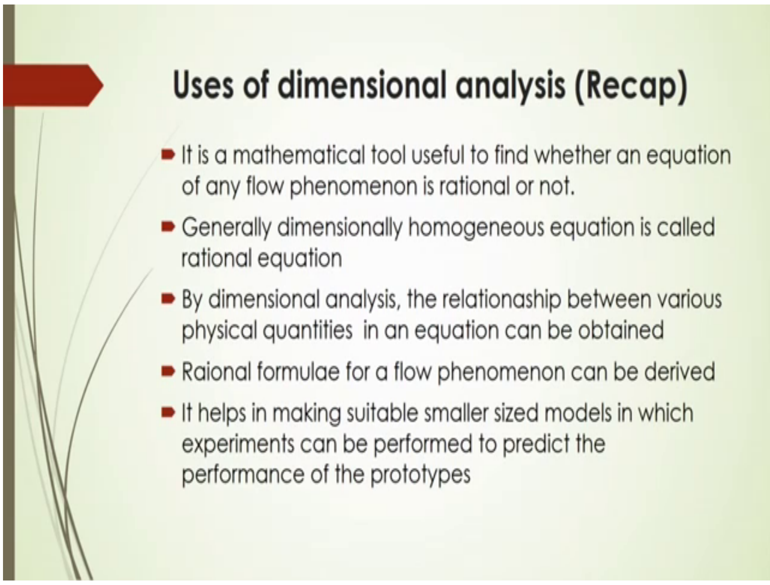

Now, as per this usefulness of this dimensional analysis we have already discussed so, many things in the previous lectures that how this dimensional analysis can be used. And, this dimensional analysis how it can be actually made there are several methods I think only two methods are there: one is Rayleigh methods and Buckingham pi methods. So, we have already discussed that Rayleigh methods; so how this dimensional analysis are useful that we have already discussed that it is a mathematical tool and it is useful to find whether an equation of any flow phenomena is rational or not.

Generally, the equations are called rational if the equations are made homogeneous in both sides of the equations that is unit of the equations should be unit in both sides. So, then equation will be called as homogeneous equation and this dimensionally homogeneous equation is called this rational equation. And, by dimensional analysis the relationship between various physical quantities which are expressed in equations can be obtained. And, also if you are making the formula whether this formula will be rational and if this rational formula are being used after derivation and then it will be helpful in making suitable smaller size models in which experiments can be performed to predict the performance of the prototypes.

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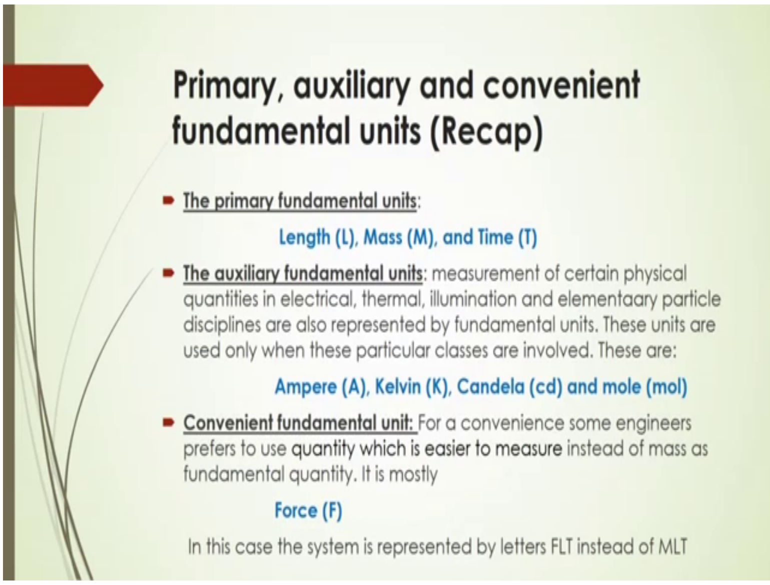

So, one have already talked about that there are units or quantities by which you can analyze this dimensional analysis for just interpretation of this basic phenomenon of the flow that happens in the experiments and even based on this experimental data in the small scale which is to be used for the larger scale after scaling up by similarities laws. Now, for that you have to have or you have to know some basic or fundamental units like here some units that will be used in all purposes and based on that unit some derived units also are formed for representing different quantities.

Now, already we told that that there are generally some fundamental units like it is called primary, auxiliary and convenient fundamental units there. So, what are those actually primary fundamental units? Generally length, mass and time are three considered as a primary fundamental units. So, length is L, mass M and time T whereas, in certain cases that case some quantities in electrical thermal even elimination and elementary particle disciplines in that case you will see those are represented by a fundamental units.

And these units are used on the when this particular classes are involved. So, those are Ampere, Kelvin, Candela and mole whereas, some time some engineer clc for design a purpose there using a some units or fundamental units as a conventional way. So, for their convenience they prefers to use the quantity which is easier to measure instead of mass as fundamental quantity it is mostly forced. So, they are using force instead of mass. So, the systems is standing as FLT instead of MLT; MLT is the Mass, Length and

Time unit whereas, FLT is the Force, Length and Time unit. So, these are actually primary, auxiliary and convenient fundamental units that of course, you have to know before doing the dimensional analysis.

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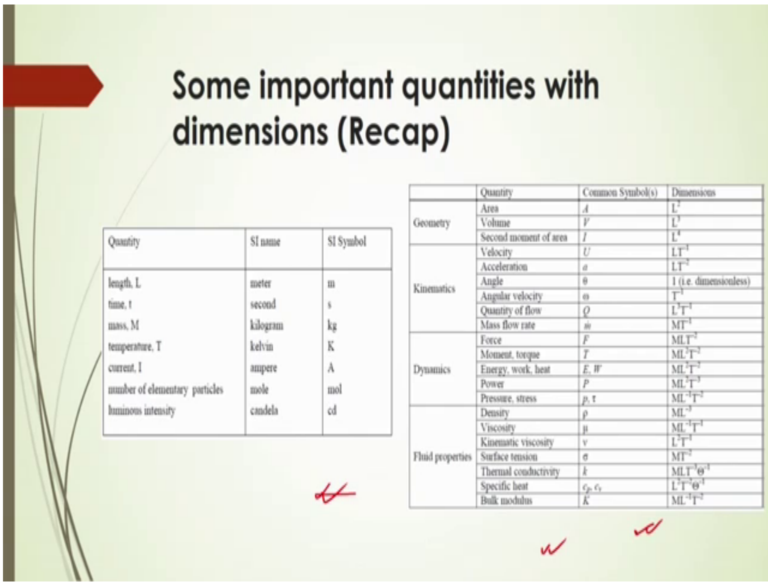

And for those if I consider that quantities then you what should be the dimensions and you know the units, but dimensions will be representing the units also. So, in that case these two tables are given here for the different quantities, what should be the dimensions like area is suppose metre square unit is metre square. So, it should be dimensions as by fundamental unit as L square.

A volume it should be L cube, similarly if I consider that velocity to be meter per second so, it should be meter as L and time should be T. So, it should be L by T or L T to the power minus 1. Like this here you will get the different quantities and what should be their corresponding dimensions it is given in table here. So, you have to use those dimensions for the dimensional analysis.

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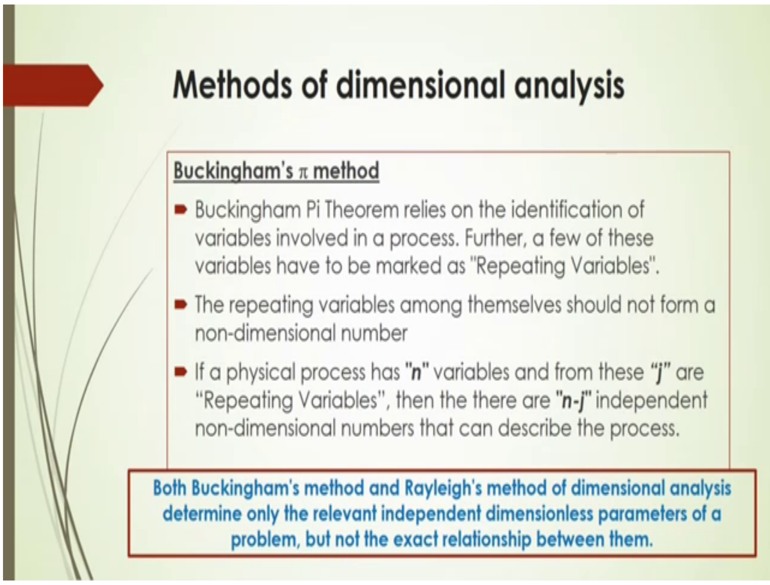

Now, coming to that method of Buckingham's pi method we have already discussed in the previous lecture what is the Rayleigh method, how the dimensional analysis can be made based on the Rayleigh method. Here another method is called Buckingham's method. So, in this methods it generally relies on the identification of variables that involved in a process and also called a few of these variables have to be marked as a repeating variables, sometimes it is called core variables.

The repeating variables among themselves should not form a non-dimensional number. So, if a physical process has n variables and from these fundamental variables like j are repeating variables, then you can say that there are n minus j independent non-dimensional numbers can be formed to describe the process. So, here also you have to first identify what are those processes for particular physical processes. Like if I see that the fluid is flowing through the pipe then in that case you will see the; what should be the resistance force during the flow. And, that resistance of the flow are depends on the pipe diameter, viscosity of the fluid, density of the fluid and also roughness of the pipe all those things.

And so, here for this physical processes we will see that there will be a some number of variables like here resistance force, density, viscosity, surface tension even you can say that there will be flow that is velocity and also that roughness of the pipe those are the variables. Now, upon these variables you will see there should be a fundamental variables or for which there should be a some fundamental units. So, all those if you consider that some variables that will be identified and in such that those should be repeatedly used for the analysis and that should be independent variables. Those independent variables to be used those are called repeating variables.

So, if I have these n variables from these, if j are the repeating variables then we can make n minus j independent that is non-dimensional numbers can be formed. And, those dimensionless numbers will be used to express the flow phenomena or physical processes just by expressing one dimensional or that is dependent dimensional number, that is here the dimensionless number made by dependent variables will be expressed as a function of other dimensional non-dimensional that is numbers. So, both in this case we can say Buckingham's method and Rayleigh's method of dimensional analysis. So, determine only the relevant independent dimensionless parameters of a problem, but not the exact relationship between them.

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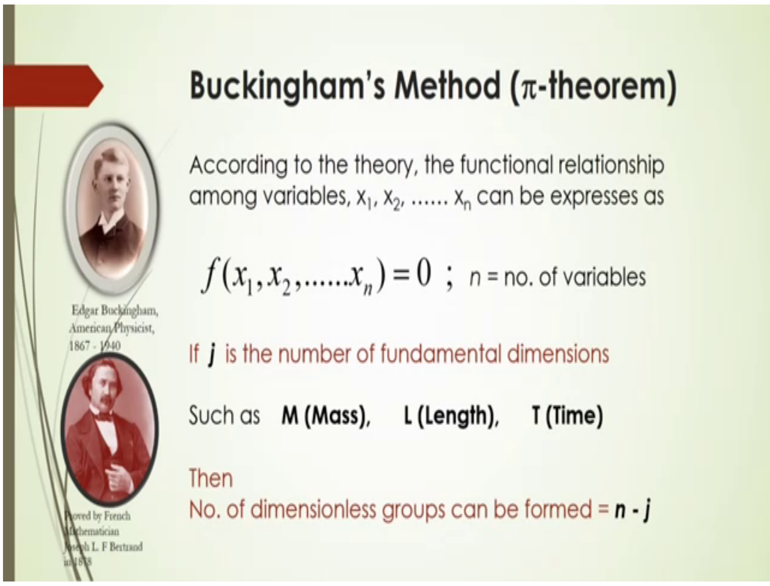

So, here we can say according to the theory the functional relationship among variables x 1 x 2 x n can be expressed as f of function of x 1 x 2 x n that will be is equal to 0. So, here x 1 x 2 up to x n are the variables and where, n is the number of variables here. If j is the number of fundamental dimensions such as M, L and T then a number of dimensionless groups can be formed is equal to n minus j.

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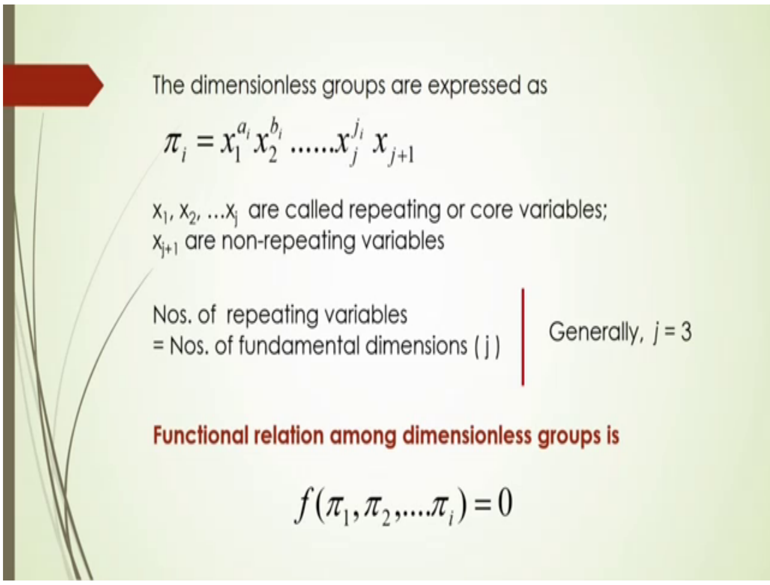

And the dimensionless groups are expressed as then pi i that would be is equal to x 1 to the power a i x 2 to the power b i and up to x j to the power j i into x j plus 1. Very interesting that this pi i is the ith number of dimensionless groups or dimensionless number. Here may be as per that formula n minus j number of dimensionless groups will be formed and x 1 x 2 and x j are called repeating or core variables and x j plus 1 are non-repeating variables. So, number of repeating variables will be number of fundamental dimensions.

So, you have to remember that how many repeating variables to be considered, only three number of the repeating variables to be considered here. Because, the number of fundamental dimensions will be exactly equal to the repeating variables there and, generally the j is equal to 3 because this number of dimensionless fundamental dimensions are 3. Now, functional relation among dimensionless groups then to be expressed as function of pi 1 pi 2 p i that will be equals to 0. So, ultimately after dimensionless groups formation then you have to make a relationship like this.

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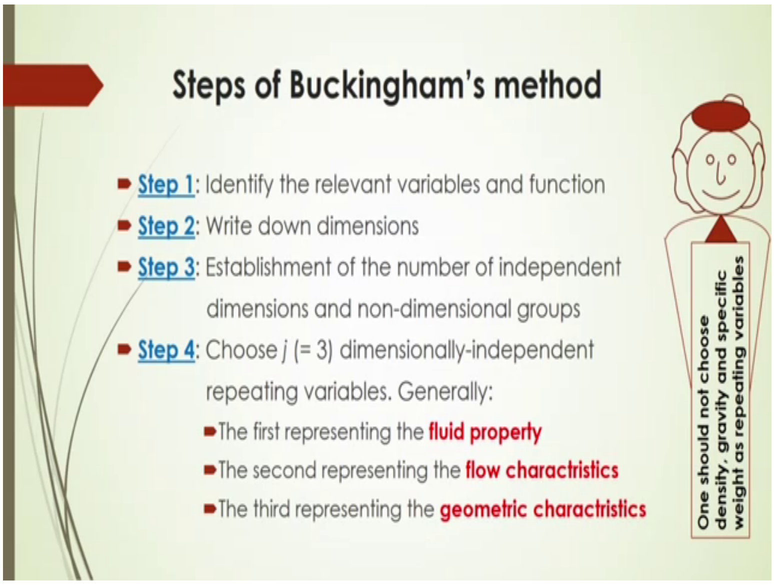

Now, what are the step by step way to this dimensional analysis that you have to remember. As a first step you have to identify the relevant variables and function and after that the step 2 just a note down the dimensions. And, then in step 3 you just establish the number of independent dimensions and non-dimensional groups. And, next step choose j, j is equal to 3 here dimensionally independent repeating variables. Generally you have to choose the repeating variables as a first repeating variables will be representing the fluid property and the second repeating variables will represent the flow characteristics.

And, the third repeating variables will be representing the geometric characteristics. So, fluid property maybe viscosity or density or surface tension either one you can select. And, the flow characteristics like velocity of the fluid and the third one is geometric characteristics maybe length of the pipe for length of the; what is that conduit or length of the unit or length of the reactor or diameter of the reactor.

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So, these are the variables such repeating variables to be selected and then step 5 what you have to do then you have to create the dimensionless groups as pi s by non-dimensionalizing the remaining variables and by solving the coefficients. And then step 6 you have to set the non-dimensional relationship and last step then you can arrange for your convenience all these dimensionless groups. Sometimes it is required to actually replace any of the pi s by a power of that pi for getting the significant dimensionless groups. Or, you can make this significant dimensionless groups for your convenience by a product with the other pi s.

Of course, whenever you are making this convenient dimensionless group for your significance you have to remember that the same number of independent dimensionless groups will be remain in the system.

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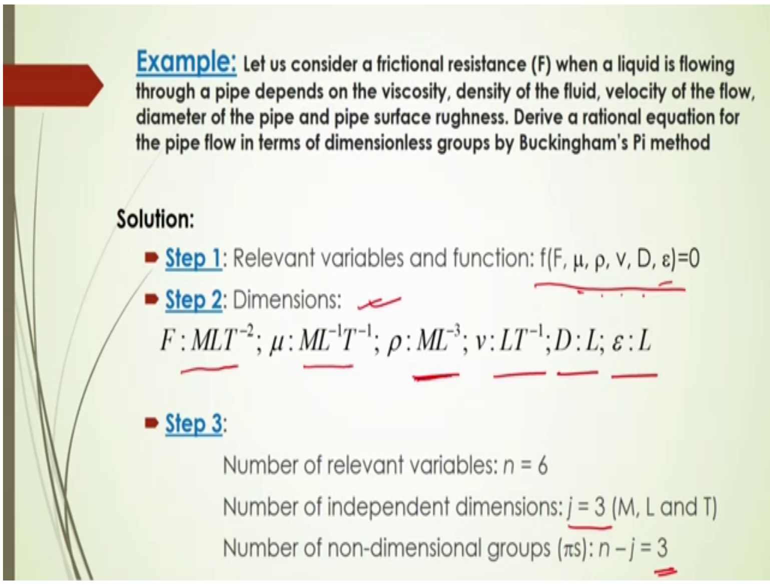

So, let us do an example here. Let us consider a frictional resistance F when a liquid is flowing through a pipe that depends on the viscosity, density of the fluid, velocity of the flow, diameter of the pipe and pipe surface rughness. Now, in this case based on these variables you have to derive a rational equation for the pipe flow in terms of dimensionless groups by Buckingham's pi method. Now, step by step you have to do all those analysis like step 1, what you have to do? Now what are the relevant variables that you have to find out and what are to the functions.

Here variables are one is frictional resistance and viscosity, density of the fluid, velocity of the flow, D is the diameter of the pipe and epsilon is given here. This epsilon is here this is pipe rughness and then we can have this function of F, mu, rho, v, D and epsilon. Now, as a step 2 what you have to do that you have to identify all the dimensions for all those variables here. Now, for F the dimensions are this, MLT to the power minus 2 as given in the table earlier and mu was the ML inverse T inverse, rho the dimensions of rho is ML to the power minus 3. Dimensions of v is LT to the power minus 1, diameter is L and rughness of course, it is a distance this will be represented by dimensions L.

Next step what you have to do you have to identify how many numbers of variables are there. There are here in this case 1 2 3 4 5 6, 6 variables are there. So, number of independent dimensions will be j is equal to 3 as we know that is ML and T and the number of non-dimensional groups will be then formed as n minus j that is 6 minus 3 that will be is equal to 3 here. So, we can expect or we can of course, have only 3 dimensionless groups based on this 6 variables of this physical process.

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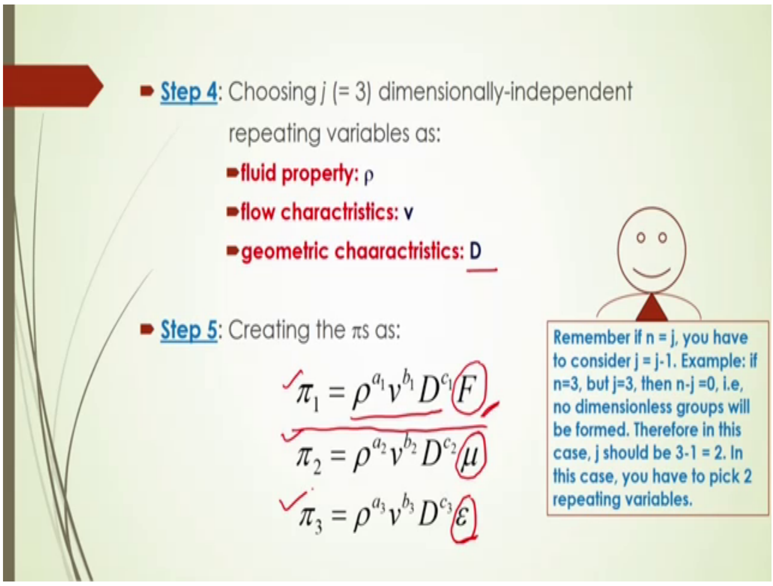

And next step what you have to do you have to choose the dimensionally independent repeating variables, that we have already told that for selection of this repeating variables you have to take one variable from fluid property, one variable from flow characteristics and one variable from geometric characteristics. So, in this case we are taking this rho that is density of the fluid as a fluid property one variables one repeating variable and then v velocity of the fluid is another repeating variables. And, third one that is geometric characteristics as diameter of the pipe D as taken as third repeating variables.

So, these three repeating variables will be considered here and the next step you have to make the pi s that is a dimensionless groups like this here pi 1 here since there are three only dimensionless groups can be formed. So, you are getting pi 1 pi 2 and pi 3. So, each groups will be related as here first thee should be repeating variables as rho v and D and you have to select their sum power like rho to the power a 1 v to the power b 1 and D to the power c 1 and then non-repeating variables here is coming F, one is F another is mu another is epsilon.

So, in this case pi 1 should be rho to the power a 1 v to the power b 1 and D to the power c 1 into F, you can take this mu instead of F here also. So, either anyway so, other non-repeating variables you can put either of this groups here. So, that finally, it will give you the ultimates 3 groups or the same 3 groups will be obtained. Now, this pi 2 will be then rho into here a 2 that is another power v to the power b 2 D to the power c 2 and then non-repeating variables here mu. And, third group as pi 3 it will be rho to the power a 3 v to the power b 3 and D to the power c 3 and here epsilon is the pipe, surface roughness the third and last non-repeating variables.

So, in this way we can have if suppose there are more than suppose 6 variables are there in the physical processes; if suppose there are 7 variables then in that case 7 minus 3 it will be 4 dimensionless groups. So, another here it will come pi 1 pi 2 and pi 3 and pi 4. So, in that case rho a 4 v to the power b 4 D to the power c 4 into some other here of course, it will be there are other non-repeating variables that will be may be other like sigma maybe what is the surface tension of the fluid. So, in that case you have to substitute a surface tension. So, in this way you can form n number of dimensionless groups based on the variables involved in the physical processes.

Now, in this case of course, you have to remember you will get some problems where you will see that for a particular process only 3 variables are involved. So, in that case how many dimensionless groups will be formed? Then 3 minus 3 it will be 0, then there will be no dimensionless groups will be formed. So, what you have to do for that cases? If n is equal to j; that means, here 3 that is number of variables will be is equal to fundamental dimensions. Then in that case you have to consider j should be is equal to j minus 1; that means, here instead of 3 you have to consider 2 fundamental dimensions. Suppose, if there is n is equal to 3 the j is equal to 3 then n minus j is equal to 0.

So, that is no dimensionless groups will be formed therefore, in this case j should be 3 minus 1 that will be is equal to 2. And, you have to pick only 2 repeating variables in this particular cases; we will come to the example for this type of problem. So, before going to that we have just finishing this problem here with this 6 variables problems, this here we are having then 3 dimensionless groups here this pi 1 pi 2 and pi 3.

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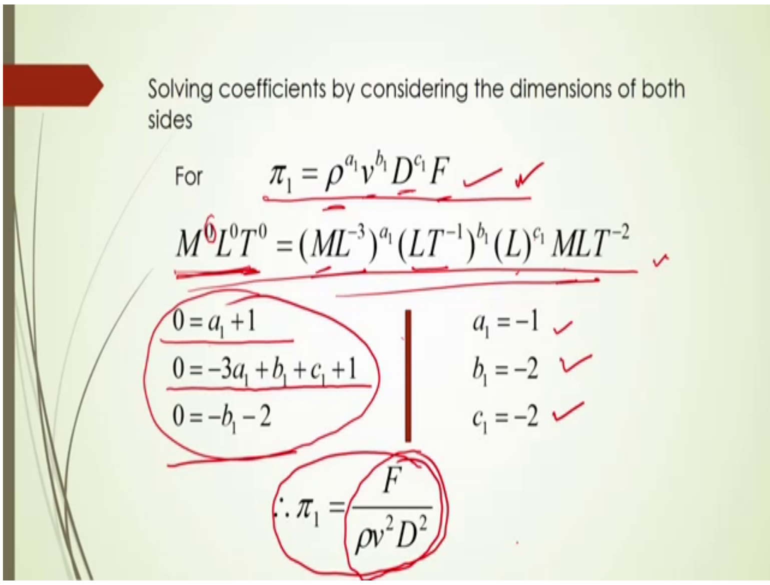

Now, next what we have to do, you have to solve the equations by equating the dimensions on the left hand side and right hand side of those equations here shown in here. And, solving the equations for your coefficients as defined in this equations for pi 1 pi 2 and pi 3. Now, for pi 1 if we consider then it will be rho a to the power 1 v to the power b 1 D to the power c 1 into F; this shown in here and then comparing this what is the dimensions.

Now, for pi 1 since it is dimensionless groups so, it is dimensions M L T here M to the power 0 L to the power 0 and T to the power 0 it will come and that there should be no dimensions here. And, in the right hand side here for this rho, what is the dimension of rho? This is M L to the power minus 3 whole to the power then it will be a 1. Similarly, what is the dimension for v to be L T to the power minus 1 whole to the power then b 1 here it will b. Similarly, for d here L to the power c 1, similarly for F it would be M L T to the power minus 2. So, you have to substitute the dimensions for all the respective variables what is given in what is obtained in your dimensionless groups pi 1.

Now, comparing this dimensions from both sides of this equation here. So, we are having if I consider M first, then what should be the dimensions power here? For M the power is 0, in the right hand side here for M the power is a 1 and another M is 1. So, it will be a 1 plus 1 in the right hand side and left hand side it will be 0. So, it will be 0 will be is equal to a 1 plus 1. Similarly, for L it will be 0 will be is equal to minus 3 a 1 plus b 1 plus c 1 plus 1. And, similarly for if we consider the power of this T from this both side of equation here, then you can get 0 will be equals to minus b 1 2. And, after solving this equations 3 equations we are getting here a 1 will be equals to minus 1 b 1 will be equals to minus 2 and c 1 will be equals to minus 2.

Now, after that what we have to do, you have to substitute this solution of this a 1 b 1 and c 1 in this equation here pi 1. So, pi 1 will be is equal to what? Rho 1 to the power minus 1 v to the power minus 2 D to the power minus 2 into F. So finally, after simplification it will come as pi 1 will be equals to F by rho v square into D square. So, you just see you just compare this right hand side of this equation this F by rho v square D square it will come as dimensionless. So, these are dimensionless group so, one dimensionless group now is form.

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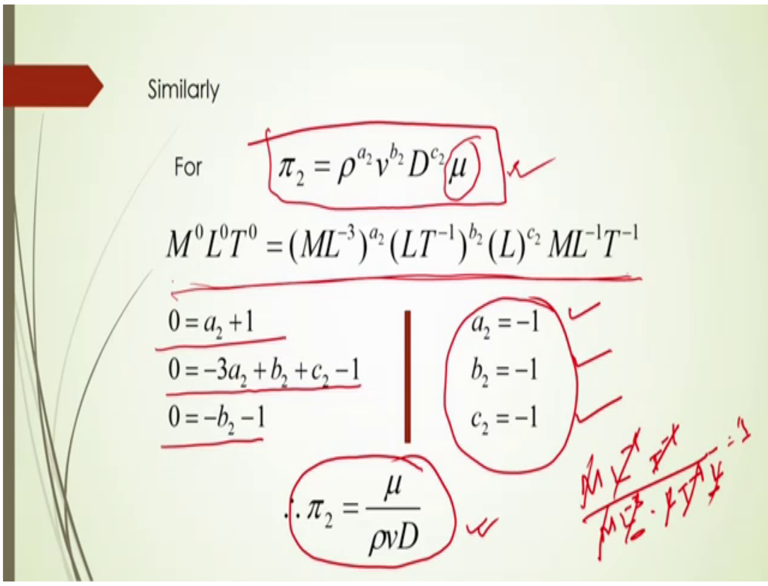

Similarly, you can do for pi 2 second dimensionless group, similarly it will be as rho to the power a 2 v to the power b 2 D to the power c 2 into mu, here mu instead of F is considered. So, again of you substitute the dimensions in the both sides of this equation then we are having M to the power 0, L to the power 0, T to the power 0, that will be equals to what? Rho rho here M L to the power minus 3, then to the power a 2; similarly, for v LT to the power minus 1 to the power b 2, similarly D L to the power c 2 and mu it will come M L to the power minus 1 T 2 the power minus 1. So, in this way we are just substituting the what should be the dimensions for this equation here for pi 2.

After that you compare this dimensions the power of this dimensions on both sides you just compare and forms this equations here 0 is equal to a 2 plus 1. Similarly, for L if you compare then it will come as 0 is equal to minus 3 a 2 plus b 2 plus c 2 minus 1. Similarly, T if you compare the power on both sides you will see you will have 0 will be equals to minus p 2 minus 1. And, after solution you can get this a 2 is equals to minus 1 b 2 will be is equal to minus 1 and c 2 will be equals to minus 1. Now, after substitution of this a 2 b 2 and c 2 in this equation pi 2 will be equals to this. Then we can have this pi 2 will be equals to mu by rho v D after simplification.

So, here again you just see verify whether this mu by rho v D is coming dimensionless or not. Very simple you can get this dimensionless because, mu is what is that ML to the power minus 1 T to the power minus 1 whereas, rho is what? Rho is M L to the power minus 3. v is what? L T to the power minus 1. And D is what? L, you just see M M should be cancelled out, L to the power minus 1, and here what is that L to the power minus 3 and L this and this will be cancelled out and this. After T to the power minus 1 T to the power minus 1, ultimately it is coming 1 that is here there will be no dimension. So, we are having this dimensionless group pi 2 as mu by rho v D.

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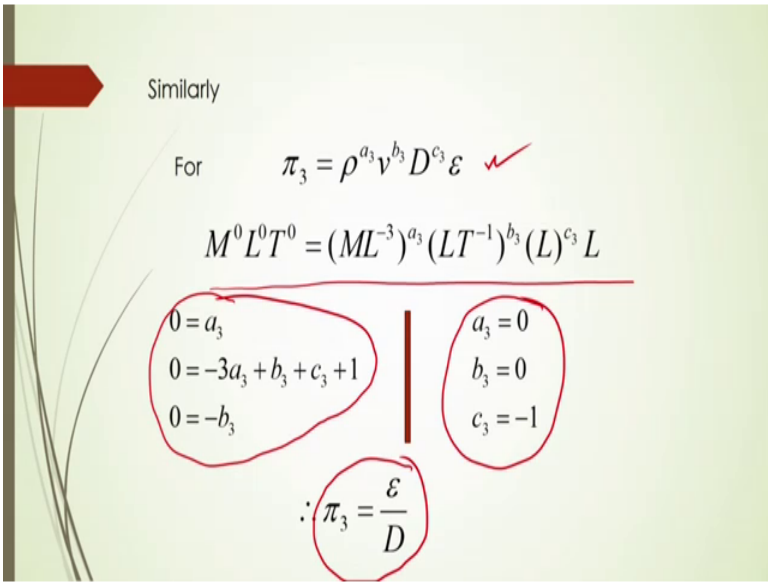

The similar manner you can obtain this pi 3 as rho a to the power rho to the power a 3 v to the power b 3 and c 3 and epsilon. And, after substitution of this dimensions and you can get this respective equations as then and after solutions; you can get a 3 equals to 0 v

3 is equal to 0 and c 3 is equal to minus 1. And after substitution of this a 3 b 3 and c 3 here in this pi 3 then you can get after simplification as pi 3 is equal to epsilon by D.

You will see this essentially by D is also a dimensionless groups because, epsilon is the dimension of epsilon is L and dimension of D diameter of the pipe is L. So, L should be cancelled and then pi 3 will come as non-dimensional group. So, in this way we are having this pi 1 pi 2 pi 3 3 dimensionless groups based on the 6 variables in this physical processes. And, the step is simple that this Buckingham pi theorem, then this is the simple way to form this dimensionless groups based on this Buckingham theorem.

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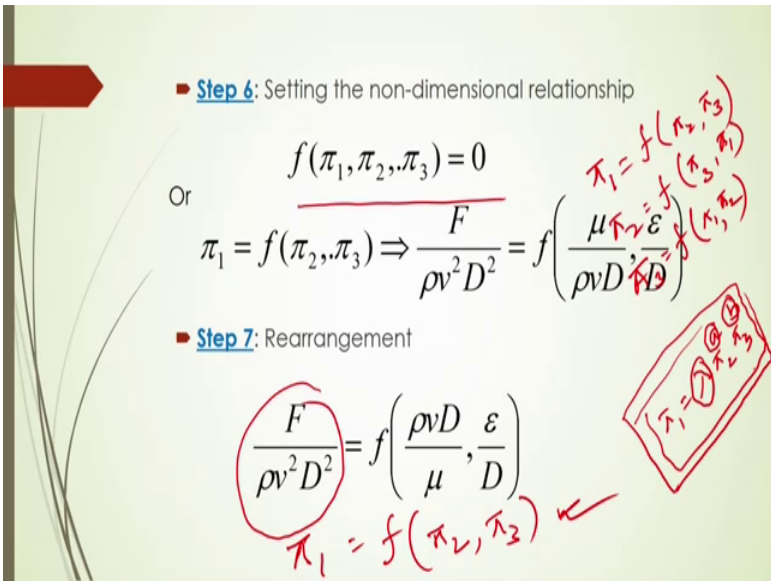

After that what you have to do, you have to setting the non-dimensional relationship as this f function of pi 1 pi 2 pi 3 that will be equals to 0. Now, you can make like this pi 1 that will be is equal to of function of pi 2 pi 3 also or you can make pi 2 as a function of pi 3 comma pi 1 in this way also.

Similarly, pi 3 will be is equal to function of you can form pi 1 pi 2. So, in this way also you can make a functionality here and next step is that what you have to do, that you have to rearrange this functionality as here like this f by what is that rho v square D square as a pi 1. This is as pi 1 function of here, this is pi 2 and this is pi 3. So, this is the functionality of this dimensionless groups.

Now, important point is that that after making the dimensionless groups and functionality, what you have to do. You can make an exact relationship among these groups. How? Then you just say here it will be pi 1 that will be is equal to sum coefficient of this here into pi 2 to the power a and pi 3 to the power b. So, this is your exact relationship. Now, this a b and this efficient lambda you can obtain this constants just by what is that fitting the experimental data and by regression analysis you can get this lambda a b coefficient.

And, then based on the experimental data what should be the exact relationship among these dimensionless groups. And from this you can then interpret what should be the frictional resistance, if you know the other variables like rho v D and epsilon there. So, it will ultimately come as a function this f will be a function of some other relationships and those coefficients after obtaining by pt with experimental data you can have the one correlation model. And, based on which you can interpret the experimental data or you can predict to the frictional resistance; if you change the variables they are all those variables for this particular physical processes.

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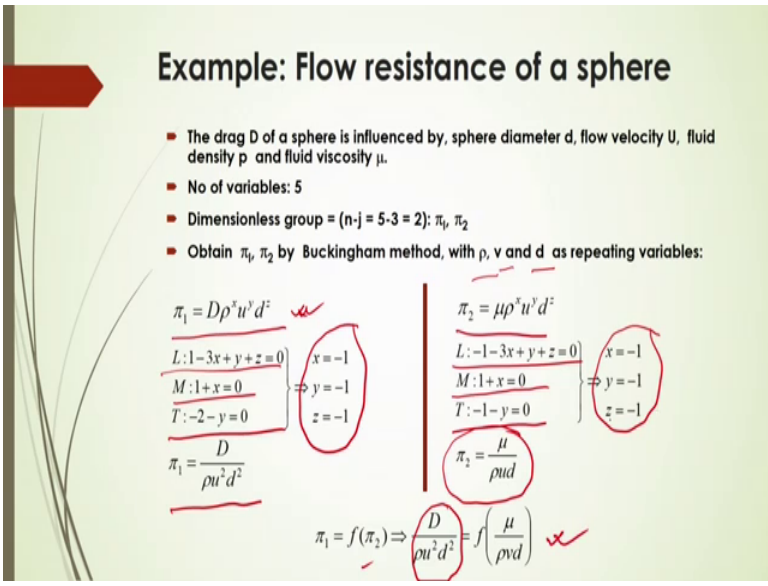

Now, let us do another example based on this Buckingham pi theorem. Let us do this it is told that flow resistance of a sphere, now in this case the drag force will be acting on the sphere and it will be influenced by the sphere diameter d, flow velocity U, fluid density rho and the fluid viscosity mu. Exact to the same way here in this case you will see that there are only 5 variables. Then how many dimensionless groups can be made? A dimensionless group will be then n minus j n is 5 n j is 3; so, 5 minus 3 it will be coming as 2. So, only 2 dimensionless groups will be formed.

Now, how to obtain dimensionless groups as Buckingham pi theorem and for that you have to choose repeating variables as here rho v and d as described earlier. So, in that case what should be the pi 1 p 1 is equal to D rho to the power x u to the power y d to the power z. So, in that case if you compare the coefficients of these a variables on both sides of this equations for this pi 1 then you are getting this equations. And, after that you have to solve this equations and then you can get this x is equals to minus 1, y is equal to minus 1 and z is equal to minus 1. And, after substitution of these x y z in equation this pi 1 you can get this pi 1 is equal to D by rho u square d square.

Similarly, for pi 2 again based on the same way same method you can say just by comparing the coefficients on the left hand right hand side we can have after solution of this x y z as pi 2 is equal to mu by rho u d. So, here since only 2 dimensionless groups are formed you have to make a functionality of these two groups as pi 1 is equal to function of pi 2. That means, here D by rho u square D square that will be equals to function of mu by rho v d here as this.

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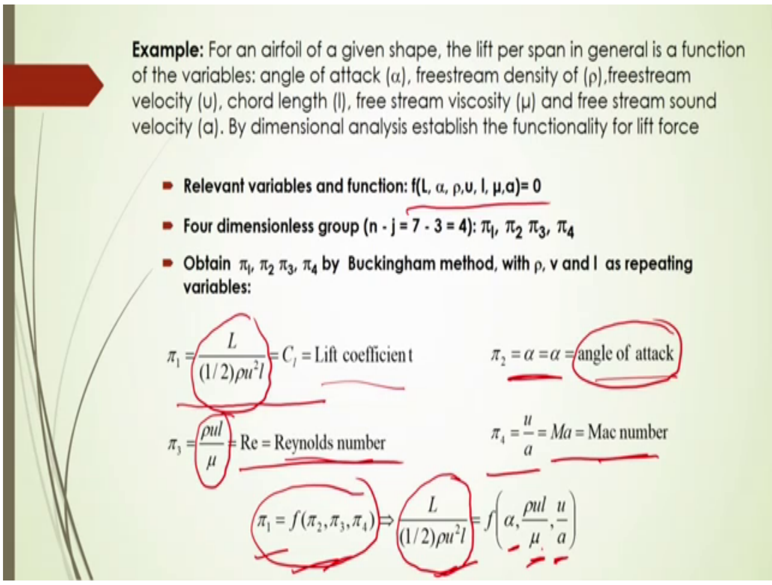

Similarly, another examples let us do here, what is that? For an airfoil of a given shape, the lift per span in general is a function of the variables like angle of attack we have already discussed in the that angle of attack. And, then free stream density of fluid and free stream velocity u and chord length is l. And, free stream viscosity of the fluid as mu whereas, the free stream sound is also to be considered here that will be as a. Now, by dimension analysis you have to establish the functionality for lift force here.

So, we have discussed these lift force, drug force in the previous even earlier lectures so, they are in the module as what is that drag and lift and cavitation there. So, in this case we are saying that this lift force will be a function of what is the an angle of attack, free stream fluid density, velocity the flow and chord length free stream viscosity and also the sound velocity there. So, we are having this relevant variables and we can make this functionality as if as a function of what is that L; that is functionality as L alpha rho ul mu a that will be is equal to 0. And, in this case how many dimensionless groups can formed here, since here having 7 variables. So, we can get 7 minus 3 that will be is equals to 4 dimensionless groups as pi 1 pi 2 pi 3 and pi 4.

So, to obtain this dimensionless groups we can use this Buckingham method with the repeating variables of rho v and l um. So, we can have this pi 1 will be is equal to here after solving all those equations by comparing the dimensions on both sides of the equations. Then we are having this pi 1 will be is equal to L by half rho u square l, that will be is equal to C l that is lift coefficient. And, pi 2 is alpha this is the angle of attack, this is here as alpha as one dimensionless groups.

And here pi 3 as what is that rho ul by mu, that will be has Reynolds number and pi 4 as u by a that will called Mac number Ma or only M sometimes it is represented. So, Mac number so, we are getting this 4 dimensionless groups as what is that lift coefficients angle of attack, Reynolds number and Mac number.

So, we have to form this functionality based on this dimensionless groups as here pi 1 will equals to pi 2 pi 3 pi 4 no, this is the function of this pi 2 pi 3 pi 4 here as this. And, then you can substitute this functional that is dimensionless groups as here that is lift coefficient and that will be is equal to function of angle of attack, Reynolds number and the Mac number here. Now, if you know the experimental data for this lift coefficient by a just varying or at different this independent variables of by changing alpha rho ul mu and a then you can have the functionality just by regression analysis with the experimental data.

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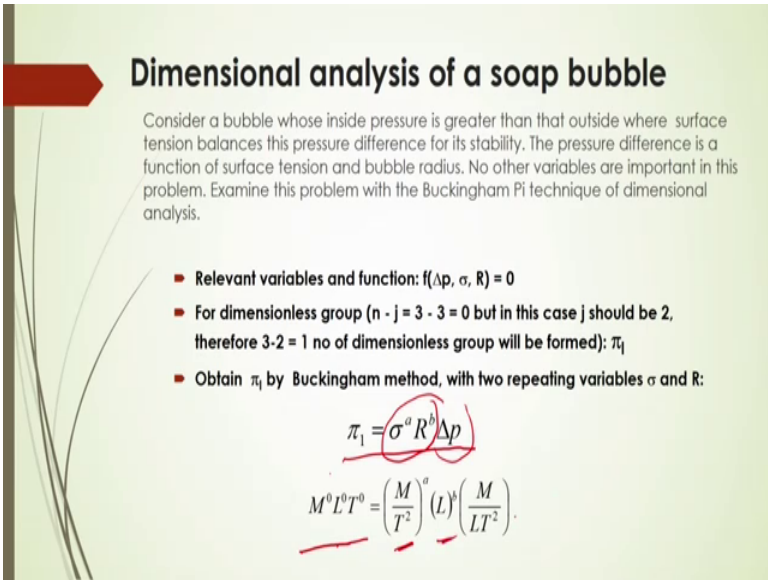

Now, and example let us do for this, if we consider the dimensionless of a soap bubble. Now, if we say that this soap bubble whose a inside pressure is greater than the outside pressure. Generally the bubble if you are making any bubble; that means, you will see that inside pressure will be always greater than the outside pressure for whose stabilized. And, this pressure difference of course, will be balanced by the surface tension force.

So, this surface tension force how actually the change the; what is that, that is a or balance that what is that pressure difference inside and outside of the bubble. And, in any certain operations if you say that that if the radius of the bubbles or size of the bubbles, in such a way that there will be a certain change of what is that surface tension or pressure inside and outside. And, at a certain time you will see the bubble will not be stabilized there. So, for a particular critical diameter you will see that bubbles will be stabilized in its medium.

So, in that case you can say that we can express that pressure difference is a function of surface tension and the bubble radius there; no other variables are important in this problem. So, if we consider so, only 3 variables are there as for this problem. So, based on the Buckingham pi technique how can actually we do the dimensional analysis here. So, relevant variables and functions are that delta p is the pressure difference and surface tension is another variable and radius of the bubble is another variable here. So, delta p sigma and R here we are getting 3 variables.

But we know that there are 3 fundamental dimensions that is 3; so how many dimensionless will group? As per definition it will be n minus j that is 3 minus 3 it will be 0, then there will be no dimensionless groups are formed; it may not be. So, for this type of problem we have already told that in this type of problem you have to consider only 2 number of fundamental dimensions; that means, here j should be considered as 2.

So therefore, 3 minus 2 here only 1 number of dimensionless group should be formed so, that will be pi 1 here. So, if any problem if you are seeing that only 3 variables are there you have to consider only two fundamental dimensions; so, here according to that principles we are getting only 1 dimensionless groups.

So, to obtain this dimensionless groups as per Buckingham method again we have to select the repeating variables. And, here the repeating variables will be as per the number of dimensionless fundamental dimensions accordingly. So, here since it is 2; then you have to select only two repeating variables and let us select this two repeating variables here sigma and R because, we are not getting any density, we are not getting any what is that viscosity here. So, you have to select the physical properties of sigma here and geometric variables as R here; so sigma R are the two repeating variables.

So, if we considered here then dimensionless group as pi 1 is equal to sigma to the power a R to the power b and these are two are repeating variables and this delta p as here non-repeating variable. So, in this case if we again they substitute that fundamental dimensions here this M to the power 0 L to the power 0 T to the power 0. Since, pi 1 is a dimensionless group and then here sigma, if you substitute the dimension has M T to the power minus 2 then it will be to the power a L to the power b and M LT to the power minus 2 so this.

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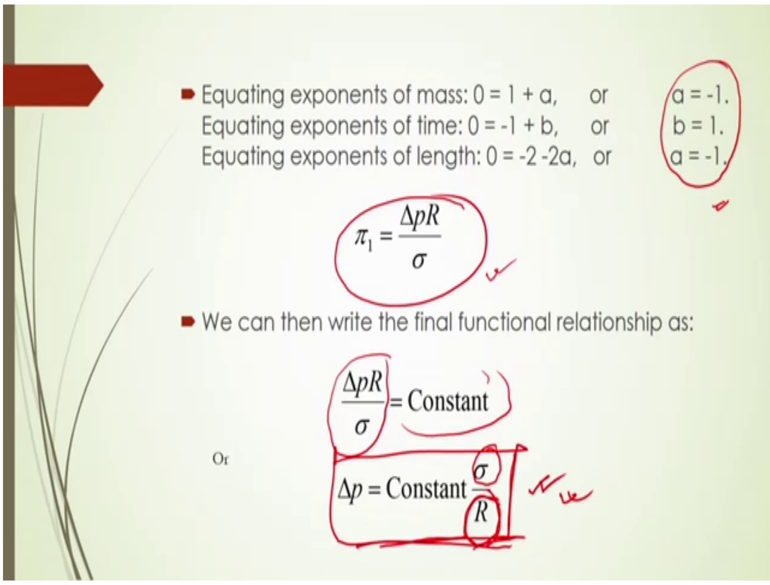

And then what you have to do equating the exponents of mass here, that will be 0 that will be 1 plus a and equating exponents of time that will be 0 is equal to minus 1 plus b and equating exponent of length that will be is equal to minus 2 minus 2 a that will be 0. So, after solution we are getting this a b and what is that this a that will be is equal to minus 1, 1 and minus 1 here. So, we can then get this pi 1 after substitution of this a and b value here, then it will be delta p R by sigma. So, we can have this dimensionless group.

Now, we can then write the final functional relationships as here, since there is no other dimensionless groups; then we can have this only dimensionless groups will be a constant. So, delta p R by sigma that will be is equal to only constant. So finally, delta p will be is equal to constant into sigma by R. So, this is your, you know that Laplace equation for this soap bubble or bubbles that are formed for stabilizing.

So, by dimension analysis we then also from the Laplace equations for expressing the what is that equation for the bubble for its stabilized condition. So, that will be delta p that is equal to constant into sigma by R. So, if you increase the radius of the bubble what will happen then the pressure difference will be reduced and if you decrease the bubble then pressure difference will be higher.

So, for small bubbles we will see there be a higher pressure high internal pressure will be there in the bubble. And of course, this surface tension is important, if you increase the surface tension you will see the pressure will be high. If you decrease the surface tension, that is why if any marbles you are producing in the water. And, if you are producing in the lower surfactant solution like what is that here if you add some surfactant in the water then your surface tension will reduce. So, in that case more finer bubbles or more smaller bubbles you can produced by adding surfactant in the solution.

So, in this lecture we actually learned then how to do the dimension analysis based on the Buckingham pi theorem. And, in the previous lecture we have got the method how to do the dimension analysis based on the relief method. So, these two methods you have to know for analysis and after analyzing this dimensional formation of this groups based on these two methods we will be analyzing the similarity law and the scaling of the systems in the next class. Next lectures we will be discussing that similarities laws that similarities laws of course, will be depending on these dimensionless groups. And, that dimensionless groups for that based on the variables and how it will be related will be discuss.

So, I would suggest to read further for example, of and I will suggest to go through this text books here so.

Thank you for this lecture today.