Fluid Flow Operations

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Module – 09
Lecture - 22
Dimensional Analysis and Law of Similarity: Part 1 Dimensional Analysis

Welcome to massive open online course on Fluid Flow Operations. So, in this lecture we will discuss about dimensional analysis and also law of similarity as a part 1 we will in this lecture will be discussing on the Dimensional Analysis.

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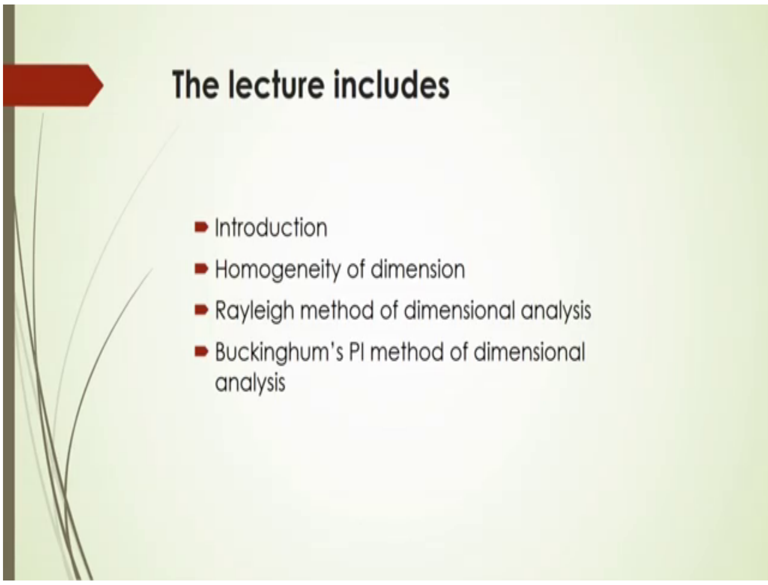

Now what is the actually that homogeneity of the dimension? What are the different methods of dimensional analysis and what will be the other methods like Buckingham's pi method of dimensional analysis to be discussed here. Also what will be the useful of this analysis of this dimension here this is important.

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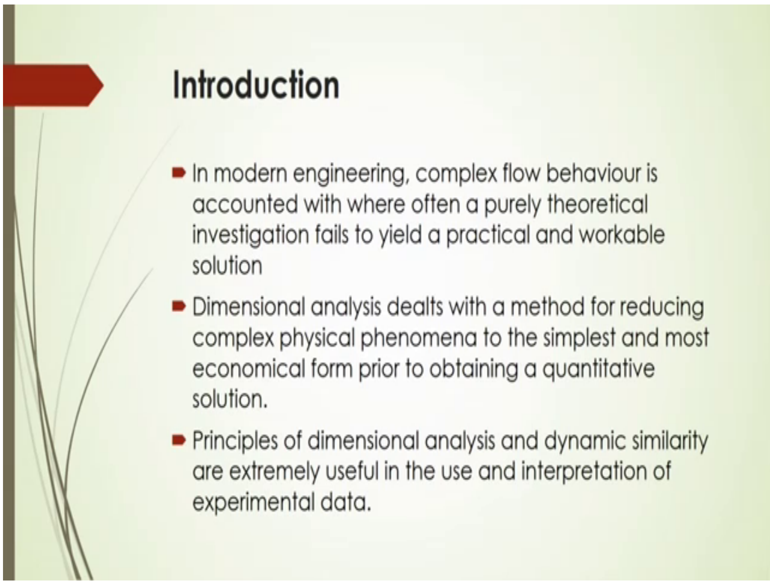

So, you know that any complex flow behaviour in the real engineering you will see that will accounted with where often a purely theoretical investigation fails to yield a practical and workable solution there. And you cannot have the theoretical solution sometimes for this complex flow phenomena for that, you have to analyze the flow phenomena in such a way that, the sometimes you have to that reduce some complex physical phenomena to the simplest and most economical form prior to obtain a quantitative solution.

In that case you have to know how to handle all those complex phenomena whenever any physical operations is taken place based on this complex flow behaviour and how to analyze on that to predict that certain output of that flow phenomena based on the flow behaviour. Now, in that case dimensional analysis and a dynamic similarity are the two components which are extremely useful to the use and interpretation of the experimental data.

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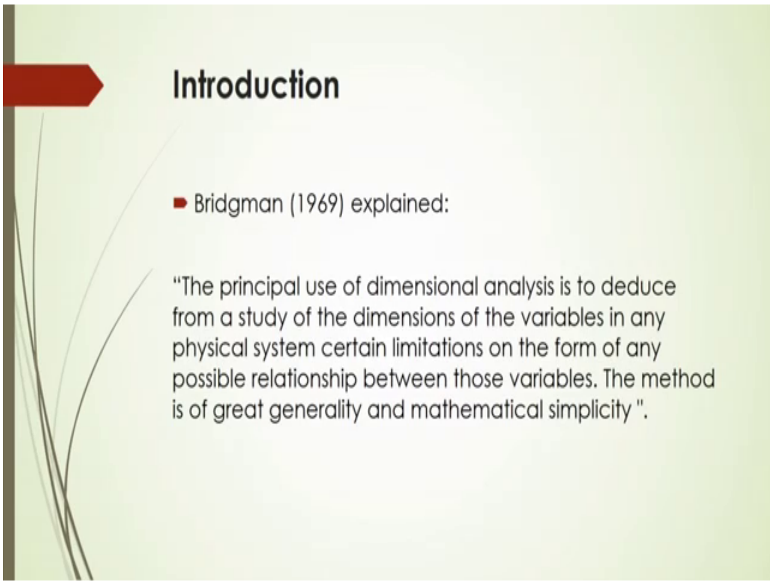

So, in that case Bridgman 1969 he explained that this is the principal use of dimensional analysis which is to be deduce from a study of the dimensions of the variables in any physical system, they are certain limitations on the form of any possible relationship that will between those variables and the method is of generality and mathematical simplicity will be there. So, dimensional analysis is to be done to just reduce the complex phenomena of the fluid flow operation to its simple form.

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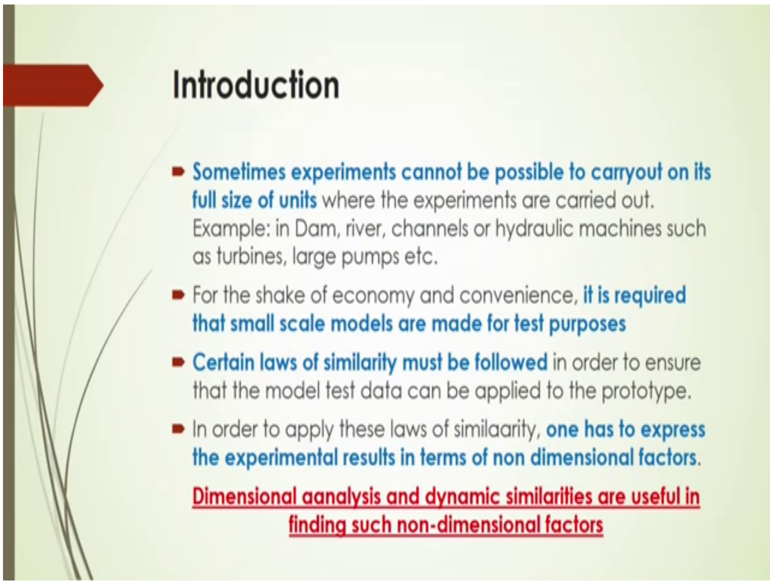

Sometimes you will see whenever you are going to do the experiments that experimental observation cannot be interpret in such a way that you will not get the exact actually what will be the phenomena, if you are considering it in a large scale that case. So, you have to do some experiments in the small scale and how to scale all those small scale phenomena to utilize for the large scale system. Now, like if you consider in dam river channels or hydraulic machines as turbines sometimes for large pumps it is very difficult to do the experiments.

So, in that case to know the phenomena of that dam river channels or hydraulic machines in that case you have to do some experiments of those flow phenomena in the small scale in laboratory. So, based on that laboratory scale physical phenomena and interpretation you can scale of those data to design or to analyze the system which in large unit like dam river and channels and other hydraulic units. So, for that you have to do in such a way that should be economical and also convenience, but it is required to know that small scale models should be tested in such a way that it will be useful for the same geometrical shape of that other large scale system.

Now, in that case certain laws of similarity of course, you have to follow in order to ensure that the model test data can be applied to the prototype or not. And, in order to apply those laws of similarities or that is or phenomena you can say that you have to express the experimental results or interpret the experimental results in terms of non-dimensional factors.

Because, that if you are having the dimensions for different, physical properties of the system or physical geometries of the systems are different then you will not be able to scale up it to its large proportion. So, in that case if you are making it non-dimensional forms then only it will be possible to just scale it up by changing any dimensions of the systems. Now, dimension analysis that is why and the dynamic similarities are useful in finding such non-dimensional factors. So, will be discussing one by one that how to do that dimensional analysis for the different methods to analyze this dimension and how to use that for the scale up in the subsequent lectures.

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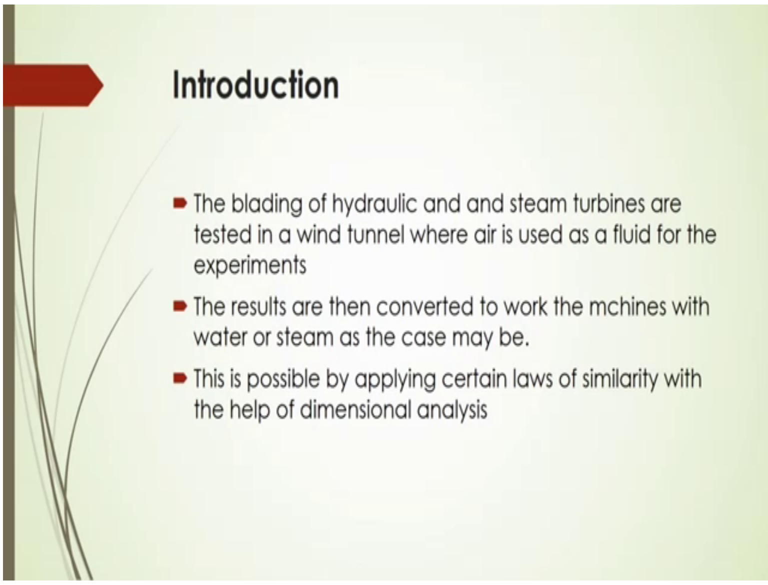

And also like if you are considering that blading of the hydraulic and steam turbines that are tested in wind tunnel, where air is used as a fluid for the experiments. Now, these results based on these experiments are then converted to work the machines with water or steam as the case may be there.

So, whatever here fluid property were using the same fluid property or same based on that experimental observations of by using that fluid like air you can use that results for the experiments which would be used for that experiments by steam or other gaseous medium. And, this is possible by applying certain laws of similarity with the help of dimensional analysis.

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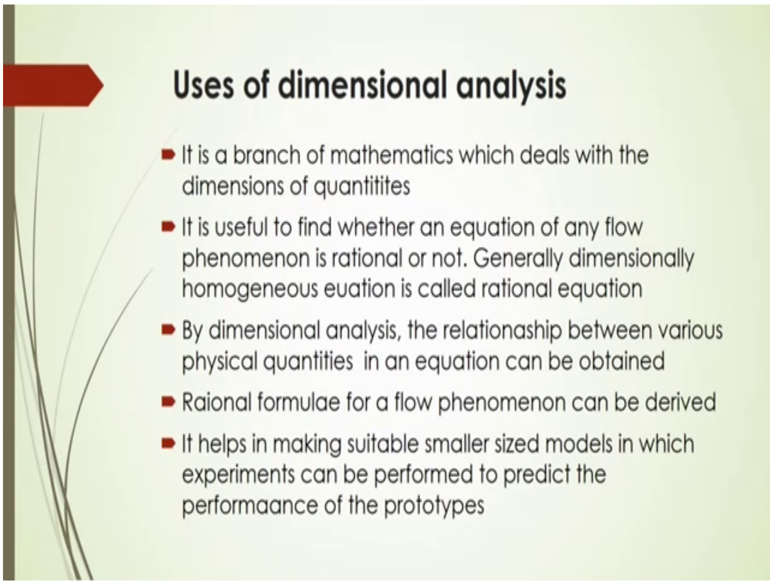

Now, it is sometimes useful to find whether an equation of any flow phenomena is a rational or not. So, this is important generally dimensionally homogeneous equation is called rational equation that case. By dimensional analysis the relationship between various physical quantities in an equation can be obtained. Now, rational formula for a flow phenomena can be derived by this dimensional analysis also, it helps in making a suitable smaller sized model in which experiments can be performed to predict the performance of the prototype.

So, dimensional analysis there will be a certain use for that we can use this analysis for the branch of mathematics which deals with the dimensional quantities and also to find out whether the equation of any flow phenomena is a rational or not. Generally, whatever dimensional analysis will be form to make that final equations you have to verify whether this equation will be operational or not or whether it will be homogeneous in dimensions or not.

That means in the both sides of the equation the dimensions will be same. So, in that case you have to be very careful whenever the dimensions were will be same or not. So, it is obvious whenever you are making the dimensional analysis and making the dimension analysis groups there you will see all the functions will be in terms of dimensions in that case all the dimensional groups will be non-dimensional in both sides whether in left side or right side that you can easily then interpret and also you can scaling up for the particular other larger scale.

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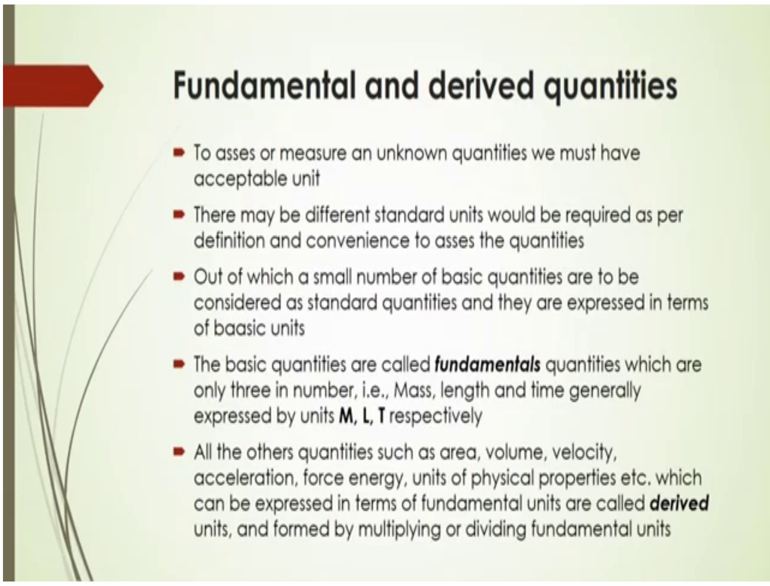

Now to asses or measure an unknown quantities we must have the acceptable unit in that case. So, whenever you are going to make any dimensional analysis you have to use some units or quantities in that case that should be acceptable and unit where are any suppose any systems your any quantities if you are going to measure that should be acceptable unit not like that any arbitrary unit should be used there, that would be validate to be a or widely acceptable of form. So, there may be different standard units you can have, but which standards to be used particular that also you have to be very particular in that case.

So, you have to have the definitions and the convenience of using those quantities in the dimensional analysis. Now, out of which small number of basic quantities that are to be considered as a standard quantities and they are expressed in terms of basic units. Now, you will see there should be a some basic units which would be used for analysis of this dimension and those basic quantities are called fundamental quantities which are only 3 number we know that mass, length and time generally those fundamental quantities are expressed by the units that is M, L and T respectively, M for mass L for length and T for time.

Now all other quantities are such area volume, velocity, acceleration, force energy and other derived units of this fundamental units what will be units of physical properties also can be expressed in terms of these fundamental units that will be called as derived units and that will be formed by either multiplication or by dividation of the fundamental units there. So, whenever suppose you are making a units for what is that velocity. So, velocity units for velocity is simply length per time. So, that case what is the unit for velocity to be L per T or meter per second in that case.

So, meter is the fundamental units and second is the fundamental units, in that case this meter per second is the derived units those are based on this what is that fundamental units of length and time. Similarly, other also other units like force what will be the force and the units for force energy and also some other physical quantities like surface tension what will be the units of surface tension this is generally Newton per meter. So, this Newton this is simply kg into meter per second square divided by meter. So finally, you are getting this derived unit of this surface tension. So, in that case others let us see other different fundamental units are there or not.

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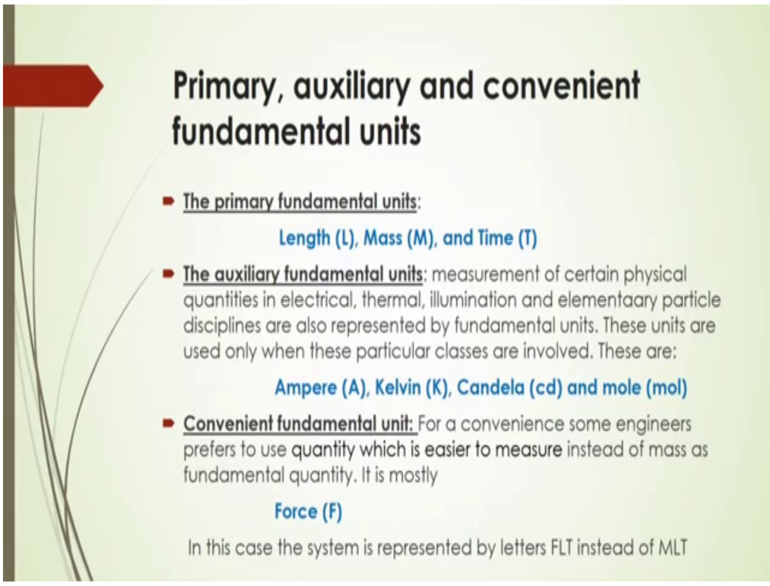

So, there are 3 conventional units that will be used for that, primary auxiliary or convenient fundamental units there. So, the primary fundamental units are generally length, mass and time whereas, auxiliary fundamental units are generally if you are going to measure a certain physical quantities like electrical, thermal or illumination or elementary particle disciples and also that are represented by the fundamental units.

And these units are used only when these particular classes are involved. So, these are ampere, Kelvin and Candela and mole. So, these are auxiliary fundamental units this is not that based on that primary fundamental unit length mass and time this is separate units that sometimes to be used for the analysis of this dimension there. So, in that case electrical units you have to use for that fundamental as a fundamental that will be ampere and the thermal in that case Kelvin illumination sometimes candela and other like concentration of these particles in a solution then you have to use that mole in terms of mole.

Now, sometimes whenever you are going to do the dimensional analysis you have to conveniently use some fundamental unit like that force for a convenience you have to sometimes some engineers they are preferring to use quantity which is easier to actually measure instead of mass as fundamental quantity in that case they are using force instead of mass. So, in this case the system is represented by letters here FLT instead of MLT where MLT means mass length and time where FLT is the force length and time instead of MLT here.

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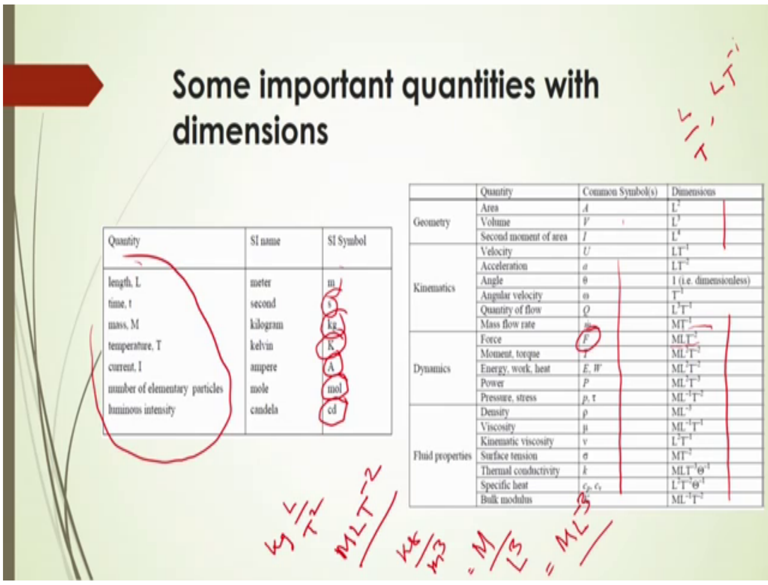

Now, some important quantities with dimensions are shown here in the slides in a table like here in this here you will see the quantity those quantities here length, time mass, temperature, current number of elementary particles even luminous intensity. These are quantities and what should be the unit for those suppose length unit for length is meter, unit for the time is second for mass kilogram temperature is Kelvin current ampere number of elementary particles in terms of mole and luminous intensity that will be unit as candela and for those what should be the symbol for S I symbol that meter it will be m second is for S.

Similarly kilogram kg, Kelvin K ampere, a mole M and candela cd like this and for those quantity we can divide those quantities into several categories like geometrical quantity like area, volume second moment of area and also kinematics quantity like velocity, acceleration angle, angular velocity and also flow and also flow quality mass flow rate all those things are kinematic.

And dynamics like you know force, momentum and also torque, energy, work heat power and pressure stress all those are dynamic quantity. Sometimes fluid properties also will be used for analyze that so in that case you have to know the quantity of that fluid properties like density, viscosity, kinematic viscosity, surface tension, thermal conductivity, specific heat, bulk modulus. So, all those are these quantities are under a different categories, in that case their common symbols which are being used to analyze in that case like area for a volume V like it is given in this table like this here, similarly what should be the dimensions respectively that you can have in this case.

So, how to area how to use this dimensions like area is meter square you can say that L square you can write here, volume V that is L cube similarly what is that area second moment of area it will be L to the power 4 like that what should be the velocity, what should be the dimensions for velocity; velocity is nothing, but meter per second meter is represented by L and velocity is second; second that means, here T so here L by T. So, sometimes it will be represented as L T to the power minus 1, so in this way dimensions are represented.

Similarly, you can say that force what should be the unit or dimensions for force? So, force is represented by F and what is the dimensions here force how to define a force in that case here MLT t to the power minus; that means, 1 kg of mass is accelerated as A, so in that case acceleration is L by T square. So, it will be MLT to the power minus 2 here. So, this is your dimensions here so in that case other also you can have like density that is kg per meter cube. So, this is nothing, but kg for M and meter cube means L to the power 3.

So, it will be coming as ML to the power minus 3, so in this way these dimensions are expressed. So, you have to remember all those dimensions also sometimes it will be used for that analysis of the dimension there. So, for dimensional analysis sometimes you have to compare with these dimensions with the other dimension. So, that is why you have to know this dimensions of each quantities here like what is the mainly density, viscosity surface tension you have to know and also velocity, mass flow rate, area or volume so it is very important. So, all those dimensions as per these tables you have to know.

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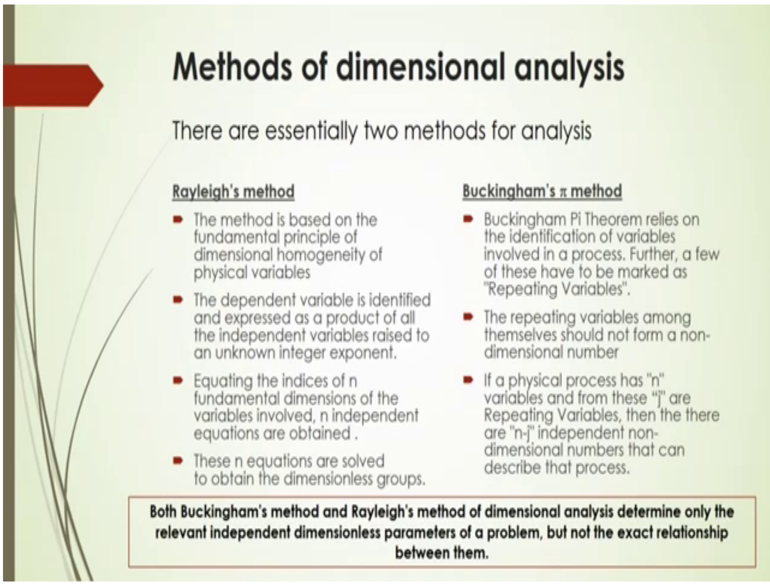

Now, before going to the dimensional analysis we have to know some basic: what are those methods that available to analyze this dimension. So, there are essentially two methods for analysis we will discuss here one is the Rayleigh's method another is Buckingham pi method. What is that Rayleigh's method? The method is based on the fundamental principle of dimensional homogeneity of the physical properties, in this case you have to have for the analysis some dependent variables which will be identified and also expressed as a product of all the independent variables raised to an unknown integer exponent.

So, equations will be made by equating the indices of n fundamental dimensions of the variables involved n independent equations that will be obtained and these n equations are solved to obtain the dimensionless groups. So, will give the examples how system in one by one step by step, how to actually use this Rayleigh's method for dimensional analysis and Buckingham pi theorem here also that relies on the identification of the variables here involved in the process in that case further a few of these have to be marked as some repeating variables or core variables there and the repeating variables among themselves should not form a non dimensional number in that case.

So, if a physical process has n variables in that case and form these j or the repeating variables then n minus j will be independent non dimensional numbers that can be described here in this process. So, you have to form the dimensions non dimensional groups by these two methods one by one then we will be discussing here. So, in this case remember that both Buckingham's method and Rayleigh's methods of dimensional analysis determine only the relevant independent dimensionless parameters of the problem, but not exact relationship between them there. So, there will be just the functional so those dimensionless groups what are formed based on the analysis will be a function each other, but you have to identify which group will be independent to the other variables there

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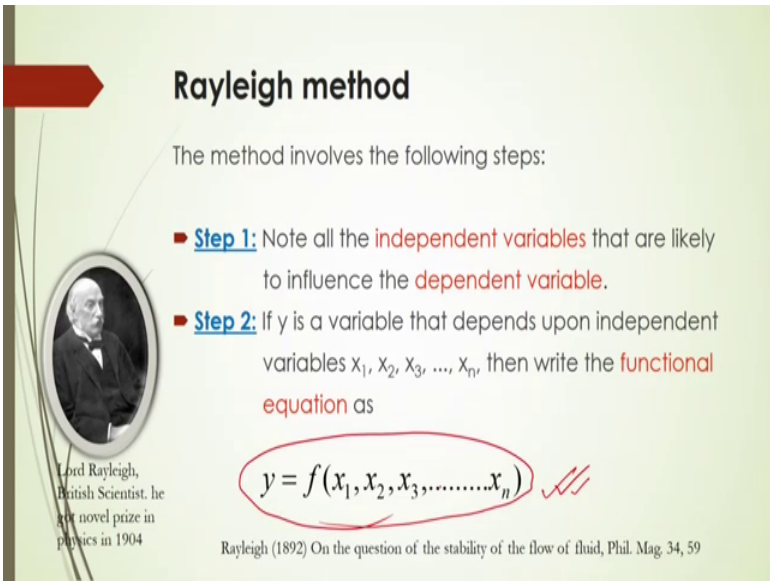

Now let us discuss that Rayleigh's method one by one what is that, let us consider a system in that case suppose any experimental observation any experiment that is done in your laboratory.

So, in that case suppose you are having any experiment like this if a flow rate will be increases what will happen in the pressure drop in the pipe will be suppose increased or you will see if you increase the diameter of the pipe there will be change of pressure drop, if you increase the length of the pipe there will be change of pressure drop, if you suppose use the different type of fluids in that case you will see there will be a change of pressure drop. And of course, same you can say that the pressure drop is a function of what is that a pipe diameter and fluid velocity and pipe length and the when you will see it is a function of a physical properties of the system.

So, whenever any fluid will be flowing through the pipe then you will see there with change of pressure drop there may be other things also will be changed suppose if any chemical processes like if you are going to do an experiment for suppose carbon dioxide gas absorption in a two phase systems in a gas liquid column. So, what will happen you will see there will be a mass transfer of this carbon dioxide gas from the gaseous medium to the liquid medium. So, there will be a certain change of this quantity of that carbon dioxide from the gaseous medium to the liquid medium. So, in that case you will see there will be a mass transfer of gas to the liquid.

Now, there will be a some coefficient to represent that mass transfer phenomena so that is called mass transfer coefficient. Now interesting is that this mass transfer coefficient will be changing with respect to what is that column diameter flow rate of the gas, flow rate of the liquid, again you will see there will be a what is the geometry of the column again you will see physical properties of the system. Suppose, if you are using sodium chloride or sodium hydroxide, potassium hydroxide or some other liquid medium, then in that case what will be the physical properties like surface tension, density, viscosity all those phenomena; so, all those physical properties that should be considered.

So, in that case mass transfer coefficient is a function of what is that column diameter here you will see that viscosity of the liquid, density of the liquid again density of the gas flow velocity so all those things. Now in that case so you have to identify based on this Rayleigh's method if we are considering here note all the variables first. Now out of which you will see for which one should be independent variables and which one should be dependent variables. So, here in this case suppose this pressure drop; pressure drop is a function of diameter of the column, pressure drop is a function of what is that viscosity of the system pressure drop is a function of velocity of the fluid.

So, in that case pressure drop is a dependent variables and independent variables are what that that will be your viscosity, column diameter or a pipe diameter and flow velocity, so these are independent variables these independent variables will change this dependent variables there. So, if we considered that if y is a variable for a particular operation if y is a variable that depends upon the independent variables like maybe n number of independent variables will be there may be 2 3 4 5 6 that depends on the experiment that experiment to experiment it will vary and a what type of a experiment you are going.

So, in that case if suppose y is a dependent variables and x 1, x 2, x 3 and x n are those independent variable that we can write a function like this y is a function of x 1, x 2, x 3 and x n. So, first step you have to identify what are those variables and out of those variables what are the independent variables and what are the dependent variables after that you have to make a functionality like this y will be is equal to function of x 1, x 2, x 3, and x n.

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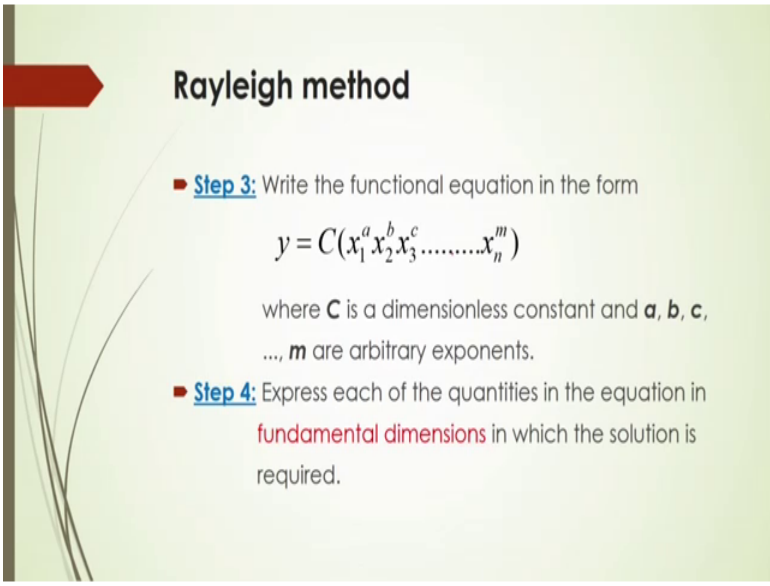

And then next step that is as a step 3 what you have to do write the functional equation in the form like this like here, y will be is equal to C into x 1 to the power a x 2 to the power b x 3 to the power c that at the x n to the power m. Here you do not know in these functions what should be the exact equality from these dependent variables with this independent variables. So, for that you have to make here there will be it will be the exact equations here what is that y will be is equal to C into x 1 to the power a x 2 to the power b x 3 to the power c x n to the power m.

So, this is the exact functionality here C is a constant here also it is also dimensionless constant and a b c and m are arbitrary exponents those exponents to be find out from the experimental data after repeating this equations there and by some regression analysis you have to do then you have to find out what will be the c a b c all those components. So, our objective is to find the dimensionless groups and making a functionality by this dimensionless groups. Now, then step 4 what you have to do express each of the quantities in the equation in fundamental dimensions.

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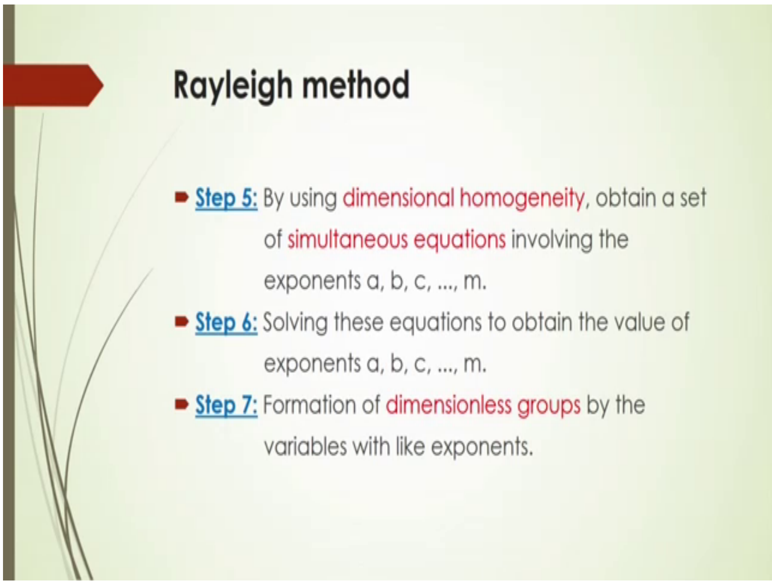

In its the solution is required in that case what you have to do by using dimensional homogeneity obtain a set of simultaneous equations involving exponents a b c and m we, we will give the examples later on one by one what to do. So, in this case you have to remember that you have to first a considered the dimensional homogeneity; that means, in the both sides of the equation the left side hand side and the right hand side the dimensions should be same. So, obtain a set of simultaneous equations that will involving the exponents of a b c and m there.

Now, solving these equations to obtain the value of exponents a b c you will get how to find out that a b c value there now formation of dimensionless groups by the variables with like exponents you have to have.

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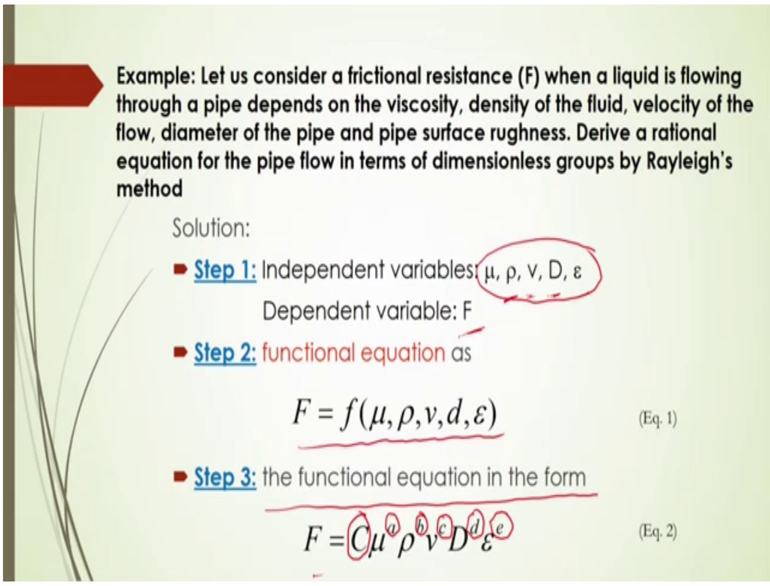

Now, let us do this example here, let us consider a frictional resistance F when a liquid is flowing through a pipe depends on the viscosity, density of the fluid, velocity of the flow, diameter of the pipe and pipe surface roughness. So, in this case you have to derive a rational equation for the pipe flow in terms of dimensionless groups by this Rayleigh's method. Now what you have to do now? First step is that you have to find out the independent variables and dependent variables.

What are independent variables here? We have seen that the frictional resistance F is flowing through the pipe that depends on viscosity, density, velocity, diameter of the pipe and pipe roughness. So, here we can then identify here viscosity is the independent variables, density is the independent variables, velocity is the independent variables, diameter of the pipe is independent variables and the surface roughness denoted by epsilon is a independent variable. Now, dependent variables is F because this frictional resistance, if I consider this F then F would be the dependent variable this F is dependent all those independent variables.

Now, next step what you have to do? You have to make the functional equation like this F is a function of that f mu, rho, v, d epsilon. Then what we have to do the functional equation in the form of this F is equals to C mu to the power a rho to the power b v to the power c D to the power d and epsilon to the power e here. So, this c is the dimensionless constants a b c d e that we have just assuming those will be your what is that exponents here you have to find out those exponents now.

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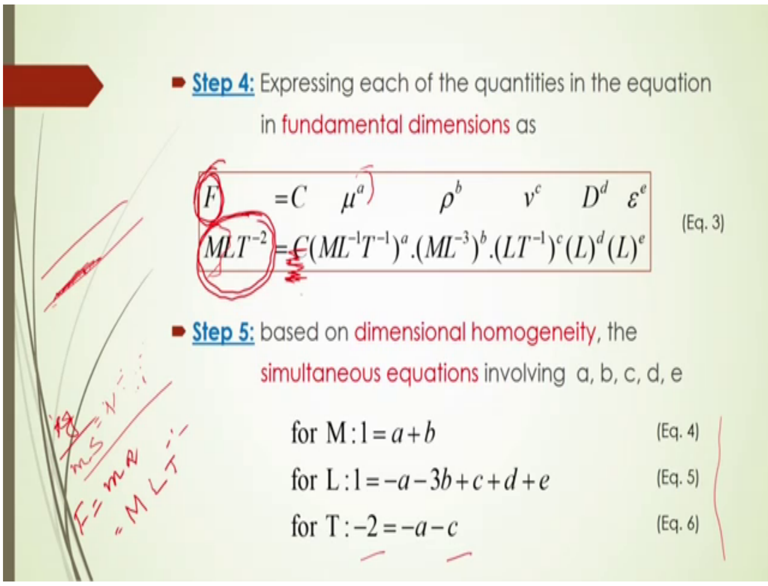

In a next step, that you have to express each of the quantities in the equation in a fundamental dimensions; now what are those? This the this is the that is functionality F will be is equal to C into mu to the power a rho to the power b v to the power c D to the power d epsilon to the power e. Now, in this case what should the fundamental dimensions of this F here, so for this F what would be the unit that expressed in terms of fundamental dimensions? That will be MLT to the power minus 2 and if you say that how it is coming this MLT to the power minus 2, that you know that what is the definition of F; F is nothing, but m into acceleration.

So, what is m; m is mass; mass should be considered as M, a is the acceleration; acceleration is nothing, but meter per second square; that means, meter is L and second per second square that is here T to the power minus 2. So, that is why it is coming as MLT to the power min us 2, here c; c would be not have this is dimensionless we do not have this dimensions here in this case c for c.

Similarly, mu; mu is the viscosity of the liquid how it will be expressed this here this MLT ML to the power minus 1 into T to the power minus 1, this is the dimensions of the viscosity how it is coming dimension the unit for viscosity is kg per metre second. So, in this case kg is what M metre is L to the power minus 1 here in this case, then M into L to the power minus 1 and then T to the power minus 1. So, it will be here simply viscosity dimensions of the viscosity ML to the power minus 1 T to the power minus 1, then whole to the power a since we are here assume that there would be exponent is a. Similarly here rho rho is MLT to the power minus 3, then 2 to the to the power whole to the power b.

Here similarly b b is the dimensions what is that dimensions for b is LT to the power minus 1, then it should be whole to the power c. Similarly d d is the diameter of the pipe, so what would be the unit that would be length the length dimension should be L to the power d. Similarly epsilon; epsilon is the roughness factor that roughness is nothing, but the how much skin or scaling inside the pipe there so that will be vary small minute. So, in this case you can say if we consider this is the pipe. So, wall of the pipe will be scaled with some chemicals so that will be your you know that there will be some roughness, so that roughness will be denoted by the what is that small length.

So, that is why its unit should be as l its dimensions should be l so L to the power minus a here. Now as a step 5 what you have do. So, based on the dimension homogeneity the simultaneous equations that will be involving a b c d e to be represented by this equations here 4 to 6 here, in this case first you have to equate this dimensions that is in the left hand side and right hand side first of all you consider the dimensions M. Now for then M in the left hand side of this equation number 3 what is the power of this M, the power of this M in the left hand side is 1 this is 1, this here in this case it will be power of M is 1.

So, in the right hand side what to be the power of this M here? You are getting here in this case M to the power a; that means, here a is that power of the same here again power of the M is b here power no other power, so it will be only a plus b. So, here one will be is equal to a plus b, similarly for dimensions L, if you equate the exponents or power of this L from this left hand side and right hand side then we can say that 1 would be is equal to minus a minus 3 b plus c plus d plus e and in the equation 6 we can again have these equations for time just equating the dimensions of this time from this left hand side and right hand side as this will be minus 2 will be is equal to minus a minus c.

Next what you have to do? You have to solve these three equations in such a way that you have express some exponents in terms of others.

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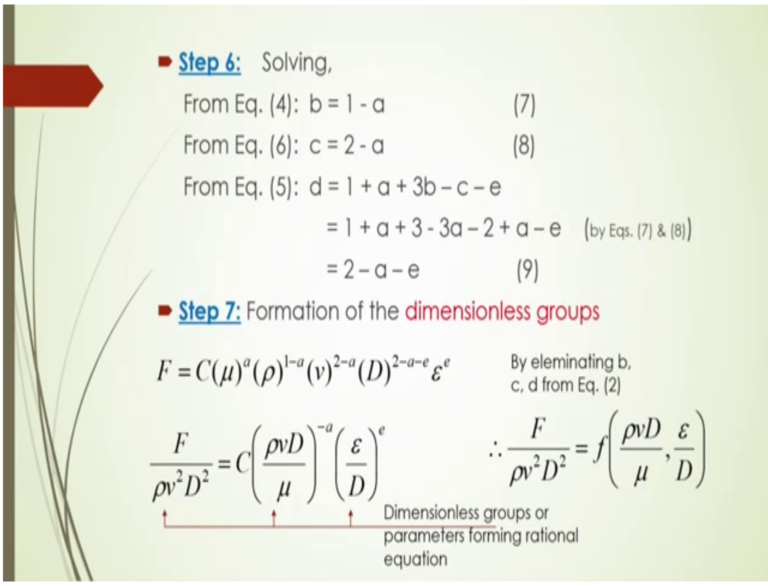

In this case you will see if we saw this b from the question number 4 then you can have b would be is equal to 1 minus a similarly from equation 6 we can have c would be is equal to 2 minus a very interesting that we are expressing these exponents in terms of certain other exponents here. So, here b is as a function of 1 minus a similarly c is a function of again a that would be 2 minus a from equation similarly from equation 5 we can have d d as a function of again a and remaining exponents here.

So, it would be 1 plus a plus 3b minus c you just substitute the value of b and c here from this equation number 7 and 8 then we can have this finally, 2 minus a minus e because we are not able to find we are not able to substitute the value of e from other equation, so e should be remain same in that equation. So, only thing is that then we have to substitute the values of b and c from this equation 7 and 8. So, we are having this value of d in terms of a as well as remaining exponents like here e. So, finally, these three equations 7 8 and 9 we will be used for the exponents b c and d in that particular that a functionality here.

So, formation of the dimensionless groups would be then here if you substitute those a b that is b c d value in these exponents then we can represent this F is equal to C mu to the power a into rho to the power here see rho to the power it was there b. So, b instead of b it will be 1 minus a similarly v to the power it was there C. So, v to the c instead of c it would be substitute 2 minus a from equation number 8, similarly for D; D to the power it was there D; D should be in terms of 2 minus a minus e that is from equation 9. Similarly epsilon to the power e; e we are not having any e equation there so we cannot express the remaining left here.

So, at least you have to from these three exponents to be represented by remaining other exponents there, even sometimes you will see there you have to be very careful that whenever you are going to express this exponents, now which exponents to be replaced by remaining exponents that you have to identify you can select the other exponent also suppose if you are representing this suppose C D e e in terms of b and e that also you can do, then you will get also similar type dimensionless groups.

Now from this equation here that a F will be is equal to here what is that. So, in this case here again you can have this F that is F that is expressed here in this case you can rearrange these based on the exponent here. So, if you rearrange these based on the exponent what would be the a exponents so all the a exponents to be here represented in this case. So, it will be coming as rho v D by mu to the power minus a and epsilon D to the power e after rearrangement of this equation and then remaining other that is variables you have to segregate from that right hand side and with this F.

So, you will see finally, F by rho v square by D square it is also 1 dimensionless groups and this one is one dimensionless groups and this one will be one dimensionless groups. So, these are dimensionless groups are formed C F by rho v square D square this is you will see just verify it this should be dimensionless verify this rho v D by mu this is dimensionless epsilon d this is also dimensionless.

So, this dimensionless groups are formed based on the rearrangement of this a b and e here a e a and e here so and remaining here in this case. So, very interesting that this dimensionless group how many dimensionless groups actually are formed here? You have to remember that that how many total variables are there in your experiment, suppose if there is a 6 variables based on this experiment there are how many variables 1 2 3 4 5 6; 6 variables.

Then what are the fundamental dimensions are there? There will be only 3 fundamental dimensions that is M L and T; that means, Mass, Length and Time. So, in that case if you subtract that number of fundamental dimensions from the total variables then you can have how many what would be the dimensionless groups will form. So, here 6 minus 3 that will be three dimensionless groups will form. So, accordingly you will see here in this case 3 dimensionless groups we are having here in this case. So, based on this dimensionless groups we can form a functionality of this the dimensionless groups here this is one dimensionless groups and this a F by rho v square D square as a function of rho v D by mu into epsilon D.

Very interesting that rho v D by mu is a Reynolds number; Reynolds number is a dimensionless groups so this is the function of Reynolds number. So, flow resistance as a dimensionless groups if you divide it by rho v square by D square that is equal to one dimensionless group this is a function of other one dimensionless group. So, in this way you can make a dimensional analysis and make a functionality of the dimensionless groups.

Thank you for this lectures today.